Pergamon

Wat. Sci. Tech. Vol. 36, No. 8-9, pp. 57–63, 1997.

0273–1223/97 $17·00 + 0·00

PII: S0273-1223(97)00618-5

A STORAGE MODEL FOR THE SIMULATION OF THE HYDRAULIC BEHAVIOUR OF DRAINAGE NETWORKS

Homayoun Motiee*, Bernard Chocat* and Olivier Blanpain**

* *Laboratoire Méthodes, INSA Lyon, Bat 304, 69621 Villeurbanne Cedex, France*
** *Laboratoire Artois mécanique et habitat, université d'Artois, 62406 Béthune Cedex, France*

ABSTRACT

This paper presents a model for the hydraulic simulation of a drainage network using the storage concept. This model is easier to use than the complete Barre de Saint Venant equations and gives better results than the usual conceptual models, i.e. the Muskingum model, or than models obtained by the simplification of the Saint Venant equations (kinematic wave model and diffusion wave model). © 1997 IAWQ. Published by Elsevier Science Ltd

KEYWORDS

Hydraulic modelling; model calibration.

INTRODUCTION

The hydraulic simulation of drainage networks is today very important for many purposes. Anyway, the diversity of the problems to be solved, makes it impossible to reach all of the objectives with only one tool. In fact two main families of models are available:

– The first family consists of physically based models. They use the Barre de Saint Venant equations. This kind of model needs some important hypotheses which are not always verified in sewers systems. Anyway, they are able, in most cases, to give a correct description of gradually varied flows. The main problems with these models are the great number of data they require, the difficulty to build stable numerical schemes, their slowness, and the difficulties in interpretation of results.

– Models of the second family can be called hydrological models. The prototype of this category is the Muskingum model. These models are very simple to use, even for people who are not hydraulicians, need very few data and are very quick to run. On the other hand, they are truly unable to represent the effects of transitional flows due to problems of hydraulics or inconsistencies of networks.

In a previous ICUSD, Yen (1993), mentioned that "*in majority of cases the change of flow in Δt is small such that the quasi (stepwise) steady flow routing model is sufficient for solving problems*". This idea is shared by numerous researchers who think that the better quality of the Barre de Saint Venant model is often counterbalanced by the difficulty of correct usage.

Several researchers, especially in the Method laboratory of INSA (Chocat, 1978), have considered that the gap between these two categories of models was wide enough to give place to a new family of models, easier to use than the Barre de Saint Venant equations, and able to take into account some of the most important effects of the flow complexity and of the network reality. Such models need to present several pragmatic qualities:

– be able to represent the actual functioning of the drainage system whatever the type of flow (free surface or surcharged, infracritical or supercritical);
– be able to represent the functioning of the drainage system whatever the kind of network (branched or looped) and whatever the nature of the special structures that can be found inside;
– be a lot quicker in running than the Barre de Saint Venant equations;
– be stable whatever the time step and the space step;
– furnish results easy to understand and to use, even for a non-hydraulician.

MODEL PRESENTATION

The proposed model (Motiee, 1996) uses the storage concept, developed by Chocat (1978) and further developed by Blanpain (1991). It uses the continuity equation :

$$\frac{\partial S(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = 0 \tag{1}$$

where S(x,t) is the wetted cross section and Q(x,t) the flow rate at the position x and the time t.

This equation can be written between two points separated by a distance (space step) Δx as follows :

$$\frac{dV(t)}{\Delta t} = Qe(t) - Qs(t) \tag{2}$$

In this equation, Qe(t) and Qs(t) are respectively the inflow and the outflow, upstream and downstream of the reach; V(t) is the total storage volume in the reach.

The second equation is the storage equation. It must link together the storage volume in the reach and the flow rates upstream and downstream:

$$V(t) = f(t, Qe(t), Qs(t)) \tag{3}$$

In fact, it is always possible to express the storage volume according to the wetted area in the reach:

$$V(t) = \int_{x_0}^{x_0+\Delta x} S(x,t)dx \tag{4}$$

If the evolution of the wetted cross section is always increasing or always decreasing along the reach, we may write:

$$V(t) = (\alpha \, Sam(t) + (1-\alpha) \, Sav(t)) \, \Delta x \tag{5}$$

Where Sam(t) is the wetted area upstream, Sav(t) the wetted area downstream and where the α coefficient depends on the flow characteristics.

Different discretizations can be made from equation (3) (Chow, 1959; Ponce, 1981). Blanpain (1993) showed that second-rate schemes, like:

$$\frac{V(t)-V(t-\Delta t)}{\Delta t} = \frac{Qe(t)+Qe(t-\Delta t)}{2} - \frac{Qs(t)+Qs(t-\Delta t)}{2} \tag{6}$$

were never stable. For this reason he suggested using the simple implicit scheme, previously built by Chocat (1978):

$$\frac{V(t)-V(t-\Delta t)}{\Delta t} = Qe(t) - Qs(t) \tag{7}$$

We used the same scheme, because Blanpain (1993) showed that the results were good. In that case, the outflow can be calculated as:

$$Qs(t) = Qe(t) - \frac{V(t)-V(t-\Delta t)}{\Delta t} \tag{8}$$

To use relation (8), we must be able to calculate V(t). Several cases are possible.

Case of free surface flow, without backwater effect

Without any backwater effect the value $\alpha=1$ can be used in equation (5) (Blanpain, 1993); In these conditions, it is easy to write an explicit equation:

$$V(t) = Sam(t)\,.\,\Delta X \tag{9}$$

$$\text{Or:}\quad V(t) = \frac{Qe(t)}{Ve(t)}\,.\,\Delta x = \frac{\Delta x}{Ve(t)}\,.\,Qe(t) \tag{10}$$

In equation (10), Ve(t) is the flow velocity upstream and Qe(t) the inflow rate. If the incoming hydrograph is discretized, it is possible, to calculate the flow velocity corresponding to each time step, for example by using the Manning-Strickler formula. The main hypothesis is that the variations of the hydrograph are slow enough to obtain a quasi steady flow at each time step.

Equation (10) can be written with a temporal parameter :

$$Tp(t) = \frac{\Delta x}{Ve(t)} \tag{11}$$

With this notation, we obtain:

$$V(t) = Qe(t).Tp(t) \tag{12}$$

Equation (12) shows that this model can be understood as a conceptual, non linear model. In this way it can be related to the Kalinin-Miljukov model. Compared with simplified hydrodynamic models, this formulation of the storage model can be related to the kinematic wave model. The patterns of the surface profile supposed by the two models are actually the same, even if the ways to carry out the calculations are very different.

Backwater effects

In a drainage network backwater effects can be observed in a large number of cases: high level of water at the outlet, due to a river or to the tide; increase of the flow rate downstream; decrease of the discharge capacity from one reach to the reach downstream (decrease of the slope, increase of the roughness, etc.); presence of a special structure disturbing the flow, etc. Backwater effects can occur either if the network operates in pressure flow or if it operates with free surface flow. Whatever the case , the method is the same. It supposes that the pattern of the surface profile, inside reaches affected by the backwater curve, can be described by the equation (13) :

$$\frac{\Delta H}{\Delta x} = J - I \text{ (I=slope of the reach, J=head losses by unit of length)} \quad (13)$$

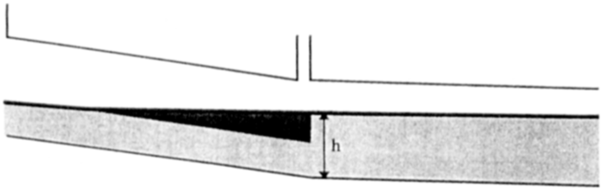

Figure 1. Pattern assumed for the surface profile in case of backwater effect.

For example, we can look at a simple case, consisting of two consecutive reaches (Fig. 1), the second one affecting the first one, due to a smaller capacity. The problem to be solved is to calculate the correct values of V1 and Qs1 so that equations (14) and (15) are both verified :

$$Qs1 = Qe1 - \frac{(V1(t) - V1(t - \Delta t))}{\Delta t} \quad (14)$$

$$V1(t) = f(Qe1) \quad (15)$$

Where Qe1 is the inflow in the first reach; Qs1, the outflow in the first reach; Qe2=Qs1 the inflow in the second reach; V1, the storage volume in the first reach.

Equation (15) represents the fact that, on the first hand, the depth of water, h, calculated at the entrance of the second reach, is directly dependent on the inflow Qe2 (hypothesis of a quasi steady flow), and, on the other hand, that the storage volume V1(t) in the upper reach is dependent on the depth. In fact equation (15) shows a continuously decreasing relation between V1 and Qs1 : The greater Qs1, the greater is h and the greater is V1; if Qs1 = 0, then h = 0 and Vs1 = 0.

Equation (14), shows a linear decreasing relation between Qs1 and V1 (if V1(t) = 0, then, Qs1 = Qe1 + V1(t-Δt)/Δt and if Qs1 = 0 then V1(t) = V1(t-Δt) + Qe1.Δt).

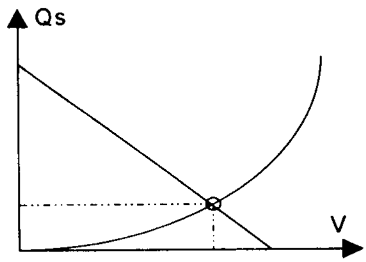

Figure 2. Graphic representation of the existence and unicity of equilibrium values of Qs1 and V1.

There is only one equilibrium value and it always exists (Fig. 2). This equilibrium value can be found by an iterative method.

Equation (13) shows that in the case of backwater effects, the storage model can be related to the diffusion wave model, which results from several simplifications of the Barre de Saint Venant equations.

Anyway, even if the two models assume a similar pattern for the surface profile, they cannot be directly assimilated because of the differences between the calculation methods.

MODEL VALIDATION

The main objective of the validation step is to verify that the results of the model are consistent with the reality it is supposed to represent. More precisely, the validation step must:
- define the validity field;
- estimate errors and uncertainties;
- choose the best values to be assigned to the parameters.

This step can be carried out by several methods, based either on theoretical assumptions or on empirical assessments. In this last case, the results furnished by the model are compared with reference results. These reference results either can be derived from measurements or can be furnished by a reference model. For the validation of the storage model, we used an empirical methodology, developed by Semsar (1995), which is presented in another paper (Chocat *et al.*, 1996). The method consists in comparing the results given by the model to be studied with the results given by the Saint Venant equations, for a set of 15 typical networks, under 12 kinds of working conditions.

The discrepancies between the results given by two of the models are measured by the discrepancies between the two hydrographs obtained at the outlet of the networks. Two different criteria are used:

- the minimum total quadratic deviation between the two discretized hydrographs (the hydrograph of reference: Q1 and the hydrograph to be compared: Q2), obtained at the outlet of the networks, called TQD:

$$TQD = \frac{\sqrt{\sum_{i=1}^{n}\left(Q1(i.\Delta t) - Q2(i.\Delta t + \delta)\right)^2}}{\sum_{i=1}^{n}\left(Q1(i.\Delta t)\right)}$$

- the time lag between the two hydrographs which minimises the TQD: δ

In his thesis, Semsar (1995), showed that the quality of the results can be said to be excellent if both of the criteria were less than 0.015, good if they were between 0.015 and 0.05, and acceptable if they were less than 0.1. It is not possible to give here all the results. We summarise them by fig. 3, which shows, for each of the 15 networks, the average global deviation, defined as :

$$(AGD) = \frac{1}{12}\sum_{i=1}^{12}\sqrt{(TQD_i)^2 + (\delta_i)^2}$$

Where TQD_i and δ_i are respectively the total quadratic deviation and the optimum time lag, obtained for the working condition i.

This figure shows that all the AGD values (except for the network number 13) are below 0.1. That means that both the TQD and δ are below 0.1; i.e. that most of the results are acceptable.

If we compare these discrepancies with those between the Muskingum model and the Barre de Saint Venant equations, we can observe a significant improvement in the results : the average value of AGD is 0.1 for the Muskingum model, and 0.063 for the storage model.

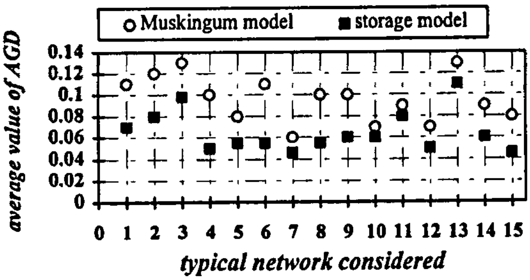

Figure 3. Average discrepancies between the results given by the Muskingum model and the storage model, compared with the Barre de Saint Venant equations.

Anyway, this improvement is not as substantial as we were expecting, probably because the use of average values smooths out the discrepancies. It is also interesting to notice that the results given by the two models (Muskingum model and the storage model) are good for the same networks. In fact for both of them, the more complex the networks (i.e. with a great number of reaches, junctions, special structures, etc.), the more the results are similar to those given by the Saint Venant equations. This conclusion concurs with some results obtained by Chocat (1978), Thibault (1981), Semsar (1995).

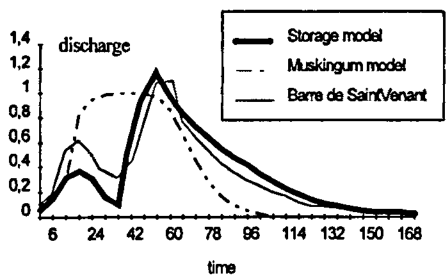

Figure 4. Comparison of hydrographs given by the storage model with those given by the Barre de Saint Venant equations and the Muskingum model.

Figure 4 shows an example of the improvement due to the use of the storage model. In this case the backwater curve is very important, and "blocks" the outflow between the 12th minute and the 42nd minute. The result is a hydrograph with two maximums, separated by a period of lower flow. The storage model gives a good idea of this kind of phenomenon, whereas the Muskingum model is unable to represent it.

Compared with the Muskingum model, another advantage of the storage model is that it is able to furnish a realistic pattern of the surface profile, even in the case of important backwater effects. Compared with the complete Saint Venant equations the main advantage of the storage model is that the calculation times are a lot less important (the average calculation time is divided by ten in the studied cases). Moreover, it is easier to use and easier to understand the results, even for a non hydraulician, and the model presents no stability problems. Compared with models obtained by simplifying the Saint Venant equations, the main advantage of the storage model is that it uses the most appropriate simplification (diffusion wave model or kinematic wave model), depending on the flow conditions.

Even if the storage model is not a panacea, able to replace all the pre-existing models, it seems to offer an interesting new possibility in a number of cases. It also confirms the previous idea (Yen, 1993), that it is not always necessary to use the complete Barre de Saint Venant equations.

REFERENCES

Blanpain O. (1993). *Intégration d'aides expertes dans un logiciel technique d'assainissement.* Thèse DO, INSA de Lyon. France.

Chocat B., Semsar Y.A., Blanpain O. (1996) *"Elaboration of assistance methods and tools in the software packages of urban drainage systems for selection of hydraulic models"*. ICUSD 96, ed. F. Sieker and H.R. Verworn, Hannover, Germany, pp 1563-1566.

Chocat B. (1978). *Un modèle de simulation des écoulements dans les réseaux d'assainissement pluvial.* Thèse DI, INSA de Lyon, France.

Chow W. T. (1959). *Open channel hydraulics.* Mc Graw-Hill book Co., New York, USA..

Motiee H. (1996). *Un modèle pour la simulation des réseaux d'assainissement fondé sur le consept de stockage.* Thèse DO, INSA de Lyon, France.

Ponce (1981). *Kinematic, dynamic and inertial waves.* International symposium on unsteady flow in open channel, Colorado state University, USA, paper 2.

Semsar Y.A. (1995). *Mise au point d'une méthodologie de'évaluation et de comparaison des modèles de simulation hydraulique des réseaux d'assainissement.* Thèse DO, INSA de Lyon, France.

Thibault S. (1979). *Eléments pour une phénoménologie en hydrologie urbaine.* Thèse DI, INSA de Lyon, France.

Yen B.C. (1993). *Is hydraulics over-used or under-used in urban drainage.* 6th ICUSD, proceedings pp 91-97.