Journal of Food Engineering **19** (1993) 247–274

The Finite-Element Method in Food Processing: A Review

V. M. Puri[a] & R. C. Anantheswaran[b]

[a]Department of Agricultural and Biological Engineering, [b]Department of Food Science, The Pennsylvania State University, University Park, PA 16802, USA

(Received 28 October 1991; revised version received 17 April 1992; accepted 25 April 1992)

ABSTRACT

The finite-element method has been successfully used to model several food-processing operations. Much of the activity to date has focused on heating and cooling, freezing and thawing, and heat and mass transfer, including drying and mechanical damage. Most of the studies have been confined to two-dimensional and/or axisymmetric regions undergoing transient response. A limited number of research results have been reported for three-dimensional and/or coupled heat, moisture, and stress analysis in food products. This review indicates that there still exists considerable potential for the use of the finite-element method in food-processing. Some other areas where the pay-off can be significant include baking, extrusion, aseptic processing, microwave heating, and optimization of food quality in terms of texture, nutrient retention, and microbial degradation during thermal processing.

INTRODUCTION

A food product undergoes numerous processing steps from production to consumption. A clear understanding of the dynamics of food products during the processing treatments is central to maintaining the desired quality, texture, and sterility. Food-processing studies therefore hold the potential for improvements in current processes and can lead to development of new products. The wide range of food materials that are subjected to an array of processing steps in the food industry represents a real challenge in the understanding and clear definition of these processes by using physical principles.

All approaches leading to a better understanding and description of various processes to date can be broadly classified as experimental and

Journal of Food Engineering 0260-8774/93/$06.00 —

analytical. In the experimental approach, the primary emphasis is to simulate the food-processing operation in laboratory or pilot-scale set-ups. This approach provides valuable data on the system performance but may lack a generalized theoretical description of the process. The correlations developed to describe the process by using the experimental approach are usually empirical or semi-empirical and hence cannot be easily generalized. However, the experimental approach is indispensable in validating the analytical methods.

The analytical approach, on the other hand, is based on physical principles. This approach has led to generalized solutions for a certain class of problems. The analytical approach can be subdivided into three categories: exact or closed-form solutions, approximate but closed-form solutions, and, more recently, numerical solutions. The exact solutions are usually idealizations and represent a significant simplification of the complex practical problems. They are invaluable in providing qualitative information related to trends and order of magnitude of variables and the associated changes rates, such as heat flux. The approximate closed-form solutions are attempts at solving real-world problems. Some of the limitations include regular geometrical shapes of food materials, essential or natural types of boundary conditions, and steady or unsteady solutions in one or two dimensions only.

The advent of computers, and more recently high-speed parallel and vector machines, has provided the designers and researchers with an opportunity to simulate real-world processes. The commonly employed numerical techniques are the finite-difference (Teixeria *et al.*, 1969; Charm *et al.*, 1972; Bakal & Hayakawa, 1973; Heldman & Gorby, 1975; Eshleman *et al.*, 1976; Hsieh *et al.*, 1977; Baird & Gaffney, 1977; Cleland & Earle, 1977, 1979*a*, *b*; Tong, 1988) and the finite-element methods. The recently developed and evolving boundary-element technique's potential contribution is significant but has been applied to only a limited number of food- and agricultural-engineering problems (Puri, 1987; Rosenfeld *et al.*, 1991). These three numerical methods (as well as others, such as finite-volume) are subsets of the much more generalized method of weighted-residual technique (Brebbia *et al.*, 1984).

The use of finite-element method for understanding, describing, and analyzing the various food-processing operations is the focus of this paper. The specific objectives of this paper were:

(i) to review the applications of the finite-element method used to solve problems of food-processing as reported in literature; and
(ii) to outline the use of finite-element method in other areas of food-processing.

Numerous textbooks summarizing the finite-element method are available (e.g. Becker *et al.*, 1981; Irons & Shriver, 1983; Desai, 1984; Segerlind, 1984; Brauer, 1988; Zienkiewicz, 1989). The reader should consult these references for details of the method. An overview of the finite-element method within the context of food-engineering is also given by DeBaerdemaeker *et al.* (1977), Singh and Segerlind (1974), and Segerlind *et al.* (1974). Some of the key advantages and disadvantages of the finite-element method compared with the finite-difference method are listed below:

- Advantages:
  1. Spatial variation of material properties can be handled with relative ease.
  2. Irregular regions can be modeled with greater accuracy.
  3. The method is better suited to non-linear problems.
  4. Element sizes can be easily varied.
  5. Spatial interpolation is much more meaningful.
  6. Mixed-boundary-value problems are easier to handle.
- Disadvantages:
  1. The element equations are usually much more mathematically complex as compared with the grid-point equations for the finite-difference method.
  2. The method is numerically intensive and can therefore take more CPU time and memory storage space, as compared with the finite-difference method, to solve the same problem.

LITERATURE REVIEW

The literature review in this paper is organized under four main subheadings: Heating and cooling, Freezing and thawing, Heat and mass transfer, and Mechanical damage. The selection of these headings represents the most suitable descriptors for organizing and reporting the work done in the food-processing area. Whenever possible, assumptions and limitations mentioned by the authors have been provided. A summary of the various finite-element models (FEM) employed in the four major areas of food processing is given in Table 1. Much of the research activity to date has focused on heating and cooling and heat and mass-transfer applications, as shown in Table 1.

Heating and cooling

Heating (for drying) and cooling (for aeration) are two critical processing steps for grains to maintain quality during storage. The usual heat-trans-

TABLE 1
Summary of the Use of the Finite-Element Method in Food-Processing

Process	*Author(s)*	*Affiliation*	*FEM Attributes* 1D*	2D*	AXI-	3D*	Time-dependent	Heat transfer	Mass transfer	Elasticity	Visco-elasticity	*Food product(s)*	*Applications*
Heating and cooling	Misra & Young (1979)	N.C. State University	✓				✓	✓				Apples	Cooling
	Gustafson & Segerlind (1977)	U. of Minnesota		✓			✓	✓				Corn kernels	Drying
	Sokhansanj & Gustafson (1979)	U. of Saskatchewan		✓			✓	✓				Rice kernels	Drying
	Alagasundaram *et al.* (1990)	U. of Manitoba				✓	✓	✓				Bulk barley, rapeseed	Drying, aeration
	Potluri (1985)	Texas A&M		✓			✓	✓				Beef carcass	Cooling
	Upadhyaya *et al.* (1986)	Cornell U.			✓		✓	✓				Whole eggs	Hot washing
	Jiang *et al.* (1987)	U. of Minnesota		✓			✓	✓				Broccoli	Cooling
	Hayakawa & Succar	Rutgers U.	✓				✓	✓				Tomato, potato	Cooling
	Pan & Bhowmik (1991)	Ohio State U.		✓			✓	✓				Tomatio	Cooling
	Stringer *et al.* (1989)	VPI State University		✓			✓	✓				Fish	Cooling
	Lin *et al.* (1989)	Penn. State U.			✓		✓	✓				Alginate gel	Microwave heating
	Chen *et al.* (1990)	Purdue U.			✓		✓	✓				Potato	Microwave heating
	Naveh *et al.* (1983*a*, *b*, *c*)	U. of Minnesota		✓	✓		✓	✓				Cream-style corn, apple sauce, canned meat	Sterility, temperature overshoots
	Sastry *et al.* (1985)	Penn. State U.				✓	✓	✓	✓			Mushrooms	Agartine degradation
	Sastry (1986)	Penn. State U.				✓	✓	✓	✓			Mushrooms	Canning
	Engelman & Sani (1983)	U. of Colardo			✓		✓	✓				Beer	Pasteurization

	McCarthy & Merson (1989)	U. of California, Davis		√				√			Fluid foods	Sterilization
	Kumar *et al.* (1990)	U. of Minnesota			√		√	√			Fluid foods	Sterilization
	DeAlwis & Fryer (1990)	Cambridge, UK		√			√				Lamb & potato in brine	Sterilization
Freezing and thawing	Comini *et al.* (1974)	Fac Ingegreria Trieste, Italy		√			√	√			Lamb and beef carcasses	Freezing
	Purwadaria & Heldman (1980)	Michigan State U.		√			√	√			Ground beef	Freezing
	Abdalla & Singh (1985)	U. of California, Davis			√		√	√			Model food substance	Freezing and thawing
Heat and mass transfer	Misra & Young (1978*a*, *b*)	N.C. State University	√				√		√		Soybean kernels	
	Chhinnan & Bakshi (1984)	U. of Georgia			√		√		√		Black-eye peas	Dehulling
	Segerlind (1984)	Michigan State U.		√							Grains	Air flow
	Khompis *et al.* (1984)	Michigan State U.				√					Grains	Air flow
drying	Talbot (1989)	U. of Florida		√		√					Grains	Air flow
	Rumsey & Fortis (1983)	U. of California, Davis		√							Walnuts	Air flow
	Marchant (1976)	AFRC, Silsoe, UK		√							Grains	Air flow
Heat and mass transfer	Zhang *et al.* (1984)	U. of Georgia		√			√		√		Rice kernels	Soaking, drying, parboiling
	Syarief *et al.* (1987)	U. of Minnesota		√			√		√		Corn kernels	Drying
mass transfer	Haghighi *et al.* (1988*a*, *b*, 1990)	Purdue U.			√		√	√	√	√	Soybean and barley kernels	Drying and thermal stresses
	Upadhyaya *et al.* (1989)	U. of California, Davis	√				√	√	√		Soybean kernels	Drying
	Hong *et al.* (1986)	U. of Georgia		√			√		√		Raisin, peanuts almonds, banana	Moisture transfer in mixtures

TABLE 1 — contd.

Process	*Author(s)*	*Affiliation*	*FEM Attributes*									*Food product(s)*	*Appli-cations*
			*1D**	*2D**	*AXI-*	*3D**	*Time-dependent*	*Heat transfer*	*Mass transfer*	*Elasti-city*	*Visco-elasticity*		
	Lu & Siebenmorgan (1991)	U. of Arkansas			√		√		√			Rice	Adsorption
Mechanical damage	Gustafson *et al.* (1977*a*, *b*)	U. of Minnesota			√					√		Fruits	Damage analysis
	Rumsey & Fridley (1977)	U. of California, Davis			√					√	√	Fruits, vegetables	Damage analysis
	Chen *et al.* (1984)	U. of California, Davis			√					√	√	Fruits	Damage analysis
	Cooke *et al.* (1977)	Cornell U.			√					√		Biological tissues	Cell damage
	Cardenas-Weber *et al.* (1989, 1991)	Purdue U.			√	√				√		Melons	Damage analysis

*1D = one-space dimension, 2D = two-space dimensions, 3D = three-space dimensions.

fer process induces stresses in the kernel, which, if not controlled, can cause kernel-cracking and hence loss in quality and yield. Gustafson *et al.* (1977*b*, 1979) studied the heating and cooling of single kernels of corn and its influence on stresses in non-homogeneous regions (endosperm and germ) of the kernel. The finite-element formulation included solution of the two-dimensional time-dependent heat-conduction equation. The key assumption in this formulation was that material properties were independent of temperature and moisture content and differential contraction of kernels due to moisture migration is negligible. For subsequent stress analysis, the grain kernel was assumed to be an elastic body. The validation of the FEM by using experimental data was not included. The following findings resulted from FEM sensitivity studies: (i) during heating, the maximum tensile stress occurs at the center of soft endosperm, and (ii) during cooling, the maximum stress occurs within the crown region of hard endosperm.

Subsequently, Sokhansanj and Gustafson (1979) studied the cracking of single rice kernels under heating, cooling, hydrating, and dehydrating environments. The key assumption in their formulation was that moisture-migration and temperature-diffusion equations are not coupled. The two-dimensional FEM used linear triangular elements to solve moisture and temperature diffusion equations. These values were then used to calculate moisture- and temperature-induced stress by using Hooke's law. Model-validation results were not included. The following determinations were made from this study: (i) maximum tensile stresses resulted soon after heating or cooling, and (ii) moisture-induced stresses were sustained for several hours.

Alagasundaram *et al.* (1990) calculated grain-temperature distribution in a bin during drying and aeration by using numerical methods. Grain temperature is one of the three key abiotic factors affecting long-term grain storage (the other two being moisture content and gas distribution). The transient heat-conduction equation was solved in three dimensions by using linear and quadratic hexahedron elements. The variational technique was to derive the non-linear, non-isotropic finite-element formulation. Comparison of the FEM predicted and measured values for barley and rapeseed showed differences ranging from 1·6 to 4·9°C for linear elements and 1·2 to 5·3°C for quadratic elements. Quadratic-element predictions, on the average, were more accurate than linear-element predictions.

Potluri (1985) used the diffusion equation (steady and unsteady state) to develop a two-dimensional time-dependent FEM to predict the cooling of loin carcasses and to estimate their surface-heat-transfer coefficients. He modified the FEM developed by Arce *et al.* (1983) for a constant heat-transfer coefficient. The FEM used linear triangular elements. The Ritz method was used to generate the capacitance and stiffness matrices. The time-dependent equation set was subsequently solved by using the Crank–Nicolson method. The main assumption in Potluri's formulation appears to be that the cooling of carcass is primarily by heat conduction, with the evaporation being handled through a modified heat-transfer coefficient. The FEM was generalized to a Universal Finite Element Model that required for its input only the weight of the carcass and its fat content. The measured and FEM-predicted values were within 1°C, with r^2 greater than 0·92. On the basis of this study, the following observations were made: (i) the FEM can be used to control the ambient temperature for carcass-cooling by using simple inputs, such as weight and fat content, (ii) the FEM can be used to predict carcass temperature and the heat-transfer coefficient.

Hot washing of eggs is an important step to clean and sanitize the egg shell. This key step is also implicated in egg-shell damage. Upadhyaya *et al.* (1986), following an earlier study by Gates *et al.* (1984), used an axisymmetric (egg being an ovaloid) time-dependent formulation to study thermally induced stresses during the egg-washing operation. They solved the coupled heat-conduction and elasticity equations to determine temperature and stress states. The following assumptions were made: (i) egg shell is linearly elastic and isotropic; (ii) egg shell is thermally isotropic; (iii) the liquid albumen and yolk are incompressible; and (iv) the air within the air cell behaves as an ideal gas. Upadhyaya *et al.* (1986) used commercial software ANSYS (Version 4.1) to model the egg-shell performance. The FEM validation was done by comparing the results from a previous study by Upadhyaya *et al.* (1985). The element types used were: shell element for egg shell with three degrees of freedom per node (STIFF 11), and isoparametric, quadrilateral elements (STIF 55) for thermal analysis. Upadhyaya *et al.* (1986) made the following observations: (i) substantial tensile stresses developed after 60 s of washing; (ii) incorporation of load at the poles greatly increased shell stresses; and (iii) the location of maximum stress varied with egg shape.

Cooling of fresh fruits and vegetables is a key intermediate-processing step to ensure their texture and quality. Misra and Young (1979) modeled the cooling of an apple in spherical co-ordinates. They simulated the temperature response of apples by using a one-dimensional time-dependent finite-element formulation. Two-node linear elements were used in the FEM formulation. The key assumptions were: (i) the ambient drying conditions, i.e. relative humidity and temperature, are constant, and (ii) no respiration and moisture exchange takes place between the apple and the ambient air. The model validation was done by comparing the FEM-predicted temperature values to a constant-property analytical solution. The comparison indicated that the FEM results were not statistically different from analytical values. Misra and Young (1979) arrived at the following conclusions: (i) FEM yielded very good approximations for transient cooling in an apple; (ii) equal and unequal elements were not sufficiently sensitive to influence FEM calculations significantly; however, equal elements yielded better results.

Pre-cooling of vegetables is a crucial step in order to maintain their freshness. Jiang *et al.* (1987) studied the pre-cooling of broccoli by using the finite-element method. Since the broccoli stalk is the slowest-cooling component, it is the most perishable. A time-dependent axisymmetric heat-conduction equation was used to model the stalk performance. The FEM was developed by using the variational approach and following the procedure outlined by Polivka and Wilson (1976). The computer program Determination of Temperature (DOT) (developed by Polivka and Wilson (1976)) was modified for studying the cooling of broccoli. Quadratic quadrilateral elements were used in this study. The respiration effect was included by using a heat-generation term. Some of the key assumptions in their study were: (i) negligible surface-evaporation cooling, (ii) no variation in thermal properties, (iii) constant initial temperature, and (iv) limited respiratory activity. Comparisons between the measured and FEM-predicted cooling curves showed that FEM adequately simulated the experimental results. The key findings of this study were: (i) the differences between the measured and predicted values were within 1·1°C, and (ii) FEM tended to deviate from measured values in the lower part of the stalk.

Hayakawa and Succar (1982) developed a one-dimensional FEM to describe heat transfer and moisture loss in spherical-shaped fresh-produce items. They used the heat-conduction equation in spherical coordinates with a temperature-dependent heat-source term to model heat generation due to respiration by the produce. The density and thermal conductivity were approximated as functions of temperature. The boundary conditions were of convective type. The total moisture loss was estimated as the integral of the evaporation at the surface of the produce. Galerkin's technique was used to formulate the solution. The model was used to study the influence of density, thermal conductivity, surface-heat conductance, transpiration rate, and relative humidity on heat transfer and total moisture loss from the produce. Good agreement was observed between the theoretical results and the experimental measurements of temperature and moisture loss in tomatoes and potatoes during a typical cooling process.

Fresh produce is immediately stored in a cooling chamber after harvest to retard the degradation rate and enhance shelf life. Cooling of tomatoes after harvest is a key step in slowing down the respiration rate. Accurate prediction of tomato temperature during cooling is therefore critical to prolonging its storage stability. Pan and Bhowmik (1991) modeled the cooling of a tomato by using a two-dimensional time-dependent heat-conduction equation. Tomato respiration rate was simulated by using a heat-generation term. The finite-element software, ANSYS, was used to determine the temperature distribution in a tomato. Triangular, isoparametric elements were used in the finite-element formulation. By using constant material-property values and constant heat generation, the FEM-predicted values were compared with measured data. The model predictions agreed with experimental measurements to within 1°C.

The time–temperature distribution during cooling of six species of fish in a rectangular box was modeled by using ANSYS (Version 4.3) (ANSYS, 1988) by Stringer *et al.* (1989). The time-dependent conduction equation was solved in two dimensions. Information relating to element type and model assumptions were not included. The transient-temperature values predicted by FEM were consistently lower than the measured values. This was attributed to the fixed-temperature-boundary condition (during simulation) not being constant throughout the test duration.

Owing to the increased usage of microwaves in food-processing and their rapid rate of heating, it is important that the interaction of the microwaves with the food product be better quantified. Lin *et al.* (1989) and Lin (1991) used the TWODEPEP software (IMSL, 1983) to determine the temperature distribution in axisymmetric model food systems during microwave heating. The transient heat-conduction equation with microwave-heat absorption as the heat-source term was solved to describe the temperature distribution during microwave heating of high-moisture solid foods of rectangular and cylindrical shapes. The microwave power absorbed at any location in the sample was derived as a function of dielectric properties and the sample geometry. Convective and evaporative heat losses at the surface of the food were incorporated as boundary conditions in addition to the initial-temperature condition. The FEM predictions compared favorably with experimentally measured temperatures in model food systems of rectangular geometry. But the FEM did not adequately describe the measured-temperature values at the center region in cylinder-shaped model food systems. This was attributed to the incomplete description of microwave propagation within cylinder-shaped products. Sensitivity analysis of the model showed that changes in thermal diffusivity, dielectric properties, and power output of the microwave oven significantly changed the temperature distribution within the food product.

Chen *et al.* (1990) used the heat-conduction equation in axisymmetric co-ordinates to model the heating characteristics of cylindrical specimens of potato. Using Lambert's law, they derived a heat-generation equation to account for the microwave interaction. Chen *et al.* (1990) modified an existing finite-element code available in the book by Segerlind (1984). The heat-generation term was added to this code. The FEM validation was done by comparing the fibre-optic-probe-measured and FEM-predicted values. Measured center (along the axis of the cylinder) temperatures were in excess of 130°C, whereas FEM-predicted values were about 115°C. For temperatures below 80°C, FEM-predicted and measured values were in good agreement.

Naveh (1982) and his co-workers (Naveh *et al.*, 1983*a, b, c*; Kopelman *et al.*, 1982) modified an existing finite-element program Determination of Temperature (DOT) developed by Polivka and Wilson (1976). The purpose of the study by Naveh and his co-workers was to predict the overshooting of temperature in cans and jars when the steam-off condition prevails and to estimate the sterility of food products such as cream style corn in a jar and meat in a can. They solved the time-dependent heat-conduction equation in two-dimensional and axisymmetric regions. Their finite-element formulation, based on the variational principle, is limited to food products described by the quasi-harmonic equation. The boundary conditions were of constant temperature and convective types. Linear and quadratic rectangular and triangular elements were used by Naveh *et al.* (1983*a*, *b*, *c*). The FEM was validated by using temperature data, for example, of apple sauce in a jar during the heating process. The following key findings were based on this study: (i) the slowest-heating zone was along the axis of symmetry in the region close to the head space; (ii) the effect of glass on product-heating rate is significant but is more pronounced in small jars. On the basis of the insight gained from the FEM results, Naveh *et al.* (1983*a*, *b*, *c*) developed a simple equation to estimate heating parameters for different-sized jars (when thermal properties are known). The effectiveness of FEM in describing a sterilization process was determined by Naveh and his co-workers througout this study.

Sastry *et al.* (1985) used the hybrid finite-element–finite-difference technique to predict the degradation of agaratine in canned mushrooms. The transient-heat-conduction and mass-diffusion equations in three-dimensional co-ordinates were simultaneously solved in an irregular domain. Time-dependent convective-type boundary conditions were used. Kinetic data on the degradation of agaratine as a function of temperature was also incorporated into the model. The predicted results of heat transfer were found to be in satisfactory agreement with the experimentally measured temperature at the center of the mushrooms. The predicted values of agaratine retention in mushrooms compared favorably with experimental measurements. However, the predicted concentrations in the brine were somewhat lower than the experimental values.

Sastry (1986) used the finite-element technique to evaluate thermal-process schedules for low-acid foods containing particulates. The temperature profiles within the heat exchanger and holding tube were determined first by using energy balance, and then the temperature profile within the particle was evaluated on the basis of the conduction of the equation with a time-dependent convective boundary condition. The solution was obtained by using the three-dimensional Galerkin–Crank–Nicolson algorithm. Good agreement between the FEM-predicted and measured temperatures was found at the cold spot within mushrooms being thermally processed in cans.

In-package pasteurization of fluid foods in bottles and cans is common in the food industry. Pasteurization prolongs shelf life by destroying the micro-organisms that cause food spoilage. Engelman and Sani (1983) modeled pasteurization of beer in a bottle by using the finite-element software FIDAP (Engelman, 1982). They solved the momentum, continuity, and energy equations in axisymmetric co-ordinates. In order to reduce the computational time, they employed the penalty-function approach to solve the continuity equation approximately, i.e. the continuity equation is proportional to pressure instead of zero (thus using the weakened constraint). Nine-node, isoparametric quadrilaterial elements were used for the velocity and temperature fields, whereas the three-node approximation (linear in the x and y directions) was used for pressure. The FEM-calculated velocity distribution was qualitatively compared with published data. The FEM-simulated velocity and pressure characteristics had good qualitative agreement with those reported in literature.

Subsequently, Kumar *et al.* (1990) also used FIDAP to model the temperature distribution and the slowest heating point (cold spot) within an upright metal can being heated in a steam retort. The continuity, momentum, and energy-conservation equations were solved for an axisymmetric case by using quadrilateral elements. A trial-and-error technique was used to refine the mesh development. The viscosity of the food product was assumed to be non-Newtonian, and the apparent viscosity at a shear rate of 0·01 s^{-1} and as a function of temperature was used in the analysis. With the assumption of side-wall heating only, the

FEM showed that natural convection heating moved the cold spot within the container to the bottom center.

Sterilization of fluid food containing particulates by conventional heating methods is not very effective because it results in most of the foods being overcooked, i.e. poor product quality. DeAlwis and Fryer (1990) studied ohmic heating as an alternative to sterilizing particulate-fluid foods by using a finite-element model. In ohmic heating, alternating electric current is passed through the food material for rapid thermal processing. As heat is generated within the material, ohmic heating thus overcomes the limitation of conventional heating. Particulate fluid foods can therefore be processed by HTST techniques. The ohmic-heating process involves the simultaneous solution of voltage and temperature fields. However, DeAlwis and Fryer (1990) solved the decoupled set of voltage and heat equations. The steady-state voltage distribution was described in the food by using Laplace's equation in two dimensions. The unsteady-state temperature field was modeled by using the diffusion equation with a heat-generation term. The heat-generation term was estimated from the voltage distribution calculated by using Laplace's equation. Linear triangular elements were used in the finite-element formulation. The FEM was validated by comparing the predicted results with the analytical solution and experimental data from lamb meat in brine and a potato piece in brine. The FEM-calculated values compared favorably with both analytical and experimental results. Through this finite-element analysis, DeAlwis and Fryer (1990) have demonstrated ohmic heating as a promising unit operation for rapid sterilization of particulate-fluid foods.

McCarthy (1987) and McCarthy and Merson (1989) modeled the steady-state two-dimensional thermal response of a free-falling film of homogeneous-fluid food in a steam environment (as encountered during steam-infusion heating) by using the finite-element method. The Galerkin method was used to solve the thermal-energy equation for steady-state heat transfer in a moving fluid with a constant-temperature-type boundary condition. The average fluid temperatures measured as a function of position compared well with the model predictions. The FEM was also used to predict the kinetics of microbial destruction as a function of time and temperature.

Freezing and thawing

Comini and his co-workers (Bonacina & Comini, 1973; Bonacina *et al.*, 1973; Comini *et al.*, 1974; Rabelleto *et al.*, 1978) used the finite-element method to model the freezing of lamb and beef carcasses in air-blast tunnels. They solved the two-dimensional time-dependent diffusion equation by using the finite-element approach. Quadratic triangular and quadrilateral elements were used. Using variable thermal conductivities for the dry-solid and liquid components, they successfully predicted the measured carcass temperature to within 1°C.

Purwadaria and Heldman (1980, 1982) and Purwadaria (1980) developed a two-dimensional time-dependent FEM to simulate the freezing of elliptical- and trapezoidal-shaped bodies. A time-dependent heat-conduction equation with a heat-generation term was used by the authors. The experimental data for FEM validation were obtained by using ground beef. The main limitation appears to be the description of the heat-generation term to include phase-change effects (freezing). Using linear triangular elements, variable thermal conductivity, and a variable local-surface heat-transfer coefficient, they observed close agreement between the measured and FEM-predicted values. They found that: (i) FEM adequately simulated temperature history and isothermal fields in elliptical and trapezoidal geometries; and (ii) FEM confirmed the significant influence of geometric size and initial temperature on freezing time.

Abdalla and Singh (1985) developed a finite-element code for freezing and thawing in axisymmetric foodstuffs. The heat-conduction equation was applied to an axisymmetric body with a convective-type boundary condition. The Galerkin method was used in the finite-element formulation. Physical properties of the material used in the FEM were assumed to be temperature-dependent. The FEM predictions compared favorably with experimentally measured temperatures during freezing and thawing of a 2·54-cm sphere made of a model food substance. The model showed that the product temperature has a considerable influence on its rate of thawing.

Heat and mass transfer

Drying

Misra and Young (1978*a*, *b*) and Misra (1977) hypothesized that shrinkage induced by moisture migration is directly responsible for grain cracking. By using the key assumptions: (i) free shrinkage is directly proportional to change in moisture content, (ii) elastic shrinkage is Hookean, (iii) dry-mass density of soybeans increases owing to shrinkage and inversely influences mass diffusivity, and (iv) material properties vary according to a specified functional form, a time-dependent finite-element formulation in one-dimensional spherical co-ordinates was developed. Linear approximation functions were used to solve the moisture-diffusion equation subjected to surface mass transfer. The FEM was applied to soybean kernels. The validated FEM was used to study states of moisture-induced stresses. Through this study, Misra and Young (1979) showed that: (i) the maximum shear stress occurs within the first four hours, and (ii) the elastic constitutive equation was inadequate for predicting stresses during drying (Misra *et al.*, 1981).

Dehulling, also referred to as decortication, is a key processing step to improve the texture and culinary properties of grain legumes. Chhinnan and Bakshi (1984) modeled the moisture transfer during the drying of black-eye peas by using the finite-element method. The seed shapes were assumed to be either spherical or cylindrical. Assumptions in the author's finite-element formulation were that: (i) the initial moisture content is uniform, (ii) seed is always in thermal equilibrium with the surroundings, (iii) the surface reaches the thermodynamic equilibrium instantaneously, (iv) volumetric change is negligible, and (v) seed is homogeneous and mass diffusivity is constant. Chhinnan and Bakshi (1984) used the finite-element formulation developed by Lamauro and Bakshi (cited in Chhinnan and Bakshi, 1984). Four-node quadrilateral elements in axisymmetric domains were used in the analysis. The finite-difference scheme, was apparently used in the time domain. A comparison of the measured and FEM-predicted values showed that the results were closer for later periods of drying.

Using forced-air convection to dry grains to acceptable short- and long-term-storage moisture contents is fairly common. In addition to the complex heat and mass transfer occurring simultaneously, knowledge of the pressure and velocity distribution is essential for optimal design. Velocity and pressure terms are coupled, which leads to a complex set of non-linear equations. The constant-coefficient version, i.e. that wherein properties do not change, has been solved by using the finite-difference (Brooker, 1961) and finite-element methods (Marchant, 1976). Segerlind (1982), using the granular-permeability coefficients, recast the non-linear equation that was solved by using the finite-element method. The variational technique was used to solve for pressure drop in a two-dimensional region. Quadratic triangular elements were used in a fully developed flow. The main assumption appears to be that the non-linear equation can be written in an isotropic quasi-linear form and then solved iteratively to obtain a converged solution. The FEM-predicted values were compared with the simulated values reported by Brooker (1961). The two data sets had good qualitative agreement. Subsequently, Segerlind and his co-workers (Khompis *et al.*, 1984) analyzed fully three-dimensional pressure- and velocity-distribution patterns in a grain-storage bin. Twenty-node quadratic three-dimensional elements were used by Khompis *et al.* (1984). Four different air-distribution systems (perforated-floor, straight-duct, square-duct, and Y-shape-duct) were analyzed. The three-dimensional results provided insights not available in two-dimensional programs, for example, the pressure distribution across a plane close to the inlet had significantly higher non-uniformity than the cross-section away from the inlet. The results of Khompis *et al.* (1984) appear to be more accurate (i.e. pressure and velocity) than the two-dimensional results reported by Marchant (1976) and Smith (1982).

Talbot (1989) examined the potential use of the commercial software ANSYS to model air flow in particulate materials. The governing equations for porous-media flow were used, which are different from the equations used by Segerlind (1982). Air-flow distribution in two-dimensional and three-dimensional regions was studied. The two regions used in the analysis were the same as those of Segerlind (1982) and Khompis *et al.* (1984). For two-dimensional simulations, four-node isoparametric elements were used, whereas eight-node elements were used for three-dimensional analysis. Results of ANSYS were similar to those obtained by other researchers. Talbot's results demonstrate the effectiveness of using the commercial software ANSYS to study air-flow patterns through porous media.

The pressure and flow distribution during drying of walnuts was studied by Rumsey and Fortis (1983). The governing equation in two dimensions was the same as the one used by Brooker (1961). Steady, isotropic air flow was modeled by using quadrilateral elements. Comparison of measured and FEM-predicted results showed good agreement in both high- and low-air-velocity regions. The FEM was then used to correct the air-flow pattern by simulating different design considerations, such as increasing static pressure under an expanded metal-screen floor, reducing the angle of the false bottom. The uniformity of drying was enhanced by these considerations. Chapman *et al.* (1989), using a two-dimensional analysis with eight-node quadrilateral elements, determined that grain surcharge in a bin does not influence air-distribution patterns.

Marchant (1976) solved the two-dimensional pressure-drop equation by using the finite-element method. Eight-node quadrilateral elements were used in this study. The finite-element formulation was achieved by using the variational technique. The FEM was validated by comparing the FEM predicted to the one-dimensional analytical solutions. Further validation was effected by comparing the FEM-predicted results with those given by Brooker (1961) and Barrowman and Boyce (1966). The FEM simulated Brooker's results adequately, but Barrowman and Boyce's results were within 0·016–0·019 m/s of the FEM-simulated values.

Mass transfer

Zhang *et al.* (1984) used the finite-element method to model simultaneous absorption and expansion of rice during the soaking process. The expansion was assumed to be linearly proportional to change in moisture content. The unsteady-state two-dimensional diffusion equation was used. Mass diffusivity as a function of concentration and moisture content and the convective boundary condition were used in the model. The predicted values of the mass-average moisture content were close to the experimentally measured values. Based on the predicted data, the average mass diffusivity of milled rice was found to decrease with an increase in moisture content. Such a model can also be used to predict moisture distribution in rice with tempering intermediate soaking periods. Subsequently, Bakshi and Chhinnan (1985) used the finite-element approach to estimate mass diffusivity as a function of the coefficient of concentration. The coefficient was estimated by using a non-linear optimization technique that minimized the squared error. The values calculated by the optimization technique in conjunction with the FEM were reasonable. This approach can also be employed to estimate transport properties during drying, soaking, and parboiling processes.

Knowledge of moisture diffusion in grain kernels during drying plays a key role in determining and maintaining the quality of the material. Moisture diffusivity depends on kernel component materials, temperature, and moisture content. Syarief *et al.* (1987) followed the indirect approach of using the FEM to determine the moisture diffusivity of corn kernels. The corn kernel was assumed to be a two-dimensional body wherein moisture migration primarily occurs by diffusion. Syarief *et al.* (1987) modified the Determination of Temperature (DOT) finite-element program developed by Polivka and Wilson (1976) for their problem solution. A key assumption in their formulation appears to be lack of coupling of the moisture diffusion with temperature. Four-node quadrilateral elements were used in the FEM formulation. The bilinear approximation function for elements was used in the FEM to determine the best values of parameters in an exponential equation for representing the diffusion coefficient. No FEM-validation data were provided, but one of the authors (Gustafson) has used the DOT program in the heating and cooling of grains (Gustafson *et al.*, 1979). From this study, Syarief *et al.* (1987) determined that: (i) diffusivity values were different for two different components, (ii) the diffusion coefficient of germ was from 3·6 to 4·9 times that of endosperms. The FEM was demonstrated to be an invaluable tool in the determination of the diffusivity coefficient.

Simultaneous heat and mass transfer invariably occurs during the drying of grains. This, in turn, is also accompanied by high compressive and tensile stresses that can cause significant damage to grain kernels. Haghighi and Segerlind (1988*a, b*) modeled the simultaneous heat and moisture transfer in an isotropic sphere by using the finite-element method. The one-way-coupled heat and moisture diffusion was used in axisymmetric co-ordinates to model the drying of grains. By using the variational approach, equations for the discretized region were developed by employing three-node triangular elements. The time discretization was effected by using the finite-difference approach. The key assumptions in the formulation appear to be: (i) moisture migration primarily takes place by diffusion, with negligible capillary and gravity effects, and (ii) moisture migration caused by kernel shrinkage is neglected. The model was validated by qualitatively comparing the FEM-predicted values with the published values. Subsequently, Haghighi *et al.* (1990) solved decoupled heat- and moisture-transfer equations in axisymmetric co-ordinates to simulate the drying of barley kernels. The constant-property FEM predictions, obtained using nine-node Lagrangian elements, were compared with published experimental data. The surface-temperature and mass-average moisture changes were modeled accurately.

Recognizing that both temperature and moisture changes are responsible for stresses in grain kernels, Haghighi and Segerlind (1988*b*) developed a viscoelastic finite-element formulation for grain kernels. They employed the thermo-hydro-rheologically simple-material assumption to derive the FEM (Haghighi, 1979). Using a one-dimensional representation (a sphere) for soybean and published data for soybean kernels, they simulated stress distribution in soybean kernels. The FEM was based on Galerkin's method with non-linear properties. No FEM validation was reported in the paper. Some of the key findings were: (i) the maximum peak shear stress and peak tangential stress at the surface are reached after about 1 hour; (ii) tangential-stress changes from the compressive to the tensile state occurred along the surface; and (iii) time to reach the peak stress is independent of change in temperature.

Upadhyaya and Rumsey (1989) used the fully coupled heat- and moisture-transfer equation in one dimension to study the heat and mass diffusion in a soybean kernel. A linear-approximation function was used in the finite-element formulation. Although they did not directly validate their model or explicitly state the problem-formulation assumption, the FEM-predicted values were in qualitative agreement with those reported by Haghighi and Segerlind (1988*a*).

Lamauro and Bakshi (1985) used FEM to predict moisture diffusion in foods stored over a period of time. Hong (1985) and Hong *et al.*

(1986) used this model to study moisture transfer during storage of mixed raisins, roasted peanuts, banana chips, and roasted almonds. They solved the transient-diffusion equation in cylindrical co-ordinates by using four-node isoparametric elements. The experimentally measured and predicted values of moisture content during storage were found to be within the range 0·28–0·84%.

Lu and Siebenmorgan (1991) studied the moisture-adsorption behavior of long-grain rice, which is the prime suspect in rice fissure. The moisture migration in single rice kernels was modeled by using the diffusion equation in axisymmetric co-ordinates. The shape of rice kernels was approximated as a prolate spheroid. The key assumptions were: (i) moisture adsorption occurs at constant temperature, (ii) the diffusion coefficient is independent of moisture, and (iii) kernel shrinkage is negligible during adsorption. Linear triangular elements were used to model the kernel geometry and the adsorption process. The FEM was employed to estimate the diffusion coefficients of endosperm bran and hull by minimizing the FEM-predicted and measured moisture distributions. The FEM adequately predicted moisture–time–space profiles. The study by Lu and Siebenmorgan (1991) showed that: (i) the FEM is capable of describing the moisture-adsorption behavior of long-grain rice, and (ii) the average errors at various temperatures and relative humidities between the measured and FEM-predicted values ranged from 0·11 to 0·23% moisture content on a dry basis.

Mechanical damage

Fresh fruits and vegetables have a complex cellular structure. There is a definite need of a rational theory that can accurately characterize the stress–strain behavior of biological materials. Recognizing this, Gustafson and Segerlind (1977) proposed a continuum solid–liquid–gas theory to describe the behavior of fruits (Gustafson, 1977). Such a theory has the potential to predict damage during a mechanical process and hence control (or minimize) this damage through appropriate measures. This theory contains seven parameters. Analysis, by using this theory, is possible only via the numerical methods. Gustafson *et al.* (1977*a*), following the variational principle, derived an FEM for the load response of fruits. Their analysis was confined to axisymmetric bodies. The FEM used eight-node quadrilateral and six-node triangular elements. The model was not validated by the authors. The performance of a hypothetical fruit with skin was analyzed by using the FEM. A good agreement was observed between the FEM-predicted and experimental values reported in the literature. A key observation was that the restraint created by the skin can produce stresses, in a cellular body with turgor pressure.

Mechanical damage to fruits and vegetables usually occurs during harvesting, handling, and processing. During these operations, the fresh produce is likely to experience point (contact) loading, which thereby induces a significant amount of damage. On the basis of the observation that fruits behave like viscoelastic bodies, Rumsey and Fridley (1977) modeled the fruit response by using the finite-element method. A two-dimensional FEM formulation for elastic and viscoelastic bodies was obtained following Herrmann's (1973) work. The results were based on an axisymmetric formulation by using quadrilateral elements. Quadrilateral elements made up of four constant-strain triangles were used. The FEM was verified by comparing it against analytical solutions. Validation of the FEM by using experimental data was not reported by the authors. The key finding of this study was that the finite-element predictions were significantly different as compared with the elastic formulation during impact when drop heights exceeded 30 cm and/or the material properties changed.

Damage to plant and animal tissues and to fruits and vegetables is usually initiated at the cellular level. Recognizing the importance of such a study, Cooke *et al.* (1976, 1977) modeled the response of elliptical (torus-shaped) guard cells found in beans. They used a finite-element program developed by Mang *et al.* (1976). On the basis of the hypothesis that linear-bending theory for thin orthotropic shells can be applied to analyze the guard-cell response, higher-order seven-node (one node at the centroid) triangular elements were used to model the displacement of guard cells under pressure. Since experimental data at the cellular level were not available for model validation, the authors performed analysis of FEM-predicted values to arrive at some significant findings. The key ones were: (i) the FEM qualitatively modeled the response of guard cells as published in the literature, (ii) guard-cell geometry is important in stomatal behavior, and (iii) the pore width may be expressed as a multi-linear function of guard-cell pressure and adjacent epidermal cells.

Chen *et al.* (1984) modified the static–viscoelastic-analysis program for two-dimensional solids developed by Herrmann (1973) and used by Rumsey and Fridley (1977). The modification included the inertial effects during impact. The finite-element model was verified by comparing the results with an analytical solution. The FEM results were in good agreement with the theoretical values. The authors used linear quadrilateral elements to model stresses in viscoelastic fruits successfully. Model validation with experimental data was not done by them. The

FEM results indicated that maximum stress during impact occurs just before the maximum deformation.

Cardenas-Weber *et al.* (1989, 1991) used ANSYS version 4.4 to estimate stresses during the mechanical handling of melons by robots. They used an axisymmetric, three-dimensional formulation with quadratic elements in their analysis. The model was validated by comparing the FEM-predicted and measured values; the FEM values were within 10% of measured values obtained by using a universal testing machine. The robot-gripper design was also optimized by minimizing the weight of the gripper (Edan *et al.*, 1989). It was found that bruising of melons was unlikely owing to the loads induced by the grippers. Elasticity equations were used in the finite-element analysis.

OTHER POTENTIAL USES OF FEM

Current uses of FEM in food-processing have been restricted to the evaluation of temperatures and moisture movement during heating/cooling/drying of food products. Studies have also been reported in the area of mechanical damage during handling of products and stress cracks during the drying of grains. A limited number of studies exist on the prediction of temperatures during the freezing and thawing of foods.

The use of FEM to describe thermal processing has been demonstrated by the limited number of studies done to date (Naveh, 1982; Sastry *et al.*, 1985; McCarthy & Merson, 1989). The numerically calculated temperature values can be used to estimate changes in quality parameters such as nutrient retention and microbial destruction in food products during thermal processing. This information can be used to optimize the thermal process with respect to food quality.

The FEM also has the potential to describe fluid flow and heat transfer during aseptic processing of particulate foods. With particulate food materials, it is essential to describe the accumulation of lethality during aseptic processing, especially within the holding-tube section. Sastry (1986) used this technique to determine the temperature distribution within a food particulate. The FEM can also be used, with some approximations, to describe the two-phase flow within this system and the heat transfer to particulates. Such an analysis will be useful in determining the effect of various operating parameters on the accumulated lethality during the aseptic processing.

Microwave heating of non-homogeneous foods of irregular shapes is another area that can be addressed efficiently by using the FEM. Simultaneous heat and mass transfer occurring in irregular-shaped foods can be analyzed during microwave heating by using the finite-element formulation. Foods of any geometry can be studied along with varying physical and dielectric properties. The FEM can describe changes in the physical and dielectric properties as a function of temperature and moisture content during heat. The effect of non-uniform microwave-field distribution on the surface of a food product, resulting in spatially dependent boundary conditions, can also be investigated by using this numerical method. The temperature information can then be utilized to analyze microbial destruction and nutrient degradation during microwave heating. The FEM can also be used to simulate heat and mass transfer during drying and to optimize the process with respect to product quality.

In the area of freezing and thawing, the scope of FEM can be expanded to optimize the process with respect to energy cost and quality of the food product. Novel approaches to the thawing of foods, such as with the use of microwave heating, can be evaluated by using this numerical technique. A simulation model can be effectively used to evaluate the feasibility of using such techniques for thawing different food products and their impact on product quality and safety.

Most food products exhibit complex rheological properties during processing. Many food products are non-Newtonian and their properties vary with processing conditions. The FEM can be used to describe the flow of such complex food materials through changing geometries and directions such as expansion, contractions, bends, and elbows. This can lead to more efficient design of fluid-flow and handling systems in a food-processing plant.

Food extrusion is another area where the FEM can be effectively implemented to simulate the extrusion process mathematically. This method can be implemented to solve the flow and heat-transfer equation within the barrel of the extruder. There are several models available to describe the rhelogical behavior of the food doughs used in extrusion (Clark, 1978). The analysis can be used to predict the product-flow rate and the temperature distribution and related changes in product quality during extrusion. This technique can be adapted to evaluate various geometries and types of screws and dies used in extruders and be used to determine the optimum operating conditions for desired product quality. The finite-element method can also be effectively used to model continuous systems, such as tubular heat exchangers. With the time-dependent simulation results, process optimization and in-line quality control are feasible.

Food-processing applications involving thermo-mechanical and thermo-hydro-mechanical interactions are efficiently modeled by using the finite-element method. Some applications include cracking of egg shells under rapid cooling or heating and moisture adsorption by hygroscopic food products. The insights gained from the FEM can be used to improve packaging and to optimize handling and processing units. FEM is also useful where experimental work is either prohibitively expensive (e.g. development of pilot plant or product test line) or difficult to perform (e.g. short duration and very high-temperature conditions).

In other areas of food-processing that involve not only heat and moisture transfer but chemical reactions as well, for example, baking, the potential to simulate the process by using FEM is significant. The ready availability of commercial software, its ever-increasing user-friendliness, and its continuously expanding scope to varied engineering applications make this software potential candidates for possible consideration and adaptation to food-processing applications. The economic benefits to the food-processing industry to be gained using the finite-element method as a research and development tool, when realized, will be staggering.

REFERENCES

Abdalla, H. & Singh, R. P. (1985). Simulation of thawing foods using finite element method. *J. Food Process Engr.*, **7**, 273–86.

Alagasundaram, K., Jayas, D. S., White, N. D. G. & Muir, W. E. (1990). Three-dimensional finite element heat transfer model of temperature distribution in grain storage bins. *Trans ASAE*, **33**, 577–84.

ANSYS (1988). *ANSYS Engineering Analysis System.* Swanson Analysis System, Houston, PA, USA.

Arce, J. A., Potluri, P. L., Schneider, K. C., Sweat, V. E. & Dutson, T. R. (1983). Modeling beef carcass cooling using a finite element technique. *Trans ASAE* **26**, 950–4, 960.

Baird, C. D. & Gaffney, J. J. (1977). A numerical procedure for calculating heat transfer in bulk loads of fruits and vegetables. *ASHRAE Trans* **82**, 525–40.

Bakal, A. & Hayakawa, K. (1973). Heat transfer during freezing and thawing of foods. *Adv. Food Res.*, **20**, 217–56.

Bakshi, A. S. & Chhinnan, M. S. (1985). Estimation of parameters for moisture transport in foods. *J. Food Process Engng.*, **7**, 143–55.

Barrowman, R. & Boyce, D. S. (1966). Air distribution from lateral ducts in barley. *J. Agr. Engng. Res.*, **11**, 243–7.

Becker, E. B., Carey, G. F. & Oden, J. T. (1981). *Finite Elements*, Volumes 1–6. Prentice Hall, Englewood Cliffs, NJ, USA.

Bonacina, C. & Comini, G. (1973). On the solution of the non-linear heat conduction equations by numerical methods. *Int. J. Heat Mass Transfer*, **16**, 581–6.

Bonacina, C., Comini, G., Fasano, A. & Primicerio, M. (1973). Numerical solution of phase change problems. *Int. J. Heat Mass Transfer*, **16**, 1825–30.

Brauer, J. R. (1988). *Finite Element Method: What Every Engineer Should Know about Finite Element Analysis*. Marcel Dekker, New York, NY, USA.

Brebbia, C. A., Telles, J. C. F. & Wrobel, L. C. (1984). *Boundary Element Techniques — Theory and Applications in Engineering*. Springer Verlag, New York, NY, USA.

Brooker, D. B. (1961). Pressure patterns in grain drying systems established by numerical methods. *Trans ASAE*, **4**, 72–4, 77.

Cardenas-Weber, M. C., Stroshine, R. L., Haghighi, K. & Edan. Y. (1989). Finite element analysis of melons handled by different robot gripper designs. ASAE Paper No. 89-1602. ASAE, St. Joseph, MI, USA.

Cardenas-Weber, M. C. Stroshine, R. L., Haghighi, K. & Edan, Y. (1991). Melon material properties and finite element analysis of melon compression with application to robot gripping. *Trans ASAE*, **34**, 920–9.

Chapman, J. E., Morey, R. V., Cloud, H. A. & Nieber, J. L. (1989). Airflow patterns in flat storage aeration systems. *Trans ASAE*, **32**, 1368–78.

Charm, S. E., Brand, D. H. & Baker, D. W. (1972). A simple method for estimating freezing and thawing times of cylinders and slabs. *ASHRAE J.*, **14** (11), 39–45.

Chen, S., Hermann, L. & Chen, P. (1984). Analysis of stresses in fruit during impact. ASAE Paper No. 84-3854. ASAE, St. Joseph, MI, USA.

Chen, D.-S.D., Singh, R. K., Haghighi, K. & Nelson, P. E. (1990). Finite element analysis of temperature distribution in microwaved particulate foods. ASAE Paper No. 90-6602. ASAE, St. Joseph, MI, USA.

Chhinnan, M. S. & Bakshi, A. S. (1984). Finite element analysis to model moisture transfer in rewetted California blackeye peas during drying. ASAE Paper No. 84-6516. ASAE, St. Joseph, MI, USA.

Clark, J. P. (1978). Dough rheology in extrusion cooking. *Food Technol.*, **32** (7), 73.

Cleland, A. C. & Earle, R. L. (1977). A comparison of analytical and numerical methods of predicting the freezing times of foods. *J. Food Sci.*, **42**, 1390–5.

Cleland, A. C. & Earle, R. L. (1979*a*). A comparison of methods for predicting the freezing times of cylindrical and spherical foodstuffs. *J. Food Sci.*, **44**, 958–63, 970.

Cleland, A. C. & Earle, R. L. (1979*b*). Prediction of freezing times for foods in rectangular packages. *J. Food Sci.*, **44**, 964–70.

Comini, G., DelGuidice, S., Lewis, R. W. & Zienkiewicz, O. C. (1974). Finite element solution of non-linear heat conduction problems with special emphasis on phase changes. *Int. J. Num. Meth. Engr.*, **8**, 613–24.

Cooke, J. R., DeBaerdemaeker, J. G., Rand, R. H. & Mang, H. A. (1976). A finite element analysis of guard cell deformations. *Trans. ASAE* **19**, 1107–21.

Cooke, J. R., Rand, R. H., Mang, H. A. & DeBaerdemaeker, J. G. (1977). A nonlinear finite element analysis of stomatal guard cells. ASAE Paper No. 77-5511. ASAE, St. Joseph, MI, USA.

DeAlwis, A. A. P. & Fryer, P. J. (1990). A finite-element analysis of heat generation and transfer during ohmic heating of food. *Chem. Engng Sci.*, **45**, 1547–59.

DeBaerdemaeker, J., Singh, R. P. & Segerlind, L. J. (1977). Uses of the finite element method in food processing. *J. Food Process Engr.*, **1**, 37–50.

Desai, C. S. (1984). *Finite Element Method*. Prentice Hall, Englewood Cliffs, NJ, USA.

Edan, Y., Haghighi, K., Stroshine, R. L. & Cardenas-Weber, M. C. (1989). Finite element analysis and optimization of a robot gripper design. ASAE Paper No. 89-7537. ASAE, St. Joseph, MI, USA.

Engelman, M. S. (1982). FIDAP — a fluid dynamics analysis package. *Adv. Engng. Soft.*, **4**, 163–6.

Engelman, M. S. & Sani, R. L. (1983). Finite element simulation of an in-package pasteurization process. *Numerical Heat Transfer*, **6**, 41–54.

Eshelman, W. D., Baird, C. D. & Gaffney, J. J. (1976). A numerical simulation of transient heat flow in irregular shaped foods. ASAE Paper No. 76-6504. ASAE, St. Joseph, MI, USA.

Gates, R. S., Cooke, J. R. & Upadhyaya, S. K. (1984). Eggshell stresses from flat plate and thermal loads. ASAE Paper No. 84-5519. ASAE, St. Joseph, MI, USA.

Gustafson, R. J. (1977). Finite element method package of learner programs. Technical Report No. UCCTR77001, University of Minnesota Computer Center.

Gustafson, R. J. & Segerlind, L. J. (1977). Continuum theory for gas–solid–liquid media — II: Finite element method. *Trans ASAE*, **20**, 1190–3, 1200.

Gustafson, R. J., Mase, G. E. & Segerlind, L. J. (1977*a*). Continuum theory for gas–solid–liquid media — I: Theory development. *Trans ASAE*, **20**, 1186–9.

Gustafson, R. J., Thompson, D. R. & Sokhansanj, S. (1977*b*). Temperature and stress analysis of corn kernel using the finite element method. ASAE Paper No. 77-5514. ASAE, St. Joseph, MI, USA.

Gustafson, R. J., Thompson, D. R. & Sokhansanj, S. (1979). Temperature and stress analysis of corn kernels — finite element analysis. *Trans ASAE*, **22**, 955–60.

Haghighi, K. (1979). Finite element formulation of the thermo-hydro-stress problem in soybeans. Ph.D. Dissertation. Michigan State University, East Lansing, MI, USA.

Haghighi, K. & Segerlind, L. J. (1988*a*). Modeling simultaneous heat and mass transfer in an isotropic sphere — a finite element approach. *Trans ASAE*, **31**, 629–37.

Haghighi, K. & Segerlind, L. J. (1988*b*). Failures of biomaterials subjected to temperature and moisture gradients using the finite element method. Parts I and II. *Trans ASAE*, **31**, 930–7, 938–46.

Haghighi, K., Irudayaraj, J., Stroshine, R. L. & Sokhansanj, S. (1990). Grain kernel drying simulation using the finite element method. *Trans ASAE*, **33**, 1957–65.

Hayakawa, K. & Succar, J. (1982). Heat transfer and moisture loss from spherical fresh produce. *J. Food Sci.*, **47**, 596–605.

Heldman, D. R. & Gorby, D. P. (1975). Computer simulation of individual-quick-freezing of foods. ASAE Paper No. 75-6016. ASAE, St. Joseph, MI, USA.

Herrmann, L. R. (1973). User's manual for viscoelastic analysis for 2-D solids program. Department of Civil Engineering Publication No. 73-1, University of California, Davis, CA, USA.

Hong, Y. C. (1985). Finite element modeling of moisture transfer during the storage of well-mixed multicomponent dried foods. M.S. Thesis. University of Minnesota, St. Paul, MN, USA.

Hong, Y. C., Bakshi, A. S. & Labuza, T. P. (1986). Finite element modeling of moisture transfer during storage of mixed multicomponent dried fruits. *J. Food Sci.*, **51**, 554–8.

Hsieh, R. C., Lerew, L. E. & Heldman, D. R. (1977). Prediction of freezing times for foods as influenced by product properties. *J. Food Proc. Engr.*, **1**, 183–97.

IMSL (1983). *TWODEPEP User's Manual.* International Mathematical and Statistical Libraries, Houston, TX, USA.

Irons, B. & Shriver, N. (1983). *Finite Element Primer.* Ellis Harwood, Chichester, Sussex, UK.

Jiang, H., Thompson, D. R. & Morey, R. V. (1987). Finite element model of temperature distribution in broccoli stalks during forced-air processing. *Trans ASAE*, **30**, 1473–7.

Khompis, V., Segerlind, L. J. & Brook, R. C. (1984). Pressure patterns in cylindrical grain storages. ASAE Paper No. 84-3011. ASAE, St. Joseph, MI, USA.

Kopelman, I. J., Pflug, I. J. & Naveh, D. (1982). On the overshooting in thermal processes. *J. Food Technol.*, **17**, 441–50.

Kumar, A., Bhattacharya, M. & Blaylock, J. (1990). Numerical simulation of natural convection heating of canned thick viscous liquid food products. *J. Food Sci.*, **55**, 1403–11.

Lamauro, C. J. & Bakshi, A. S. (1985). Finite element analysis of moisture diffusion in stored foods. *J. Food Sci.*, **50**, 392–6.

Lin, Y. E. (1991). Heating characteristics of simulated solid foods in a microwave oven. Ph.D. thesis, Pennsylvania State University, University Park, PA, USA.

Lin, Y. E., Anantheswaran, R. C. & Puri, V. M. (1989). Modeling temperature distribution during microwave heating. ASAE Paper No. 89-6506. ASAE, St. Joseph, MI, USA.

Lu, R. & Siebenmorgan, T. J. (1991). Finite element simulation of moisture adsorption of long-grain rough rice. ASAE Paper No. 91-6061. ASAE, St. Joseph, MI, USA.

Mang, H. A., Gallagher, R. M. & Kanodia, V. L. (1976). Finite element shell instability (FESIA) report. Department of Structural Engineering, Cornell University, Ithaca, NY, USA.

Marchant, J. A. (1976). The prediction of air flows in crop drying systems by the finite element method. *J. Agr. Engng Res.*, **21**, 417–29.

McCarthy, K. L. (1987). Steam infusion sterilization of a free-falling film. Ph.D. dissertation. University of California, Davis, CA, USA.

McCarthy, K. L. & Merson, R. L. (1989). A finite element method to model steam infusion heat transfer to a free falling film. *J. Food Process Engng*, **11**, 43–54.

Misra, R. N. (1977). Failure and stress analysis in soybean subjected to temperature and moisture gradients. Ph.D. dissertation, North Carolina State University, Raleigh, NC, USA.

Misra, R. N. & Young, J. H. (1978*a*). Finite element analysis of simultaneous moisture diffusion and shrinkage of soybeans during drying. ASAE Paper No. 78-3056. ASAE, St. Joseph, MI, USA.

Misra, R. N. & Young, J. H. (1978*b*). Finite element procedures for estimating shrinkage stresses during soybean drying. ASAE Paper No. 78-3559. ASAE, St. Joseph, MI, USA.

Misra, R. N. & Young, J. H. (1979). Finite element approach for solution of transient heat transfer in spheres. *Trans ASAE*, **22**, 944–9.
Misra, R. N., Young, J. H. & Hamman, D. D. (1981). Finite element procedures for estimating shrinkage stresses during soybean drying. *Trans ASAE*, **24**, 751–5.
Naveh, D. (1982). Analysis of transient conduction heat transfer in thermal processing of foods using the finite element method. Ph.D. dissertation, University of Minnesota, Minneapolis, MN, USA.
Naveh, D., Kopelman, I. J., Zachman, I. & Pflug, I. J. (1983*a*). Transient cooling of conduction heating products during sterilization: temperature histories. *J. Food Processing &Preservation*, **7**, 259–73.
Naveh, D., Pflug, I. J. & Kopelman, I. J. (1983*b*). Transient cooling of conduction heating products during sterilization: sterilization values. *J. Food Processing &Preservation*, **7**, 275–86.
Naveh, D., Kopelman, L. J. & Pflug, I. J. (1983*c*). The finite element method in thermal processing of foods. *J. Food Sci.*, **48**, 1086–93.
Pan, J. C. & Bhowmik, S. R. (1991). The finite element analysis of transient heat transfer in fresh tomatoes during cooling. *Trans ASAE*, **34**, 972–6.
Polivka, R. M. & Wilson, E. L. (1976). Finite element analysis of nonlinear heat transfer problems. *User's Guide for Determination of Temperature (DOT)*. Department of Civil Engineering, University of California, Berkeley, CA, USA.
Potluri, P. L. (1985). Modeling temperature and surface heat transfer coefficient in beef carcass cooling using finite element technique. Ph.D. dissertation, Texas A & M University, College Station, TX, USA.
Puri, V. M. (1987). Earth tube heat exchanger performance correlation using boundary element method. *Trans ASAE*, **30**, 514–20.
Purwadaria, H. K. (1980). A numerical prediction model for food freezing using finite element method. Ph.D. thesis, Michigan State University, E. Lansing, MI, USA.
Purwadaria, J. K. & Heldman, D. R. (1980). A finite element method for prediction of freezing rates in food products with anomalous shapes. ASAE Paper No. 80-6015. ASAE, St. Joseph, MI, USA.
Purwadaria, H. K. & Heldman, D. R. (1982). A finite element model for prediction of freezing rates in food products with anamolous shape. *Trans ASAE*, **25**, 827–32.
Rabellato, L., DelGuidice, S. & Comini, G. (1987). Finite element analysis of freezing process in foodstuffs. *J. Food Sci.*, **43**, 239, 243, 250.
Rosenfeld, D., Shmulevich, D. & Rosenhouse, G. (1991). Three-dimensional simulation of acoustic response of fruits for firmness sorting. ASAE Paper No. 91-6046. ASAE, St. Joseph, MI, USA.
Rumsey, T. R. & Fortis, T. (1983). Improving airflow distribution in a batch walnut dryer. ASAE Paper No. 83-6541. ASAE, St. Joseph, MI, USA.
Rumsey, T. R. & Fridley, R. B. (1977). Analysis of viscoelastic contact stresses in agricultural products using the finite element method. *Trans ASAE*, **20**, 162–7, 171.
Sastry, S. K., Beelman, R. B. & Speroni, L. J. (1985). A three-dimensional finite element model for thermally induced changes in foods: application to degradation of agaritine in canned mushrooms. *J. Food Sci.*, **50**, 1293–9, 1325.
Sastry, S. K. (1986). Mathematical evaluation of process schedules for aseptic

processing of low-acid foods containing discrete particulates. *J. Food Sci.*, **51**, 1323–8.
Segerlind, L. J. (1982). Solving the nonlinear airflow equation. ASAE Paper No. 82-3017. ASAE, St. Joseph, MI, USA.
Segerlind, L. J. (1984). *Applied Finite Element Analysis*. John Wiley, New York, NY, USA.
Segerlind, L. J., Singh, R. P., DeBaerdemaeker, J. G. P. & Gustafson, R. J. (1974). Theoretical aspects of finite element method. ASAE Paper No. 74-5501. ASAE, St. Joseph, MI, USA.
Singh, R. P. & Segerlind, L. J. (1974). The finite element method in food engineering. ASAE Paper No. 74-6015, ASAE, St. Joseph, MI, USA.
Smith, E. A. (1982). Three-dimensional analysis of air velocity and pressure in beds of grain and hay. *J. Agr. Engng Res.*, **27**, 101–17.
Sokhansanj, S. & Gustafson, R. J. (1979). Finite element prediction of rice cracking during drying. ASAE Paper No. 79-3547. ASAE, St. Joseph, MI, USA.
Stringer, L. J., Diehl, K. C., Wilson, J. M. & Hackney, C. R. (1989). Cooling rates for boxed fish. ASAE Paper No. 89-6120. ASAE, St. Joseph, MI, USA.
Syarief, A. M., Gustafson, R. J. & Morey, R. V. (1987). Moisture diffusion coefficients for yellow-dent corn components. *Trans ASAE*, **30**, 522–8.
Talbot, M. T. (1989). Pressure patterns established by commercial finite element package. ASAE Paper No. 89-6039. ASAE, St. Joseph, MI, USA.
Teixeria, A., Dixon, J. R., Zahadrink, J. W. & Zinswester, G. E. (1969). Computer determination of spore survival distributions in thermally processed conduction-heated foods. *Food Technol.*, **23**, 352–4.
Tong, C. (1988). Microwave heating of baked dough products with simultaneous heat and moisture transfer. Ph.D. thesis, University of Wisconsin, Madison, WI, USA.
Upadhyaya, S. K., Cooke, J. R. & Rand, R. H. (1985). A fluid filled spherical shell model of thermal elastic behavior of avian eggs. *J. Agr. Engng Res.*, **32**, 95–109.
Upadhyaya, S. K., Cooke, J. R., Gates, R. S. & Rand, H. M. (1986). A finite element analysis of the mechanical and thermal strength of avian eggs. *J. Agr. Engng Res.*, **33**, 57–78.
Upadhyaya, S. K. & Rumsey, T. R. (1989). A finite element model for coupled equations. ASAE Paper No. 89-6578. ASAE, St. Joseph, MI, USA.
Zhang, T., Bakshi, A. S., Gustafson, R. J. & Lund, D. B. (1984). Finite element analysis of nonlinear water diffusion during rice soaking. *J. Food Sci.*, **48**, 246–50, 277.
Zienkiewicz, O. C. (1989). *The Finite Element Method*. McGraw-HIll, New York, NY, USA.