# Synthesis of Hydrologic and System Sciences in the Development of Rainfall-Runoff Models*

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ABSTRACT

This paper presents a discussion on the synthesis of hydrologic science and systems theory in the development of rainfall-runoff models. A major problem appears to be that conceptual rainfall-runoff models contain structural peculiarities which make the application of model-identification and parameter-estimation methods and other useful system-theoretic concepts difficult. Recent developments which have been intended to overcome some of these difficulties are discussed, and areas requiring further research are suggested.

## INTRODUCTION

The common definition of hydrology is "a science which deals with the processes governing the depletion and replenishment of the water resources of the land areas of the earth and treats the various phases of the hydrologic cycle" [33]. The emphasis here is on acquiring knowledge and understanding. On a more practical level, however, hydrology is considered to be an engineering discipline, i.e., the objective is "to make inferences from hydrologic data about the future distribution of water resources in space and time in order to effectively manage them" [16].

In order to achieve this objective, hydrologic engineering (in the last two decades) has evolved, in part, as a synthesis of hydrologic science and techniques borrowed from systems theory. This evolution has certainly not

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been entirely painless. On the one hand, we must deal with processes of enormous complexity for which completely adequate models are yet to be developed (Kartevelishvili suggested that "the development of an adequate casual theory of hydrologic processes may be much more demanding than was the development of theory of relativity or quantum theory" [16]). On the other hand, systems theory is still a young (though powerful) science whose ability to deal with complex nonlinear systems (e.g., hydrologic processes) is as yet rather limited. These, however, are the realities we must deal with, and the challenge is to make the most effective use of what we have.

One type of model that appears to have a great deal of potential for on-line streamflow forecasting is the so-called "conceptual" rainfall-runoff (CRR) model [e.g., Stanford Watershed Model (SWM), U.S. National Weather Service Soil Moisture Accounting Model (SMA-NWSRFS)]. CRR models incorporate within their structure the general physical mechanisms (i.e., interception, infiltration, percolation, evapotranspiration, surface and subsurface runoff, etc.) which govern the soil-moisture phase of the hydrologic cycle. Important features of these models are that (a) they keep track of the present state of moisture conditions of the watershed, and (b) they model some of the dominant nonlinearities of the system, such as those associated with saturation of the soil mass. In spite of the sophistication of CRR models, their potential for providing accurate streamflow forecasts has not been realized, leading many engineers to question their usefulness and cost effectiveness. A major problem appears to be that CRR models contain structural peculiarities which make the application of model-identification and parameter-estimation methods and other useful systems-theoretic concepts extremely difficult [27, 8]. Recent results, however, indicate that many of the systems techniques that are potentially useful have not been exploited. In the first part of this paper, we focus on these issues and discuss some of the recent developments related to CRR modeling. Having provided this perspective, some of the areas are discussed in which the author feels future research should be directed.

## GENERAL IDENTIFICATION PROBLEM

Consider a watershed system, $W$, for which various characteristics such as inputs $u$ (e.g., precipitation, thermal radiation), states $x$ (e.g., various soil moisture components), and outputs $z$ (e.g., streamflow, potential evapotranspiration) can be observed. The *general identification problem* can be broadly stated as that of finding a model $M$ whose input-state-output behavior is as close as possible to that of the watershed. The solution of this problem involves two major stages: (1) identification of a suitable structure (the

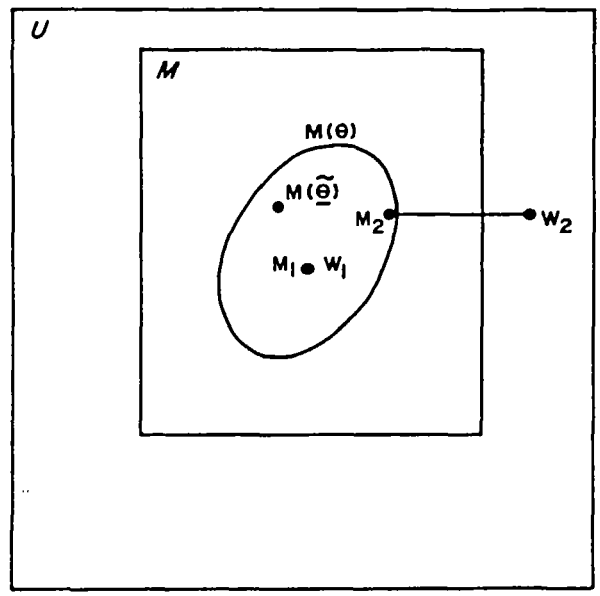


FIG. 1. Geometric interpretation of the identification problem.

mathematical functions relating the inputs, states, and outputs) for the model, and (2) calibration of the model parameters. This notion is geometrically demonstrated in Figure 1. Let $\mathscr{U}$ represent the universal set. If $\mathscr{M}$ represents the set of all mathematical models ($\mathscr{M} \subset \mathscr{U}$), then structure selection is equivalent to choosing a specific subset $M(\theta)$ of $\mathscr{M}$, where $\theta$ represents the parameters of the specified model set ($\theta \in \Theta$, $\Theta$ = feasible set). Having done this, we must then select particular values for the parameters (say $\theta \in \Theta$) such that the model $M(\theta)$ is, in some sense, "closest" in its behavior to the watershed $W$. Clearly, we would like to choose those parameter values for which the model exactly reproduces the behavior of the watershed $W$ under all circumstances. Referring to Figure 1, we see that this is only possible if the watershed $W$ ($W \subset \mathscr{U}$) is contained within the restricted model set $M(\theta)$, $\theta \in \Theta$ (e.g., $M_1 = W_1$ in Figure 1). Given the extreme complexity of the watershed process, it is clear that $W$ will not in general be contained in the set $M(\theta)$ (e.g., $W = W_2$ in Figure 1), and our interest therefore lies in finding that model $M(\theta)$ whose behavior is closest to that of $W$ (e.g., $M_2$ is "closest" to $W_2$). As the geometric interpretation of Figure 1 suggests, the more realistic (less restricted) the model set $M(\theta)$, the "closer" our identified model will be to watershed $W$. Note that, in general, the "closeness" of $M(\theta)$ to $W$ will be evaluated using some suitably chosen estimation criterion that mea-

sures the difference between the model and watershed outputs. The success of the identification process clearly depends on appropriate treatment of both of the stages discussed above. For the purpose of simplicity, let us first assume that a suitable model structure has been successfully identified. Let us go so far as to assume that the selected model set actually contains the watershed process $W$ in which we are interested. We shall later relax this (unrealistic) assumption. Therefore, our problem is to establish a methodology which, based on available information such as historical precipitation and streamflow observations, will enable us to find those parameters $\theta$ for which the model $M(\theta)$ is identical in behavior to $W$. This entails:

(1) Specification of a measure of "closeness" (called the estimation criterion) between the model and the watershed. This is usually defined in terms of the differences between the model and watershed outputs when both are subjected to the same inputs.

(2) Selection of a method for identifying those parameter values which "optimize" (minimize or maximize as appropriate) the chosen estimation criterion. Since the estimation criterion is usually nonlinear in the parameters, this usually involves implementation of an iterative optimization algorithm.

Unfortunately, the above has proved in practice to be not entirely straightforward.

## ESTIMATION CRITERION

First of all, the selected estimation criterion must take into consideration the stochastic nature of the errors present in the measured data. Although this issue was first pointed out by Clarke [4] the use of subjectively chosen estimation criteria such as the simple least squares (SLS) continues to be widespread. Sorooshian [24] and Sorooshian and Dracup [26] first proposed the use of maximum-likelihood (ML) theory as an appropriate framework for parameter estimation. They developed estimation criteria for two kinds of error structures commonly believed to exist in hydrologic data—correlated (systematic) and heteroscedastic (nonconstant-variance) errors. More recently, Gupta [8] has proposed a methodology which accounts for both kinds of errors simultaneously, based on the nonlinear structure of the stage discharge relationship. The ML methods have proved to be successful in practice. Our work with the U.S. National Weather Service's model (SMA-NWSRFS), which was reported by Sorooshian, Gupta, and Fulton [30], indicated that the ML approach is superior to the SLS method. In particular, it was observed that the effects of heteroscedasticity in the data were more severe than those of autocorrelation, at least for daily streamflow measurements. The ML

estimator for the heteroscedastic error case (HMLE) provided parameter estimates that (a) were more conceptually realistic and (b) provided consistently superior forecasts to those obtained using either the SLS or the ML autocorrelated error procedure (AMLE). The HMLE also was more efficient in terms of its ability to extract relevant information from the data set. HMLE parameter estimates seemed to be remarkably unaffected by the nature and quality of data used for the calibration. Further, good parameter estimates were obtained using just one year of data; use of longer periods such as two and three years served only to marginally improve the estimates. Some of these conclusions have recently been confirmed by independent researchers: Lemmer and Rao [17], Ibbitt and Hutchinson[13], and Delleur et al. [6]. For convenience the ML estimators are presented in the Appendix.

## OPTIMIZATION ALGORITHM

The algorithms most frequently used to identify the parameters of CRR models have been those belonging to the class of "direct-search" procedures such as the simplex method [21] and the pattern-search method [23] (see e.g. [5, 20, 3, 12, 14, 29, 30]). As is well known, such techniques are not highly efficient, since they are based on trial-and-error testing schemes in order to determine a feasible direction of movement. Derivative-based techniques have seldom been used. The reason seems to have been a general belief that the values of the derivatives of the model equation with respect to its parameters cannot explicitly be obtained, due to the presence of threshold-type parameters (e.g., [14, 19]). Some researchers have compared the performance of direct-search algorithms with those wherein the gradients are approximated using finite-difference techniques (e.g., [3, 12, 14], among others). In most cases, the direct-search procedures were reported to be superior; Johnston and Pilgrim [14] suggested that this was probably due to inaccuracies arising in the numerical gradient approximation procedures.

The problem of explicitly computing exact values for the derivatives of models containing threshold parameters has been recently addressed by Gupta [8]. He proposed a method based on a state-space analysis of the behavior modalities of such models, which does *not* require replacing the threshold structures by smoothing functions. As an example, consider the simple discrete-time storage reservoir depicted in Figure 2, which is a common element in many watershed models. Depletion ($S_t$) of the reservoir from below occurs at a rate $K$ (dimensions: inverse time, $T^{-1}$) proportional to its contents. Its capacity is limited to a maximum value $M$ (dimensions: length, $L$). When the precipitation input is such that the storage capacity is exceeded, the excess water ($R_t$) flows over the top of the reservoir (i.e., total

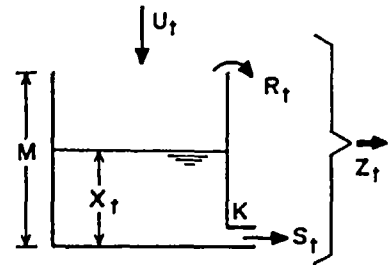


FIG. 2. A simple two-parameter linear reservoir model with threshold parameter $M$.

outflow consists of both depletion from below, $S_t$, and excess-water spillover, $R_t$). In this simple reservoir model, the two parameters are $K$ and $M$. Notice that the model operates in two modes. Let

$x_{t-1}$ = state of model at beginning of time interval $t$,

$u_t$ = precipitation input during time interval t,

and

$$x_t = x_{t-1} + u_t. \tag{1}$$

$x_t$ represents the intermediate model state, i.e., the state of the model increased by the amount of input into the reservoir, and prior to the outputs being computed and depleted from the reservoir. When $x_t \leqslant M$, the model operates in mode 1 and the equations are

$$x_t = (1 - K)(x_{t-1} + u_t), \tag{2}$$

$$z_t = S_t = K(x_{t-1} + u_t). \tag{3}$$

When $x_t > M$, the model operates in mode 2 and the equations are

$$x_t = (1 - K)M, \tag{4}$$

$$z_t = S_t + R_t = x_{t-1} + u_t - (1 - K)M. \tag{5}$$

The source of this modality of behavior is the "threshold parameter" $M$. The

model is clearly linear in each mode ($z_t \propto u_t$), but the proportionality constant is equal to $K$ in mode 1, while it is equal to 1 in mode 2. Mode 2 is interpreted as the behavior of the system under conditions of "saturation." Using Gupta's method, the equation of the above model can be written as

$$x_t = T_{1t}[(1-K)(x_{t-1}+u_t)] + T_{2t}[(1-K)M] \tag{6}$$

$$z_t = T_{1t}[K(x_{t-1}+u_t)] + T_{2t}[x_{t-1}+u_t-(1-K)M] \tag{7}$$

where the threshold functions, $T_{1t}$ and $T_{2t}$, are defined as

$$T_{1t} = \begin{cases} 1 & \text{for } x_{t-1} \leqslant M-u_t, \\ 0 & \text{for } x_{t-1} > M-u_t, \end{cases} \tag{8}$$

$$T_{2t} = \begin{cases} 0 & \text{for } x_{t-1} \leqslant M-u_t, \\ 1 & \text{for } x_{t-1} > M-u_t. \end{cases} \tag{9}$$

Using the chain rule of calculus, it can be shown that the required differential equations are

$$\frac{\partial x_t}{\partial K} = T_{1t}\left[-(x_{t-1}+u_t)+(1-K)\frac{\partial x_{t-1}}{\partial K}\right] + T_{2t}[-M], \tag{10}$$

$$\frac{\partial x_t}{\partial M} = T_{1t}\left[(1-K)\frac{\partial x_{t-1}}{\partial M}\right] + T_{2t}[(1-K)], \tag{11}$$

and

$$\frac{\partial z_t}{\partial K} = T_{1t}\left[(x_{t-1}+u_t)+K\frac{\partial x_{t-1}}{\partial K}\right] + T_{2t}\left[\frac{\partial x_{t-1}}{\partial K}+M\right], \tag{12}$$

$$\frac{\partial z_t}{\partial M} = T_{1t}\left[K\cdot\frac{\partial x_{t-1}}{\partial M}\right] + T_{2t}\left[\frac{\partial x_{t-1}}{\partial M}-(1-K)\right], \tag{13}$$

where the initial conditions to start the recursions in Equations (10) through (13) are

$x_0$ = some initial assumed or known constant value $\quad (0 \leqslant x_0 \leqslant M)$

and hence

$$\frac{\partial x_0}{\partial K} = 0,$$

$$\frac{\partial x_0}{\partial M} = 0. \tag{14}$$

If necessary, the second derivatives can be computed in a similar fashion.

Gupta [8] tested the usefulness of the method by comparing the performance of a Gauss-Newton type (derivative-based) and the simplex (direct-search) optimization algorithms on a 6-parameter CRR model. His results indicate that the derivative-based method is more efficient and uses less computer time, especially when the number of parameters to be optimized is large.

## ISSUES RELATED TO MODEL IDENTIFIABILITY

### *Model Structure*

The work reported above demonstrates how systems-theoretic concepts such as maximum-likelihood theory, state-space form, and derivative-based optimization methods can be effectively adapted to the problems of calibrating hydrologic models. Attempts to apply these methods to CRR models have revealed, however, that major problems associated with determination of unique and consistent parameter estimates still remain [30, 10]. Sorooshian and Gupta [27, 10] examined this issue in some detail and concluded that a major source of the problem is poor structural identifiability of CRR models. That is, the particular subprocess equations which make up the structure of the model hinder the application of systems-theoretic parameter-estimation techniques.

To illustrate this, let us consider again the simple discrete-time linear reservoir with threshold parameter depicted in Figure 2 and described in the previous section. As before, we assume that this model is a perfect representation of the watershed process. This enables us to assume a true set of parameters (say $M^*, K^*$) and, for a hypothetical sequence of inputs, to generate the "true" watershed output. Under these idealized circumstances, the calibration of this model should be rather simple. We find, however, that the calibration results are quite dependent on the parameter values chosen to initiate the optimization. The reason for this becomes clear if we examine the general shape of the response-surface contours for the estimation criterion. The shape of these contours is influenced by the structural equations con-

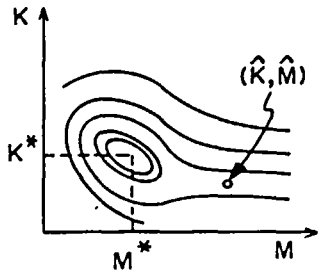


FIG. 3. Response-surface contours of Figure 2.

stituting the model and will be similar to that shown in Figure 3. Note that for large values of the threshold parameter $M$, the response surface has an elongated valley. Clearly if the initial value of $M$ is selected in the region of this valley, any search algorithm, even though partially successful in the $K$ direction, will make little progress (if any) in the $M$ direction. As a result, the search will terminate at a nonoptimal point such as $(\hat{K}, \hat{M})$.

For this simple problem, the solution is rather obviously to choose a small initial value of $M$ so that the search remains in the region of elliptical contours and progresses towards the optimum $(K^*, M^*)$. In a full-scale CRR model, however, the structural equations are far more complicated and the choice of appropriate initial parameter estimates is not as obvious. Sorooshian and Gupta [27], for example, reported convergence to the bottom of an extremely long extended valley while attempting to calibrate the SMA-NWSRFS model. The existence of this valley made it impossible to select a unique "best" set of parameters for the model. The problem was found to be related to the particular structural form of the percolation equation. Gupta and Sorooshian [9] demonstrated that this problem could be treated by the choice of an appropriate nonlinear reparametrization of the equation.

The above examples served to illustrate the fact that some serious problems associated with structural identifiability of CRR models need to be resolved before the effectiveness of systems-theoretic techniques can be fully realized. A major issue to be tackled here is how to detect structural nonidentifiability. Sorooshian and Gupta [28] have recently attempted to establish a mathematical basis for such study. They define "a model $M$ parameterized by $\theta$ to be *globally* identifiable if, and only if, different parameter values of $M$ give rise to different model output vectors." In a similar manner, "a model structure is [defined to be] *locally* identifiable at $\theta^*$ if and only if there exists an open neighborhood of $\theta^*$ in which it is identifiable." On the basis of these definitions, a mathematical framework for studying local structural identifi-

ability was proposed. In particular, a measure called the *sensitivity ratio* was developed and shown to be effective in determining poorly identifiable parameter combinations in multiparameter vector spaces. In brief, the region in the parameter space around $\theta$ for which the model output sequences are considered to be indistinguishable is approximated using a hyperellipsoid. The sensitivity ratio $\eta_i$ for the $i$th parameter is then computed as

$$\eta_i = \frac{\mathrm{PS}_i(\theta)}{\mathrm{CPS}_i(\theta)}, \tag{15}$$

where

$\mathrm{PS}_i(\theta)$ = parameter sensitivity index for $\theta_i$;

$\mathrm{CPS}_i(\theta)$ = conditional parameter sensitivity index for $\theta_i$.

A two-dimensional example is presented in Figures 4a and b. As can be seen, $\mathrm{PS}_i(\theta)$ represents the maximum that the parameter $\theta$ can vary (allowing other parameters to vary freely) while remaining within the hyperellipsoid. Similarly, $\mathrm{CPS}_i(\theta)$ represents the amount that $\theta_i$ can vary if all other parameters are assumed fixed at their chosen values. By taking the ratio of $\mathrm{PS}_i(\theta)$ to $\mathrm{CPS}_i(\theta)$, we get a nondimensional measure $\eta_i$ of the amount by which the other model parameters ($\theta_j \neq \theta_i$) compensate for the changes in model output caused by perturbations in the parameter $\theta_i$. Note that, when $\eta_i = 1$, as in Figure 4b, there is no compensation for the effects of the parameter $\theta_i$ on the model output by the other parameters. As $\eta_i$ gets larger, this indicates poorer and poorer identifiability of $\theta_i$ in relation to other model parameters. The mathematical details can be found in [28].

The study of global identifiability is made difficult by the nonconvexity and nonlinearity of the problem. Clearly, much more work needs to be done on these issues.

*Calibration Data*

The second aspect of calibration related to model identifiability is the appropriate choice of the calibration data set. It is clear that the success of the parameter-estimation phase is ultimately dependent on the quantity and quality of the data available. It has often been suggested or implied in the literature that the data used should be "representative" of the various phenomena experienced by the watershed. Many researchers have attempted to satisfy this requirement by using as large a data set as possible, without demonstrating superior results. Sorooshian, Gupta, and Fulton [30] have pointed out that, rather than length, it is the quality of information contained in the data which is important. They also stated that the data sequences

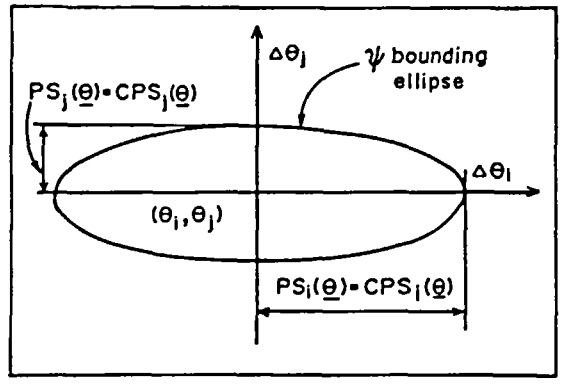


FIG. 4. Two-parameter example of an indifference region: (a) some parameter interactions.

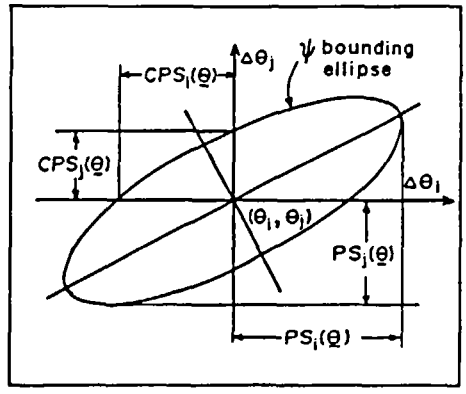


FIG. 4b. No parameter interactions.

which contain greater "hydrologic variability" are more likely to activate the various operational modes of the model sufficiently to result in reliable parameter estimates. However, the issue of how to measure "hydrologic variability" was not addressed in that paper. Work on the latter issue has recently been presented by Gupta and Sorooshian [10, 11]. A theoretical investigation into the relationships between the data, the model structure, and the precision of the parameter estimates was conducted. Some interesting results, particularly related to threshold-parameter identifiability (see the parameter $M$ in Figure 2) were obtained. It was shown, for example [10] that the precision to which the threshold parameter $M$ can be identified is (a) directly proportional to the number of times the reservoir switches from the

nonoverflow mode ($R_t = 0$) to the overflow mode ($R_t \neq 0$), and (b) independent of the duration of the overflow mode. This has interesting implications to the choice of calibration data, since it implies that a data set containing a few large storm events may be less informative than a data set containing many storms of moderate size. In a follow-up study, Gupta and Sorooshian [11] investigated the informativeness of single storms of varying intensity in the context of a 6-parameter model. Once again, it was found that a storm of moderate intensity contained information of better quality than similar storm of greater or lesser intensity. The important consideration was shown to be the degrees of activation of various modes of the model. In the same paper, we also proposed a procedure for evaluating the relative worth of different data sets based on the principle of best *a priori* information. We are currently in the process of further verifying these ideas using the SMA-NWSRFS model.

## IDENTIFIABILITY IN THE PARAMETER-DATA SPACE

As is clear from the above two subsections, the structural identifiability of the model parameters depends on two factors: (a) the location (in the feasible parameter space) of the optimal parameter values, and (b) the quality of the

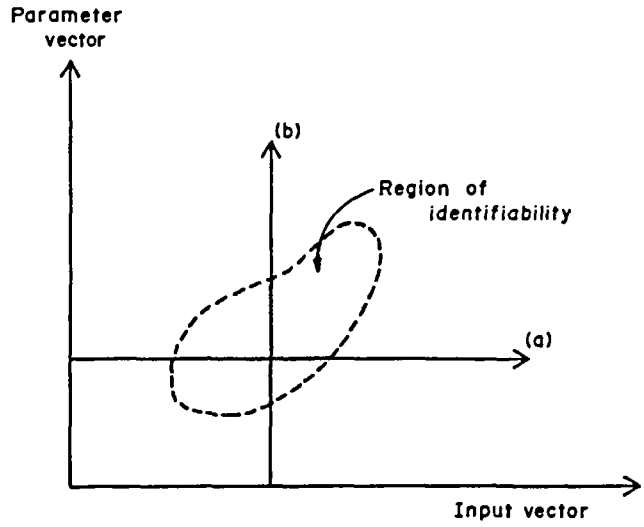


FIG. 5. Region of identifiability in the parameter data space: identifiability studies in (a) the input space conditioned on a chosen parameter set, (b) the parameter space conditioned on a chosen input data set.

input data set. To date, the limited studies on these issues have ignored the interrelationship between the two (i.e., parameter space and input space). A comprehensive evaluation of the structural identifiability of a model should aim at determining that region of the combined parameter–input-data vector space in which the calibration procedure can be expected to be successful. The conceptual representation of this concept is presented in Figure 5 where the hatched area represents the hypothetical region of identifiability (note that in practice the boundaries of such regions will not be clearly defined, due to the fuzziness inherent in the measurement of identifiability). It should be pointed out that the feasibility of such an analysis depends heavily on our ability to recognize the important characteristics of input data sequences and to describe them using a small number of parameters (or statistics). For example, a typical rainfall sequence may perhaps be described (crudely) by a normalizing transformation parameter, a mean level, a variance term, and an autocorrelation coefficient. Other methods for describing the feasible input space based on concepts of pattern recognition may prove to be superior. As yet, these ideas are offered only as food for thought in order to stimulate interest for further research.

## THE ISSUE OF MODEL SELECTION

So far, the discussion has centered around the problem of identifying the parameters of a watershed model when the structure of that model is known. Clearly, we must be confident in our ability to calibrate a model under ideal conditions if we are to begin to address the issue of model identification in the (nonideal) real world. Let us now examine a more important issue, that of choosing an appropriate structure for a model of a particular watershed. By "appropriate," we mean here that the model should be capable of reproducing adequately the various aspects of the output hydrographs which are of interest. Note that this is not necessarily the same as having the smallest forecast mean squared error.

The literature of the last two decades reveals two clear trends in the area of on-line streamflow forecasting. On the one hand, there has been the development of "conceptual" type models which purport to be "physically based." The second trend has been towards developing "systems-theoretic" models [25], i.e., linear or quasilinear input-output (rainfall-runoff) models that are designed around the powerful theoretical base of linear systems theorem (for examples of this type of models, see [32]). As discussed earlier, even though conceptual models are believed to be inherently more accurate in their representation of watershed behavior, calibration difficulties often limit their usefulness. In contrast, systems-theoretic models (e.g., time-series models,

state-space form relations, etc.) are usually easier to construct and calibrate, but have been strongly criticized as employing unrealistic assumptions about the nature of the physical system (e.g., ignoring the nonlinear dynamics of the watershed process). It has sometimes been argued that, from an engineering point of view, the usefulness of a watershed model (be it conceptual or systems-theoretic) need not depend on its conceptual realism so much as on its capability to reproduce input-output behavior. Some researchers have attempted to compare certain conceptual and systems-theoretic models from this point of view. More often than not, the published results have supported the systems-theoretic models. The reasons for this are probably quite varied and have not been clearly discussed in the literature. Sorooshian [25] mentioned two important points, however. First, the state of the art of parameter estimation in conceptual models has not been adequately refined, whereas the solution techniques available for systems-theoretic models are comparatively efficient. The second reason is that the comparisons are rarely carried out under conditions which would highlight the inadequacies of either type of model. As suggested by Linsley [18], the most important property of a model (and the least often tested) is its inherent accuracy, i.e., it should not be a question of prediction accuracy under average or slowly changing conditions but one of model credibility under extreme or rapidly varying conditions. Kitanidis and Bras [15] found, for example, that under rapidly changing hydrologic conditions, the conceptual SMA-NWSRFS model performed significantly better than an ARMAX (autoregressive moving average with exogeneous inputs) linear stochastic model with on-line adaptively estimated parameters and states. Though the ARMAX model was found to forecast satisfactorily in the recession limb, the conceptual model was found to be more reliable in forecasting the most important features of the hydrograph, such as the beginning of the rising limb, the time to and height of peak, and the total water volume. Similar results have been noted by Todini and Wallis [31], Andjelic and Szollosi-Nagy [1], and O'Connell and Clarke [22].

The problem of whether to employ a systems-theoretic or a conceptual model is really just a smaller facet of a larger problem, i.e., how to decide on a level of *complexity of model structure* appropriate to the modeling of a given watershed. Of the various issues related to this problem, one that has not received the attention it deserves is that of system scale. Consider the conceptual diagram presented in Figure 6 which illustrates the fact that the variability in the hydrologic output of a watershed system is influenced by two major factors: (a) variability in the inputs, and (b) properties associated with the physical structure of the watershed (e.g., expansion and compression of time scales, damping and attenuation, nonlinearities, input-dependent system modality, losses, etc.). It seems entirely possible that, in certain watersheds, most of the variability in the outputs can be related, in a fairly

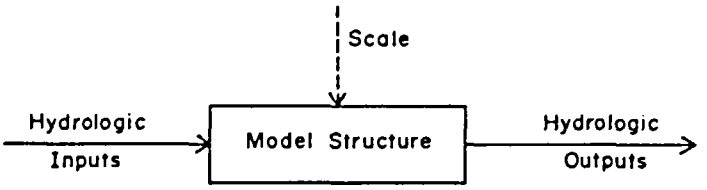


FIG. 6. Factors contributing to variability in hydrologic outputs.

simple manner, to variability in the input. That is, the influence of input variability on output behavior predominates over that of system structure, so much so that the latter influences are difficult to separate out. An example might be a small watershed whose response is dominated by surface runoff (due, for example, to minimum infiltration loss or groundwater-flow contribution, etc.). At the other extreme, we might have a watershed in which the effects of the input variable are completely damped by the system. In such a case, the characteristics of system structure obviously control properties of the output hydrograph. An example might be a very large watershed with little or no surface runoff (due to high infiltration and/or channel loss rate, etc.). The former watershed seems to be a prime contender for the systems-theoretic modeling approach, while with the latter we might benefit more from a conceptual approach.

In the above examples, we have referred to the spatial aspects of scale. In a similar manner, the time scale on which the watershed is to be modeled will also have an important role in the selection of an appropriate model structure. This area of research deserves a great deal of attention. If we can establish some means of identifying the relative importance of the hydrologic inputs versus the system structure (with respect to simulating system outputs), the problem of selecting an appropriate model (conceptual or systems-theoretic or something in between) should be much easier to deal with.

## SUMMARY

Successful hydrologic modeling requires careful synthesis of hydrologic science and systems science. Due to the enormous complexity of the hydrologic processes, this synthesis is proving to be an extremely challenging task. In this paper, I have attempted to discuss some recent results related to the development of models for on-line streamflow forecasting. These results include (a) the development of maximum-likelihood theoretic techniques and derivative-based optimization methodologies for parameter estimation, and (b) techniques for investigation of issues related to model structural identifi-

ability. Also discussed was the problem of model selection. It was pointed out that the current discussion regarding the relative merits of systems-theoretic versus conceptual models is really just a small facet of a larger problem, i.e., how to decide on an appropriate level of complexity of model structures. It was suggested that an important aspect of this problem which needs to be addressed is that of system scale. It is hoped that this discussion will serve to motivate further research into the issues addressed in this paper. Finally, I would like to emphasize that the issues discussed here are not unique to hydrologic modeling, and I hope that this paper will help to stimulate a dialogue between hydrologists and modelers in other scientific disciplines.

## APPENDIX. DESCRIPTION OF MAXIMUM-LIKELIHOOD ESTIMATORS

### 1. *The Maximum-Likelihood Estimator for the Autocorrelated-Error Case (AMLE)*

We have

$$\min_{\theta,\rho,\sigma_\nu^2} \text{AMLE} = \frac{n}{2}\ln(2\pi) + \tfrac{1}{2}\ln\frac{\sigma_\nu^{2n}}{1-\rho^2} - \tfrac{1}{2}\rho^2\sigma_\nu^2\varepsilon_1^2 + \frac{1}{2\sigma_\nu^2}\sum_{t=2}^{n}(\varepsilon_t - \rho\varepsilon_{t-1})^2, \tag{1a}$$

where

$$\sigma_\nu^2 = \frac{1}{n} - \rho^2\varepsilon_1^2 + \sum_{t=2}^{n}(\varepsilon_t - \rho\varepsilon_{t-1})^2, \tag{1b}$$

and $\rho$ is estimated from the implicit equation

$$\varepsilon_1^2 - \sum_{t=2}^{n}\varepsilon_{t-1}^2\rho^3 + \left(\sum_{t=2}^{n}\varepsilon_t\varepsilon_{t-1}\right)\rho^2 + \left(\sigma_\nu^2 - \varepsilon_1^2 + \sum_{t=2}^{n}\varepsilon_{t-1}^2\right)\rho - \sum_{t=2}^{n}\varepsilon_t\varepsilon_{t-1} = 0, \tag{1c}$$

where $\varepsilon_t = q_{t,\text{obs}} - q_{t,\text{sim}}$ = (residual at time $t$) with $q_{t,\text{obs}}$, $q_{t,\text{sim}}$ the measured and the simulated flows at time $t$, $\theta$ is the set of parameters to be estimated,

$n$ is the number of data points, $\sigma_\rho^2$ is a constant variance term, and $\rho$ is the first-lag autocorrelation coefficient. The above estimator is developed based on the assumption that the output errors are Gaussian with a constant variance and correlated according to a first-lag autoregressive scheme (for details, see [26]). Note that in the case $\rho = 0$ the estimates obtained using AMLE are equivalent to those of the simple least-squares (SLS) criterion

$$\min_{\theta} \text{SLS} = \sum_{t=1}^{n} \varepsilon_t^2. \tag{2}$$

2. *The Maximum-Likelihood Estimator for the Heteroscedastic-Error Case (HMLE)*

We have

$$\min_{\theta,\lambda} \text{HMLE} = \left\{ \sum_{t=1}^{n} w_t \varepsilon_t^2 \right\} \left\{ n \left[ \prod_{t=1}^{n} w_t \right]^{1/n} \right\}^{-1}, \tag{3a}$$

where $w_t$ is the weight at time $t$, computed by

$$w_t = f_t^{2(\lambda-1)}, \tag{3b}$$

where $f_t$ is the expectation of $q_{t,\text{true}}$ (either $q_{t,\text{obs}}$ or $q_{t,\text{sim}}$) and $\lambda$ is the unknown transformation parameter which stabilizes the variance. The implicit expression to estimate $\lambda$ is

$$\left[ \sum_{t=1}^{n} \ln(f_t) \right] \left[ \sum_{t=1}^{n} w_t \varepsilon_t^2 \right] - n \left[ \sum_{t=1}^{n} w_t \ln(f_t)\, \varepsilon_t^2 \right] = 0. \tag{3c}$$

Briefly, the HMLE estimator is derived based on the assumption that the errors are Gaussian with mean zero and covariance matrix $V$, where $V_{t,t} = \sigma_t^2$ and $V_{t,t+s} = 0$ for $s \neq 0$.

Stabilization of the variance is attempted through the use of the Box-Cox [2] power transformation which relates the variance of each error to its associated output value (see [26]). In this study, $f_t = q_{t,\text{obs}}$ was used in the computation of the weights (the original procedure reported in the aforementioned papers used $f_t = q_{t,\text{sim}}$). Fulton [7] has shown that this results in a more stable estimation scheme. It is interesting to note that if the variances of the additive errors are homogeneous (independent of the time or magnitude

of the associated flows), then the procedure will automatically select the value of $\lambda = 1.0$. This results in $w_t = 1$ for all $t$, and the estimation criterion reduces to the SLS. If, however, the variance of the errors is proportional to a power function of the magnitude of the flows, then the procedure will select a value of $\lambda = 1.0$. Pertinent to our problem is the case where the error variance increases as the flow values get larger (Sorooshian and Dracup [26] have discussed the underlying reasons at great length). In this situation, the ML estimate of $\lambda$ will be less than unity, and this ensures that, in the estimation criterion, the errors associated with lower flows (which contain more reliable information) are weighted more heavily.

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