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Flow of Slurries in Pipelines

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Pressure drop correlations for flow of slurries in pipelines were developed for each of the following four flow regimes: flow with a stationary bed, saltation flow, heterogeneous flow, and homogeneous flow. A total number of 2 848 data points, comprised of 1 912 collected from the published literature together with 936 taken using our own test pipelines and relating to ranges of the pertinent variables extensive enough to span all four flow regimes were used as the basis of these correlations. Also, these data were used in developing an associated quantitative regime delineation scheme. The correlations provide an improved predictive capability over previously available procedures and are also broader in scope. The delineation procedure developed here permits straightforward classification of the data according to the flow regime prevailing, and it is moreover inclusive of all the data and is self-consistent.

SCOPE

The principal objective of this study was to explore possible effective schemes for correlating data relating to flow of slurries in pipelines. The immense practical implications of this mode of solid transport have motivated a resurgence of activity which is continuing to contribute to an expanding storehouse of raw data. Some unifying framework capable of enforcing order in this vast maze of data, and hopefully of providing a quantitative means for predicting the prevailing flow regime, is needed. A number of different flow regimes, depending upon the dynamic conditions of the flow and the nature of the slurry, are known to occur in such flows, and therefore we have developed pressure drop correlations pertaining to each of the following four regimes: flow with a stationary bed, saltation flow, heterogeneous flow, and homogeneous flow. Quantitative criteria for ascertaining the flow regime prevailing under given conditions of flow were also developed, and these are used in conjunction with the pressure drop correlations. The estimation of pressure drop in slurry pipeline design is an important practical problem which, owing to the complexity of the process involved, has been subject to considerable uncertainty. A comparison of the few available slurry flow correlations with the present results was made using the entire body of data collected for establishing our correlations as a basis. Such a comparison, besides establishing the improved predictive ability of our new correlations, serves also to provide the proper perspective needed in the selection of the most suitable design procedure in any given situation. In some of the previous empirical correlations, significant portions of the data have been excluded on the grounds that they do not belong to flow regimes being considered. However, the criteria used for such exclusion can not definitively identify the demarcation between flow regimes since, at the very least, in reality the transition from one flow regime to another must take place gradually and not in the abrupt fashion implied by the use of a transition number. Indeed, the descriptive designation of the various regimes is an essentially subjective procedure. Nonetheless, it is important to emphasize that the criteria developed in the present work for ascertaining the appropriate flow regimes are not used to exclude portions of the experimental data but to insure that the most nearly suitable correlation is used in pressure drop estimation. Furthermore, these quantitative criteria are consistent with the expected progressive transition from one flow regime to the next, and, in fact, even in those very few cases in which the flow regime number configuration appears to be anomalous, these criteria do provide a fully satisfactory explanation of the observed behavior.

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CONCLUSIONS AND SIGNIFICANCE

Correlations were established for predicting the friction loss for flow of slurries in a pipeline. Because in this flow various types of flow regimes are known to arise, individual correlations pertaining to individual flow regimes were developed. In addition, these correlations were used to devise a flow regime delineation scheme which is comprehensive enough to account for the entire body of experimental data used as a basis for the correlations. The datum base used was sufficiently broad to span flow behavior extending from bed load flow to suspended flow, and the significance of these results derives in part from the fact that no part of the data (or flow regime) is excluded and in part from the fact that the present correlations do a better job of predicting friction loss than previously available results. Furthermore, the present study belongs to a small body of work which has sought to establish some scheme to unify slurry flow data and transform them into a more easily usable form. The results of the present work enable one to carry out calculations of pressure drop for pipeline flow of a slurry in a completely straightforward manner.

There are a number of difficult problems relating to the present status of knowledge in the area of pipeline transport of slurries. First, because the variables of significance in such flows are vast in number as well as range, the body of raw data is necessarily large and rapidly increasing, yet a comprehensive correlative scheme unifying this body of data and rendering it interpretable does not exist. Second, information concerning industrial practice in this area generally pertains to specific commodities and is, at any rate, frequently disclosed in incomplete form because of proprietary considerations. Third, because often the data are collected without regard to their possible ultimate utility in developing correlations, they are not always sufficiently detailed. This last fact is another good (though not the primary) reason justifying the need to redirect some energy to the exploration of effective correlating procedures in the hope that such activity will alert experimenters to the need for more completely detailed information. With these general observations in mind, we turn to a review of specific pieces of previous work.

An exhaustive review of previous work relating to hydraulic transport would simply be too long because the subject possesses an extensive literature, and at any rate it would not materially enhance the presentation of the present results. Consequently, we restrict our review here mainly to those few works in which predictive correlations for pressure drop were proposed and refer the more inquisitive reader to Yuan's (1971) thesis or to the compilation of papers in the volume edited by Zandi (1971). In later sections, moreover, we cite additional references used as sources for the data collection we compiled for our correlation work.

One of the earliest attempts at developing a pressure drop correlation for slurry flow was made by Blatch (1906) who proposed the relation

$$i = i_w + AC \tag{1}$$

In Equation (1), A is taken to be a constant, although Blatch, who used sand slurries in 2.54 cm pipe, asserted that it depended on pipe diameter; later Howard (1939), using sand slurries in a 10.2 cm pipe, showed that A depended on particle size as well. These results are hardly surprising considering the primitive nature of Equation (1). Wilson (1942) used the more rational approach of summing the work needed to maintain the solids in suspension together with the energy dissipation for the flowing water and derived the relation

$$i = i_w + A' \frac{v_s}{v} C' \tag{2}$$

Wilson's procedure does not account for interactions among the various energy contributions in the flow. Newitt et al. (1955) used Wilson's approach, and, based on experiments with solids of sizes from −240 mesh to 4 760 μ and a specific gravity between 1.18 and 4.6 in a 2.54 cm pipe, they developed the following relations:

For homogeneous flow: $v \geqq v_H = (1\,800\ gDv_\infty)^{1/3}$

$$\frac{i - i_w}{i_w C} = 0.6\ (s - 1) \tag{3}$$

For heterogeneous flow: $v_B < v < v_H$

$$\frac{i - i_w}{i_w C} = 1\,100 \frac{gD}{v^2} \frac{v_\infty}{v} (s - 1) \tag{4}$$

For moving-bed flow: $v \leqq v_B = 17v_\infty$

$$\frac{i - i_w}{i_w C} = 66 \frac{gD}{v^2} (s - 1) \tag{5}$$

The transition velocities $v_H = (1\,800\ gDv_\infty)^{1/3}$ and $v_B = 17v_\infty$ are used to delineate the three flow regimes; the former is obtained by equating Equations (3) and (4) and the latter by equating Equations (4) and (5). The transition criteria depicted above imply that the transition from one regime to the next is abrupt, which is not so in reality. Neither this inadequacy nor the fact that the three regimes designated may not be sufficient to characterize the full range of flow behavior observed is terribly important in the present context. The central issue here is whether Equations (3) to (5) can be used to provide a satisfactory estimate of the head loss and certainly not whether they can faithfully establish the nature of the flow regime prevailing. The predictive effectiveness of these equations will be examined later on.

The research of Durand and his co-workers (1951 to 53) in the early fifties represents a milestone in the area of slurry flow. They used sand and gravel slurries with particle sizes ranging from 0.2 to 25 mm, pipe diameters from 3.8 to 58 cm, and solids concentrations up to 60% by volume. A principal result of the work by Durand's group is the correlation

$$\frac{i - i_w}{i_w C} = K \left[\frac{v^2}{gD(s-1)} \sqrt{C_D} \right]^m \tag{6}$$

where C_D is the drag coefficient for settling of the particle at its terminal velocity in the stagnant unbounded liquid, and for a sphere with a diameter d it is given by

$$C_D = \frac{4}{3}\frac{gd(s-1)}{v_\infty^2} \qquad (7)$$

Based on 310 data points, the constant K was found to have the value 84.9 and the exponent $m = -1.5$, although it has since been found that the values of K and m need to be adjusted for different slurry systems. This correlation is based on experimental data spanning all flow regimes, but experience has shown that it has limitations. Thus, Ellis and Round (1963) reported that the Durand form would not provide a satisfactory correlation for data on nickel slurries ($s = 8.9$, $d = 106\ \mu$) except in the homogeneous flow regime, where $v > 3.09$ m/s, and then provided the value of K is changed to 385. Murphy et al. (1954), using data on eight different materials including glass, steel, and lead in a 1.27 cm pipeline, and also Hayden and Stelson (1971), using fine and coarse sands in 2.54 and 5.08 cm pipelines, found it necessary to modify both values of K and m to achieve satisfactory correlation. Also Babcock (1971) undertook a critical reexamination of Durand's correlation, and to do so he carried out a systematic series of experiments designed to provide an assessment of the role of each of the groups appearing in the correlation. The principal conclusions drawn from this important work are that the groups included in Durand's correlation do not account sufficiently for the influences of the concentration, the particle size, and the pipe diameter, among others.

In a very significant and ambitious work, Zandi and Govatos (1967) collected a total number of 2 549 data points from various sources. After scrutinizing the data for "possible duplication, gross variation from pattern of behavior, obvious discrepancies in the reporting of the data, absence of independent variables, and all other defects that could have made the data look suspicious . . ." they ended up with 1 452 data points. Among the eliminated data at this first stage were also all those pertaining to solids concentrations of less than 5%, purportedly to exclude any data in which drag reduction effects could have been important. From the remaining 1 452 data they next excluded another 462 on the grounds that these belonged to either the saltation flow or flow with a stationary bed regimes, using as an indicator the fact that for these rejected data a so-called number N_I obeyed the inequality

$$N_I = \frac{v^2\sqrt{C_D}}{CDg(s-1)} < 40 \qquad (8)$$

Using the remaining 990 data points, for which $N_I \geqq 40$, they modified Durand's correlation to the following form:

$$\frac{i - i_w}{i_w C} = 280\,\psi^{-1.93} \text{ for } \psi < 10 \qquad (9)$$

$$\frac{i - i_w}{i_w C} = 6.30\,\psi^{-0.354} \text{ for } \psi > 10 \qquad (10)$$

with

$$\psi = \frac{v^2}{gD(s-1)}\sqrt{C_D} \qquad (11)$$

It is clear that despite the fact that Zandi and Govatos's correlation does, as it should, do a somewhat better job in predicting the head loss for the remaining 990 data points than that of Durand and Condolios, the need to exclude so large a number of data points as 462 is a limitation. The transition criterion depicted by Equation (8) is, as in the case of other correlations, arbitrary and does not represent a definite indicator of the demarcation between flow regimes. Indeed, it is not established that the parameter given in Equation (8) is even adequate at all for such a test. Thus, of the original 2 549 data points, 1 452 data points were eliminated for pathological reasons, but the additionally excluded 462 data points are presumably as healthy as the chosen 990. Nonetheless, it must be admitted that Zandi and Govatos's work represents the first important attempt (since the contributions of Durand and co-workers) to establish useful correlations of the existing body of data, and these shortcomings are primarily a reflection on the nature of the data and do not negate the significance of their contribution. The results of a detailed comparison of the various correlations will be presented later.

A parameter of considerable importance in slurry transport is the so-called critical velocity v_c, below which solids begin to settle to the bottom of the pipe. This velocity represents the demarcation between flow with a stationary bed and all modes of suspended flow. For estimating v_c, Durand (1953) presents the relationship

$$v_c = F_L\sqrt{2gD(s-1)} \qquad (12)$$

in which F_L is given graphically as a function of particle diameter and solids concentration. Spells (1955) confined his attention to small particle sizes (mixtures containing appreciable proportions of particles larger than 1 000 μ being excluded), and, based on published data, he developed the correlation

$$v_c^{1.225} = 0.0251\,gd\,(s-1)\left(\frac{D\rho_m}{\mu}\right)^{0.775} \qquad (13)$$

Another correlation for v_c can be deduced from the N_I criterion of Zandi and Govatos (1967) given by Equation (8), which yields

$$v_c^2 = 40\,\frac{CDg\,(s-1)}{\sqrt{C_D}} \qquad (14)$$

Other correlations for v_c are due to Hughmark (1961), Gregory (1927), Thomas (1962), and Wasp et al. (1970) and are more or less of the same nature. Using data on fine and coarse sand slurries, and on fine gravel slurries in 2.54 and 5.08 cm pipes, Hayden and Stelson (1971) found that the correlations of Durand, Hughmark, and Zandi yielded approximately the same degree of agreement, which was fair. Spells correlation yielded poor results, but the particle sizes of the solids used in the experiments were larger than those Spells used in deriving his correlation. In addition, it must be borne in mind that the experimental determination of v_c depends upon the subjective judgment of the observer and is therefore susceptible to uncertainties.

In more recent work, Vocadlo and Sagoo (1973) developed a head loss correlation using the idea of summing contributions purported to be due to the carrier liquid, to the solid, and to the interaction between them. The idea of viewing the head loss to be comprised of additive contributions is not new; yet its periodic rediscovery seems to give rise to renewed hopes of new theoretical insight into the problem. The Vocadlo and Sagoo entry is, like other correlations including our own, an empiricism, but its weakness is rooted in the rather meager data base used.

Perhaps one of the most important centers of activity in the area of pipeline transport of slurries is the effort that has been mounted during the past decade and a half by the group at the Saskatchewan Research Council

TABLE 1. PROPERTIES OF GLASS BEADS USED IN SLURRY FLOW EXPERIMENTS IN THIS WORK

Average diameter (μ)	Density (g/cm^3)
90	4.434
505	4.430
1 015	4.400
29.6	2.417
475	2.486
1 340	2.977
4 380	2.964

(SRC) in cooperation with the University of Saskatchewan. So far only a small number of publications resulting from this effort have appeared [see, for example, Smith et al. (1973), Schriek et al. (1974), and Husband et al. (1976)], but a large body of as yet uncorrelated data is contained in SRC reports. The scope of the Saskatchewan effort is broad, and it is bound to have a significant impact on this field.

An interesting work relating to slurry transport is Dina's (1976) report on the operation of the Mohave Power Station which receives slurried coal through the Black Mesa Pipeline. This pipeline, running between Arizona and Nevada, is an 45.7 cm, 439 km line with a design capacity of 5½ million tons of coal per year delivered as a 50% by weight water slurry. The size consist of the solid ranges from plus 8 mesh down to −325, which end comprises fully 20% of the total weight of solid. The inclusion of so large a fraction of fines is required by the need to formulate a so-called stabilized slurry which resists settling and facilitates pipeline restart upon shutdown. It is clear from Dina's account that very elaborate dewatering and clarification schemes are required in order to recover these fines from the delivered slurry.

There is a large body of opinion which regards coal slurry pipelining as a significant component in a comprehensive energy transportation strategy of the future. Besides important technical problems, vital economic, environmental, and social factors must be confronted and resolved before major initiatives are undertaken. A very interesting view of the intricacies of the public policy aspects of this problem can be surmised from the report on recent hearings held by the United States Senate.* This report is also a rich storehouse of information on slurry pipelining.

SLURRY FLOW EXPERIMENTS

The slurry flow equipment used in our experiments was described by Turian et al. (1971) in a previous article. It consisted of a pipeline loop with test sections of approximately 10.7 m in length and test pipes with nominal diameters of 5.08, 2.54, and 1.27 cm. The slurry flow rate could be measured by either a calibrated magnetic flowmeter or a weigh tank.

In our slurry flow experiments, we determined flow rate, pressure drop through straight sections of the test pipes, and the discharge concentration of solids. The slurries consisted of glass beads suspended in water. Properties of the glass beads used are listed in Table 1. Complete details relating to the experimental work and a listing of the data are given by Yuan (1971).

TABLE 2. LISTING OF LITERATURE SOURCES USED IN COMPILING DATA COLLECTION

Author	Suspended solids	Pipe diameter (cm)	Number of data points
Ambrose (1952)	Sand	14.1	24
Blatch (1906)	Sand	2.5	312
Bonnington (1959)	Sand	3.8	22
Craven (1951)	Sand	5.1; 14	87
Durand (1951)	Sand	10.4	18
Durand and Condolios (1952)	Sand	15	86
Ellis and Round (1963)	Nickel	2.5	67
Homayounfar (1965)	Coal	2.5	26
Howard (1939)	Sand; gravel	10	41
Mikumo et al. (1933)	Copper ore	3.8	41
Murphy et al. (1954)	Glass; steel; lead	1.26	604
Newitt et al. (1955)	Sand; gravel	2.5	135
O'Brien and Folsom (1937)	Sand	7.6	156
Rother (1959)	Sand	2.5; 4.1; 5.1; 7.6	51
Shih (1964)	Wood	7.6	20
Smith (1955)	Sand	5.1; 7.6	107
Soleil and Ballads (1952)	Sand	58; 70	16
Worster and Denny (1955)	Coal	7.6; 15	99

DATA COLLECTION

The results presented in this article are based on a total number of 2 848 data points. Of this total number, 1 912 data points were collected from the literature, while the remaining 936 data were taken from our own slurry pipeline consisting of a 5.08, 2.54, and 1.27 cm pipe. The sources of the data collected from the literature are listed in Table 2. The ranges of variables represented by the entire body of data points are as follows:

Pipe diameter:	1.26 to 69.9 cm
Solid density:	1.16 to 11.3 g/cm^3
Particle size:	29.7 to 38 000 μ
Solid concentration:	0.006 to 42% by volume
Mean velocity:	0.009 to 6.7 m/s

These ranges are extensive enough to span the four principal flow regimes considered in this work, namely, flow with a stationary bed, saltation flow, heterogeneous flow, and homogeneous flow. A complete listing of the data collection, including details relating to each data set, are given by Yuan (1971).

PRESSURE DROP CORRELATIONS

For pipeline flow of a given slurry, a plot of the pressure gradient against the mean velocity on logarithmic scales, that is, log $(-\Delta p/L)$ *vs.* log v, typically results in a curve similar to that shown in Figure 1. The flow regime designations depicted in Figure 1 are qualitative and may indeed constitute an incomplete description of the variation in flow behavior that takes place. Even if these four regimes represented the complete situation, it is certain that the boundaries between regimes are not well defined. In the present work, we assume that the situation depicted in Figure 1 is valid, since aside from

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* "Organizational Conflict of Interest in Government Contracting," Report on Hearings before the Subcommittee on Energy Research and Water Resources of the Committee on Interior and Insular Affairs of the U.S. Senate, Nov. 17, 21, and Dec. 5, 1975. The Honorable James Abourezk, Senator from South Dakota, Chairman. 954 pp. (1976).

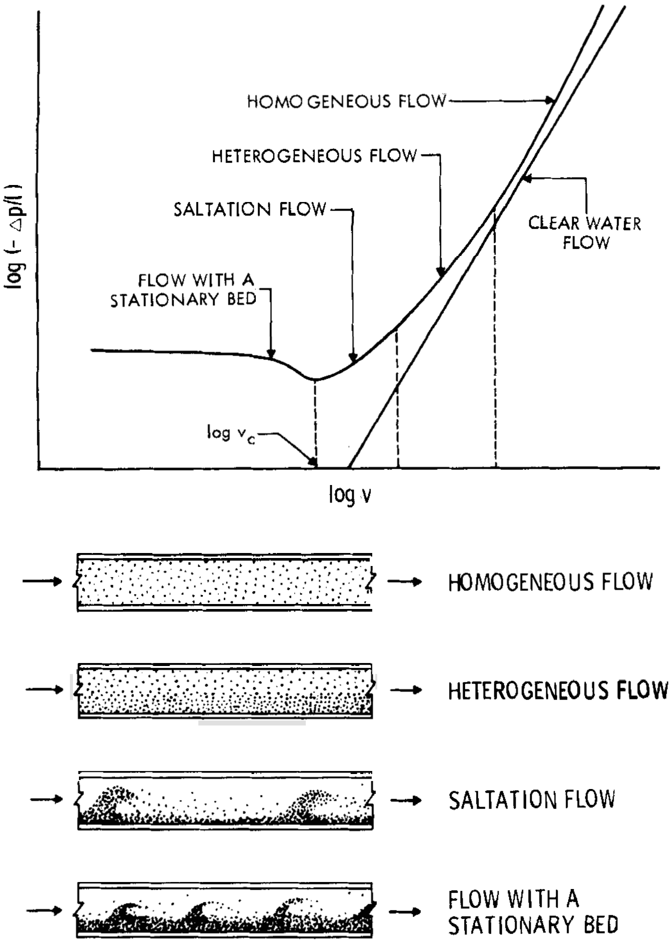


Fig. 1. Representative plot of pressure drop for slurry flow.

providing at least a coarse representation of the actual situation, the nature of the data in slurry transport is usually inconsistent with any greater refinement than this. It is evident, however, that the various forces affecting the flow must play different roles in the different flow regimes. Consequently, it is expedient to develop a separate correlation for each of the four flow regimes and to use them to delineate the various flow regimes. We devote the remainder of this section to a description of the process we used in establishing our correlations.

In a previous article (Turian et al., 1971), using a somewhat limited data base, we found that the empirical form given by

$$f - f_w = K C^{\alpha} f_w^{\beta} C_D^{\gamma} \left[\frac{v^2}{Dg(s-1)} \right]^{\delta} \qquad (15)$$

could be used to provide satisfactory correlation of slurry flow data. The constants K, α, β, γ, and δ are determined for each flow regime by fitting the experimental data for the particular regime. In Equation (15), f and f_w are the friction factors for the slurry and for water at the same mean velocity and are defined by the equation

$$f = \frac{1}{2} \frac{D}{\rho v^2} \left(-\frac{\Delta P}{L} \right) \qquad (16)$$

The form in Equation (15) is empirical, but the choice of the various groups is motivated by dimensional analysis as described in detail by Turian et al. (1971) and also by Yuan (1972). Thus, dimensional analysis suggests that the Reynolds numbers based on pipe velocity and diameter, as well as the one based on particle settling velocity and diameter, are pertinent variables. Accordingly, these are accounted for through introduction of the friction coefficients f_w and C_D, which are obviously fully defined by the respective Reynolds numbers, and which moreover seem the more appropriate in view of the fact that what is being correlated is itself a friction coefficient. The use of the difference form $(f - f_w)$ and the introduction of the concentration C help to insure that the proper limiting behavior is attained when C approaches zero. The dimensionless group $[V^2/Dg(s-1)]$ gives a relative measure of inertial to gravitational forces prevailing in the flow. This is clearly one of the most important variables in this sort of flow, also recognized by virtually all other researchers. As an empirical equation, however, the efficacy and range of the correlation which is ultimately established will depend upon nature and the range of the data base used. In this rather complex flow, the number of variables is so large that identification of the optimum number of variables needed to enforce a reasonable correlation is a valid subject of enquiry in itself. One essential distinction of our work derives from the fact that we assign a separate adjustable exponent for each dimensionless group instead of lumping the variables together as others have done. The nature of the process under consideration here is such that the data are subject to enormous variability, and this has been adequately demonstrated by Zandi and Govatos (1967). The uncertain, and possibly diminishing, returns in improvement in correlation deriving from this, and also the uncertainties regarding the identification of all critically pertinent variables, would seem to discourage consideration of more elaborate forms than Equation (15) at the present time.

In order to derive criteria for ascertaining flow regimes, we proceed as follows. Suppose that the correlations for two contiguous flow regimes a and b are given by Equation (15), with the corresponding adjustable constants K, α, β, γ, and δ labeled with subscripts a and b, respectively. Since the two regimes are contiguous, the values of $(f - f_w)$ from the two correlations are equal at the boundary between them, and at the transition point then

$$\frac{v^2}{Dg(s-1)} = K_t C^{\alpha_t} f_w^{\beta_t} C_D^{\gamma_t} \qquad (17)$$

in which

$$K_t = (K_b/K_a)^{1/(\delta_a - \delta_b)} \qquad (18)$$

$$\alpha_t = (\alpha_b - \alpha_a)/(\delta_a - \delta_b) \qquad (19)$$

$$\beta_t = (\beta_b - \beta_a)/(\delta_a - \delta_b) \qquad (20)$$

$$\gamma_t = (\gamma_b - \gamma_a)/(\delta_a - \delta_b) \qquad (21)$$

Using Equation (17), we define for convenience a dimensionless group termed the regime number R_{ab}:

$$R_{ab} = \frac{v^2}{K_t C^{\alpha_t} f_w^{\beta_t} C_D^{\gamma_t} Dg(s-1)} \qquad (22)$$

The transition point between flow regimes a and b is given by $R_{ab} = 1$. Moreover, since as v increases the change in f_w is relatively small in the turbulent flow region, it can further be inferred that $R_{ab} > 1$ corresponds to the faster flow regime, while $R_{ab} < 1$ corresponds to the slower flow regime. It will be shown later that the regime number defined above can be used in a unified method for delineating flow regimes.

In order to develop individual correlations for each of the flow regimes, the data must be grouped according to flow regime. Available data in the literature are not usually identified on this basis. The grouping must be accomplished according to some sort of criterion. As a first attempt, Newitt's criteria, Equations (3) to (5), and Durand's critical velocity correlation, Equation (12), were used. It was found that these relations did not consistently result in a satisfactory demarcation of the data, since they yielded results which were inconsistent with the part of the data for which the flow regime was known. In fact, the relations failed even with sand slurries, which represents the type of slurries on which they are based. It is possible that the conditions pertaining to the data tested exceeded the ranges of validity of these correlations. Under these circumstances, we used the following alternative procedure. For each slurry a plot of $(-\Delta p/L)$ vs. v was made as in Figure 1. Fortunately, almost half of the data sets were presented in this form already. From each plot, three transition velocities are chosen. In carrying out this step, the individual investigator's description of his data is taken into consideration. Also, whenever applicable, Newitt's criteria were used. The procedure is admittedly subject to personal judgment. But the dependence on the subjective element is hopefully ameliorated when the actual correlations are established, because it is the data which fall near the boundary between two flow regimes which are subject to uncertainty. The character of the particular correlation relating to each regime is, however, determined by the bulk of data which do not belong to the fringes of the regime.

The sets of data grouped according to the foregoing demarcation procedure were used to determine the adjustable constants in the correlation, Equation (15). From the resulting correlations, a set of transition criteria, in accordance with Equations (17) and (22), were derived which were then used to redemarcate the data. The redemarcated data were used to determine the correlations for each flow regime anew, and these were then used to derive a new set of transition criteria. This iterative process was terminated when satisfactory delineation of flow regimes, and also satisfactory pressure drop prediction, were achieved. The final forms of the correlations are the following:

Flow with a stationary bed (regime 0):

$$f - f_w = 0.4036\, C^{0.7389} f_w^{0.7717} C_D^{-0.4054} \left[\frac{v^2}{Dg(s-1)} \right]^{-1.096} \tag{23}$$

Saltation flow (regime 1):

$$f - f_w = 0.9857\, C^{1.018} f_w^{1.046} C_D^{-0.4213} \left[\frac{v^2}{Dg(s-1)} \right]^{-1.354} \tag{24}$$

Heterogeneous flow (regime 2):

$$f - f_w = 0.5513\, C^{0.8687} f_w^{1.200} C_D^{-0.1677} \left[\frac{v^2}{Dg(s-1)} \right]^{-0.6938} \tag{25}$$

Homogeneous flow (regime 3):

$$f - f_w = 0.8444\, C^{0.5024} f_w^{1.428} C_D^{0.1516} \left[\frac{v^2}{Dg(s-1)} \right]^{-0.3531} \tag{26}$$

TABLE 3. REGIME NUMBER CONFIGURATIONS

Regime	$(R_{01}$-1)	$(R_{12}$-1)	$(R_{23}$-1)	Flow regime (equation number)	Number of data points
0	−	−	−	Stationary bed (23)	361
1	+	−	−	Saltation (24)	1 230
2	+	+	−	Heterogeneous (25)	493
3	+	+	+	Homogeneous (26)	645

From the adjusted correlation constants for the various flow regimes, depicted in Equations (23) to (26), it can be surmised that the values for β, γ, and δ do, on the whole, follow a consistent trend, interpretable in terms of the presumed variation in the relative roles of the various forces predominating during transition from bed load flow to homogeneous flow. Thus, the exponents of the term $[v^2/Dg(s-1)]$ suggest that the gravitational forces become relatively less important as the homogeneous flow regime is approached. Similar interpretations can be attached relative to the exponent values for the f_w and C_D terms.

FLOW REGIME DELINEATION

The regime number defined by Equation (22) is based on a reduced or generalized form of the dimensionless group $v^2/Dg(s-1)$ which characterizes the ratio between the inertial and net gravitational forces in the flow. The regime numbers $R_{i(i+1)}$ corresponding to transition between the i^{th} and $(i+1)$st regimes are determined using Equations (18) to (22) together with the appropriate values of the adjusted constants given in Equations (23) to (26). These are given as follows:

$$R_{01} = \frac{v^2}{31.93\, C^{1.083} f_w^{1.064} C_D^{-0.06160} Dg(s-1)} \tag{27}$$

$$R_{12} = \frac{v^2}{2.411\, C^{0.2263} f_w^{-0.2334} C_D^{-0.3840} Dg(s-1)} \tag{28}$$

$$R_{23} = \frac{v^2}{0.2859\, C^{1.075} f_w^{-0.6700} C_D^{-0.9375} Dg(s-1)} \tag{29}$$

The transition between flow with a stationary bed (0) and saltation flow (1) takes place at $R_{01} = 1$, that between saltation flow (1) and heterogeneous flow (2) at $R_{12} = 1$, and that between heterogeneous flow (2) and homogeneous flow (3) at $R_{23} = 1$. Equations (27) to (29) conform to the regime delineation associated with the actual experimental data. Furthermore, based on the experimental data, the sequence of signs of the array $[R_{i(i+1)} - 1]$ with $i = 0$, 1, and 2, which we designate here as the regime number configuration, follows that depicted in Table 3 by all data except for 119 out of the total 2 848 points. The configuration for these apparently anomalous 119 data points is shown in Table 4. We shall demonstrate shortly that the seemingly odd behavior exhibited by these data points can be fully resolved, and indeed that these data can also be subsumed under the regime delineation scheme developed here.

The comparison between experiments and correlations is shown in Figures 2 to 5, which show the data delineated according to the criteria in Table 3, together with the correlations Equations (23) to (26). In Figures 4 and 5 the scatter of the data appears to be quite considerable. However, because the ordinate values are differences (that is, f-f_w), these plots tend to convey an exaggerated impression of the deviation for small values of $(f$-$f_w)$, which is the range corresponding to these two regimes.

We now return to an examination of the 119 data points which exhibited an anomalous flow regime configuration. Since these 119 data points clearly belong to

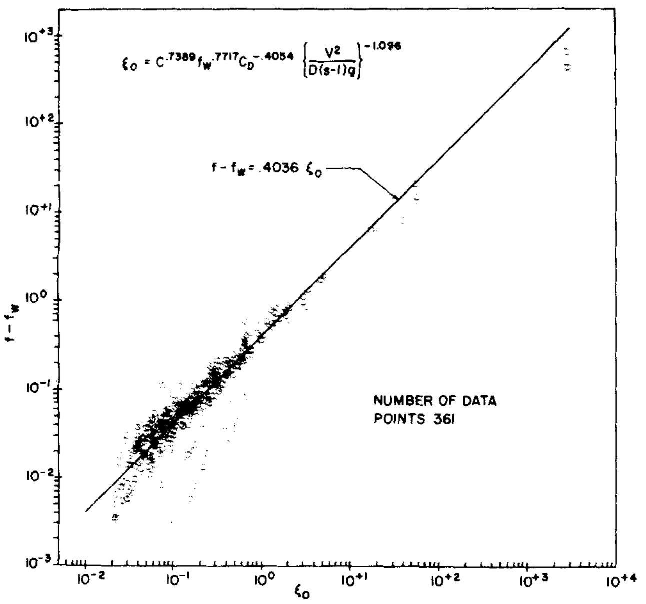


Fig. 2. Friction factor correlation for slurry flow in the regime with a stationary bed.

the parameter ranges for which the slurry flow correlations developed in this work apply, the failure of the regime number configuration scheme to accommodate them might seem to constitute a serious inadequacy. It will, however, be demonstrated that these data points can indeed be included in an extended configuration scheme, and that the difficulty resides in the fact that the regime delineation scheme developed up to this point represents only a part of the overall situation. It will be convenient, as in the earlier discussion, to designate the stationary bed, the saltation, the heterogeneous, and the homogeneous flow regimes by the numbers 0, 1, 2, and 3, respectively.

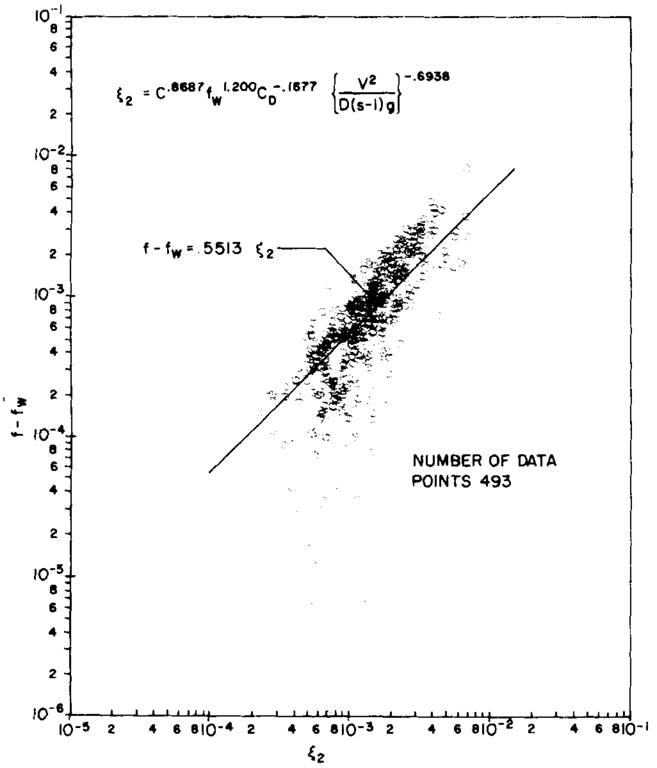


Fig. 4. Friction factor correlation for slurry flow in heterogeneous flow regime.

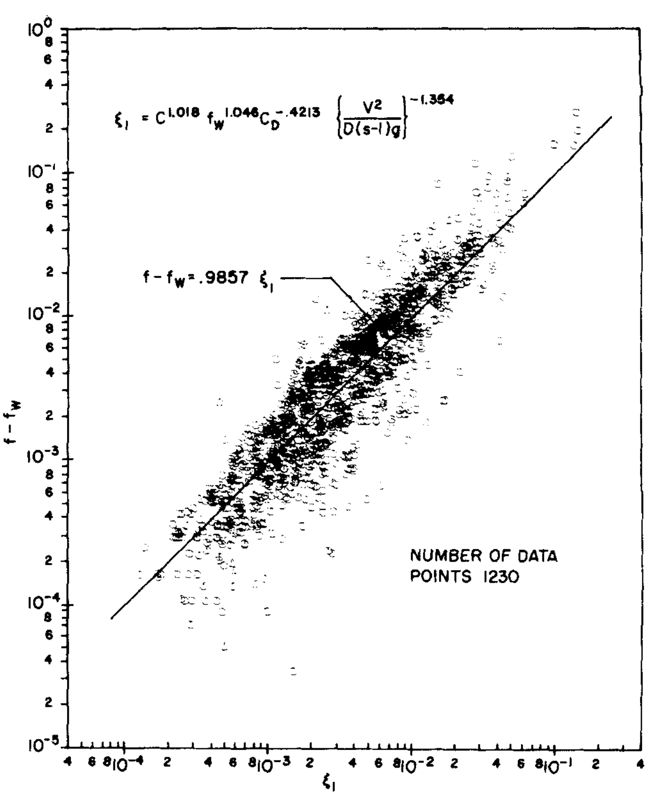
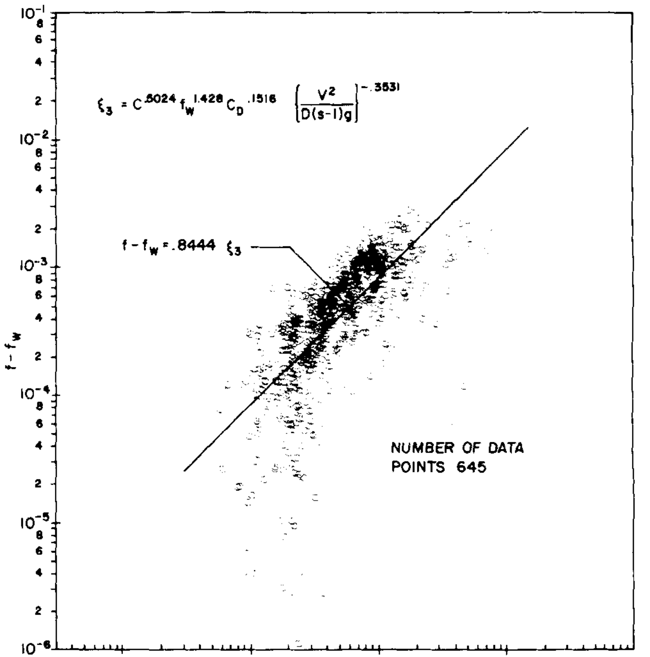


Fig. 3. Friction factor correlation for slurry flow in saltation regime.

A very convenient means for visualizing flow regime transitions [Equations (27) to (29)] is to plot solid particle size *d* against mean slurry velocity *v* in a given pipe with all other variables fixed. In such a plot, the curves corresponding to the three transition equations will have either the form depicted in Figure 6 or the one in Figure 7. A slurry flow system described by a point on any of the solid lines is considered to be in a state of transition between flow regimes. The area below and to the right of this point, that is, corresponding to a region of smaller particle size and higher mean velocity, repre-



16 %

Fig. 5. Friction factor correlation for slurry flow in homogeneous flow regime.

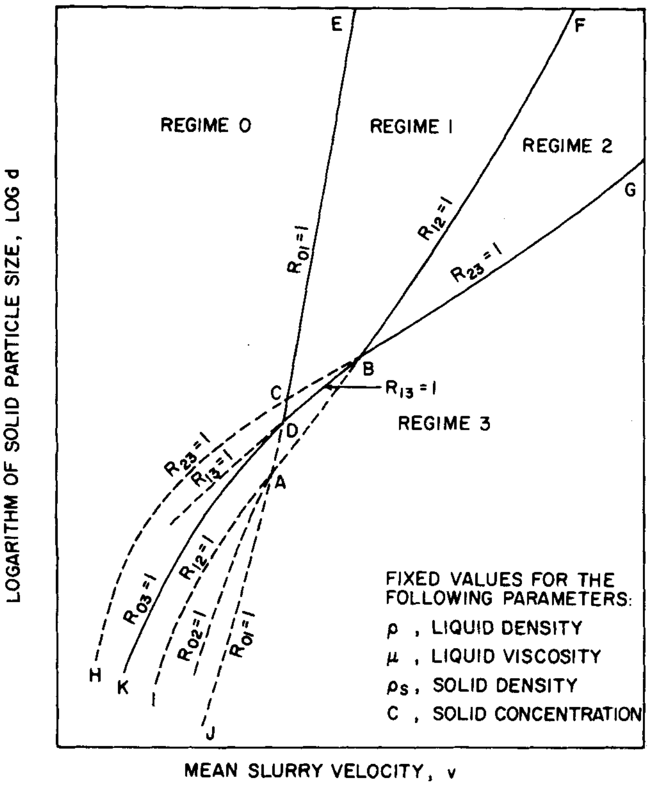


Fig. 6. Illustration of flow regime delineation for a given pipe.

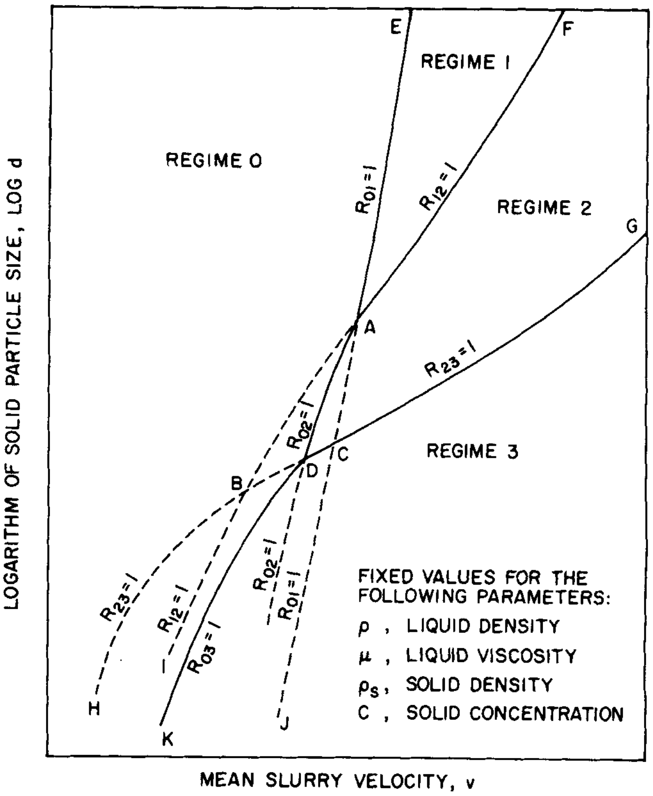


Fig. 7. Illustration of flow regime delineation for a given pipe.

TABLE 4. ANOMALOUS REGIME NUMBER CONFIGURATIONS

Regime*	$(R_{01}$-1)	$(R_{12}$-1)	$(R_{23}$-1)	Flow regime	Number of data points
A	−	−	+	?	10
B	−	+	+	?	1
C	−	+	−	?	0
D	+	−	+	?	108

* Letters are used merely for identification.

TABLE 5. AREAS CORRESPONDING TO DATA POINTS WITH ANOMALOUS REGIME NUMBER CONFIGURATION

Configuration	Corresponding area circumscribed by (in Figure 6)	(in Figure 7)
A	H-C-A-I	H-B-I
B	I-A-J	I-B-C-J-
C	—	A-B-C-A-
D	A-B-C-A	—

sents flow regimes more to the homogeneous side than the area above and to the left of the point. The locations of these various areas with respect to the particular transition curve clearly obey the following scheme:

(1) $R_{i(i+1)} = 1$ on the transition curve
(2) $R_{i(i+1)} < 1$ in the area to the left of the curve
(3) $R_{i(i+1)} > 1$ in the area to the right of the curve

The foregoing properties can be verified by actually checking the calculated value of regime number within each area. By virtue of these properties, the regime number configuration of any given area in the *d-v* space can be readily determined. For example, the area HCAI in Figure 6 has regime numbers of the following magnitude: $R_{01} < 1$, $R_{12} < 1$, and $R_{23} > 1$; excluding the boundary, this area has exactly the configuration (−, −, +) which is configuration A as defined in Table 4. The corresponding area in Figure 7 is that circumscribed by the curves HB and BI. Thus, the four flow regimes are identified, and all the areas circumscribed by the dashed curves HB, BA, and AJ in both illustrations are seen to be related to the odd configurations, as shown in Table 5.

The foregoing clarification falls short of identifying the flow regime corresponding to any given point located within the odd regime number areas. This can, however, be resolved easily. Consider, for example, the triangular area ABC in Figure 6. The friction loss equations for regimes 0 and 2 [that is, Equations (23) and (25)] do not apply in this area. It is an intermediate region between regime 1 and regime 3; the transition between these two regimes is determined by equating the corresponding friction loss equations, that is, Equations (24) and (26). There results the following new regime number:

$$R_{13} = \frac{v^2}{1.167\, C^{0.5153} f_w^{-0.3820}\, C_D^{-0.5724}\, Dg(s-1)} \tag{30}$$

The corresponding transition condition is described by

$$R_{13} = 1 \tag{31}$$

It is obvious that the line defined by Equation (31) will pass through the point of intersection B (in Figure 6) of the lines $R_{12} = 1$ and $R_{23} = 1$.

Through similar arguments, the following additional regime numbers are obtained:

$$R_{02} = \frac{v^2}{0.4608\, C^{-0.3225} f_w^{-1.065}\, C_D^{-0.5906}\, Dg(s-1)} \tag{32}$$

$$R_{03} = \frac{v^2}{0.3703\, C^{0.3183} f_w^{-0.8837}\, C_D^{-0.7496}\, Dg(s-1)} \tag{33}$$

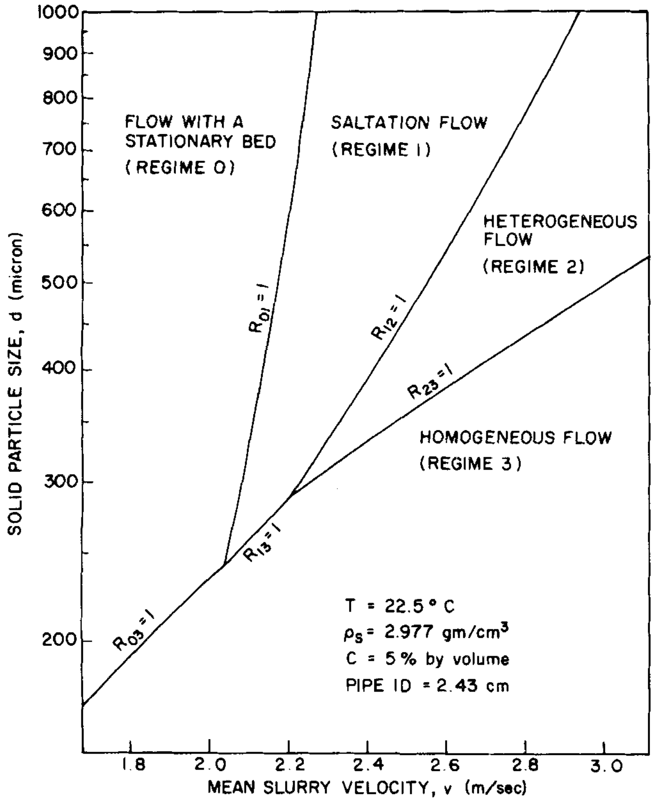


Fig. 8. Flow regime diagram for solid-water flow in 1 in. PVC pipe.

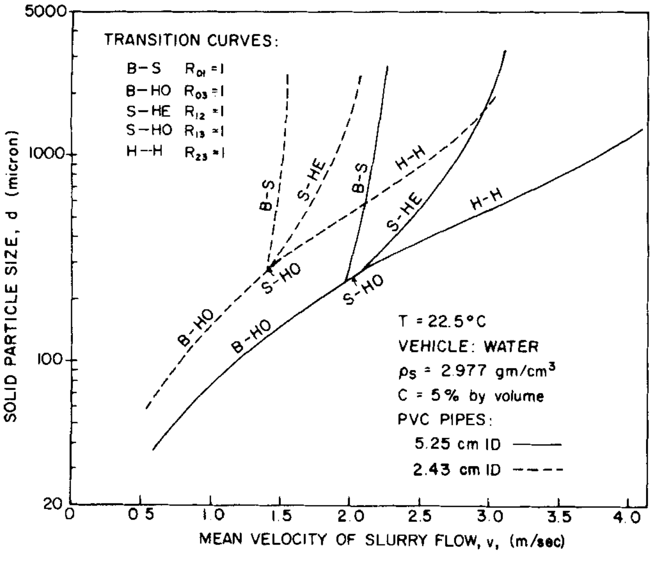


Fig. 9. Effect of pipe diameter on flow transitions for slurry flow.

The corresponding transition conditions are given by

$$R_{02} = 1 \tag{34}$$

$$R_{03} = 1 \tag{35}$$

The status of the confused areas in the d-v space is thus resolved by the scheme just described and illustrated in Figures 6 and 7. These two figures were drawn qualitatively for convenience. A quantitative illustration of the d-v plot is provided in Figure 8 for a slurry system with $\rho_s = 2.977$ g/cm³, $C = 5\%$ by volume in a 2.54 cm PVC pipe at $T = 22.5°C$. The variation of flow characteristics with d, as well as v, is calculated by the various transition equations as indicated in the plot. In Figure 9, the effect of pipe size on flow regime transitions is illustrated by comparing the curves for 5.08 and 2.54 cm pipes. The rules in our delineation procedure are conveniently summarized in Table 6.

CALCULATION OF PRESSURE DROP

The estimation of pressure drop using the correlations in this work is rather straightforward. It entails determination of the flow regime prevailing and the selection of the corresponding correlation for computing the friction factor f. The pressure drop is then calculated from

$$-\frac{\Delta p}{L} = 2f\frac{\rho v^2}{D} \tag{36}$$

The determination of the flow regime is made by calculating the regime numbers R_{01}, R_{12}, and R_{23}, using Equations (27), (28), and (29). The flow regime is then ascertained by comparison of the resulting configuration with the pattern shown in Table 6. If, at this stage, it is discovered that the configuration is odd, then the regime numbers R_{13}, R_{02}, and R_{03} are also calculated using Equations (30), (32), and (33), and the regime is again ascertained by comparison with the pattern in Table 6.

The value of the drag coefficient C_D is needed in the slurry flow correlations. To avoid the need for a trial-and-error procedure, resulting from the fact that v_∞ is unknown, we use the correlation described in our previous article (Turian et al., 1971). These equations are

$$\log_{10} N_{Re} = -1.38 + 1.94 \log_{10}\Lambda$$
$$-8.60 \times 10^{-2} (\log_{10}\Lambda)^2 - 2.52 \times 10^{-2} (\log_{10}\Lambda)^3$$
$$+9.19 \times 10^{-4} (\log_{10}\Lambda)^4 + 5.35 \times 10^{-4} (\log_{10}\Lambda)^5 \tag{37}$$

in which

$$\Lambda = N_{Re}\, C_D^{1/2} = \left\{ \frac{4}{3} \frac{g d^3 \rho(\rho_s - \rho)}{\mu^2} \right\}^{1/2} \tag{38}$$

Table 6. Criteria for Flow Regime Delineation

Configuration	R_{01}-1	R_{12}-1	R_{23}-1	R_{02}-1	R_{03}-1	R_{13}-1	Corresponding flow regime
0	−	−	−				0
1	+	−	−				1
2	+	+	−				2
3	+	+	+				3
A	−	−	+		−		0
	−	−	+		+		3
B	−	+	+		−		0
	−	+	+		+		3
C	−	+	−	−			0
	−	+	−	+			2
D	+	−	+			−	1
	+	−	+			+	3

TABLE 7. COMPARISON OF FRICTION LOSS CORRELATIONS FOR SLURRY FLOW

Correlation	Number of data points applicable	Average abs. % deviation	Mean value of f_{exp}/f_{calc}	Standard dev. of distribution, f_{exp}/f_{calc}
	(0) Flow with a stationary bed (366 data points)			
Durand and Condolios [Equation (6)]*	366	49.3%	0.992	0.657
Zandi and Govatos [Equations (9) and (10]	25	59.9%	1.52	0.659
Present work [Equation (23]	366	26.7%	1.07	0.358
	(1) Saltation flow (1 287 data points)			
Durand and Condolios [Equation (6)]*	1 287	24.5%	1.13	0.340
Zandi and Govatos [Equations (9) and (10]	1 050	23.0%	0.914	0.301
Newitt et al. [Equations (3) to (5)]	1 287	26.4%	1.17	0.609
Present work [Equation (24)]	1 287	20.2%	1.10	0.311
	(2) Heterogeneous flow (493 data points)			
Durand and Condolios [Equation (6)]*	493	11.4%	1.11	0.122
Zandi and Govatos [Equations (9) and (10]	493	6.48%	0.991	0.085
Newitt et al. [Equations (3) to (5)]	493	6.12%	1.00	0.082
Present work [Equation (25)]	493	5.85%	1.02	0.080
	(3) Homogeneous flow (702 data points)			
Durand and Condolios [Equation (6)]*	702	10.7%	1.11	0.152
Zandi and Govatos [Equations (9) and (10]	702	7.64%	1.07	0.141
Newitt et al. [Equations (3) to (5)]	702	7.32%	1.07	0.129
Present work [Equation (26)]	702	4.91%	1.02	0.122

* With $K = 84.9$ and $m = -1.5$.

COMPARISON OF PRESENT WORK WITH OTHER CORRELATIONS

The slurry flow correlations developed in this study are compared with the correlations of others using the experimental data collection described previously as a basis. The results of such a comparison are shown in Table 7, in which the following definitions are used:

Mean value:

$$\bar{y} = \sum_{i=1}^{n} \frac{y_i}{n} \tag{39}$$

Standard deviation of distribution:

$$S = \left\{ \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n} \right\}^{1/2} \tag{40}$$

where $y = f_{exp}/f_{calc}$, and n is the number of data points.

It is clear that on the whole the present correlations represent an improvement over those previously proposed. In addition, for the flow with a stationary bed, all data points are excluded by Newitt's demarcation criteria, and only 25 out of 366 data points survive Zandi and Govatos's criterion. In the saltation flow regime, 237 data points are eliminated by Zandi and Govatos's criterion. These comparisons also demonstrate that all correlations do an increasingly better job of predicting friction loss as homogeneous flow conditions are approached, and this is probably due largely to the fact that the experimental data are least reproducible for bed load flows.

CONCLUSIONS

The results presented in the preceding pages demonstrate that it is possible to develop empirical correlations capable of providing a fairly satisfactory prediction of pressure drop as well as of serving as the basis of a rather useful classification scheme in pipeline transport of slurries.

In a previous article (Turian et al., 1971) our examination of the problem served to define a number of the pertinent variables governing the flow, while in the present effort we have extended the correlation scheme to provide considerably broader coverage in relation to both variable ranges and flow behavior. The need for regime delineation schemes, similar to the one developed here, is motivated primarily by the fact that the markedly different behavior characterizing the various flow regimes observed is associated with the correspondingly markedly different roles played by the various effects influencing the flow. However, in this type of problem the ordering of the data according to flow regime does have an important practical benefit. It permits classification of the data into groupings of relatively more manageable size considering the immense number and range of variables governing the flow. Indeed, in the future it may be necessary to examine possible effective classification schemes based upon the nature of the particle size distribution, because most correlations, including our own, are incapable of tolerating broad particle size distributions. Needless to say, as the data base is expanded and refined, so will the correlation procedures, but it is clear that the data must be complete and detailed and that there must be a reasonably unambiguous identification of the nature of the prevalent flow regime. Research needs in slurry transport, and in related areas such as studies on pumping-feeding systems and on solid-liquid separation and drying, abound and are vitally important, but the problem of developing effective correlation methods utilizing sound and comprehensive data bases must remain one of the major thrusts.

NOTATION

A = parameter [Equation (1)]
A' = parameter [Equation (2)]
C = mean solid concentration [Equation (1)], also discharge concentration
C' = spatial concentration of solids in a pipe [Equation (2)]
C_D = $(4/3)\ gd(s-1)/v_\infty^2$, drag coefficient for free falling sphere
D = inside diameter of pipe
d = diameter of solid particle
F_L = parameter [Equation (12)]
f = $(-\Delta p/L)\ (D/2\rho v^2)$, friction factor for pipe flow
g = gravitational acceleration
i = head loss for pipe flow, in feet of water per foot of pipe
K = constant [Equation (6)]
K = constant [Equation (15)]
L = pipe length
m = constant [Equation (6)]
N_I = $v^2\ C_D^{1/2}/[CDg(s-1)]$ [Equation (8)]
N_{Re} = $d\rho v_\infty/\mu$, Reynolds number for free falling spheres
n = number of data points
$-\Delta p$ = pressure drop in length L of pipe
R_{ab} = regime number defined by Equation (22)
S = standard deviation [Equation (40)]
s = ρ_s/ρ, relative density
v = mean velocity
v_B = $17v_\infty$ [Equation (5)]
v_c = critical velocity of slurry (Figure 1)
v_H = $(1\,800\ gDv)^{1/3}$ [Equations (3) and (4)]
v_s = settling velocity of particles inside a pipe [Equation (2)]
v_∞ = terminal velocity of sphere settling in an unbounded fluid
y = $f_{\exp}/f_{\text{calc}}$
$\bar{y}$ = mean value of y [Equation (39)]

Greek Letters

$\alpha, \beta, \gamma, \delta$ = constants, Equation (15)
Λ = $N_{Re}\ C_D^{1/2}$, Karman number
μ = viscosity of liquid
ξ = parameter (Figures 2 to 5)
ρ = density of liquid
ψ = $v^2\ C_D^{1/2}/[gD(s-1)]$ [Equations (9) to (11)]

Subscripts

calc = calculated
exp = experimental
i = i^{th}
m = liquid-solid mixture
t = transition between flow regimes
w = related to water or liquid
0 = regime of flow with a stationary bed
1 = saltation flow regime
2 = heterogeneous flow regime
3 = homogeneous flow regime

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Use of Trajectory Analysis to Study Stability of Colloidal Dispersions in Flow Fields

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Trajectories have been computed for two equal sized spherical particles in simple laminar shearing and in uniaxial extensional flows. Effects of interparticle attraction, electrostatic repulsion, and hydrodynamics were included.

The results are pertinent to questions of colloidal stability under various conditions of flow. Particulate dispersions can react in several different ways as the intensity of shearing is increased from zero: the dispersion can remain stable; it can be redispersed, if it had been initially flocculated into a weak secondary minimum in the interparticle potential curve; it can be flocculated into a strong primary minimum in the potential curve; or, in extreme cases, it can be redispersed from the primary minimum. Results are presented which illustrate criteria for flocculation or stability to both laminar shearing and extensional flow. It is shown that hydrodynamic effects can significantly alter the criteria developed for stability of dispersions to Brownian coagulation.

SCOPE

Colloidal stability, by which one means the tendency of a suspension to remain dispersed and to resist flocculation, is dependent on many factors. These include the actions of Brownian motion, London attractive forces, electrostatic repulsive forces, and hydrodynamic forces, the latter being generated by bulk motion of the suspension. It is desirable to know how these various forces combine to promote stable or unstable dispersions. The factors which affect colloidal stability are important to those, such as manufacturers of polymer latices, who wish to insure stability, and also to those who wish to flocculate particulate suspensions, thereby separating finely divided solids from a liquid.

In the present study, the trajectories of two equal sized spherical particles in simple laminar shearing and in a uniaxial extensional flow were analyzed. In addition to the action of hydrodynamic forces, the effects of London attraction and electrostatic repulsion were calculated. Brownian motion was excluded from the trajectory analysis, but its effects were considered separately,