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Modelling the Effects of Imperfect Mixing on the Performance of Anaerobic Reactors for Sewage Sludge Treatment

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Abstract: An approach to the modelling of suspended-growth anaerobic digestion systems based on the assumption of an incompletely mixed reactor is presented. The mathematical model developed describes the dynamic behaviour of anaerobic sludge digesters under non-ideal mixing conditions. The microbial kinetic model for the anaerobic digestion of waste-activated sludge distinguishes the processes of death and lysis of activated sludge cells, hydrolysis of particulate material, fermentation of soluble substrates, volatile fatty acids utilisation and methane formation. The interaction of two microbial groups is considered, i.e. acid-formers and methanogens. Their growth is assumed to depend on Monod kinetics for the substrates. Death and lysis, hydrolysis and biomass decay are described by first order reactions. The biokinetic expressions were linked to a simple mixing model which considered the reactor volume split into two sections: the flow-through and the retention regions. The transfer of material between regions was assumed to be limited. Deviations from an ideal completely mixed regime were represented by changing the relative volume of the flow-through region (α) and the turnover time of material in the vessel (τ). The dynamic model described the effects of the retention time and reactants' distribution, resulting from the mixing condition, on process performance. Computer simulations under different conditions showed a considerable decline in methane production and treatment efficiency due to incomplete mixing. The COD removal efficiency increased by extending the retention time and the degree of mixing. The evaluation of the impact of the mixing parameters showed that α has a far more significant effect on the performance of anaerobic digestion than τ does. Nevertheless, both are important and the overall efficiency is a complex function of both parameters. The results obtained confirm and emphasise the importance of considering mixing when simulating anaerobic digestion, calculating process conversion efficiency, and during anaerobic reactor design.

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Key words: dynamic modelling; anaerobic reactor; imperfect mixing; sewage sludge treatment

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NOTATION

A	Volatile acids concentration (g COD dm^{-3})
b^A	Decay coefficient for acid-formers (day^{-1})
b^M	Decay coefficient for methanogens (day^{-1})
COD	Chemical oxygen demand (g dm^{-3})
f_d	Net biodegradable fraction of active biomass (dimensionless)
k^A	Maximum specific substrate utilisation rate for acid-formers (g COD utilised g^{-1} COD biomass day^{-1})
k^M	Maximum specific substrate utilisation rate for methanogens (g COD utilised g^{-1} COD biomass day^{-1})
K_c^M	Half-velocity coefficient for methanogens (g COD dm^{-3})
k_d	Activated-sludge cell death rate coefficient (day^{-1})
k_h	Hydrolysis rate coefficient (day^{-1})
K_s^A	Half-velocity coefficient for acidogenesis (g COD dm^{-3})
M	Methane concentration (g COD dm^{-3})
P	Degradable particulate COD concentration (g dm^{-3})
Q	Volumetric flow rate (dm^3 day^{-1})
R	Reaction rate (g dm^{-3} day^{-1})
S^A	Soluble substrate COD concentration (g dm^{-3})
t	Time (day)
V	Working volume of reactor (dm^3)
X_a^A	Active acidogenic microorganism concentration (g COD dm^{-3})
X_a^{AS}	Viable activated-sludge biomass COD concentration (g dm^{-3})
X_v^M	Total methanogenic biomass (g dm^{-3})
Y^A	Yield coefficient for acid-formers (g COD biomass g^{-1} COD utilised)
Y^M	Yield coefficient for methanogens (g COD biomass g^{-1} COD utilised)
α	Ratio of the volume in the flow-through region to the total reactor volume (dimensionless)
$(1-\alpha)$	Relative volume of the retention region (dimensionless)
γ	Cell soluble degradable COD immediately released (dimensionless)
θ	Hydraulic (also solid) retention time (day)
τ	Interchange parameter (day)

Superscripts

A	Acidogenic phase
AS	Activated sludge
M	Methanogenic phase

Subscripts

exch	Exchange between zones
i	Initial conditions
o	Influent
1	Flow-through region
2	Retention region

1 INTRODUCTION

Anaerobic mesophilic digestion is the predominant method for sewage sludge treatment in many countries. Anaerobic stabilisation of organic wastes involves the bioconversion of complex compounds to acetic acid, carbon dioxide and hydrogen, which are subsequently converted to methane by decarboxylation of acetate and by reduction of carbon dioxide using hydrogen. Microorganisms responsible for this bioprocess are very sensitive to many environmental factors. A detailed reaction scheme for the anaerobic digestion of domestic sludge has been described by Gujer and Zehnder.[1]

Besides the production of flammable biogas, the high degree of organic matter stabilisation that can be achieved makes the product sludge suitable for disposal to agricultural land. Population growth and more comprehensive sewage treatment are generating larger quantities of sludge, while current legislation demands effective sludge treatment before disposal. There is a clear need for efficient anaerobic sludge digestion. Mixing is an important factor for the optimal performance of anaerobic digesters.

The need for efficient mixing of sludge in anaerobic reactors has been stressed in the technical literature.[2] Good mixing promotes the transfer of substrates and heat to the microorganisms, maintains uniformity in other environmental factors and assures the effective use of the entire reactor volume by preventing stratification. Conversely, incomplete mixing jeopardises the efficiency of the treatment process and, therefore, the stability of the product sludge. Experimental studies have shown that mixing has significant effects on the treatment efficiency of anaerobic reactors.[3–5] However, most of the results obtained are empirical correlations between the impeller speed and the chemical oxygen demand (COD) destruction or methane production and refer to small-scale laboratory experiments. This approach does not contribute greatly to understanding the mechanisms by which mixing influences anaerobic digestion performance and neither does it provide criteria on which full-scale bioreactor design or strategies of operation can be based. Mathematical modelling has the potential to provide a rational explanation of the links between mixing and anaerobic digestion kinetics.

Because of the inherent complexity of biological processes, besides the bioreactor engineering involved, it is difficult to develop mathematical models reflecting reality. Therefore, modellers resort to simplifications. One of such widely-used simplifying hypotheses is that of uniformity. The implications of this hypothesis on modelling have been recently reviewed by Aris.[6] The

assumption of ideal completely mixed reactors may be valid in some cases, when due to the small scale of the experimental reactors used, perfect mixing may effectively be achieved or when the characteristic time constants for the kinetic parameters are much larger than the mixing and mass transfer time constants.[7] However, the difficulty in achieving complete mixing increase with reactor scale and therefore, as a result of inevitable compromises such as increasing costs, loss of equipment efficiency, etc., mixing in large reactors may not be as good as in small ones. Residence time distributions (RTD) studies conducted in full-scale primary digesters have shown actively mixed volumes as low as 23% of the total volume.[8] It is recognised that inhomogeneities in the medium can have profound influence, especially, on the production of metabolites.[7] While imperfect mixing patterns are more common than ideal ones in real reactors, anaerobic digestion models often assume complete mixing conditions. Therefore, their applicability appears to be limited. The need for a better understanding of the relationship between mixing and anaerobic digestion performance and the lack of mathematical models addressing this subject led to this study.

2 MATHEMATICAL MODEL

2.1 Biological reaction kinetics

The dynamic model for the anaerobic digestion of biological sludge was adapted from the steady-state equations of a kinetic model presented by Pavlostathis and Gossett.[9] The conceptual representation for the anaerobic digestion of waste-activated sludge on which the kinetic model is based is shown in Fig. 1.

The biodegradable fraction of waste-activated sludge is considered to consist almost exclusively of the biodegradable portion of activated sludge microorganisms.

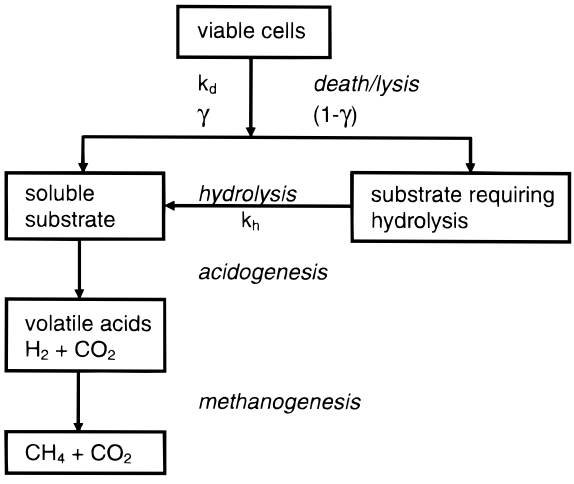

Fig. 1. Conceptual model for the anaerobic digestion of biological sludge (after Pavlostathis and Gosset[9]).

The kinetic model distinguishes five processes: death/lysis of the activated-sludge cells, hydrolysis of dead cell particulates, formation and consumption of soluble substrates, production and consumption of volatile fatty acids, and methane production. The interaction of two microbial groups is considered: acid-formers and methanogens. The consumption of soluble substrates and volatile acids as well as the growth of anaerobic microorganisms are assumed to obey Monod-type kinetics. Processes of death and lysis, hydrolysis and biomass decay are described by first order reactions. Table 1 shows the biokinetic expressions which describe the anaerobic digestion of waste-activated sludge. Details about the assumptions considered and the experimental findings supporting them can be found elsewhere.[9]

2.2 Mixing model

The microbial kinetics shown in Table 1 were coupled to a simple mixing model referred to as the two-regions model. A conceptual representation of the two-regions mixing model is illustrated in Fig. 2.

This mixing model assumes the reactor volume split into two sections: the flow-through and the retention regions. Both regions are assumed to be perfectly mixed but the transfer of material between zones is limited. The retention region has features of the behaviour shown by a stagnant zone. Different levels of mixing are accomplished by adjusting the relative volume of the flow-through region (α) and the exchange rate between regions expressed as the turnover time of material in the vessel (τ). Despite its simplicity, this classical model is used in chemical engineering for the description of retention time distributions in 'real' reactors[10] and has proved to be a useful tool for the theoretical study of the effects of inhomogeneity in chemical and biological systems. For example, the use of this mixing model allowed the prediction for the first time of chaotic transient behaviour in the Belousov–Zhabotinsky reaction[11]

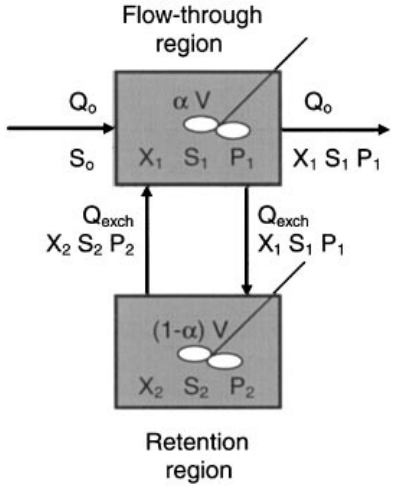

Fig. 2. Two-regions mixing model.

TABLE 1
Kinetic Rate Expressions of the Anaerobic Digestion of Biological Sludge

1. Death and lysis of the viable activated-sludge biomass	
$R(f_d X_a^{AS}) = f_d k_d X_a^{AS}$	$f_d = 0{\cdot}73, \quad k_d = 2{\cdot}0\ \text{day}^{-1}, \quad X_{ai}^{AS} = 11{\cdot}31\ \text{g dm}^{-3}$
2. Hydrolysis of death cell particulates	
$R(P) = k_h P$	$k_h = 0{\cdot}15\ \text{day}^{-1}, \quad P_i = 0\ \text{g dm}^{-3}$
3. Decay of acidogenic biomass	
$R(b^A) = b^A X_a^A$	$b^A = 0{\cdot}1\ \text{day}^{-1}$
4. Decay of methanogenic biomass	
$R(b^M) = b^M X_v^M$	$b^M = 0{\cdot}015\ \text{day}^{-1}$
5. Fermentation of soluble substrates	
$R(S^A) = \dfrac{k^A S^A X_a^A}{k_s^A + S^A}$	$k^A = 8{\cdot}0$ g COD utilised g^{-1} COD biomass day^{-1}
	$K_s^A = 0{\cdot}045\ \text{g COD dm}^{-3}, \quad S_i^A = 0\ \text{g dm}^{-3}$
6. Volatile fatty acids utilisation	
$R(A) = \dfrac{k^M A X_v^A}{k_c^M + A}$	$k^M = 6{\cdot}2$ g COD utilised g^{-1} COD biomass day^{-1}
	$K_c^M = 0{\cdot}045\ \text{g COD dm}^{-3}, \quad A_i = 0\ \text{g dm}^{-3}$
7. Growth of acidogenic biomass	
$R(X_a^A) = \dfrac{Y^A k^A S^A X_a^A}{K_s^A + S^A}$	$Y^A = 0{\cdot}2$ g COD biomass g^{-1} COD utilised
	$X_{ai}^A = 0{\cdot}1\ \text{g COD dm}^{-3}$
8. Growth of methanogenic biomass	
$R(X_v^M) = \dfrac{Y^M k^M A X_v^M}{K_c^M + A}$	$Y^M = 0{\cdot}057$ g COD biomass g^{-1} COD utilised
	$X_{vi}^M = 0{\cdot}12\ \text{g COD dm}^{-3}$
9. Methane generation	
$R(M) = \dfrac{k^M A X_v^M}{K_c^M + A} - \dfrac{Y^M k^M A X_v^M}{K_c^M + A}$	$M_i = 0\ \text{g COD dm}^{-3}$

and the explanation of experimental values for the continuous culture of *Aerobacter cloacae*, which were not adequately described by an ideal mixing model.[12] The scheme of macromixing assumed by the model may resemble the observed mixing patterns of anaerobic digesters in only some cases. In a real reactor, of course, one does not have two homogeneous zones of different composition but a composition that varies throughout the reactor volume.

2.3 Model development

Material balances applied to the boundaries depicted by the two-regions mixing model (Fig. 2) yield the set of ordinary differential equations which constitute the dynamic model as follows.

Mass balance on degradable portion of viable activated-sludge microorganisms ($f_d X_a^{AS}$) gives

$$\frac{df_d X_{a1}^{AS}}{dt} = \frac{f_d(X_{a0}^{AS} - X_{a1}^{AS})}{\alpha\theta} + \frac{f_d(X_{a2}^{AS} - X_{a1}^{AS})}{\alpha\tau} - R(f_d X_{a1}^{AS}) \tag{1}$$

$$\frac{df_d X_{a2}^{AS}}{dt} = \frac{f_d(X_{a1}^{AS} - X_{a2}^{AS})}{(1-\alpha)\tau} - R(f_d X_{a2}^{AS}) \tag{2}$$

Mass balance on particulate solids requiring hydrolysis (P) gives

$$\frac{dP_1}{dt} = \frac{P_0 - P_1}{\alpha\theta} + \frac{P_2 - P_1}{\alpha\tau} + (1-\gamma)R(f_d X_{a1}^{AS}) - R(P_1) \tag{3}$$

$$\frac{dP_2}{dt} = \frac{P_1 - P_2}{(1-\alpha)\tau} + (1-\gamma)R(f_d X_{a2}^{AS}) - R(P_2) \quad (4)$$

Mass balance on soluble substrate for acid-formers (S^A) gives

$$\frac{dS_1^A}{dt} = \frac{S_0^A - S_1^A}{\alpha\theta} + \frac{S_2^A - S_1^A}{\alpha\tau} + R(P_1) + \gamma R(f_d X_{a1}^{AS}) - R(S_1^A) \quad (5)$$

$$\frac{dS_2^A}{dt} = \frac{S_1^A - S_2^A}{(1-\alpha)\tau} + R(P_2) + \gamma R(f_d X_{a2}^{AS}) - R(S_2^A) \quad (6)$$

Mass balance on degradable portion of acidogenic biomass ($f_d X_a^A$) gives

$$\frac{df_d X_{a1}^A}{dt} = \frac{f_d(X_{a0}^A - X_{a1}^A)}{\alpha\theta} + \frac{f_d(X_{a2}^A - X_{a1}^A)}{\alpha\tau} + f_d[R(X_{a1}^A) - R(b_1^A)] \quad (7)$$

$$\frac{df_d X_{a2}^A}{dt} = \frac{f_d(X_{a1}^A - X_{a2}^A)}{(1-\alpha)\tau} + f_d[R(X_{a2}^A) - R(b_2^A)] \quad (8)$$

Mass balance on volatile fatty acids for methanogens (A) gives

$$\frac{dA_1}{dt} = \frac{A_0 - A_1}{\alpha\theta} + \frac{A_2 - A_1}{\alpha\tau} + R(S_1^A) - R(X_{a1}^A) + f_d R(b_1^A) - R(A_1) \quad (9)$$

$$\frac{dA_2}{dt} = \frac{A_1 - A_2}{(1-\alpha)\tau} + R(S_2^A) - R(X_{a2}^A) + f_d R(b_2^A) - R(A_2) \quad (10)$$

Mass balance on methanogenic biomass (X_v^M) gives

$$\frac{dX_{v1}^M}{dt} = \frac{X_{v0}^M - X_{v1}^M}{\alpha\theta} + \frac{X_{v2}^M - X_{v1}^M}{\alpha\tau} + R(X_{v1}^M) - R(b_1^M) \quad (11)$$

$$\frac{dX_{v2}^M}{dt} = \frac{X_{v1}^M - X_{v2}^M}{(1-\alpha)\tau} + R(X_{v2}^M) - R(b_2^M) \quad (12)$$

Mass balance on methane (M) gives

$$\frac{dM_1}{dt} = \frac{M_0 - M_1}{\alpha\theta} + \frac{M_2 - M_1}{\alpha\tau} + R(A_1) - R(X_{v1}^M) \quad (13)$$

$$\frac{dM_2}{dt} = \frac{M_1 - M_2}{(1-\alpha)\tau} + R(A_2) - R(X_{v2}^M) \quad (14)$$

where:

$\theta = V/Q_0$: hydraulic retention time

$\tau = V/Q_{exch}$: turnover time

$\tau/\theta = Q_0/Q_{exch}$: relative turnover time, also relative interchange rate

In the set of equations above, equations with odd numbers apply to the flow-through zone whereas those with even numbers apply to the retention zone. The development of the mass balances on products (i.e. volatile acids and methane) is based on the reasoning that COD is conserved in the anaerobic digestion system. Thus, the utilised substrate COD is either converted into cellular material or to products. Therefore, the difference between the substrate utilised and the net cellular COD produced represents the amount of product COD. In expressing eqns (13) and (14) it has been assumed that methane flows with the liquid phase of the digester contents (i.e. no slip velocity between gas and liquid). Therefore, we speak here of a *methane equivalent concentration* in each region. Of course this does not occur in a real reactor but, since methane concentration does not influence biokinetics, process rates are not affected by this assumption.

3 COMPUTER SIMULATIONS

Model equations were solved simultaneously using the commercial software MATLAB® on a Hewlett Packard Work-Station HP 9000/720. A third order Runge–Kutta–Fehlberg numerical method was applied. The numerical values of the kinetic parameters and the initial condition of the state variables are shown in Table 1.

Computer simulations were conducted in order to evaluate the effect of incomplete mixing upon anaerobic digestion performance through changes on the characteristic mixing parameters α and τ. The COD removal efficiency was calculated as $(COD_{methane}/COD_0) \times 100$. The values of mixing parameters applied were selected on the basis of information found in the literature. Tracer studies conducted in full-scale anaerobic sludge digesters have revealed well-mixed portions of digester volumes ranging widely from 23% to 88%, while the balances have been stagnant volumes.[8,13] There is less evidence regarding average interchange rates of contents in anaerobic digesters. Smith *et al.*[14] applied the same tracer technique used by Monteith and Stephenson[8] to evaluate the liquid mixing characteristics of a pilot-scale contact process anaerobic digester. In one study they found mixed and 'dead' volumes of 49% and 51%, respectively; whereas the ratio of the feed flowrate to the flowrate of material flowing through

the 'dead' zone was equal to 3. Using a simulation model with features similar to those of the two-regions mixing model described in this study Smith *et al.*[14] analysed the same tracer-response curve for the same anaerobic digester. With this method the calculated values of the characteristic mixing parameters were respectively, 87%, 13% and 10. Differences in reactor design and operational conditions prevent any comparison between anaerobic systems. Also tracer studies are often inexact. Therefore, this information should be considered only for indicative purposes.

3.1 Limiting cases

By definition, for a relative volume in the flow-through region close to unity (i.e. $\alpha \approx 1$) and, for any value of α, with an interchange rate of material between regions approaching infinity (i.e. $\tau \approx 0$) the dynamic model produces results closely approaching those of a completely mixed reactor. Otherwise, for any α, with $\tau \approx \infty$ (i.e. no interchange of material between regions) the system consists of a reactor with a completely dead zone of volume $(1 - \alpha)V$. For values of mixing parameters other than those mentioned above, the mathematical model simulates the performance of an imperfectly mixed digester.

4 RESULTS AND DISCUSSION

4.1 Effect of mixing on anaerobic digestion performance under unsteady conditions

Figure 3 shows an example of simulation of the anaerobic digestion process applied to waste-activated sludge with no pretreatment. A standard version of the model, based on a completely mixed reactor ($\alpha = 1$), was used. The dynamic behaviour of an anaerobic sludge digester under the same conditions except that mixing is assumed to be incomplete, as portrayed by the two-regions model, is shown in Fig. 4.

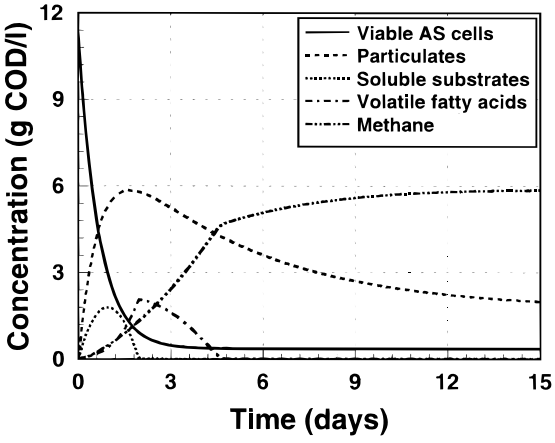

Fig. 3. Dynamic simulation of the anaerobic digestion of waste-activated sludge in a continuous-flow, perfectly mixed reactor ($\theta = 15$ days).

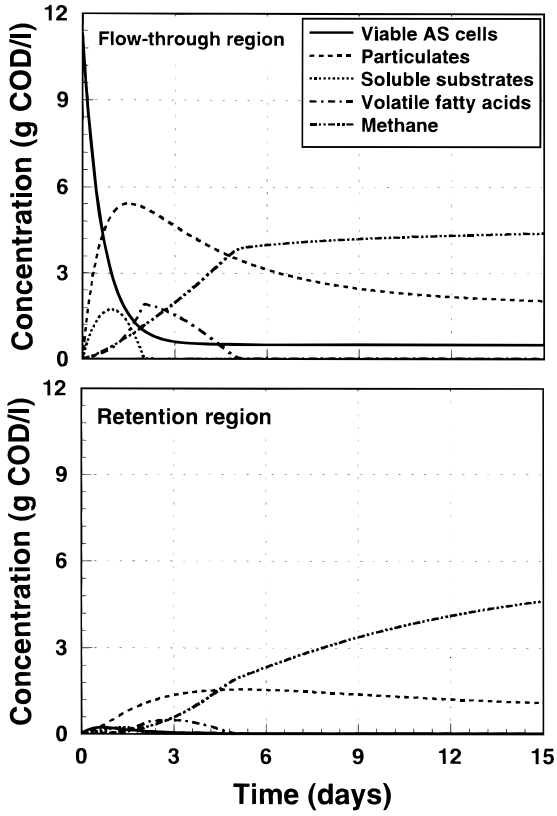

Fig. 4. Dynamic simulation of the anaerobic digestion of waste-activated sludge in a continuous-flow, incompletely mixed reactor ($\theta = 15$ days; $\alpha = 0{\cdot}7$; $\tau = 30$ days).

Significant differences between the concentration patterns shown by both systems arose due to the different degrees of mixing considered. Seeding of microorganisms and initial substrate in the incompletely mixed reactor was assumed to occur in the flow-through zone. A good degree of mixing would quickly distribute those materials to the retention zone. The limited interchange between zones resulted in non-homogeneous distribution of components in the reactor and less volume available for active digestion. The methane concentration of the three-phase (solid–liquid–gas) stream flowing out of the incompletely mixed reactor, which has the same composition as the flow-through zone content, was 24·9% lower than that of the well-mixed digester. Conversely, the concentrations of raw sludge and volatile acids in the effluent of the incompletely mixed reactor were respectively 37·9% and 37·3% higher than those of the completely mixed digester. This supports the view that deviations from the ideal mixing regime result in declined performance of anaerobic reactors. The methane concentration in the retention region after one hydraulic retention time was slightly higher than that in the flow-through zone, although lower than the methane concentration in the perfectly mixed reactor (Fig. 3). The constant feeding of the retention zone with substrate readily available for

methanogens allowed a high production of methane in that zone. Conversely, the flow-through region was continuously fed with untreated raw sludge. However, it should be noted that the retention region does not contribute directly to the materials flowing out of the system and it has a volume much smaller than that in the flow-through zone.

An average value for the turnover time in well mixed anaerobic sludge digesters is about 30 min.[15] Therefore, this value was used to simulate a two-regions reactor with minimal limitation on the exchange of material between zones. The modelling results are shown in Fig. 5.

Because of the short turnover time applied, the material seeded and fed to the flow-through region flowed immediately to the retention zone. As a result, the medium concentrations in both regions were exactly the same between them throughout the duration of the simulation. Conversely to what happened in the digester with limited interchange (Fig. 4), significant hydrolytic and acidogenic as well as methanogenic activities were observed in the retention region of the reactor with a much shorter turnover time (Fig. 5). The methane production in the latter reactor was slightly higher than in the former (10·9%). The resulting homogeneous medium concentration throughout the volume of the reactor due to the high interchange rate used shows the ability of the two-regions model to simulate anaerobic reactors with ideal and non-ideal mixing.

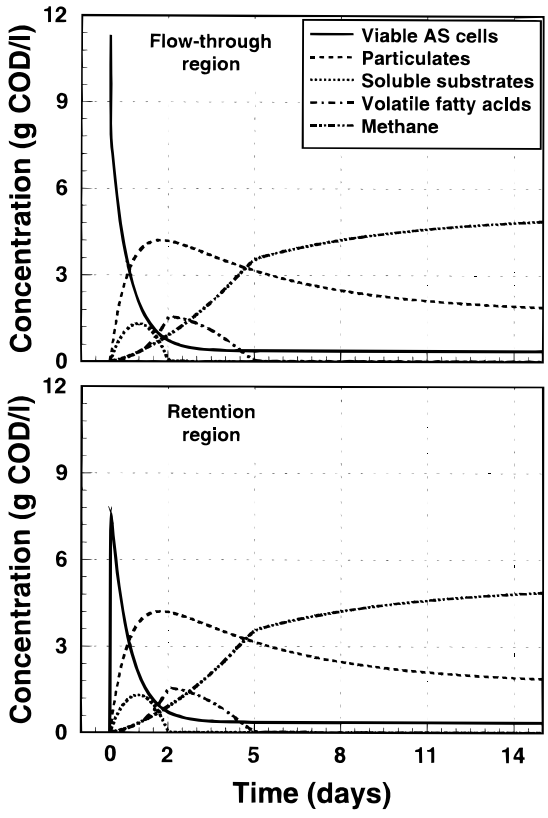

Fig. 5. Dynamic simulation of the anaerobic digestion of waste-activated sludge in a two-regions reactor with non-limited interchange between zones ($\theta = 15$ days; $\alpha = 0{\cdot}7$; $\tau = 0{\cdot}02$ days).

4.2 Effect of the hydraulic retention time

The effect of the hydraulic retention time (θ) on the sludge treatment efficiency attained by anaerobic digesters under different mixing conditions was evaluated. The steady-state results are shown in Fig. 6. The COD removal efficiency increased with retention time and the degree of mixing. For $\theta = 15$ days the treatment efficiency showed by the poorest mixed digester ($\alpha = 0{\cdot}7$ and $\tau/\theta = 5$) was 8·2% lower than that attained by the perfectly mixed reactor ($\alpha = 1$). As shown in Fig. 6, extending the retention time could improve the treatment efficiency of the incompletely mixed digester up to a value similar to that of the perfectly mixed reactor. However, about six extra days would be necessary, which means about 40% additional digester volume would be required for the treatment of the same quantity of waste. Thus, even a small loss in efficiency can result in the need for significantly larger and more costly equipment.

4.3 Effect of the characteristic mixing parameters α and τ on COD removal efficiency

The performance of anaerobic digestion, measured in terms of the COD removal efficiency attained, as a function of time and the relative volume of the flow-through zone (α) is shown in Fig. 7. As expected, sludge treatment efficiency increased with time and changes in α strongly influenced the results. It is obvious from physical considerations that increasing volumes of the flow-through region result in more volume available for immediate anaerobic digestion activity and, therefore, in higher organic matter removal.

Figure 8 shows the evolution of the COD removal achieved by an anaerobic reactor as a function of time

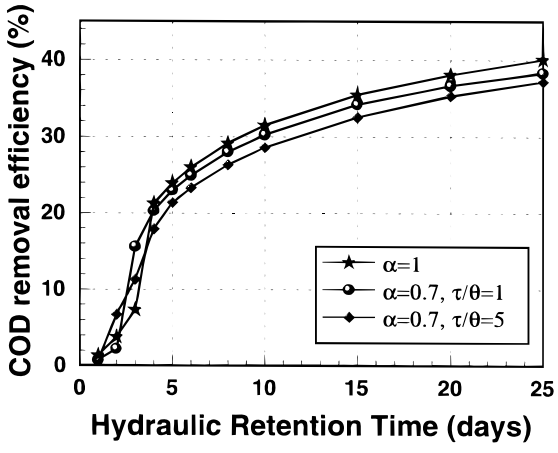

Fig. 6. Effect of the hydraulic retention time on the COD removal efficiency of anaerobic sludge digestion.

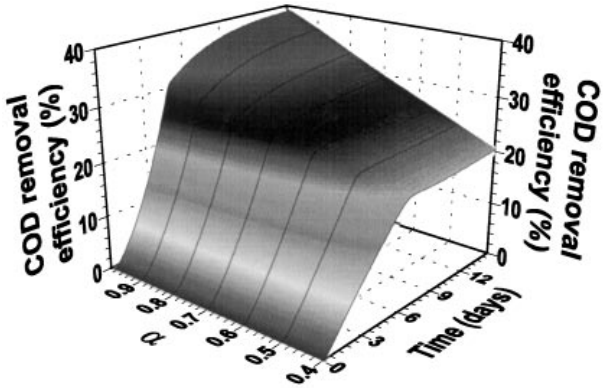

Fig. 7. Effect of the relative volume of the flow-through region (α) on the COD removal efficiency of anaerobic sludge digestion under unsteady state conditions ($\theta = 15$ days, $\tau/\theta = 2$).

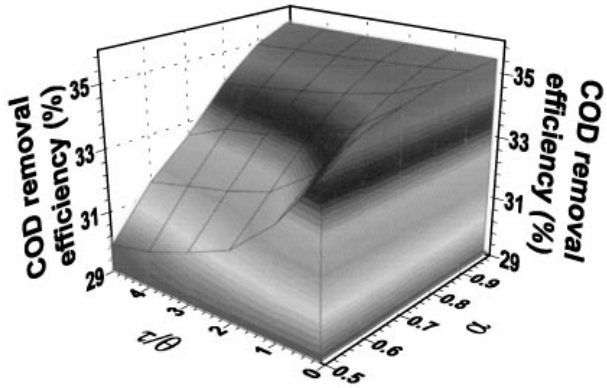

Fig. 9. Effect of the relative volume of the flow-through zone (α) and the relative turnover time (τ/θ) on the COD removal efficiency of anaerobic digestion at steady state conditions ($\theta = 15$ days).

and τ/θ (this can be thought of as a relative turnover time—the ratio of the flow-through zone feed rate to the internal exchange). In the range of values evaluated, the relative turnover time showed a far lower impact on the anaerobic digestion performance than the flow-through zone volume did (Fig. 7). For values of τ/θ between 0·1 and 2, decreasing interchange rates resulted in only slightly lower COD removals. However, for values of τ/θ above that range lower interchange rates resulted, somewhat unexpectedly, in a recovery of the treatment efficiency. This may be because for long turnover times (i.e. low exchange rates) biological reactions majorly occur before the initial contents of the flow-through zone are displaced to the retention zone. Longer retention times allow higher COD removal efficiencies.

The combined effect of the relative volume of the flow-through region (α) and the relative turnover time (τ/θ) on the COD removal efficiency at steady-state is shown in Fig. 9. A considerable decline of the COD removal resulted from non-ideal mixing conditions. For high values of α and low values of τ/θ (i.e. high interchange rate) the anaerobic system shows the features of a well-mixed reactor. As mentioned for the limiting cases, this results in independence between the effects of the mixing parameters and COD removals under conditions close to a perfectly mixed anaerobic reactor. Conversely, low values of α and high values of τ/θ characterize poorly mixed reactors with the consequent decline of the anaerobic digestion performance.

The steady-state COD removal shown by the anaerobic reactor at the worse scenario evaluated (Fig. 9) was 16% lower than that attained by a completely mixed digester. This value appears to be not significant when compared with the typically high conversion efficiencies showed by high-rate anaerobic systems (i.e. wastewater treating reactors). However, for conventional anaerobic sludge digesters this figure may be significant; particularly when the associated sanitary quality of the product sludge is jeopardised. It should be considered that the 'dead' zone accounted for by the model still contributes to the anaerobic digestion process. The effect of a truly dead zone on the anaerobic reactor performance would be expected to be more severe.

The results obtained with the mathematical model developed indicate that mixing affects the residence time distribution as well as the distribution of components in the reactor and thus influences the rates of the anaerobic digestion process. This influence on rates is a result of the number of non-linear rate expressions and the substrate-dependent Monod relationship by which anaerobic digestion is represented.

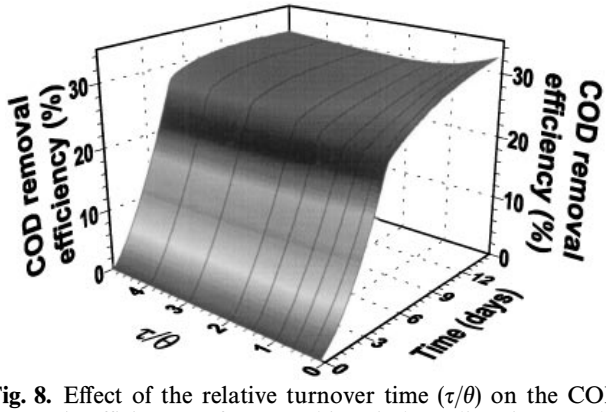

Fig. 8. Effect of the relative turnover time (τ/θ) on the COD removal efficiency of anaerobic sludge digestion under unsteady state conditions ($\theta = 15$ days, $\alpha = 0{\cdot}7$).

4.4 Potential applicability of the two-regions model of liquid mixing

The kinetic model described retains the validity originally evaluated by Pavlostathis and Gosset.[9] However, the proposed dynamic model requires experimental verification in order to assess its applicability. The measurement of the characteristic mixing parameters is particularly needed. Nevertheless, the results obtained are qualitatively in agreement with what could be expected theoretically and with published experimental observations.[3–5]

Some changes to the model could improve its simulation features. It is expected that extending the kinetic expressions to represent the effects of factors such as pH, temperature inhibition, stripping of biogas, etc. on anaerobic digestion could improve the accuracy of the description of the relationship between mixing and process performance. Similarly, an improved mixing model could enhance the predictive features of the mathematical model. However, caution should be exercised because the difficulties in measuring a greater number of mixing parameters as required by a sophisticated model could greatly restrict the applicability of such a simulation model, particularly in pilot- and full-scale reactors where mixing effects are more significant.

Recently, simple models have been shown to be of sufficient accuracy to simulate liquid mixing patterns in pilot-scale anaerobic reactors.[16,17] The characteristic mixing parameters of these models, which are similar to those required by the two-region mixing model, were calculated from experimental tracer-response curves by means of the mathematical models themselves, fitting experimental data to model using the least-squares method. Furthermore, Reinhold *et al.*[16] found a good agreement between the values of the mixing parameters calculated with that method and those measured experimentally by means of probes. This suggests the possibility of calculating α and Q_{exch} using the same approach. It also supports the view of the potential applicability of simple liquid mixing models, such as the two-region model described, for the simulation of anaerobic reactors under non-ideal mixing conditions. Imperfect mixing models may also be a useful tool during reactor scaling-up.

5 CONCLUSIONS

A dynamic model has been used to investigate some of the effects of mixing on anaerobic digestion performance. Simulation of anaerobic reactors under mixing conditions closely approaching ideality as well as poorly mixed digesters was possible. This shows the potential applicability of the proposed liquid mixing model for a wide range of mixing levels. Incomplete mixing of anaerobic reactor contents results in lower methane generation and waste treatment efficiency. COD removal efficiency was shown to increase with greater retention time in the system and with increased degree of mixing. Completely mixed reactors require a shorter retention time than incompletely mixed digesters to achieve the same sludge treatment efficiency. This clearly has an impact upon the treatment costs. Even a small drop in efficiency can lead to the need for significantly larger and more expensive equipment.

The evaluation of the impact of the characteristic mixing parameters α and τ on anaerobic digestion showed that the relative volume of the flow-through region (α) has a more significant effect than the turnover time (τ, i.e. interchange rate) does. However, waste treatment efficiency is a complex function of both parameters.

With macro-mixing as depicted by the two-regions model, the degree of liquid mixing affects the residence time distribution and the distribution of components in the reactor. Consequently, the kinetic rates of the anaerobic digestion process are influenced.

There is a need for further research and improvement of the model to establish the link between real reactors configuration and the mixing model as for its experimental verification. Work is ongoing towards this goal.

The results obtained emphasise the importance of considering mixing when simulating anaerobic digestion and, consequently, during reactor design. The two-region mixing model could be used for the simulation of anaerobic reactors whose mixing patterns resemble such a mixing scheme.

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