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Chapter 17

Spatially Distributed Modeling: Conceptual Approach to Runoff Prediction

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"As scientists we are intrigued by the possibility of assembling our concepts and bits of knowledge into a neat package to show that we do, after all, understand our science and its complex interrelated phenomena." (W. M. Kohler, 1969)

Abstract. The relationship between perceptual models, conceptual models and physically based distributed models of hydrological processes is analyzed. It is shown that physically-based models must be considered as conceptual models at the scale at which they are used. A simplified distributed model (TOP MODEL) that can take account of heterogeneity in catchment topography and soils is introduced. A likelihood based procedure for estimating the uncertainties associated with the predictions of complex distributed models is described.

1. Discussion

1.1. On Models of Runoff Production

The processes of runoff production within a stream catchment are dependent upon transfers of energy and mass that are variable over space and time within a complex open system. In any particular field or modeling study of those processes, there are practical limits to the way in which we can observe or represent those processes. For example, we must apply some boundary to the system under study where both the position of that boundary and any boundary fluxes will be subject to considerable uncertainty, particularly for any groundwater and evapotranspiration fluxes. Such problems of observability will mean that our understanding of the operation of a catchment system must necessarily be limited and subject to simplification and generalization.

However, within every hydrologist there is a qualitative picture of the nature of hydrological processes within a catchment, a picture that reflects his or her preconceptions, training, and field experience (if any), together with those of teachers and colleagues. It may be highly individual to the hydrologist concerned, and some of the problems of hydrological prediction have arisen as a result of misconceived applications of an inappropriate set of processes

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descriptions to a new environment (see Beven, 1987a). It is worth noting in this context that despite the widespread application of the 'Hortonian' infiltration excess model of runoff production, Robert Horton may have observed widespread overland flow in his garden hydrology laboratory in Voorheesville, New York, only rarely.

This subjective and personal picture of catchment processes comprises what may be called a *perceptual model* of the catchment. It is of little use for prediction purposes. Many of the features of this picture may be highly complex and may not easily be described in a mathematical form suitable for quantitative prediction. It is therefore necessary to make simplifying assumptions, generalizing from the perceptual model to create a set of mathematical statements comprising a *conceptual model* of the catchment. Many of those assumptions may already be implicit in the perceptual model, others will be stated explicitly in the specification of the conceptual model. However, not all conceptual models are suitable for quantitative prediction. Hydrological processes are (perceived as being) nonlinear and (stochastically) variable in both space and time, so that it may not be possible to obtain an analytical solution to the mathematical description. In that case a further level of assumptions or approximation will be necessary to yield a *procedural* model that can actually be programmed for a computer to give quantitative predictions.

There are clearly many choices involved in moving from a personal perceptual model to a predictive procedural model. Individual hydrological modelers will make different choices, even though they may have the aim of producing a model of general applicability. This has resulted in a plethora of procedural models in hydrology, without the concomitant development of tools that would allow a rigorous choice between them. Simulating the 'loss' and 'delay' processes that shape the catchment hydrograph is not actually all that difficult if the parameters of the procedural model are calibrated against observed responses. All hydrological models work (to some extent); equally, all are incompatible with the hydrologist's perceptual model of catchment response (to some extent).

1.2. A Perceptual Model of Runoff Production

In order to place the following discussion of conceptual models of runoff production in perspective, it is worth considering a personal perceptual model of runoff production in a catchment that has not been subjected to major disturbance by man. The upper boundary condition will be taken as the surface of the vegetation canopy. Boundary fluxes during a storm rainfall may be highly variable in space and time resulting from the interaction of the wind field and storm characteristics with the structure of the vegetation canopy and catchment topography.

The effect of the vegetation cover will generally be to introduce greater variability in flux intensities at the soil surface as a result of drip and stemflow processes (Crabtree and Trudgill, 1985). Stemflow may account for as much as 15 percent of the incident rainfall, which will be concentrated into a limited area around the base of the plants. Such local intensity concentrations may be important in governing the flow of water into the soil surface or, in some cases, the production of surface runoff. Localized variability in soil properties may also be important. It is known that soil hydraulic characteristics can vary markedly in space, whilst in the vertical the presence of surface crusting, roots, cracks and other macropores, or an organic mat may also play an important role in governing infiltration. Infiltration into the larger structural voids of the soil may occur at application rates of the order of 1 mm/hr, even on quite permeable soils. The presence of structural voids, macropores, or channels will result in a wetting front that moves into the soil in a highly irregular manner (Beven and Clarke, 1986), while even in the matrix, instabilities in the wetting front may result in the development of "fingering" (see Hillel, 1988).

Locally, application rates to the ground surface may exceed the infiltration capacity of the soil. This will tend to occur first on parts of the catchment where the hydraulic conductivity of the soil surface is lowest or application rates are highest. This partial contributing area of infiltration excess overland flow will expand with increasing rainfall intensity or duration. The controls on the production of overland flow in this manner may involve only a thin layer at the surface of the soil, particularly where a surface crust has developed on a bare soil surface. On vegetated surfaces, "surface" flow may take place through a vegetation mat, and have quite low mean velocities (for example, see Beven, 1978). Only over very smooth surfaces will overland flow take place as a sheet flow; on most natural surfaces there will be concentrations of flow induced by the microtopography and vegetation (for example, see Emmett, 1978), and in some cases this will lead to rilling. Areas producing overland flow regularly will tend to be linked to the perennial channel network by ephemeral rills or channels allowing surface runoff to be shed efficiently (eg. Beven, 1978).

Within the soil there may be local breaks in the hydraulic conductivity profile, of the matrix or the structural porosity, that may lead to localized concentrations of moisture and perhaps to saturated pockets of soil, possibly perched above the base of the soil profile. These pockets will tend to expand during storm conditions. Downslope linkages of such saturated pockets may be important in controlling subsurface runoff responses. Anisotropy of the soil hydraulic conductivity can also lead to localized concentrations of moisture in certain parts of hillslopes as has been shown by Zaslavsky and Sinai (1981). The build up of pockets of saturated soil at the base of the profile will also be affected by the permeability and irregularity of the bedrock surface. Huff et al. (1982) have shown for example that subsurface outflows to a stream channel may be concentrated by bedrock steps coming closer to the surface.

Draining water in the soil will tend to flow downhill. The topographic form of hillslopes should, therefore, be expected to have an important effect on spatial patterns of soil moisture and soil saturation, particularly on soils overlying a relatively impermeable bedrock. Hillslopes that are convergent in plan will tend to lead to higher moisture levels at their base relative to straight or divergent slopes. Hillslopes of low angle will also tend to result in higher moisture contents because of lower hydraulic gradients (Kirkby, 1969; Beven and Kirkby, 1979; O'Loughlin, 1981,1986). High water tables can be expected to result in fast subsurface responses, particularly where there is a well developed capillary fringe in the matrix above the water table. Wave speeds associated with saturated subsurface flows are inversely proportional to the difference between the saturated and actual moisture content close to the water table (for example, see Beven, 1982). Within the capillary fringe this difference may be very small, so that the wave speeds for initial responses to infiltrated water reaching the capillary fringe may be very high. Rapid vertical fluxes within the structural voids, perhaps generated by localized concentration of inputs at the soil surface, may be important in providing water to the capillary fringe via preferential flow pathways.

Where the soil saturates to the surface, overland flow may be generated, both from further rainfall inputs and from subsurface "return flow" seeping back to the surface. There will be an interaction between this "saturation excess" surface runoff production, primarily controlled by subsurface flow processes, and the dynamic expansion of the channel network outlined in Hewlett's variable contributing area concept (Hewlett, 1974).

1.3. On Conceptual Models of Runoff Production

The perceptual model outlined above is one of highly complex, dynamic interacting flow pathways. As such it is a model that is beyond detailed physical analysis. The physics is too complex and the characteristics of individual flow pathways are too variable in space and time. The perceptual model must, therefore, be simplified to a practically useful conceptual model in which the processes of catchment response are represented at an appropriate scale.

There is some hope that this can be done in a physically realistic way, despite the complexity evident at small scales. The catchment is a naturally dissipative open system. Dissipative systems lead to a natural integration of complexity and heterogeneity (or disorder) resulting in order in their physical structure and relatively simple impulse-response functions (Stengers and Prigogine, 1984). We see this order in the forms of hillslopes and the structure of dendritic channel networks. We see the simplified response function in the similarity of hydrographs when scaled by area and lag time and in the success of the unit hydrograph concept (Dooge, 1977). The experience of two decades of hydrological modeling suggests that not more than four or five parameters can be justified in simulating a continuous discharge record from a catchment (see for example Kirkby, 1976a). Any more will result in the identifiability problems associated with overparameterization (see for example Blackie and Eeles, 1985), although it is clear that not all conceptual representations of catchment processes have adhered to such a limit (see for example Fleming, 1975).

The same of course applies to distributed models, although in this case there is the possibility of increasing the number of parameters if observations of distributed state variables are available. This is often true in the case of distributed groundwater models, and there is a considerable literature on the so-called inverse problem of using such data to calibrate groundwater models (see Yeh, 1986). It is rarely true in the case of distributed catchment models, albeit that because of the greater nonlinearity inherent in near-surface processes such additional data would be extremely valuable in modeling catchment response. Thus distributed models should also have only a few parameter values that require calibration.

1.4. Physically-based Distributed Models as Conceptual Models

The justification for using distributed catchment models cannot, therefore, be based on the availability of distributed state data (although in some rare cases semi-distributed models have been created to make use of distributed *input* data, see for example Wilson et al., 1979). The development of distributed models has usually been based on the argument that a model which more closely represents the physics involved in the processes of catchment response must be inherently superior to a model that is lumped at the catchment scale (Beven and O'Connell, 1982; Beven, 1985; Abbott et al., 1986a). The equations that describe these processes involve one or more space dimensions and will provide distributed predictions of say, the dynamics of contributing areas (see, for example, Freeze and Harlan, 1969; Freeze, 1972; Beven, 1977; Bathurst 1986a,b; Sharma et al., 1987; Binley et al., 1989a,b, Figure 1). The parameters of those equations have a physical interpretation and can, in principle, be measured in the field. Because the parameters are accepted as physically meaningful, we may have greater faith in modifying the values to simulate the effects of future land use changes which may occur only over part of the catchment, and for which no measurements will be available for calibration.

Most physically-based distributed models are based on the partial differential equations describing surface and subsurface flow processes. Commonly, as in the Système Hydrologique Européen (SHE) model (Abbott et al., 1986a,b) and Institute of Hydrology Distributed Model (IHDM) (Beven et al., 1987) these are the Richards equation for subsurface flows and a kinematic wave equation for surface flows. To obtain a procedural model with realistic boundary conditions, these equations must usually be solved by approximate numerical methods, with state variables predicted at a grid of points in space and time. Parameter values (and initial and boundary conditions) may be allowed to vary throughout this grid to reflect the spatial, and if necessary temporal, variability in catchment characteristics. This results in a very large number of parameter values, far more than is justified on identifiability grounds if those parameters are to be calibrated by comparing observed and predicted discharges. Beven (1989) has pointed out that resort to an argument that the

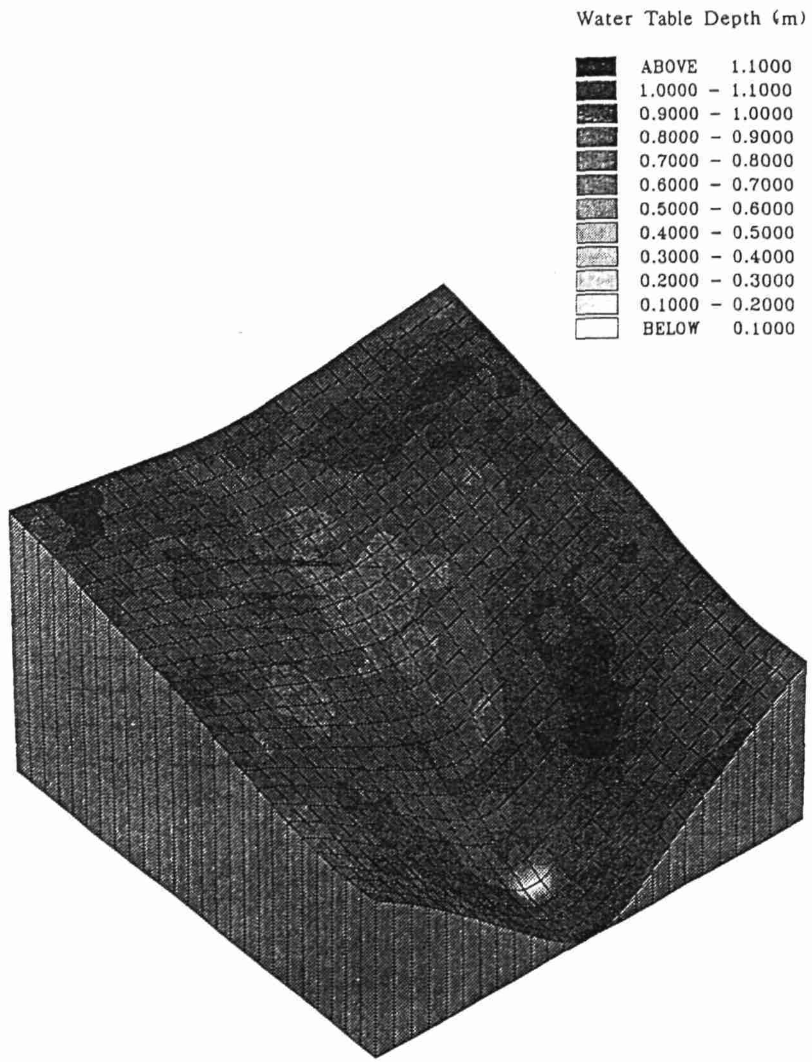

Figure 1. Predicted water table depths for a three-dimensional variably saturated heterogenous hillslope hollow. Calculations involved 12000 nodes in 12 vertical layers and were performed on a Cyber 205 supercomputer using the finite element code of Binley et al. (1989a,b).

physical nature of the parameters allows physical reasoning to be used in calibrating the parameters does not circumvent the problems of overparameterization unless additional observations of parameter values or distributed state variables are available *at an appropriate scale*. In this sense then, physically-based distributed models are no different from any other conceptual model.

Indeed, you will have realized that, despite the claims for such models being physically based, the equations on which these models are based are extreme simplifications of the procedural model described above. The equations are, in fact, descriptive equations of a different system; the physics on which they are based is the physics of small-scale homogeneous systems. In real applications of physically-based models we are forced to lump up the small scale physics to the model grid scale, for example the 250 x 250m used in the application of the SHE model to the Wye catchment (Bathurst, 1986a). There is no theoretical justification for assuming that the equations apply equally at the model grid scale, at which they are representing the lumped aggregate of heterogeneous nonlinear sub-grid processes. It is merely *assumed* that, in defining the conceptual model, the same small-scale equations can be applied with the same parameters.

The conceptual nature of these models can be demonstrated most clearly in terms of the variables used, for example capillary potential. The model will predict capillary potential at a grid point, representing the average value over some grid element. What does a grid element average capillary potential mean over an area of 250 by 250 m? It is not a physical variable in the sense that we can measure it. How can we compare it with a tensionmeter measurement within the grid square which may have a zone of influence of a few cm^3. Worse still, what does a grid square average capillary potential *gradient* mean when it is calculated from nodes 0.05m apart in the vertical for an area of 62500 m^2. Certainly great care should be taken in interpreting such "physically-based" variables.

Yet we know from field evidence that the processes of catchment response show important spatial patterns, primarily arising from topographic variability and the tendency of water to flow downhill. Therefore, it should surely be possible to define a distributed conceptual model that can improve hydrological predictions by simulating such patterns, but in a parsimonious way with a limited number of parameters. There have been two important initiatives to try and develop such a model, both based on very similar quasi-steady state simplifications of "physically-based" equations. The first, TOPMODEL, has appeared in a variety of forms (see for example Beven and Kirkby, 1979; Beven and Wood, 1983; Beven et al., 1984; Hornberger et al., 1985; Beven, 1986a,b, 1987b; Sivapalan et al., 1987; Wood et al., 1988) and will be described in some detail below. The second is based on the work of O'Loughlin (1986) and is described in Moore et al. (1986).

1.5. TOPMODEL: A Distributed Procedural Model of Runoff Production

The presentation that follows is developed in terms of local and mean depths to water table within the catchment, but it can be equally developed in terms of local and mean storage deficits (see Beven and Kirkby, 1979 and other papers cited above). It is assumed that at any point i on a hillslope, downslope saturated subsurface flow rate q_i may be described by:

$$q_i = T_o \tan\beta \exp(-fz_i) \tag{1}$$

where T_o is profile transmissivity when the soil is just saturated to the surface; z_i is the local depth to water table; f is a parameter dependent upon the rate of change of transmissivity with depth; and $\tan\beta$ is the local slope angle and is taken as an approximate hydraulic gradient for

downslope flow. Beven (1986a) has shown how Equation (1) can be related to an equivalent exponential decline in hydraulic conductivity with depth:

$$K_s = K_o \exp(-fz) \tag{2}$$

where K_o is hydraulic conductivity at the soil surface. Beven (1984) gives data to suggest that Equation (2) is a suitable form for a variety of soil types with a range of K_o from 0.01 to 100 m/hr and f from 1 to 12 m^{-1}. In soils for which Equation (2) holds and hydraulic conductivity at depth is small, $T_o \approx K_o/f$.

Under a steady and spatially homogeneous input to the water table, r:

$$q_i = a_i r \tag{3}$$

where a_i is the area of hillslope per unit contour length draining through point i. The value of a_i will be high for points at the base of convergent hillslopes and low for points near the divide and on straight and divergent hillslopes. Combining Equations (1) and (3):

$$z_i = -\frac{1}{f} ln \frac{(a_i r)}{T_i \tan\beta} \tag{4}$$

Integrating over the hillslope or catchment area "A," we can obtain an expression for mean depth to water table, given r, as:

$$\bar{z} = \frac{1}{A}\int_A -\frac{1}{f} ln \frac{(a_i r)}{T_i \tan\beta} \tag{5}$$

Note that Equation (5) assumes that f is a constant and that the relationship Equation (1) holds for the case where water is ponded on the surface ($z_i < 0$). Given the exponential nature of Equation (1), and the relatively slow rates of surface flow to be expected for vegetated surfaces, this assumption may be acceptable in many catchments. Using Equations (4) in (5) to eliminate $ln(r)$ and rearranging:

$$\bar{z} = \frac{1}{f}[z_i + \gamma - ln(a_i/T_o \tan\beta)] \tag{6}$$

where:

$$\gamma = \frac{1}{A}\int_A ln(a_i/T_o \tan\beta) \tag{7}$$

Defining:

$$ln(T_e) = \frac{1}{A}\int_A ln(T_o) \tag{8}$$

Equation (6) can be rearranged into the form:

$$f(\bar{z} - z_i) = [\lambda - ln(a_i/\tan\beta)] + [ln(T_e) - ln(T_o)] \tag{9}$$

where:

$$\lambda = \frac{1}{A}\int_A ln(a_i/\tan\beta) \tag{10}$$

is a topographic constant for the catchment.

Equation (9) expresses, in dimensionless form, the deviation of the local depth to water table from the catchment mean, scaled by the parameter *f*, in terms of a deviation in the logarithm of transmissivity from its areal integral value, and a deviation in the local topographic index away from its areal integral value. For any given value of $\bar{z}$, Equation (9) can be used to predict the *pattern* of water table depths over the catchment. Points for which the right hand side deviations yield values of z_i less than the depth of a near saturated capillary fringe are of particular interest since they represent the predicted saturated contributing area, taking account of heterogeneity in both topography and soil transmissivity. Under assumptions of a homogeneous soil cover, the transmissivity deviation goes to zero ($T_o = T_e$ everywhere) and the pattern will depend on the topographic index alone. Figure 2 shows the patterns of soil saturation predicted from digital elevation data for the River Wye catchment at Plynlimon, mid-Wales.

In the procedural version of TOPMODEL the relationship (Equation [9]) (or a variant of it) is used at any time step to predict the saturated contributing area. Any rainfalls falling on this area, or in excess of that required top fill the storage deficit associated with small values of z, form the predicted saturation excess runoff production. Application of Equation (9), which is based on steady state assumptions, at successive time steps implies that a value of $\bar{z}$ is updated at each time step and that subsurface wave speeds are such that redistribution of inputs to the saturation zone is relatively rapid compared with the rate of change of $\bar{z}$. Updating of $\bar{z}$ requires continuous accounting of inflows and outflows for the saturated zone. An expression for subsurface outflows to the stream channel can be derived from Equation (1) as:

$$Q_b = Q_o\, exp(-f\bar{z}) \tag{11}$$

where $Q_o = A\,T_e \exp\{-\lambda\}$ (see Beven, 1986a,b; Sivapalan et al., 1987). Inflows to the saturated zone will depend upon infiltration rates and redistribution in the saturated zone, which in turn will depend on local values of K_o and z_i.

Infiltration rates will commonly be equal to the input flux rates at the ground surface, but for high input intensities and low local hydraulic conductivities there may be ponding at the surface and infiltration excess runoff production. Beven (1984) has provided an infiltration model based on Green-Ampt assumptions that is consistent with the assumption of an exponentially declining hydraulic conductivity. Sivapalan et al. (1987) and Wood et al.(1988) have also used a Philip equation infiltration model within the TOPMODEL structure.

It is very difficult to predict redistribution of infiltrated water in the unsaturated zone above the water table in a way that is both consistent with the perceptual model cutlined above and reflects the pattern of soil properties and depths to water table in the catchment, but does not involve a large number of parameter values. Two forms, that add only one additional parameter, have been used in the TOPMODEL structure. In Beven and Wood (1983), which was formulated in terms of patterns of storage deficit, local vertical flux was predicted as:

$$q_v = S_{uz}/S_i\, t_d \tag{12}$$

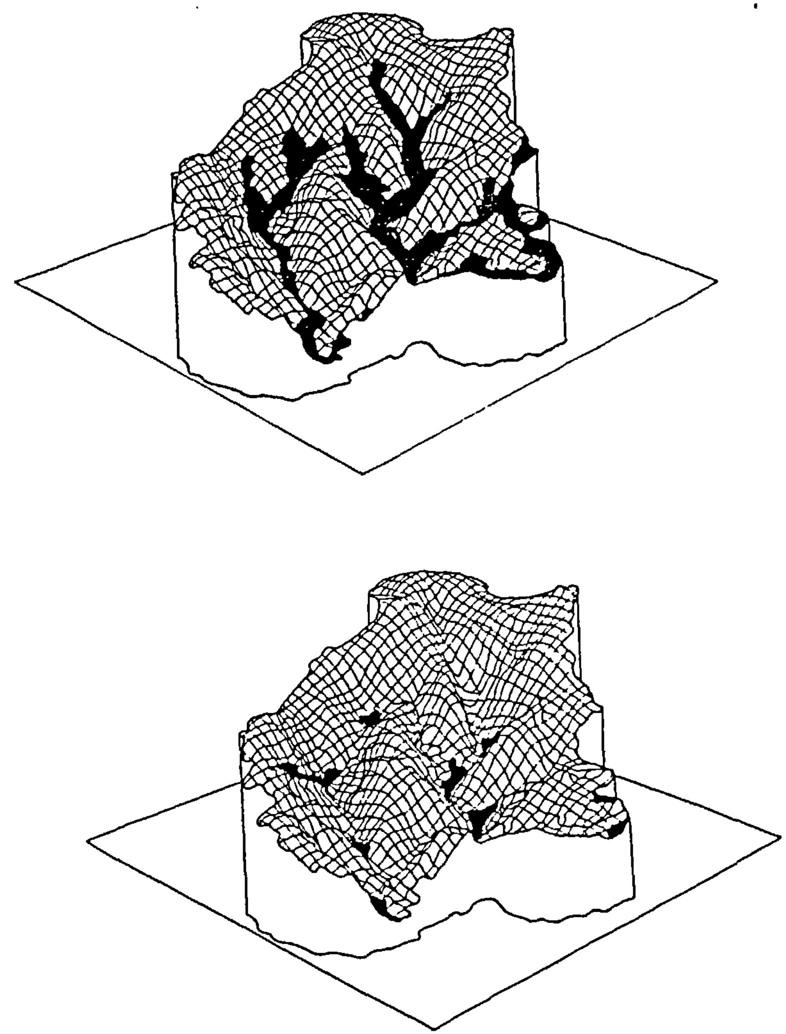

Figure 2. Predicted saturated areas at two different moisture deficits based on digital elevation data for the 10 km² Wye catchment Plynlimon.

where S_{uz} is local storage of mobile water in the unsaturated zone, S_i is local storage deficit, and t_d is a time delay per unit of storage deficit. Alternatively in Beven (1986a) the rate of flow into the saturated zone was set equal to the saturated hydraulic conductivity at the water table multiplied by a parameter, α, representing the hydraulic gradient ie.:

$$q_v = \alpha \, K_o \exp(-fz_i) \quad (13)$$

In fact, Beven (1986a) sets α equal to 1, eliminating this as a parameter. Both forms allow the local delay in the unsaturated zone to vary with the depth of the water table. In the original Beven and Kirkby (1979) form of the model all the fast unsaturated zone drainage was assumed to reach the water table within one time step.

A similar problem arises in the prediction of evapotranspiration rates. Evapotranspiration takes place from both ground and plant surfaces and involves withdrawals of water throughout the rooting zone, dynamically controlled by meteorological and plant controls as well as the moisture status of the soil. The TOPMODEL structure, in common with many other conceptual hydrological models, assumes that if actual rates of evapotranspiration cannot be specified as a boundary condition, then the relationship between potential and actual rates is a function of root zone moisture status. The simplest form, involving only one parameter, allows evapotranspiration to take place at the potential rate wherever the soil is saturated to the surface or there is remaining storage of freely draining water in the unsaturated zone. When this storage is depleted, further evapotranspiration from the root zone is allowed at a rate determined by the ratio of a soil moisture deficit in the root zone to some maximum deficit, ie:

$$E_a/E_p = 1 - S_{rz}/S_{rmax} \quad (14)$$

where E_a is actual evapotranspiration rate, E_p is the potential rate, S_{rz} is a root zone moisture deficit, and S_{rmax} is a maximum allowable deficit which is an additional parameter of the model. Areal differences in evapotranspiration rates are predicted by the model depending upon the predicted contributed areas and unsaturated zone storage.

Beven and Kirkby (1979) pointed out that measured flow velocities of surface runoff on vegetated surfaces are such that the shape of the hydrograph may be significantly influenced by overland flow routing. They used a constant routing velocity proportional to local gradient. This allows, for any given value of the saturated contributing area, a unique time delay histogram to be derived from the basin topography and a single velocity parameter.

Channel routing is also based on a linear algorithm (although other network algorithms can be easily implemented), similar to that used by Surkan (1969) and Kirkby (1976a), under the assumption of a constant wave velocity. Beven (1979) has justified the use of a constant wave velocity on the basis of field data for average reach velocities in the 10 km^2 Severn catchment in mid-Wales.

The TOPMODEL structure outlined above represents a simple distributed model with a minimum number of parameters. It may be viewed, in fact, not so much as a fixed model structure but as a set of concepts that can be tested against the perceptual model of the user in any application to a particular catchment. The distributed nature of TOPMODEL is important in this respect. Even at the very simplest level, TOPMODEL can predict, in space, where areas of saturated or near saturated soil will be found. A ready check can be made on such predictions in the field as a check of whether the concepts used are valid or need to be modified. Such comparisons between the predictions of a readily understood model and the hydrologist's *ideas* of how a catchment is working have been useful in a number of studies.

Perhaps the most important aspect of TOPMODEL and the Moore et al. (1986) model is the ability to take account of heterogeneity in soil and topographic characteristics.

Information about the spatial heterogeneity in soil characteristics is, of course, rarely available, but in the application of Equation (9) the deviations associated with the topographic variable are always likely to dominate the deviations associated with soil transmissivity. Therefore, in general little loss in accuracy would be expected in neglecting soil heterogeneity in the prediction of saturated contributing areas. Soil transmissivities are, however, more important in the determination of the drainage function, Equation (11), but here the estimate of the geometric mean transmissivity will be far more important than the estimate of the variability around that mean. Similar arguments cannot be used for catchments in which the prediction of infiltration excess overland flow is important, where the spread of the contributing area will be more directly related to the variability in hydraulic conductivities.

The model calculations may be carried out in either grid square or, since all points with the same value of $ln(a/T_0 \tan\beta)$ are assumed to respond in a hydrological similar way, discrete distribution function forms. The distribution function form allows highly efficient computations, even though the predicted spatial patterns may still be mapped back onto the catchment topography if required. In either case, the catchment can be treated as a lumped unit for calculation of z and the contributing areas dependent on z, or where rainfall patterns are known to be highly varied in space, as a number of subcatchments, where each subcatchment may vary in its input, topographic or other characteristics. A spatially scaled version of the TOPMODEL structure has been developed by Sivapalan et al. (1987), while Beven (1986a,b, 1987b) has used versions of the model with a random rainstorm generator to predict flood frequency characteristics. Figure 3 shows the results of an application of the version of the model described here to predicting flows from the River Wye catchment at Plynlimon, mid-Wales. A more complex form of TOPMODEL, designed to provide hydrological

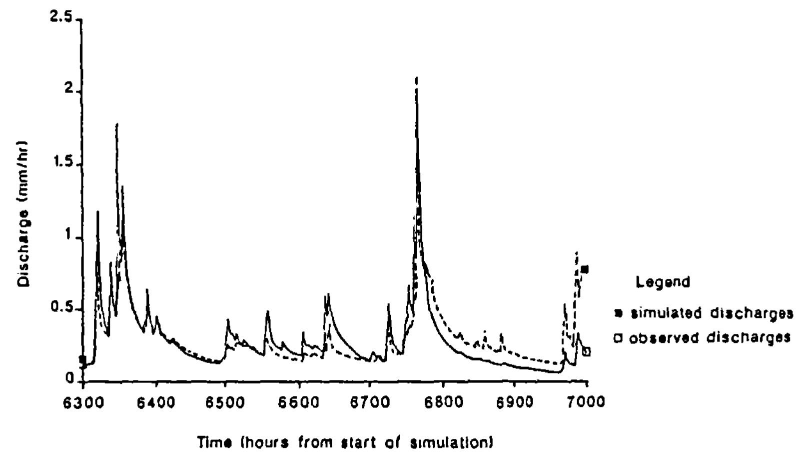

Figure 3. Validation run of TOPMODEL in a split record test for the River Wye catchment at Plynlimon.

predictions for use in multi-compartment geochemical calculations and involving 13 parameters, was evaluated by Hornberger et al. (1985) in an application to the White Oak Run catchment in Virginia, using the methods of regionalized sensitivity analysis proposed by Hornberger and Spear (1981). A number of different objective functions were evaluated in conjunction with a Rosenbrock automatic optimization routine to obtain an "optimum" set of parameter values for each objective function. The results indicate that all the objective functions were indifferent to all but a small number of the 13 parameters, but that the most sensitive parameters were different for different objective functions. Some interdependence between different parameter values was demonstrated, being evident as curved "valley" forms in the response surface. They also showed that it might be better, given such an over-parameterized model, to change the model structure to eliminate some of the parameters rather than simply fixing the values of insensitive parameters. They concluded:

> *"It is obvious that several of the parameters that were specifically included in the full model to account for processes that were observed in the field could not be estimated from the inflow-outflow data. The simpler model that we implemented gave almost an equally good fit to the data. We face the dilemma posed by Beck (1983): a 'small' (ie. rigorously calibrated) model may miss one or more key aspects of a system's behavior because the pertinent modes of behavior were not excited during the calibration period, but a "large" model, while theoretically capable of reproducing a wider range of behaviors can only be calibrated with a good deal of uncertainty."* (p.1849)

A similar dilemma must inevitably face all attempts at spatially distributed modeling due to the number of parameters that arise in any distributed description of a hydrological system.

1.6. The Future of Distributed Modeling: Likelihood and Predictive Uncertainty

It is clear that there are inherent practical problems associated with distributed modeling in hydrology. We have a highly complex dynamic system that can be represented only crudely by the current generation of models. Even these, however, require the specification of large numbers of spatially distributed parameters. These parameters must inevitably be intercorrelated in their effect on the model simulations with the result that they will be very difficult to identify on the basis of comparisons between observed and simulated behavior. Measurements of internal state variables will help in this respect, but only to a limited extent since our measurement techniques tend to be at much smaller scales than is required for comparison with model variables.

It seems inevitable, in fact, that the hydrological systems that we are intent on simulating must remain unknowable in their distributed characteristics, particularly of soil properties. This "unknowability" must lead to predictive uncertainty, even where detailed spatial information on soils and their hydrological response is available for "points" in the catchment. There are also other causes of uncertainty, in particular unknown errors in model input data and unknown errors in model structure and boundary conditions. Studies of predictive uncertainty to date have been limited to restricted studies of uncertainty due to parameter variations around some "optimum" parameter set, and to the effects of errors in input data. It is argued here that a wider concept of uncertainty is required that reflects the ultimate "unknowability" of the system.

It follows from recognition of a concept of "unknowability," that there can be no ultimate validation of a model structure and set of parameters as a simulator of a catchment system (see Stephenson and Freeze, 1974, for an early discussion of the infeasibility of validation of hydrological models). In a sense, therefore, any model and set of parameters is in contention

as an appropriate predictor for the real system. There are many reasons to consider simulation models in this light. For example, even under ideal hypothetical circumstances in which the correct model structure is known a priori, it can be shown that the introduction of errors into input data, or in sampling of spatial characteristics, can lead to a number of parameter sets giving close to "optimal" simulations. Also, Loague and Freeze (1985) have shown that in some circumstances there may be little to choose between alternative model structures in simulating catchment discharges. In addition, experience suggests that while a particular simulation may give poor results relative to others for one period, it may, over a longer period prove to be a better predictor. Consequently, it is suggested that the idea of an "optimal" simulation model and set of parameter values be replaced by the concept of likelihood value in which the value of every simulation of the behavior of the system under study is expressed as a likelihood index.

These likelihood values may be considered as an assessment of the probability of having a "correct" simulation. The likelihood function will have a value for each "set" of parameter values and as such will reflect the effects of any parameter interaction associated with the model structure. The likelihood values may be used in a number of ways; in particular, they may be used as probability weights to assess the predictive uncertainty associated with the model given the uncertainty in parameter values and input data, as conditioned by the available observations. Note that although the simulations may result from a nonlinear model or models, the combination of simulations in this way to calculate sensitivities and uncertainties is a linear process. The additional computations required to experimentally manipulate different likelihood functions should, therefore, be trivial.

Such a procedure also focuses attention on the value of observations in constraining uncertainty. Additional observations should impose additional constraints on the likelihood function and consequently on the uncertainty. This may be particularly important in the simulation of spatially distributed systems with observations of internal states, which may not be directly equivalent to simulated variables in the model. It should be noted that the definition of the likelihood function is necessarily subjectively chosen, particularly where different types of observations (for example discharges and water table levels) contribute, as some form of weighted sum, to the definition. Consequently, the predictive uncertainty resulting from the likelihood function will reflect this subjectivity of choice. It is an advantage of the proposed methodology that this dependence is made explicit.

2. Conclusions

I have argued elsewhere (Beven, 1987a, 1989) that the practical application of physically-based models must be accompanied by a realistic assessment of the uncertainty associated with the predictions. The generalized likelihood methodology proposed above provides a means for doing so in a readily understood way. In requiring multiple simulations to obtain values of the likelihood function it is a very computationally intensive procedure for many spatially distributed models. However, it is a procedure that would be readily implemented in parallel processing computers and may in the future become a valuable technique in the hydrological modeler's toolkit.

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