Sensitivity of Runoff to Climate Change: A Hortonian Approach

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Abstract

This Horton Memorial Lecture reviews the approach of Robert Horton (1875–1945) to key problems in hydrology, meteorology, and geography and then speculates on how Horton might today approach the problem of analyzing the sensitivity of catchment runoff to climate change. It is suggested that the techniques used by Horton can give us an insight into the nature of the latter problem through a partial analysis of the phenomena involved.

1. The nature of Horton's contribution

Robert Horton was born in 1875 and died in 1945. From 1889 to 1911 he was in public employment and from 1911 until his death was in private practice as a consulting engineer. He was the author of 9 monographs and over 150 scientific and technical papers. Though the Citation Index only came into being 30 years after his death, that bibliometric record lists 566 citations of Horton's work in the years between 1974 and 1990. His work on surface runoff was the basis for the design of airport drainage used by the Corps of Engineers during the Second World War and for decades afterward. His work on infiltration and other land surface fluxes was the basis of the scientific design of soil conservation works in the 1950s and 1960s. His work on drainage networks was seminal for the development of hydrology at the catchment scale. A breakdown of the entries in the Citation Index year by year reveals no falling off in the relevance of his classical papers to modern scientific research and modern engineering practice.

Robert Horton was an interesting personality. The late Walter Langbein, in his acceptance speech as the first recipient of the Horton Medal of the American Geophysical Union, said of Horton: "He was a private man, . . . he was a creative man, . . . he was a broad-gauged man."

Robert Horton was a graduate of a small liberal college. Though he did no formal postgraduate studies and never held an academic appointment, his contribution to research outshone all of his contemporaries in hydrological research. Though practicing as a consulting engineer, he was one of the first to base hydrological opinion on laboratory measurements. Figure 1, taken from an early paper on "Rainfall Interception" in *Monthly Weather Review,* shows the layout of the experimental facilities in the grounds of his house at Voorheesville in New York State (Horton 1919). His papers reveal that he made good use of the range of facilities shown in that figure.

Robert Horton in his writings stressed the need to establish hydrology on a firm scientific basis. In a wide-ranging article on the field and scope of hydrology, Horton (1931) stated the essence of hydrological concern as follows:

> The field of hydrology, treated as a pure science, is to trace out and account for the phenomena of the hydrological cycle.

In that same paper, he stressed the fundamental importance of basing hydrological conclusions on the long-term water balance and the importance of verifying this balance by measuring all the elements of it. In his own words:

> There is a simple basic fact involved in the hydrological cycle
>
> Rainfall = Evaporation + Runoff
>
> Quantitative proof of this law could only come after quantitative determinations of rainfall, evaporation and runoff had been made.

Most textbook representations of the hydrological cycle consist of idealized pictures of slopes, trees, and clouds. It is interesting that Horton represented the hydrological cycle in the form of a circular pattern as shown on Fig. 2 (Horton 1931). His emphasis on the hydrological cycle as a balanced closed cycle has been too often lost sight of in the welter of complex analysis and multiparameter simulation so prevalent in modern hydrology.

Before considering how Robert Horton might have tackled the question of the response of catchment runoff to climate change, it is necessary to review the manner in which Horton approached the hydrological problems of his day and the techniques that he used in relation to them. An examination of his more important papers, which are still cited half a century after they were written, reveals some pervading characteristics. As noted above, he took the lumped form of the continuity equation as the fundamental point of departure in the analysis of any hydrologic problem. Since the continuity equation is linear in form, it can be expressed at any scale of interest without the need to parameterize at the macro scale any of the elements of the equation as written at the micro scale. The same is not true of the nonlinear equations of motion formulated at the micro scale, and it is the manner in which this problem is overcome that characterizes the different approaches to the analysis of hydrologic phenomena at the higher scales represented by a plot or a catchment basin or a climatic region.

Horton's approach was to postulate on the basis of observations a simple relationship between the relevant variables and to construct a model of the macroscale dynamics that he hoped would be both sufficiently realistic and reasonably simple. The skill with which he used this approach is the reason why his papers are still cited so widely. It is therefore of interest to consider a small number of these widely cited papers dealing with problems relative to the modeling of land surface fluxes, which is one of the key areas in climate modeling and one that requires improvement if reliable predictions are to be made of the impact of climate change.

2. Horton's analysis of hydrologic processes

While the upward flux of evaporation from open water and from the ground has been studied in the context of large-scale water balances for more than 300 years (Halley 1651; Dalton 1802), the scientific study of the flux of moisture downward into the soil in the form of infiltration dates from the pioneering work of Horton less than 60 years ago (Horton 1933). In this classical paper, Horton distinguished between infiltration as the physical process by which liquid water entered the soil at the surface and percolation or water penetration in the lower soil levels below the surface. He introduced the concept of infiltration capacity as "the maximum rate at which rain can be absorbed by a given soil when in a given condition" (Horton 1933, p. 446). He applied this concept to the analysis of the hydrograph of runoff following rainfall and of the interrelation between infiltration on the one hand and such factors as soil moisture content, vegetation, and groundwater on the other. These same elements must be simulated in a realistic land surface component of a catchment model used to predict runoff or a general circulation model used to study climate change.

The manner in which Horton analyzed the surface runoff due to the excess of precipitation over infiltration capacity is a good example of his approach to hydro-

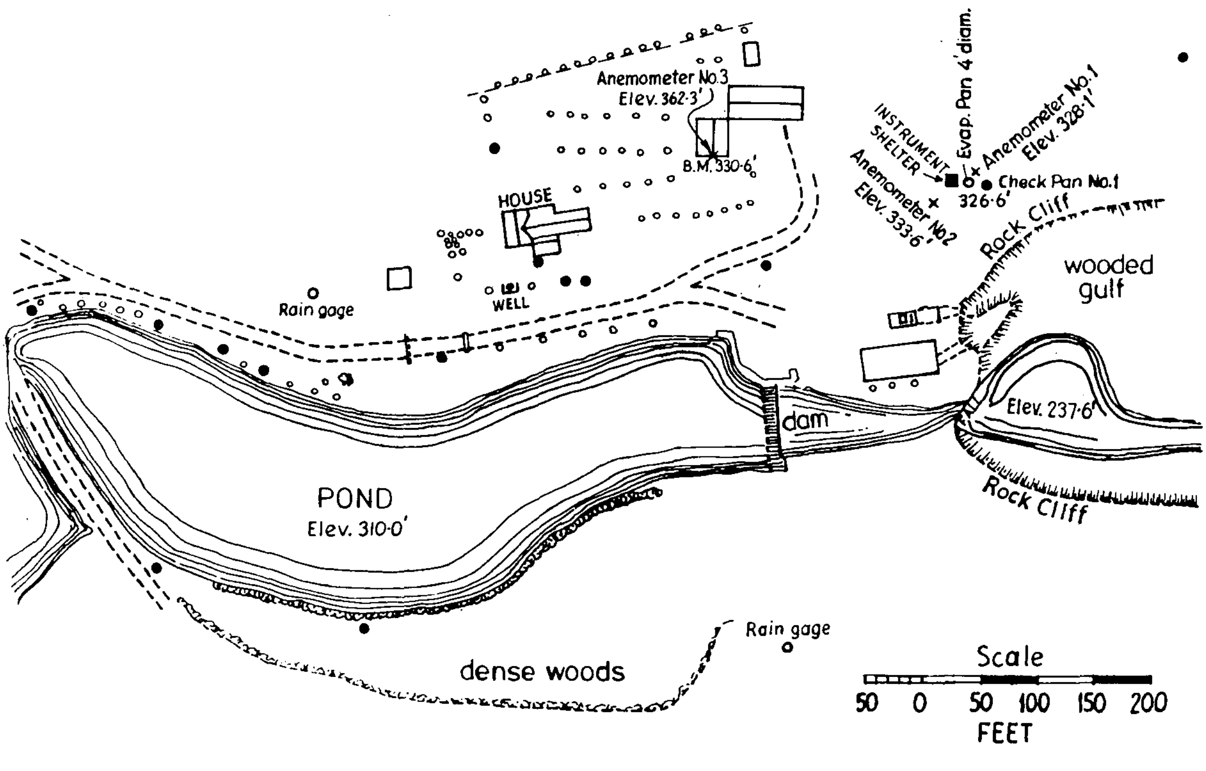


FIG. 1. Hydrological laboratory of Robert E. Horton.

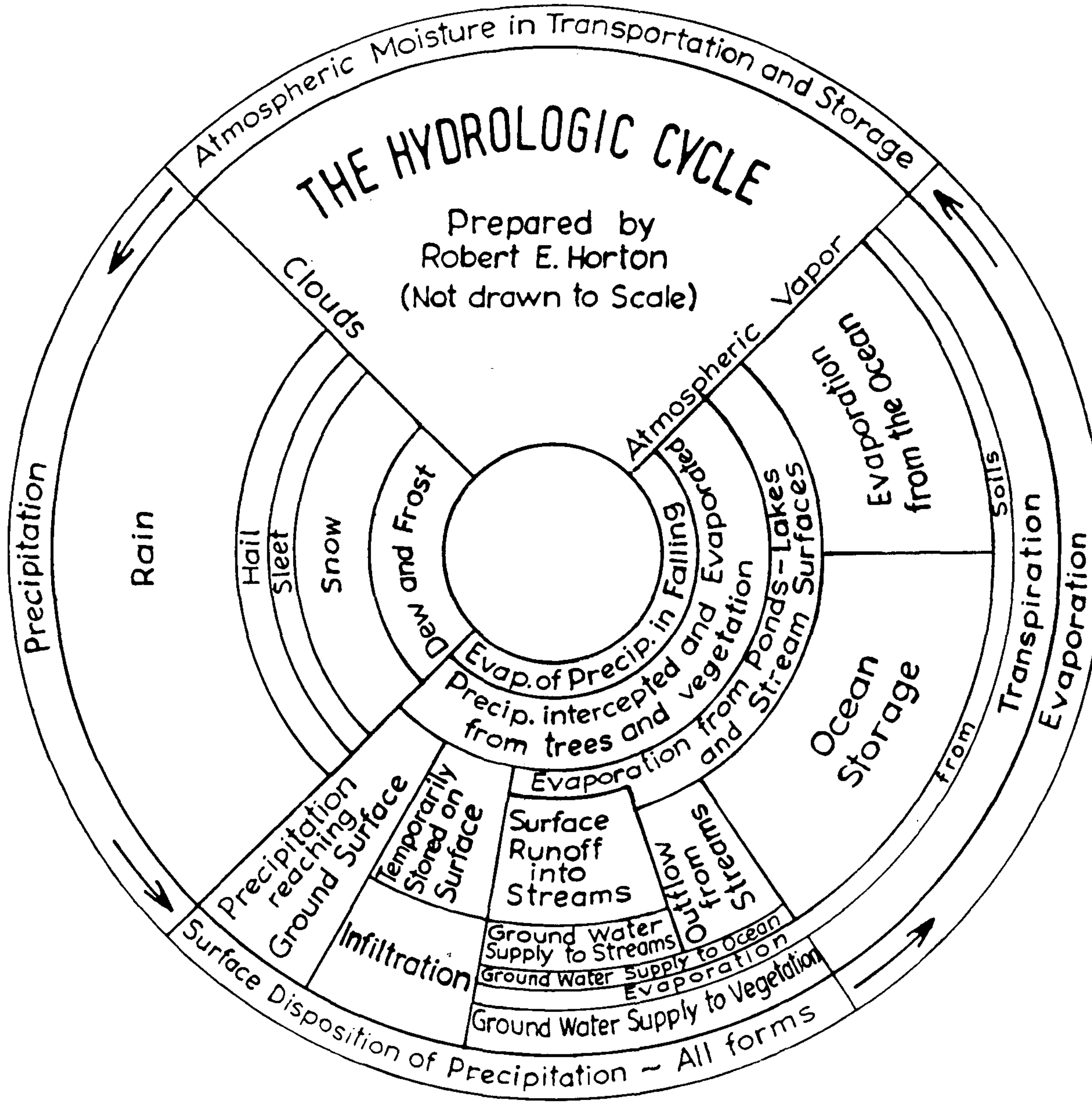


FIG. 2. Horton's hydrological cycle.

logic problem solving (Horton 1935, 1938). The basic problem tackled by Horton was that of predicting the rising hydrograph of surface flow from a sloping plane area, from an initially dry condition to a steady state of runoff under the influence of a uniform rate of rainfall excess. His starting point was to assume the form of the final steady-state relationship between the surface runoff per unit width (q_e) and the volume per unit width retained on the plane surface (V_e) as:

$$q_e = a(V_e)^M, \qquad (1)$$

where a and M are parameters depending on the nature of the surface and the flow conditions. This simple relationship for a steady state of overland flow can be shown to correspond to the reduction of the complete nonlinear equation for steady variable flow with lateral inflow to its kinematic form and was found to be a good approximation to the results of a large number of plot experiments carried out by the Soil Conservation Service during the 1930s.

Horton's next step was to make the exploratory assumption that the above power relationship between outflow and storage could be applied not only at the final equilibrium state of steady flow but also at the earlier stages of unsteady flow when the outflow at the downstream end would be less than the inflow. Since the outflow at equilibrium is equal to the steady rate of inflow, the latter assumptions allows the equation for unsteady flow to be written as

$$\frac{dV}{dt} = q_e - aV^M, \tag{2a}$$

where $V(t)$ is the storage per unit width on the surface of the plane. Rearrangement of Eq. (2a) enables us to arrive at the solution of this equation for the rising hydrograph in quadrature form as

$$\frac{q_e t}{V_e} = \int \frac{d(V/V_e)}{1 - (V/V_e)^M}. \tag{2b}$$

Through the use of partial fractions, the expression on the right-hand side of Eq. (2b) can be integrated for any rational value of M. However, the resulting analytical expression becomes complex if one goes beyond the first few integral values of M. In any case, the problem remains of inverting the resulting implicit function, which expresses time in terms of discharge into the preferable form of an expression for the discharge $q(t)$ as a function of time.

If we now turn to the question of the relationship between the value of the power M in Eqs. (1)–(3) and the physical conditions of overland flow, we find both from theory and experiment that the value of M should lie between 1.0 and 3.0. For rough turbulent flow with Manning friction, the theoretical value is $^5/_3$, and for laminar flow the theoretical value is 3.0. Horton (1938, 1945a) reproduced plot measurements for equilibrium overland flow indicating values of M between 1.0 and 2.0. Horton interpreted the intermediate values of M as representing the common condition whereby the flow over a natural surface such as a grassed slope is partially laminar and partially turbulent, and remarked that the integral of Eq. (2b) can be readily integrated for the case of $M = 2$ (Horton 1937, p. 415). For this case, Eq. (2b) can be both integrated and inverted to give an explicit expression for the surface retention $V(t)$:

$$\frac{V}{V_e} = \tanh\left[\frac{2q_e t}{V_e}\right], \tag{3a}$$

which immediately leads to an explicit expression for the runoff:

$$\frac{q}{q_e} = \tanh^2\left[\frac{2q_e t}{V_e}\right] \tag{3b}$$

as the equation of the rising hydrograph for 75% turbulent overland flow.

For the case of the recession from equilibrium, q_e can be made equal to zero in the basic equation for unsteady overland flow of Eq. (2a) and a general solution obtained for all values of M. This solution can readily be shown to be

$$\frac{q}{q_e} = \left[\frac{1}{1 + (M-1)\frac{q_e t}{V_e}}\right]^{1-1/M}. \tag{4}$$

The special case of $M = 1$, which is the lower limit for physically realistic solutions, can be obtained either a) by treating $M = 1$ as a special case, or b) by using the standard mathematical expression for the limiting form of Eq. (3) obtained as the value of M approaches the value 1.0; in either case the recession is given by

$$\frac{q}{q_e} = \exp\left(-\frac{q_e t}{V_e}\right). \tag{5}$$

Thus, Eq. (4) can be considered as the general expression of the recession hydrograph overland flow for all values of M.

3. Horton's laws of quantitative geomorphology

Horton's ability to formulate new hydrologic laws at the macro scale and contribute to problems on the interface between hydrology and other sciences is amply exemplified by his paper on the quantitative morphology of a drainage basin, published by the Geological Society of America just prior to his 70th birthday and one month before his death (Horton 1945b). This paper opens typically with a reference to the classical qualitative statement made 150 years previously by Playfair on the adjustment between rivers and their valleys (Horton 1945b, p. 280). He goes on to remark (Horton 1945b, p. 281):

> In spite of the general renaissance of the present century, physiography as related in particular to the development of land forms by erosional and gradational processes still remains largely qualitative. . . . One purpose of this paper is to describe two sets of tools which made an attack on the problems of the development of land forms, particularly drainage basins and their stream nets, along quantitative lines.

On the basis of the measurement of the drainage nets of 10 basins in New York State and one in Michigan, Horton postulated the following laws of drainage composition (Horton 1945b, p. 291):

1. Law of stream numbers: the numbers of streams of different orders in a given drainage basin tend closely to approximate an inverse geometric series in which the first term is unity and the ratio is the bifurcation ratio.

2. Law of stream lengths: the average length of streams in each of the different orders in a drainage basin tend closely to approximate a direct geometric series in which the first term is the average length of streams of the 1st order.

He adds the comment (Horton 1945b, p. 291):

> Playfair called attention to the nice and single "adjustment" between the different streams and valleys of a drainage basin but chiefly with reference to their declivities. These two laws supplement Playfair's law and make it more definite and more quantitative. They also show that the nice adjustment goes far beyond the matter of declivities.

The fundamental significance of this last paper of Horton's is reflected in the fact that the Citation Index, which only commenced 20 years after the paper was written, lists 274 citations of it.

It is reasonable to assume that if Horton were, like the present generation of hydrologists, confronted with the task of analyzing the impact of climate change on catchment runoff, he would tackle it in the same manner as he tackled the key hydrologic problems of his own day. Accordingly, it would appear an appropriate memorial to him to examine that difficult problem using Horton's prescriptions of a firm reliance on the equation of continuity, a search for insight using simple but plausible models founded on field observations, and the use of mathematical analysis of special cases to indicate the nature of the solution and to form a basis for comparison with more detailed observations. An attempt is made to do this in the remainder of this lecture.

4. Long-term equilibrium of land surface fluxes

The main difference between the hydrological problems tackled by Robert Horton and those confronting hydrologists today is one of scale. In the last Horton Memorial Lecture delivered on the occasion of the first Symposium on Global Change, Pete Eagleson pointed out (Eagleson 1991):

> The historical evolution of hydrological science has been in the direction of ever increasing space and time scale, from small catchment to large river basin to earth system, and from storm events to seasonal cycle to climatic trend.

In tackling hydrological problems at a regional and global scale, it is necessary to take into account the interaction of the hydrologic processes with vegetation and with climate variation and change. In the same way that Horton tackled the problem of unsteady overland flow on the basis of a simple model derived from the equilibrium condition, the question of changes in hydrology and vegetation due to climate change can be subjected to an initial partial analysis by starting from conditions of equilibrium between these factors.

The concept of geographic zonality has been put forward in qualitative terms for over a century. Thus, Marsh (1864) states:

> In countries untrodden by man, the proportions and relative positions of land and water, the atmospheric precipitation and evaporation, the thermometric mean, and the distribution of vegetable and animal life are subject to change only from geological influences so slow in their operation that the geographical conditions may be regarded as constant and immutable.

In operating on this large scale, the variability between the individual ecosystems is subsumed in the concept of the biome as the largest useful unit of biological activity that may contain many ecosystems and many succession stages.

The key hydrologic factor linking large-scale atmospheric models with large-scale hydrologic models is the value of the actual evaporation. It is the actual evaporation and not the potential evaporation that combines with runoff to balance the precipitation in the long-term water-balance equation. On the other hand, it is the potential evaporation that is predicted by the atmospheric component of the climate model and later reduced to actual evaporation through some form of soil moisture accounting. Under the principle of geographical zonality, one would expect that the long-term actual evaporation at biome scale would depend only on the long-term average of the climatic factors, the vegetation, and the average soil conditions. The simplest hypothesis that can be made is that, at less than geological time scales, the soil and vegetation conditions are dependent only on the long-term atmospheric factors of precipitation and potential evaporation. If the only three variables in the long-term equilibrium are the precipitation (P), the potential evapotranspiration (PE), and the actual evapotranspiration (AE), then a trivial application of dimensional analysis indicates that we can express the ratio of actual to potential precipitation as some function of the ratio of precipitation to potential evapotranspiration; i.e.,

$$\frac{\mathrm{AE}}{\mathrm{PE}} = \phi\left(\frac{P}{\mathrm{PE}}\right), \qquad (6)$$

where P/PE can be considered as a humidity ratio and is the reciprocal of the aridity index introduced by Budyko (1950). Since Budyko suggested that the value of the aridity index could be used to specify the type of biome, the simplified assumption of Eq. (6) can be referred to as the Budyko hypothesis.

The next step in the Hortonian approach would be to set out simple relationships based on such observations as are available. In this connection Horton warned against the dangers of crude empiricism when he wrote (Horton 1931, p. 201):

> It may happen that several different types of formulas will give excellent fit to the data within the limits of observation, but it is important to use the formula which not only fits the data but which gives rational results for the limiting or boundary conditions. Hydrologic literature contains many empirical generalizations which if extended could lead to absurdities.

For the type of relationship indicated by Eq. (6) above, the limiting cases can be readily formulated for the case of uniform values of precipitation and potential evaporation throughout the year and from year to year. Under the latter assumption, the moisture content of the soil would have a constant value less than saturation for all values of the humidity index less than 1 and would be at the saturation value for all values of the humidity index greater than 1. For values of the humidity index less than 1, the evaporation process would be moisture limited rather than energy limited, and accordingly, the constant value of long-term actual evapotranspiration would be equal to the constant long-term value of precipitation minus the subsurface runoff corresponding to the constant subsurface moisture content. For the case of the humidity index greater than 1, the actual evaporation would be energy limited rather than moisture limited, and hence the actual evaporation would always be equal to the potential evapotranspiration.

A number of formulations of Eq. (6) based on observations appear in the climatological literature. Thus, Schreiber (1904), on the basis of annual values for precipitation, potential evapotranspiration, and runoff in a number of catchments in central Europe, suggested the relationship

$$\frac{\mathrm{AE}}{\mathrm{PE}} = \frac{P}{\mathrm{PE}}\left[1 - \exp\left(-\frac{\mathrm{PE}}{P}\right)\right]. \tag{7}$$

A few years later, Ol'dekop (1911) suggested for Russian rivers the formula

$$\frac{\mathrm{AE}}{\mathrm{PE}} = \tanh\left(\frac{P}{\mathrm{PE}}\right). \tag{8}$$

It can be readily verified that both of these empirical expressions satisfy Horton's criterion of being asymptotic to the limiting conditions for very small and very large values of the humidity index P/PE, if subsurface runoff is taken as negligible. Budyko and Zubenok (1961), on the basis of measurements from over a thousand catchments in the Soviet Union, found that 90% of the catchments would fall within the area limited by these two expressions, and suggested that the geometric mean of these two expressions could be used as a representation of the general trend. The expression put forward on the basis of measurements in African catchments by Turc (1954) as later modified by Pike (1964) gives results very close to this geometric mean and takes the simpler form

$$\frac{\mathrm{AE}}{\mathrm{PE}} = \frac{P/\mathrm{PE}}{[1 + (P/\mathrm{PE})^2]^{1/2}}, \tag{9}$$

which also satisfies the limiting conditions. The empirical formulations of Schreiber (1904), Ol'dekop (1911), and Turc (1954)–Pike (1964) are shown on Fig. 3.

5. Sensitivity to climate change

In accordance with the Hortonian approach, the above formulations for long-term actual evaporation can be used to analyze the sensitivity of runoff to climate change without the use of complex computer-based models with a large number of parameters of uncertain provenance. This task is simplified by taking advantage of the circumstance that, for any relationship of the type given by Eq. (6), the general expression for the sensitivity of the long-term runoff (R) to changes in the long-term precipitation (P), and the long-term potential evapotranspiration (PE) is given by

$$\frac{\Delta R}{R} = \psi\frac{\Delta P}{P} - (\psi - 1)\frac{\Delta \mathrm{PE}}{\mathrm{PE}}, \tag{10}$$

where the single sensitivity factor ψ can be shown to have the form:

$$\psi = \frac{P/\mathrm{PE}\,[1 - \phi'(P/\mathrm{PE})]}{P/\mathrm{PE} - \phi(P/\mathrm{PE})} \tag{11}$$

where $\phi'(\ )$ is the derivative of the function $\phi(\ )$ relating the evaporation ratio to the humidity index in accordance with Eq. (6). We have already seen that, for very large values of the humidity index, AE approaches PE so that for such high values of the index ϕ' approaches unity, and we have

$$\psi = \frac{P}{P - \mathrm{AE}}, \tag{12}$$

which asymptotically approaches the value unity as P/PE approaches infinity. In the other limiting case of vanishingly small values of the humidity index, AE approaches P so that both the numerator and the denominator of Eq. (11) vanish, thus requiring the continued use of l'Hopitals' rule to obtain the value of the sensitivity factor for very low values of the humidity index.

The intermediate values of the sensitivity factor ψ for the empirical formulations shown on Fig. 3 are given in Table 1 for the values of the humidity index used by Budyko as the dividing points between the major biome types. As can be seen from the table, the maximum magnification ratio occurs in each case as the value of the humidity index approaches zero. However, it is notable that in the case of the Ol'dekop formula and the Turc–Pike formula the maximum value is 3.0, whereas for the Schreiber formula the limiting value of the sensitivity factor approaches infinity. This is a clear warning that empirical formulations of the same general shape as shown on Fig. 3 can indicate widely different sensitivities of actual evaporation, and hence of the catchment runoff, to changes in climatic factors.

The existence of high values of the sensitivity factor at low values of the humidity index in the case of the Schreiber formula can be seen directly if the functional value from Eq. (7) is substituted into Eq. (11). When this is done, the exponential term cancels in both numerator and denominator to give the simple expression of the sensitivity factor as

$$\psi = \frac{P/PE + 1}{P/PE}, \tag{13}$$

which is immediately seen to give a value of unity for an infinitely large value of the humidity index, and a value of infinity for an infinitesimally small value of that same humidity index.

A closer analysis of the continued operation of l'Hopitals' rule to Eq. (11) reveals that for any function that, for a vanishing value of the argument, has a limiting value of zero and a limiting first derivative of unity, the sensitivity factor for a vanishing value of the argument (i.e., for total aridity) will be given by

$$\psi(0) = k, \tag{14a}$$

where k is an integer whose value is determined by the order of the lowest derivative, which is nonzero for a zero value of the argument, that is,

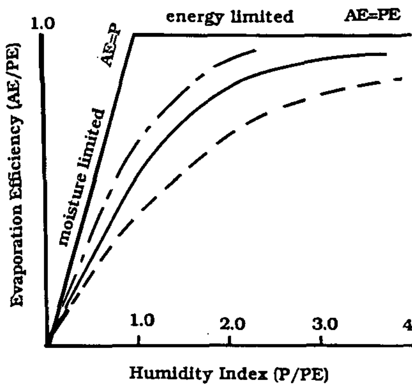
$$\phi^k(0) \neq 0. \tag{14b}$$

It can be shown in the case of the Schreiber formula given by Eq. (7) that no such nonzero value of the derivative occurs for a zero value of the argument; all terms in all the derivatives of the Schreiber function ϕ are indeterminate and remain so even if l'Hopitals' rule is applied an indefinite number of times.

It would obviously be preferable if the limiting forms of the relationship between the evaporation ratio and the humidity index were to be members of the same family of functions rather than two quite distinct functions such as those of Eqs. (7) and (8), which were recommended as limiting functions by Budyko and Zubenok (1961). One such family of functions that meets Horton's criterion of physically reasonable behavior for the limiting values of zero and infinite humidity index is that due to Bagrov (1953), who proposed the differential equation

$$\frac{dAE}{dP} = 1 - \left(\frac{AE}{PE}\right)^n, \tag{15a}$$

which for all values of the parameter n and will be asymptotical to solution AE = P for vanishingly small



——— Turc - Pike $\frac{AE}{PE} = \frac{P/PE}{\sqrt{1+(P/PE)^2}}$

— — — Ol'dekop $\frac{AE}{PE} = \tanh\left(\frac{P}{PE}\right)$

— — — — Schreiber $\frac{AE}{PE} = \frac{P}{PE}\left[1 - \exp\left(-\frac{PE}{P}\right)\right]$

FIG. 3. Empirical expression for evaporation.

TABLE 1. Range of sensitivity factor for empirical formulations.

Humidity index	Biome type	Value of ψ		
		Ol'dekop	Turc–Pike	Schreiber
0		3.0	3.0	∞
	Desert			
1/3		2.9	2.8	4.0
	Semidesert			
1/2		2.8	2.7	3.0
	Steppe			
1		2.4	2.2	2.0
	Forest			
3		1.5	1.4	1.3
	Tundra			
∞		1.0	1.0	1.0

values of P (and hence of AE) and asymptotic to a horizontal line, i.e., independent of P, for very large values of P when AE would be equal to PE. Since Bagrov's equation is written as a relationship between AE and P for a fixed value PE, it can be rearranged and written as

$$d(P/\mathrm{PE}) = \frac{d(\mathrm{AE/PE})}{1 - (\mathrm{AE/PE})^n}. \qquad (15b)$$

The latter equation would have been immediately recognizable by Horton, as it is precisely the same form as the quadrature solution for overland flow from a plane surface studied by him (Horton 1935, 1938) and given previously as Eq. (2b). Accordingly, we can recognize that for the value of $n = 2$ the solution in the case of overland flow is the Horton solution given by Eq. (2), and the solution for actual evapotranspiration is the Ol'dekop formula given by Eq. (8), both of which involve the tanh function.

It can be shown that for the Bagrov family of curves defined by Eq. (15), the limiting value of the sensitivity factor is given by

$$\psi(0) = n + 1, \qquad (16)$$

where n is the shape parameter in Eq. (15). Any attempt to define an empirical relationship between the evaporation ratio and the humidity index on the basis of year-to-year variations in order to evaluate $\phi(P/E)$ and hence $\phi'(E/\mathrm{PE})$ for use in Eq. (11) would be doomed to failure because the accuracy of the observations would render it difficult to determine ϕ and impossible to determine ϕ' with the requisite accuracy. Even if the Bagrov formulation were confirmed as reasonable, the determination of n for existing conditions would still be difficult.

While the empirical approach cannot be used to complete the task of examining the sensitivity of catchment runoff to climate change, the preliminary results obtained do give us some guidelines; in particular, that there is a contrast between the variation of the evaporation ratio and the sensitivity factor between the more arid environments ($P/E < 1$) and the more humid environments ($P/E > 1$). The empirical formulations all agree that catchment runoff is more sensitive to variations in average precipitation (P) and average potential evapotranspiration (PE) in the case of more arid environments ($P/E < 1$) than in more humid environments ($P/\mathrm{PE} > 1$), as is indicated both by common-sense reasoning and by the results from computer simulation of complex catchment models. However, even a simple modeling of seasonal effects indicates the danger of assuming that increased aridity always leads to greater sensitivity to climate change.

6. Use of a simple conceptual model

The second step in the Hortonian approach to a problem such as the sensitivity of catchment runoff to climate change would be the analysis of the behavior of simple conceptual models whose operation is physically reasonable. The key to such an analysis of overall catchment behavior is some method of soil-moisture accounting in which the variation with time of the volume of soil moisture under the given atmospheric inputs is simulated. The basic water balance of the soil can be written as

$$\frac{dW}{dt} = I(t) - \mathrm{AE}\,(t) - Q_b\,(t), \qquad (17)$$

where $W(t)$ is the volume of soil-moisture storage, $I(t)$ is the rate of downward infiltration of water through the surface into the soil, $\mathrm{AE}(t)$ is the upward flux of moisture from the soil into the atmosphere, and $Q_b(t)$ is the subsurface outflow from the soil directly into the drainage system of the catchment. For a complete soil-moisture accounting, it is necessary to apply the above storage equation both during wet periods when the soil is substantially saturated and during dry periods when the soil is substantially below effective saturation.

Remembering Horton's circular representation of the hydrological cycle as shown in Fig. 2, it is not too fanciful to suggest that he would have approved of the representation of the four basic phases of surface flux involved in the form of the circular diagram shown in Fig. 4. The latter diagram illustrates the switching of the control of the surface flux between atmosphere and the soil as a result of the alternation between wet and dry periods. All four cases can be initially analyzed in terms of simple linear equations on the basis of plausible physical assumptions (Zhao and Dooge 1990). The succession of calculations is described below, starting with the ending of a long period of rain (point A on Fig. 4). Since the soil would then be saturated, the rate of evaporation would be energy limited and thus occur at the potential rate. Accordingly, the flux would be upward and atmosphere controlled and the water balance in the soil would be given by

$$\frac{dW}{dt} = -\mathrm{PE}(t) - Q_b\,(t). \qquad (18)$$

Hydrologists have long assumed that, as a first approximation, the streamflow recession during long dry periods can be represented as a negative exponential function. This assumption implies that the rate of outflow is proportional to the amount of storage from

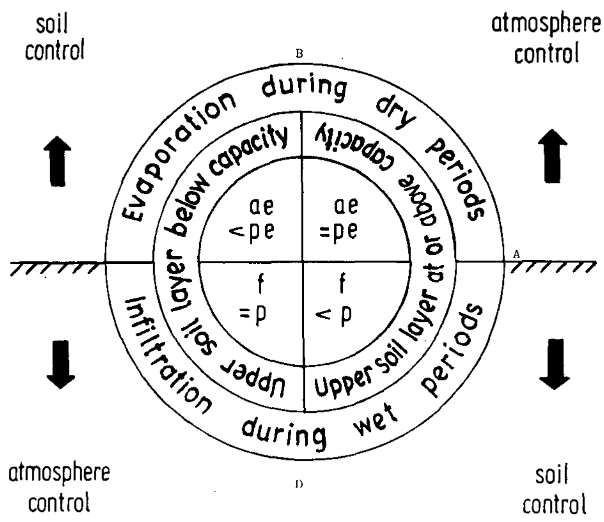


FIG. 4. Control of surface fluxes.

which the outflow is drawn, so that we can for this simple model write

$$\frac{dW}{dt} = -\text{PE}(t) - bW(t); \tag{19}$$

which is linear in form and can be readily solved for any given PE(t).

The combined effect of soil-moisture depletion by evaporation and subsurface outflow will eventually produce a condition at which water can no longer be transferred to the surface at a rate equal to the rate of potential evapotranspiration (point B on Fig. 4). The simplest assumption for the reduced rate of evapotranspiration is that it is proportional to the volume of subsurface moisture, which means that in the quadrant from point B to point C the equation of soil-moisture accounting is written as

$$\frac{dW}{dt} = -a(t)W(t) - bW(t) \tag{20}$$

where the parameter $a(t)$ is the ratio of the potential evapotranspiration to the so-called field capacity W_o; that is,

$$a(t) = \frac{\text{PE}(t)}{W_o}. \tag{21}$$

For the case of constant potential evapotranspiration, Eq. (20) can be readily integrated to give an exponential decline in the soil moisture content $W(t)$ and hence an exponential decline in both the rate of groundwater outflow $Q_b(t)$ and the rate of evapotranspiration AE(t).

If rain occurs at the end of a long dry period, the rate of infiltration into the surface of the ground will be equal to the rate of precipitation until the soil moisture is replenished to the extent that water can no longer percolate downward into the soil at a rate equal to the rate of infiltration at the surface (point C to point D on Fig. 4). Accordingly, during this phase the equation of soil-moisture accounting becomes

$$\frac{dW}{dt} = P(t) - bW(t), \tag{22}$$

which is essentially of the same form as Eq. (19) for the case of upward flux under atmospheric control. From point D onward, the rate of infiltration is subject to soil control and the simulation of the changes must be based on the principle of soil physics. For the classical case of a uniform initial moisture profile and instantaneous ponding of the surface, a first approximation to the rate of infiltration is given by (Philip 1969):

$$I(t) = \frac{A}{t^{1/2}} + B. \tag{23}$$

It can be shown (Dooge 1973) that the initial high rate of infiltration given by this approximate solution is equivalent to assuming that the rate of infiltration is inversely proportional to the volume of soil moisture so that the equation of soil-moisture accounting can be written as

$$\frac{dW}{dt} = \frac{c}{W} - bW, \tag{24}$$

where c is a parameter that depends on the soil properties and the initial moisture content. Though Eq. (24) is not linear in the soil moisture content W, it is linear in W^2, and hence an explicit solution can be obtained by linear mathematics.

The fact that all four phases of surface fluxes can be solved in closed form for constant values of the atmospheric inputs P and PE should facilitate on-line interaction between the atmospheric and the hydrological components of a climate model without undue use of computer time or computer storage. Meanwhile, these simple formulations can be used to probe the problem of the sensitivity of catchment runoff to long-term variations in precipitation and potential evapotranspiration.

7. Effect of seasonality

The analytical results referred to in the previous paragraph can be used to explore the effect on average evapotranspiration of the alternation between wetting and drying conditions. The main cause of such a seasonal alteration in higher latitudes is the annual cycle of evapotranspiration giving an excess of precipitation over potential evaporation in winter and an excess of potential evaporation over precipitation in summer. This effect has been incorporated in a simple water-balance model by Schaake and Liu, and the sensitivity of runoff to precipitation change estimated by them for the United States is shown on Fig. 5 (Schaake and Liu 1989; Schaake 1990).

The main cause of such a seasonal alteration in many low-latitude countries is the existence of distinct wet and dry seasons. A simplified version of the latter case will be used here for purposes of illustration. For this simplified presentation, the distinction between atmosphere-controlled infiltration and soil-controlled infiltration will be ignored and the amount of infiltration taken equal to the amount of precipitation. The effect of this simplification would be to overestimate the average infiltration, but this effect would be offset to some degree by the resulting overestimation of groundwater outflow. The result for a simple two-season model with a uniform rate of precipitation in the wet season and a uniform rate of potential evapotranspiration during the dry season and plausible parameter values is shown on Fig. 6.

For low values of the humidity index (P/PE), the soil would not become saturated even at the end of the rainy season. The solution of Eq. (20) with a constant value of the parameter a gives the declining rate of evapotranspiration during the dry season of the year (D) and the solution of Eq. (22) with a constant value of P gives the rate of rising evapotranspiration during the rainy period (1–D). For the equilibrium condition that would develop in the absence of interannual variation, the initial moisture content and the final moisture content must be the same for each year. This condition can be used to show that the average actual evapotranspiration for low values of the humidity index is given by

$$\frac{\mathrm{AE}}{\mathrm{PE}} = \left(\frac{a}{a+b}\right) \cdot \frac{P}{\mathrm{PE}}, \qquad (25)$$

where a is the parameter of evapotranspiration efficiency in Eq. (20) and b the parameter for groundwater outflow in Eq. (22). It should be noted that only for the case $b = 0$ (i.e., no subsurface drainage) can the limiting case give us the result AE = P that characterizes the empirical formulations of Eqs. (7), (8), and (9) above. The value of a is defined by Eq. (21) above as the ratio of annual evapotranspiration to field capacity. In humid areas, estimates of the value of the subsurface drainage parameter b can be made on the basis of established curves for base flow recession. In arid areas an estimate of this parameter would be more problematic.

Substitution of the result of Eq. (25) into Eq. (11) for the sensitivity index gives us

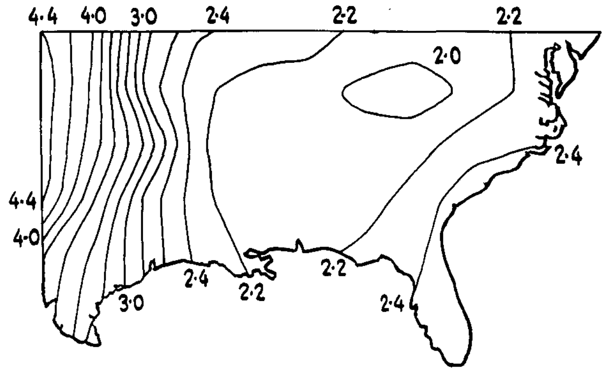


FIG. 5. Sensitivity of runoff to precipitation (Schaake and Liu 1989).

$$\psi(\mathrm{arid}) = 1 \qquad (26)$$

for all positive values of the parameters a and b. The result in Eq. (26) does not correspond to the values of the sensitivity index for vanishingly small values of the humidity index derived for the empirical distributions (and shown on Table 1) whose limiting asymptotes for arid conditions imply a complete absence of subsurface drainage, but is logical for many arid regions. For this simple model the maximum value of the sensitivity index does not occur under highly arid conditions but rather in the neighborhood of the discontinuity corresponding to the value of the humidity index for which the soil becomes saturated for a very short period each year. The range of values for which the sensitivity index is equal to unity can be computed by calculating the value of P/PE for which the initial moisture content is field capacity and the rainfall is just sufficient to restore the moisture to field capacity at the end of the year. This breakpoint value is given by

$$\left(\frac{P}{\mathrm{PE}}\right)_B = \left(\frac{W}{W+D}\right)\left(\frac{a+b}{a}\right) \times \left[\frac{1-\exp\{-(a+b)(W+D)\}}{1-\exp\{-(a+b)(W)\}}\right] \qquad (27)$$

and the effect of the duration of the wet period W is clearly shown on Fig. 6, which refers to the case of $a = 10$ (year^{-1}) and $b = 2$ (year^{-1}). For the purpose of illustration, if we can take a plausible value of field capacity as $W_o = 200$ mm and the annual evapotranspiration as 2000 mm, then the value of evapotranspiration parameter would be given as $a = 10$ year^{-1}. An assumption of $b = 2$ year^{-1} would correspond to a degree of subsurface drainage that would reduce the outflow to 37% of its starting value during a 6-month dry period.

For values of the humidity index greater than the critical value, the annual value of actual evapotranspiration can be computed by starting with the moisture content at field capacity and estimating the actual evapotranspiration in the three periods of a) reduced evaporation during the drying period, b) increasing evapotranspiration during the recharge period, and c) steady evapotranspiration at the potential rate when the moisture content is at or above field capacity. The ratio of actual to potential evaporation for these higher humidity ratios increases quite slowly from the breakpoint value in Eq. (27) to a maximum value of

$$\left(\frac{\mathrm{AE}}{\mathrm{PE}}\right)_{\max} = \left(\frac{W}{W + D}\right) + \left[\frac{1 - \exp\{-(a + b)(D)\}}{(a + b)(W + D)}\right] \quad (28)$$

for an infinitely large value of P/PE. For very low values of $(a + b)$, Eq. (28) gives a ratio close to unity, while for large values of $(a + b)$ Eq. (28) gives a ratio close to the proportion of wet conditions—that is, the ratio of W to $(W + D)$.

The total relationship between evaporation ratio (AE/PE) and the humidity index (P/PE) for the case of $a = 10$ and $b = 2$ is shown on Fig. 6 for three different durations of the dry season. For any given division of the year between periods of constant rainfall and constant evapotranspiration, the relationship is seen to consist of an initial linear relationship up to a critical point, and a very slowly increasing value of the evaporation ratio above this point. Since, for any particular length of dry season (D), the slope of the curve above

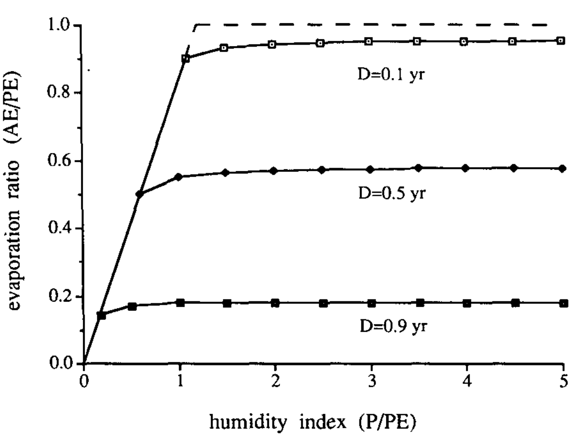


FIG. 6. Effect of length of dry period (PE = 2000 mm yr^{-1}, W_o = 200 mm, and K = 0.5 yr).

the critical break point is a small positive value, we would expect from Eq. (25) that the value of the sensitivity index (ψ) would be closely bounded by

$$\psi_{\max}\,(\text{humid}) \le \left(\frac{a + b}{b}\right). \quad (29)$$

For the case of $a = 10$, $b = 2$ shown on Fig. 6, this limiting value would be 6.0 and computations on a pocket calculator indicate actual maximum values of 4.7, 3.8, and 3.9 for values of $D = 0.1$, 0.5, and 0.9, respectively.

The importance of the parameter b in the above equation raises the serious question of the relevance to the question of the sensitivity of runoff of outputs from computer models whose subsurface component does not contain an element of subsurface outflow whether they rely on a simply Budyko box or on the much more elaborate simulation of evapotranspiration. The bound given by Eq. (29) indicates the critical importance of the degree of subsurface drainage as represented by the parameter b. The effect of the value of this parameter on the maximum value of the sensitivity index can be obtained by comparing the values of the sensitivity index for different values of the parameters a and b.

A number of attempts have been made to estimate the sensitivity of catchment runoff to precipitation and potential evapotranspiration on the basis of hydrologic models of varying degrees of complexity. The above preliminary analysis by the Hortonian method would suggest that one element in such a prediction would be the manner in which the subsurface outflow was modeled. The only studies known to the author that follows the Hortonian approach in any way are those of Schaake on the relationship between climate and water resources (Schaake & Liu 1989; Schaake 1990). As a result of these studies, a simple water-balance model was calibrated on the basis of its ability to predict a mean annual runoff, and the sensitivity of mean annual runoff to a 10% change in either precipitation or potential evapotranspiration was studied with the calibrated model. The results for the sensitivity index of runoff to precipitation due to Schaake and Liu (1989) are shown above on Fig. 5.

8. The legacy of Robert Horton

The legacy of Robert Horton consists of more than the money that he left in his will to the American Meteorological Society and the Americal Geophysical Union. His name is commemorated by this memorial lecture and by the Horton Award and the Horton Medal of the American Geophysical Union. Walter Langbein, in his acceptance speech when he became the first recipient of the Horton Medal of the American Geophysical Union, said of Robert Horton:

> It seems only proper to recall why our generation sees fit to honor his memory. He was a man of the nineteenth century with its highest sense of the public responsibility of the able and the gifted; a man of the nineteenth century with ideas sufficient for the twentieth.

The main thesis of this memorial lecture has been not only to reinforce these words of Langbein but to suggest that the clear thinking of Horton and his methodical approach to hydrologic problems remains relevant to some of the key problems that we will face in hydrology as we move to the end of the twentieth century, grappling with the global problems that will be key issues in the early part of the twenty-first century. There is ample evidence that Horton's papers are still being read for the light they throw on the many problems in hydrology that he tackled during a long professional career. Equally important may be a realization that his methodology can provide an insight into problems that have only arisen in the decades since his death in 1945. The best memorial to Robert Horton would be not just an annual lecture but a continual readiness to follow his example of promoting a fruitful interaction between well-planned and careful observations on the one hand and simple but rigorous analysis on the other. The key problem of the effect of climate change on the elements of the water balance and on water-resources development presents an excellent opportunity for such an endeavor.

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