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THERMAL DISPERSION EFFECTS IN NON-DARCIAN CONVECTIVE FLOWS IN A SATURATED POROUS MEDIUM

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ABSTRACT

The importance of thermal dispersion on non-Darcian convective flow in a saturated porous medium is discussed in this paper. A new theoretical model taking into consideration thermal dispersion effects in non-Darcian convective flows is proposed. A method for the determination of the longitudinal and transverse dispersion coefficients from experimental data is discussed.

Introduction

The problem of non-Darcian convective flow about a heated vertical surface in a porous medium has been studied theoretically by Plumb and Huenefeld [1] and experimentally by Cheng et al. [2]. A comparison of experimental data with Plumb and Huenefeld's theory shows that the theoretical model over-estimates the heat transfer rate while underestimates the temperature distribution in non-Darcian flows [2]. This discrepancy is probably due to the fact that the Plumb-Huenefeld theory does not take into account of the thermal dispersion effect which is important in non-Darcian flows. In the following a new model taking into consideration this effect is proposed. The values of the longitudinal and transverse thermal dispersion coefficients are obtained by matching the theory with experimental data.

The New Model

It is known that the effective thermal conductivity of a saturated porous medium can be considered as the sum of the stagnant thermal conductivity (due to molecular diffusion) and the thermal dispersion coefficient (due to mechanical dissipation) [3]. Thus, for a two-dimensional flow with x denoting the longitudinal direction and y the transverse direction, the effective thermal conductivities are

$$k_{xx} = k_d + k_{pxx} , \text{ and } k_{yy} = k_d + k_{pyy} \tag{1}$$

where k_d is the stagnant thermal conductivity which is isotropic for a homogenous porous medium, and k_{pxx} and k_{pyy} are the longitudinal and transverse dispersion coefficients which are unknown functions of velocities. As a first approximation, we shall assume that the dispersion coefficients take on the following simple form

$$k_{pxx}/k_d = A_L|u| , \text{ and } k_{pyy}/k_d = A_T|v| \tag{2}$$

where u and v are the Darcian velocities in the x- and y-directions. Experimental results by Cheng et al. [2] suggest that the thermal dispersion effect becomes important only in non-Darcian flows. For this reason, we may assume that

$$A_L = a_L\lambda/\nu , \text{ and } A_T = a_T\lambda/\nu \tag{3}$$

where λ is the Forchheimer's non-Darcy coefficient, ν is the kinematic viscosity, and a_L and a_T are dimensionless constants to be determined by matching with experimental data. Substituting Eq. (3) into Eq. (2) leads to

$$k_{pxx}/k_d = a_L\lambda|u|/\nu , \text{ and } k_{pyy}/k_d = a_T|v|/\nu \tag{4}$$

We now consider the problem of non-Darcian steady two-dimensional convective flow about a heated isothermal vertical plate with x and y being the coordinates along and perpendicular to the plate. The governing equations with the Boussinesq approximation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

$$u + \frac{\lambda\rho u^2}{\mu} = \frac{K}{\mu}\left[\frac{\partial p'}{\partial x} - \rho g\beta(T-T_\infty)\right] \tag{6}$$

$$v + \frac{\lambda\rho v^2}{\mu} = -\frac{K}{\mu}\frac{\partial p'}{\partial y} \tag{7}$$

$$(\rho c)_f \left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial x}\left[(k_d+k_{pxx})\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[(k_d+k_{pyy})\frac{\partial T}{\partial y}\right] \tag{8}$$

where ρ, μ, β, $(\rho c)_f$ and T are the density, viscosity, thermal expansion coefficient, the heat capacity, and temperature of the fluid; p' the pressure difference between the actual static pressure and the local hydrostatic pressure; g the gravitational acceleration and K the intrinsic permeability of the porous medium. The boundary conditions for this problem are

$$y = 0: \quad v = 0,\ T = T_w = T_\infty + A \tag{9a}$$

$$y \to \infty: \quad u = 0,\ T = T_\infty \tag{9b}$$

where A is a positive constant; T_w and T_∞ are the temperatures of the wall and at infinity. It is convenient to introduce a stream function such that

$$u = \frac{\partial\psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial\psi}{\partial x} \tag{10}$$

Substituting Eq. (10) into Eqs. (6-8) and Eq. (4) into Eq. (8) and invoking the boundary layer approximations such that $\frac{\partial^2 T}{\partial x^2} \ll \frac{\partial^2 T}{\partial y^2}$ and $\frac{\partial^2\psi}{\partial x^2} \ll \frac{\partial^2\psi}{\partial y^2}$,

it can be shown that the resulting equations admit a local similarity solution with a_L, a_T, $\hat{Gr}$ and C_x as dimensionless parameters of the problem, where $\hat{Gr} = \lambda K\beta g(T_w-T_\infty)/\nu^2$ and $C_x = (\lambda/x)(\alpha/\nu)$ with $\alpha = k_d/(\rho c)_f$. For the experimental data presented in Ref. 2, the values of $\hat{Gr}$ and C_x can be evaluated. The values of a_L and a_T can then be obtained by matching experimental data for θ versus η [where $\theta = (T-T_\infty)/(T_w-T_\infty)$ and $\eta = \sqrt{Ra_x}\,y/x$ with Ra_x denoting the local Rayleigh number defined as $Ra_x = \rho\beta gxK(T_w-T_\infty)/\alpha\nu$] as well as for $Nu_x/(Ra_x)^{1/2}$ versus $\hat{Gr}$ at constant C_x where Nu_x is the local Nusselt number defined as $Nu_x = x\frac{\partial T}{\partial y}(x,0)/(T_w-T_\infty)$. Computations were carried out for different assumed values of a_L and a_T. It is found that the values of a_L = 3000 and a_T = 700 match with experimental data well, as is shown in Figs. 1 & 2. The extension of the analysis for non-Darcian mixed convection about a flat plate is currently under investigation.

References

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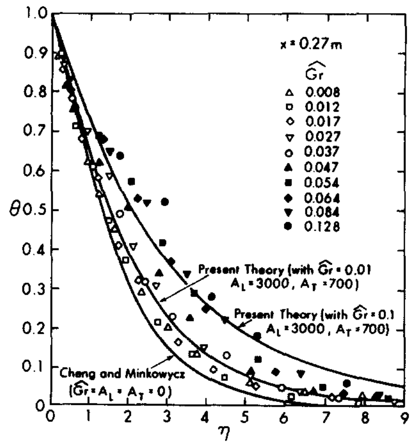

Fig. 1
Dimensionless Temperature Profiles

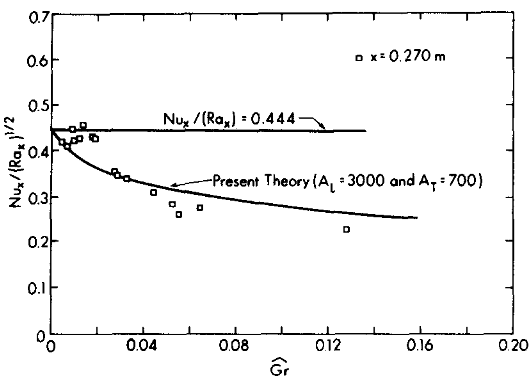

Fig. 2
Heat Transfer Results