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# FREE CONVECTION FROM A VERTICAL PLATE IN A POROUS MEDIA SUBJECTED TO A SUDDEN CHANGE IN SURFACE TEMPERATURE

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ABSTRACT

The unsteady heat transfer process involved in free convection flow along a vertical surface embedded in a porous medium is investigated. An analytical solution has been obtained for the temperature/velocity field for small times in which the transport effects are confined within an inner layer adjacent to the plate. Then, a numerical solution of the full boundary-layer equation is obtained for the whole transient from the initial unsteady state to the final steady state. Detailed results of the effect of the temperature inputs on the transient process are given. 

## Introduction

Convective heat transfer process is of fundamental importance in a variety of practical applications, ranging from mechanical engineering to geophysics and recent reviews by Nield and Bejan [1] and Nakayama [2] give the extent of the research information on this area. The unsteady convective heat transfer problems encountered in these applications are rather complex and they can be solved either analytically or numerically. Numerical techniques, such as finite differences or boundary elements, are commonly used for complex problems, while analytical methods leading to exact solutions are preferred for their simplicity in engineering applications. In spite of extensive effort to analyse the transient process during cooling or heating of a surface which is embedded in a porous medium, it appears that the literature is lacking of simple solutions which determine the heat transfer characteristics from such surfaces.

In this paper, we present a method for determining the heat transfer quantities for a vertical surface embedded in a fluid-saturated porous medium which is subjected to an impulsive change of the temperature of the plate. Thus, it is assumed that for time $\bar{\tau} < 0$ a steady temperature/velocity has been attained in the boundary-layer which occurs due to a uniform temperature $T_1$ of the plate. Then at time $\bar{\tau} = 0$ the temperature of the plate is suddenly changed to $T_2$ and maintained at this value for $\bar{\tau} > 0$. The analytical and numerical results show that the transient process is strongly affected by the levels of the existing energy inputs.

## Basic Equations

Consider a vertical flat plate embedded in a porous medium which is at a constant temperature $T_\infty$ and the plate is maintained at a uniform temperature $T_1$. At time $\bar{\tau} = 0$ the temperature of the plate is suddenly changed to $T_2$ and is maintained at this constant value for $\bar{\tau} > 0$. Using the Darcy-Boussinesq and boundary-layer approximations, the conservation equations for the unsteady free convective flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u = \frac{g\beta K}{\nu}(T - T_\infty) \tag{2}$$

$$\sigma\frac{\partial T}{\partial \bar{\tau}} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

which have to be solved subject to the boundary conditions

$$\left.\begin{array}{c} u(x,\infty,\bar{\tau}) = 0, \quad T(x,\infty,\bar{\tau}) = T_\infty \\ u(0,y,\bar{\tau}) = v(0,y,\bar{\tau}) = 0, \quad T(0,y,\bar{\tau}) = T_\infty \\ v(x,0,\bar{\tau}) = 0, \quad T(x,0,\bar{\tau}) = T_2 \end{array}\right\} \tag{4}$$

for $\bar{\tau} > 0$ and $0 \le x, y \le \infty$. Here $u$ and $v$ denote the velocity components along the $x$ and $y$ directions, with $x$ being measured along the plate starting at the leading edge and $y$ measured normal to it, $T$ is the fluid temperature and the other quantities are defined in the Nomenclature.

We shall now proceed to transform Equations (1)-(3). For $\bar{\tau} > 0$ the non-dimensional, reduced streamfunction, $f$, and the temperature, $\theta$, are defined as

$$\psi = U_c\,\delta(x)\,f(\eta,\tau), \quad \theta(\eta,\tau) = \frac{T - T_\infty}{\Delta T_1} \tag{5}$$

where $\Delta T_1 = T_1 - T_\infty$ and

$$\eta = \frac{y}{\delta(x)}, \quad \delta(x) = x\left(\frac{2}{Ra_x}\right)^{\frac{1}{2}}, \quad \tau = \frac{\alpha \bar{\tau}}{\sigma\left(\delta(x)\right)^2}, \qquad U_c = \frac{\alpha}{x}Ra_x, \quad Ra_x = \frac{g\beta K \Delta T_1 x}{\alpha\nu} \quad (6)$$

Here $\delta(x)$ is the boundary-layer thickness at $\tau = 0$ and $\psi$ is the streamfunction which is defined by $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$. Substitution of the expressions (5) into Equations (1)–(3) yields the following equation for $f$:

$$\frac{\partial^3 f}{\partial\eta^3} + \left(-1 + 2\tau\frac{\partial f}{\partial\eta}\right)\frac{\partial^2 f}{\partial\eta\partial\tau} + \left(f - 2\tau\frac{\partial f}{\partial\tau}\right)\frac{\partial^2 f}{\partial\eta^2} = 0 \tag{7}$$

where $\frac{\partial f}{\partial\eta} = \theta$. Equation (7) has now to be solved for $\tau > 0$, subject to the boundary conditions

$$f(0,\tau) = 0, \quad \frac{\partial f}{\partial\eta}(0,\tau) = \frac{\Delta T_2}{\Delta T_1}, \quad \frac{\partial f}{\partial\eta}(\infty,\tau) = 0 \tag{8}$$

where $\Delta T_2 = T_2 - T_\infty$. For the steady boundary-layer at $\tau = 0$ one can write $f(\eta, 0) = f_0(\eta)$, so that, from Equation (7), $f_0(\eta)$ satisfies the ordinary differential equation

$$f_0''' + f_0 f_0'' = 0 \tag{9}$$

along with the boundary conditions

$$f_0(0) = 0, \quad f_0'(0) = 1, \quad f_0'(\infty) = 0 \tag{10}$$

where primes denote differentiation with respect to $\eta$.

## Small Time Solution, $\tau \ll 1$

In this case there exists an inner boundary-layer and in order to study this layer it is convenient to use the new variables

$$f(\eta,\tau) = \tau^{\frac{1}{2}}F(\xi,\tau), \quad \xi = \frac{\eta}{2\tau^{\frac{1}{2}}} \tag{11}$$

as in [3, 4]. Equation (7) then becomes

$$\frac{1}{4}\frac{\partial^3 F}{\partial\xi^3} + \tau\left(-1 + \tau\frac{\partial F}{\partial\xi}\right)\frac{\partial^2 F}{\partial\xi\partial\tau} + \left(\frac{1}{2}\xi - \tau^2\frac{\partial F}{\partial\tau}\right)\frac{\partial^2 F}{\partial\xi^2} = 0 \tag{12}$$

which has to be solved subject to the boundary conditions

$$F(0,\tau) = 0, \quad \frac{\partial F}{\partial\xi}(0,\tau) = 2\frac{\Delta T_2}{\Delta T_1} \tag{13}$$

The solution in this growing inner layer is taken to match the outer steady boundary-layer, which at small $\eta$ can be approximated by the series expansion

$$f_0(\eta) \sim \eta + \frac{1}{2}a\eta^2 - \frac{1}{24}a\eta^4 - \frac{1}{120}a^2\eta^5 + \mathrm{O}(\eta^6) \tag{14}$$

where $a = f_0''(0) = -0.62756$, see [5]. The substitution of expression (11) into Equation (14) yields, for large values of $\xi$,

$$F(\xi,\tau) \sim 2\xi + 2a\xi^2\tau^{\frac{1}{2}} - \frac{2}{3}a\xi^4\tau^{\frac{3}{2}} - \frac{4}{15}a^2\xi^5\tau^2 + \mathrm{O}(\tau^{\frac{5}{2}}) \tag{15}$$

The behaviour of the inner layer as $\xi \to \infty$ is to be matched with the steady outer solution (15). Therefore, the solution of Equation (12) within the inner layer results as follows:

$$F(\xi,\tau) = F_0(\xi) + \tau^{\frac{1}{2}}F_1(\xi) + \tau^{\frac{3}{2}}F_2(\xi) + \tau^2F_3(\xi) + \mathrm{O}(\tau^{\frac{5}{2}}) \tag{16}$$

The functions $F_i(\xi)$, $i = 0, 1, 2, \cdots$, can easily be obtained from Equations (12), (13) and (15). The resulting expression for $\frac{\partial F}{\partial \xi}$ is given by

$$\begin{aligned}\frac{\partial F}{\partial \xi} = 2 - 2\left(1 - \frac{\Delta T_2}{\Delta T_1}\right) erfc(\xi) + 4a\xi\tau^{\frac{1}{2}} - a\Bigg\{\left(1 - \frac{\Delta T_2}{\Delta T_1}\right)\Big[3\xi^2\frac{1}{\sqrt{\pi}}e^{-\xi^2} \\ + \left(\tfrac{3}{2}\xi - \tfrac{5}{3}\xi^3\right) erfc(\xi)\Big] + \tfrac{8}{3}\xi^3\Bigg\}\tau^{\frac{3}{2}} - \tfrac{4}{3}a^2\xi^4\tau^2 + \mathrm{O}(\tau^{\frac{5}{2}})\end{aligned} \tag{17}$$

where *erfc* is the complementary error function. Under the transformation (11) the resulting velocity/temperature function is given at small times $\tau$ by

$$\frac{\partial f}{\partial \eta} = \frac{df_0}{d\eta} - \left(1 - \frac{\Delta T_2}{\Delta T_1}\right)\left[\left(1 + \frac{3}{8}a\eta\tau - \frac{5}{48}a\eta^3\right) erfc\left(\frac{\eta}{2\tau^{\frac{1}{2}}}\right) + \frac{3}{8}\frac{a}{\sqrt{\pi}}\eta^2\tau^{\frac{1}{2}}e^{-\frac{\eta^2}{4\tau}}\right] + \mathrm{O}(\tau^{\frac{5}{2}}) \tag{18}$$

Also, the non-dimensional heat flux from the plate can be calculated from the expression

$$q_w(\tau) = \left.\frac{\partial^2 f}{\partial \eta^2}\right|_{\eta=0} = a + \left(1 - \frac{\Delta T_2}{\Delta T_1}\right)\left(\frac{1}{\sqrt{\pi}}\tau^{-\frac{1}{2}} - \frac{3}{8}a\tau\right) + \mathrm{O}(\tau^{\frac{5}{2}}) \tag{19}$$

## Numerical Solution

The governing partial differential Equation (7), or its equivalent form (12), are parabolic and can be integrated numerically using a step-by-step method similar to that described by Merkin [6], provided that the coefficient of $\frac{\partial^2 f}{\partial \eta \partial \tau}$ or $\frac{\partial^2 F}{\partial \xi \partial \tau}$ remain positive. This marching method gives a complete solution for $\tau \leq \tau^*$, where $\tau^*$ is the maximum value of $\tau$ reached in the numerical scheme, which is less than the exact time $\tau = \frac{\Delta T_1}{2\Delta T_2}$. The matching of the solution at $\tau = \tau^*$ to the asymptotic steady state solution may now be achieved using a variation of the method first described by Dennis [7].

In order to accurately evaluate the non-dimensional heat flux from the plate initially we apply the step-by-step scheme to Equation (12) and begin the numerical solution at the small time $\tau = \tau_0$ using the profile given by expression (17). The evolution of the function $\Phi = \frac{\partial F}{\partial \xi}$ is governed by the integro-differential equation

$$\tau(1-\tau\Phi)\frac{\partial \Phi}{\partial \tau} = \frac{1}{4}\frac{\partial^2 \Phi}{\partial \xi^2} + \frac{\partial \Phi}{\partial \xi}\left[\frac{1}{2}\xi - \tau^2\int_0^{\xi}\frac{\partial \Phi}{\partial \tau}(\xi',\tau)d\xi'\right] \tag{20}$$

which has to be solved subject to the boundary conditions

$$\Phi(0,\tau) = 2\frac{\Delta T_2}{\Delta T_1}, \quad \Phi(\xi_\infty,\tau) = 2\frac{df_0}{d\eta}\left(2\sqrt{\tau}\xi_\infty\right) \tag{21}$$

where the undisturbed state $f_0'$ is enforced at a large value of $\xi$, denoted by $\xi_\infty$. The step-by-step procedure of Merkin [6] is now applied to Equation (20) for $\Phi(\xi,\tau)$ in precisely the form described by Harris *et al.* [4]. Thus the finite-difference equation

$$\begin{aligned} &S^{\xi}_{i+1,j+\frac{1}{2}} - 2S^{\xi}_{i,j+\frac{1}{2}} + S^{\xi}_{i-1,j+\frac{1}{2}} + (h^{\xi})^2\lambda\Delta\tau_j\left(\lambda S^{\xi}_{i,j+\frac{1}{2}} - \frac{4}{\Delta\tau_j}\right)\left(S^{\xi}_{i,j+\frac{1}{2}} - 2\Phi_{i,j}\right) \\ &\quad + (h^{\xi})^2\left(S^{\xi}_{i+1,j+\frac{1}{2}} - S^{\xi}_{i-1,j+\frac{1}{2}}\right)\left[i - 1 - \tfrac{1}{2}\lambda^2\Delta\tau_j\left(\Omega^{\xi}_{i,j+\frac{1}{2}} - 2\Theta^{\xi}_{i,j}\right)\right] = 0 \end{aligned} \tag{22}$$

represents an approximation to Equation (20) at $\xi = (i-1)h^{\xi}$ and $\tau = \tau_j + \frac{1}{2}\Delta\tau_j$, where

$$\left.\begin{aligned} S^{\xi}_{i,j+\frac{1}{2}} = \Phi_{i,j+1} + \Phi_{i,j}, \quad \Omega^{\xi}_{i,j+\frac{1}{2}} = \tfrac{1}{2}\left(S^{\xi}_{1,j+\frac{1}{2}} + S^{\xi}_{i,j+\frac{1}{2}}\right) + \sum_{i'=2}^{i-1} S^{\xi}_{i',j+\frac{1}{2}} \\ \Theta^{\xi}_{i,j} = \tfrac{1}{2}\left(\Phi_{1,j} + \Phi_{i,j}\right) + \sum_{i'=2}^{i-1}\Phi_{i',j}, \quad \lambda = \frac{2\tau_j}{\Delta\tau_j} + 1 \end{aligned}\right\} \tag{23}$$

$h^{\xi} = \frac{\xi_\infty}{N^{\xi}}$ and the boundary conditions require that

$$S^{\xi}_{1,j+\frac{1}{2}} = 4\frac{\Delta T_2}{\Delta T_1}, \quad S^{\xi}_{N+1,j+\frac{1}{2}} = 2\frac{df_0}{d\eta}\left(2\sqrt{\tau_j}\xi_\infty\right) + 2\frac{df_0}{d\eta}\left(2\sqrt{\tau_j + \Delta\tau_j}\ \xi_\infty\right) \tag{24}$$

This system of nonlinear algebraic equations is solved iteratively using the Newton-Raphson method and by employing the method proposed by Doolittle to decompose the associated Jacobian matrix into the product of a lower-triangular and an upper-triangular matrix. This iterative process is repeated until the absolute difference between successive approximations reaches a value less than some tolerance $\epsilon_1$. The initial time increment $\Delta\tau_0$ at time $\tau = \tau_0$ is set to some prescribed small value and a time step doubling procedure is adopted. Each time step is covered using first one and then two time increments. If the absolute difference between the two solutions is less than a preassigned tolerance $\epsilon_2$ then the time step is doubled.

The size of the discretised $\xi$–space under consideration, $0 \le \xi \le \xi_\infty$, increases with time. If we regard $\eta = \eta_\infty$ to correspond to $\eta = \infty$ then at the nearest time $\tilde{\tau}$ below $\tau = \left(\frac{\eta_\infty}{2\xi_\infty}\right)^2$ it is necessary to transfer to a version of the step-by-step procedure which uses a constant mesh width, provided that this time $\tilde{\tau}$ is less than $\tau = \frac{\Delta T_1}{2\Delta T_2}$ or, equivalently, $R = \frac{\Delta T_2}{\Delta T_1} < 2\left(\frac{\xi_\infty}{\eta_\infty}\right)^2$. In such cases we retain the current parameter values for the time increment and tolerances and apply the step-by-step method to the non-dimensional temperature function $\theta = \frac{\partial f}{\partial \eta}$ satisfying the integro-differential equation

$$(1-2\tau\theta)\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} + \frac{\partial \theta}{\partial \eta}\int_0^\eta \left(\theta(\eta',\tau) - 2\tau\frac{\partial \theta}{\partial \tau}(\eta',\tau)\right) d\eta' \tag{25}$$

and subject to the boundary and initial conditions

$$\theta(0,\tau) = \frac{\Delta T_2}{\Delta T_1}, \quad \theta(\eta_\infty,\tau) = 0, \quad \theta(\eta,\tilde{\tau}) = \frac{1}{2}\Phi(\xi,\tilde{\tau})\Big|_{\xi=\frac{\eta}{2\tilde{\tau}^{\frac{1}{2}}}} \tag{26}$$

where the initial profile for $\eta > 2\xi\tilde{\tau}^{\frac{1}{2}}$ remains the undisturbed state. The finite-difference equation

$$\begin{aligned} S^\eta_{i+1,j+\frac{1}{2}} - 2S^\eta_{i,j+\frac{1}{2}} + S^\eta_{i-1,j+\frac{1}{2}} + (h^\eta)^2\left(\lambda S^\eta_{i,j+\frac{1}{2}} - \frac{2}{\Delta\tau_j}\right)\left(S^\eta_{i,j+\frac{1}{2}} - 2\theta_{i,j}\right) \\ +(h^\eta)^2\left(S^\eta_{i+1,j+\frac{1}{2}} - S^\eta_{i-1,j+\frac{1}{2}}\right)\left[\tfrac{1}{4}(1-2\lambda)\Omega^\eta_{i,j+\frac{1}{2}} + \lambda\Theta^\eta_{i,j}\right] = 0 \end{aligned} \tag{27}$$

can then be derived to approximate Equation (25), where $S^\eta_{1,j+\frac{1}{2}} = 2\frac{\Delta T_2}{\Delta T_1}$, $S^\eta_{N+1,j+\frac{1}{2}} = 0$, $h^\eta = \frac{\eta_\infty}{N^\eta}$ and the remaining terms are defined analogously to expressions (23). The resulting nonlinear system of equations is then solved as described for Equation (22).

The matching of the steady state similarity solution

$$\theta(\eta,\infty) = Rf_0'(\eta R^{\frac{1}{2}}) \tag{28}$$

where $R = \frac{\Delta T_2}{\Delta T_1}$, at large times with that which is valid at $\tau = \tau^*$ is now achieved using an adaptation of the method of Dennis [7]. The system of equations

$$\frac{\partial f}{\partial \eta} = \theta, \quad \frac{\partial^2 \theta}{\partial \eta^2} + p\frac{\partial \theta}{\partial \eta} = q\frac{\partial \theta}{\partial \tau} \tag{29}$$

where

$$p(\eta,\tau) = f - 2\tau\frac{\partial f}{\partial \tau}, \quad q(\eta,\tau) = 1 - 2\tau\theta \tag{30}$$

must now be solved subject to the boundary conditions

$$\left.\begin{aligned} &\theta(\eta,\tau^*) = \theta^*(\eta), \quad \theta(\eta,\tau_\infty) = Rf_0'(\eta R^{\frac{1}{2}}), \quad f(\eta,\tau_\infty) = R^{\frac{1}{2}}f_0(\eta R^{\frac{1}{2}}) \\ &f(0,\tau) = 0, \quad \theta(\eta_\infty,\tau) = 0, \quad \theta(0,\tau) = R, \quad \tau^* < \tau < \tau_\infty \end{aligned}\right\} \tag{31}$$

where $\tau_\infty$ is some large but finite time corresponding to $\tau = \infty$. By replacing $\eta$ derivatives by central-differences and $\frac{\partial \theta}{\partial \tau}$ by either a backward or forward difference depending on whether $q(\eta,\tau) > 0$ or $q(\eta,\tau) < 0$, respectively, Equation (29) becomes

$$\left(1 + \frac{1}{2}\tilde{h}p_{i,j}\right)\theta_{i+1,j} + \left(1 - \frac{1}{2}\tilde{h}p_{i,j}\right)\theta_{i-1,j} - \left(2 + \frac{\tilde{h}^2}{\tilde{k}}|q_{i,j}|\right)\theta_{i,j} = \frac{\tilde{h}^2}{\tilde{k}}q_{i,j}\theta^*_{i,j} \tag{32}$$

where $\theta^*_{i,j} = \theta_{i,j+1}$ if $q_{i,j} < 0$ and $\theta^*_{i,j} = -\theta_{i,j-1}$ if $q_{i,j} > 0$. The iterative solution of the system (32) throughout the domain is achieved using precisely the procedure described by Harris *et al.* [4].

## Results and Conclusions

The restriction to finite dimensional $\xi$ and $\eta$ spaces was achieved by taking $\xi_\infty = 10$ and $\eta_\infty \approx 10$, respectively, where the precise value of $\eta_\infty = 2\sqrt{\tilde{\tau}}\xi_\infty$ is dependent on the final time $\tilde{\tau}$ reached in the first step-by-step solution. The effect on the numerical schemes of variations from these values of $\xi_\infty$ and $\eta_\infty$, while keeping $h^\xi$ and $h^\eta$ constant, respectively, was investigated and it was concluded that any larger values of $\xi_\infty$ and $\eta_\infty$ produced results which were indistinguishable from those presented in the figures. Thus the application of the step-by-step method in $\eta$ and $\tau$ variables is only required when $R < 2$. The values of the tolerances $\epsilon_1$ and $\epsilon_2$ as average errors over the unknown grid points were taken to be $\epsilon_1 = 10^{-4}$ and $\epsilon_2 = 10^{-7}$. The initial time $\tau_0$ and first time increment $\Delta\tau_0$ were assigned the values $\tau_0 = 5 \times 10^{-5}$ and $\Delta\tau_0 = 10^{-6}$. More restrictive values of these parameters were considered and discovered to produce numerical results which did not show any significant variation. The main source of deviation in the solutions for the fluid temperature arises by considering changes in the number of grid spaces $N^\xi$ and $N^\eta$. It was observed that as $N^\xi$ and $N^\eta$ increased, and consequently $h^\xi$ and $h^\eta$ decreased, the initial development of the numerical solution for the non-dimensional heat flux from the plate approached that of the small time solution.

Figure 1 shows the variation of the profile of $\theta(\eta, \tau)$ at various times $\tau$ calculated using $h^\xi = 0.0125$, $h^\eta = 0.025$, $N^\xi = 800$ and $N^\eta = 400$ for ratios of surface temperatures, $R = 0.5$ and 2, where refinements in the spatial mesh produced an insignificant improvement in the accuracy of the solution. The evolution of the surface heat flux $q_w(\tau)$ with time $\tau$ is illustrated in Figure 2 for $R = 0.5$ and 2. The numerical, transient solution is shown to develop closely following the small time solution (19) and is graphically almost identical when $\tau < 0.9$ and $\tau < 0.2$ for $R = 0.5$ and 2, respectively.

The matching method solution over the finite region $\tau^* < \tau < \tau_\infty$ is achieved by enforcing the steady state solution to apply at $\tau_\infty = 12$ and the convergence criterion was set by assigning the value $\epsilon_3 = 5 \times 10^{-9}$ for the average absolute error in $\theta$. No significant improvement in accuracy was achieved by either increasing $\tau_\infty$ or reducing $\epsilon_3$. The optimum value of the relaxation parameter was found to be $\omega = 1.5$. Solutions for $\theta(\eta, \tau)$ were found for the grid spacings $\tilde{h} \approx 0.025$, $\tilde{k} \approx 0.05$; $\tilde{h} \approx 0.025$, $\tilde{k} \approx 0.025$ and, for $R = 2$, $\tilde{h} \approx 0.0125$, $\tilde{k} \approx 0.025$. As the mesh size was reduced, and consequently the computational time increased, the numerical solution was observed to change only marginally over the solution domain. Thus $\tilde{h} \approx 0.025$ and $\tilde{k} \approx 0.05$ were used so that $n = 400$ and $m = 221$ and 235 for $R = 0.5$ and 2, respectively.

The evolution of the heat flux at the surface $q_w(\tau)$ from $\tau = \tau^*$ to $\tau = \tau_\infty$ is displayed in Figure 2 and shown to proceed past local minimum and maximum values near $\tau = \tau^*$ to the steady state solution at large times for $R = 0.5$ and 2, respectively. A single oscillation of $q_w(\tau)$ was observed for the case $R = 2$, a feature which is imperceptible in Figure 2 but whose existence in such problems was remarked upon by Harris *et al.* [4].

## Nomenclature

| | |
|---|---|
| $f$ | non-dimensional, reduced streamfunction |
| $F$ | transformed function |
| $h$ | step length for $0 < \tau \leq \frac{\Delta T_1}{2\Delta T_2}$ |
| $\tilde{h}$ | step length in the $\eta$–direction for $\frac{\Delta T_1}{2\Delta T_2} < \tau < \tau_\infty$ |
| $\tilde{k}$ | non-dimensional time increment for $\frac{\Delta T_1}{2\Delta T_2} < \tau < \tau_\infty$ |
| $K$ | permeability of the porous medium |
| $m$ | number of grid spacings in the $\tau$–direction for $\frac{\Delta T_1}{2\Delta T_2} < \tau < \tau_\infty$ |
| $n$ | number of grid spacings in the $\eta$–direction for $\frac{\Delta T_1}{2\Delta T_2} < \tau < \tau_\infty$ |
| $N$ | number of grid spacings for $0 < \tau \leq \frac{\Delta T_1}{2\Delta T_2}$ |
| $p, q$ | variable coefficients in the governing equation for $\frac{\Delta T_1}{2\Delta T_2} < \tau < \tau_\infty$ |
| $q_w$ | non-dimensional heat flux at the plate |
| $R$ | ratio of the final surface temperature to the initial surface temperature |
| $Ra_x$ | local Rayleigh number |
| $S$ | sum of numerical solutions for temperature over consecutive time steps |
| $T$ | temperature of the fluid |
| $T_1$ | initial surface temperature ($\tau < 0$) |
| $T_2$ | final surface temperature ($\tau \geq 0$) |
| $\Delta T$ | characteristic temperature |
| $u$, $v$ | velocity components along $x$– and $y$–axes, respectively |
| $U_c$ | characteristic velocity |
| $x$, $y$ | Cartesian coordinates along the plate and normal to it, respectively |

*Greek Symbols*

| | |
|---|---|
| $\alpha$ | effective thermal diffusivity of the fluid-saturated porous medium |
| $\beta$ | coefficient of thermal expansion |
| $\delta$ | boundary-layer thickness |
| $\epsilon_1, \epsilon_2, \epsilon_3$ | tolerances in the numerical schemes |
| $\xi, \eta$ | similarity variables |
| $\lambda, \Theta, \Omega$ | expressions defined in Equation (23) |
| $\theta$ | non-dimensional temperature function |
| $\nu$ | kinematic viscosity |
| $\Phi$ | non-dimensional temperature function |
| $\sigma$ | ratio of composite material heat capacity to convective fluid heat capacity |
| $\bar{\tau}$ | time |
| $\tau$ | non-dimensional time |
| $\tilde{\tau}$ | exact value of $\tau$ where transfer to the step-by-step method in $\eta$ takes place |
| $\Delta\tau$ | non-dimensional time increment for $0 < \tau \leq \frac{\Delta T_1}{2\Delta T_2}$ |
| $\omega$ | relaxation parameter |

$\psi$ streamfunction

*Subscripts*

| | |
|---|---|
| 0 | value at $\tau = 0$ |
| $i, j$ | evaluated at the $i$th and $j$th nodal points in the $\eta-$ and $\tau-$directions, respectively |
| $w$ | evaluated at the wall |
| $\infty$ | ambient condition |

*Superscripts*

| | |
|---|---|
| $*$ | point where the step-by-step numerical solution breaks down |
| $\eta$ | associated with the step-by-step numerical solution in the $\eta$ and $\tau$ variables |
| $\xi$ | associated with the step-by-step numerical solution in the $\xi$ and $\tau$ variables |

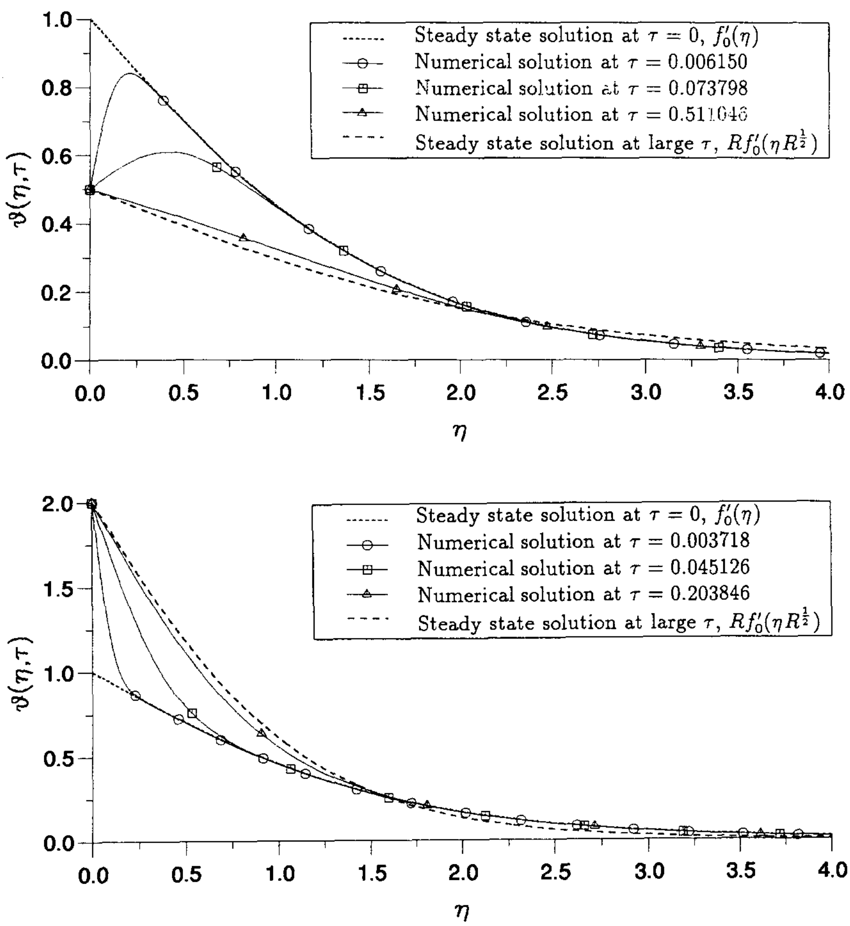


FIG. 1

Variation of the non-dimensional temperature $\theta(\eta, \tau)$ as a function of $\eta$ at various values of $\tau$ and the steady state solutions at $\tau = 0$ and $\tau = \infty$.

(a) $R = \frac{q_2''}{q_1''} = 0.5$, (b) $R = \frac{q_2''}{q_1''} = 2$.

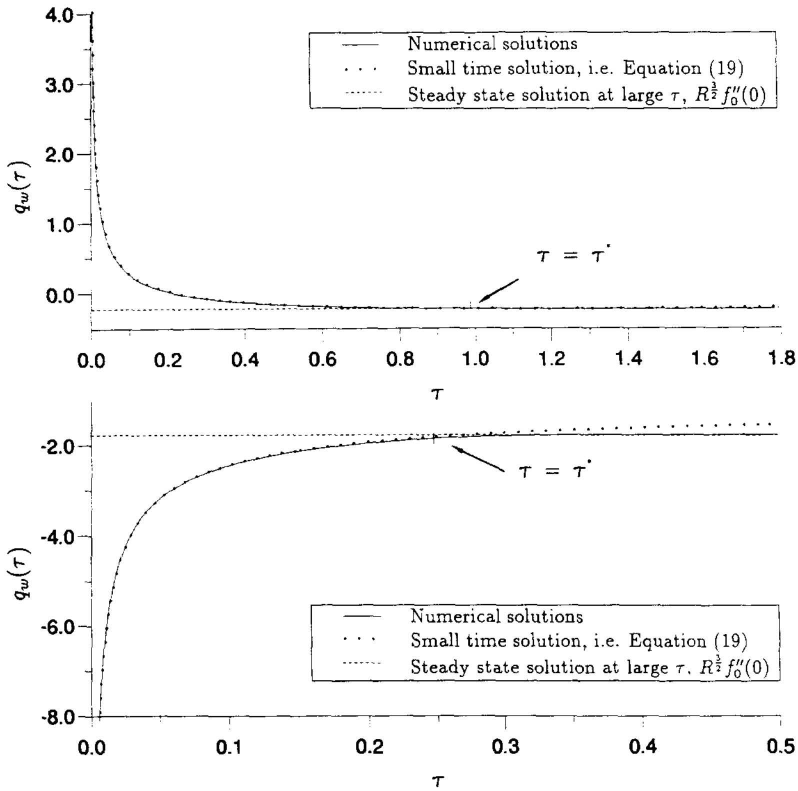


FIG. 2
Variation of the non-dimensional heat flux from the plate $q_w(\tau)$ as a function of $\tau$, the small time solution and the steady state solutions at $\tau = 0$ and $\tau = \infty$, where the transition between solution methods occurs at the indicated times.

(a) $R = \frac{q_2''}{q_1''} = 0.5$, (b) $R = \frac{q_2''}{q_1''} = 2$.

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