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THE DIFFUSIONAL DEPOSITION OF AEROSOLS IN FIBROUS FILTERS

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Abstract—The diffusional deposition of aerosol particles on an isolated circular cylinder in a laminar flow has been investigated. A numerical procedure has been presented for solving the fluid flow equations which is accurate, fast and stable for Reynolds numbers up to 100 and which can easily be extended to other geometries. Expressions for the collection efficiency for any external flow and on any shaped body has been derived when the Peclet number is very large and much larger than the Reynolds number. In this case the collection efficiency depends only on the fluid vorticity on each fibre surface.

INTRODUCTION

Langmuir (1942) was the first to make an approximate calculation of the number of aerosol particles which precipitate from a viscous stream, due to diffusion, on an isolated circular cylinder which is perpendicular to the uniform flow of the fluid. He assumed that the flow was slow and that all the particles within a volume bounded by a certain streamline are able to diffuse to the surface of the cylinder. From the work of Langmuir Natanson (1957) deduced the expression for the collection efficiency, ε, to be given by

$$\varepsilon = 1.71\kappa^{-1/3}\mathrm{Pe}^{-2/3}, \tag{1}$$

where Pe is the Peclet number, $= 2Ua/D$, and U is the undisturbed fluid velocity, a the radius of the cylinder, D the coefficient of diffusion and for slow flow the hydrodynamic factor, κ, is

$$\kappa = 2 - \log_e \mathrm{Re}. \tag{2}$$

The Reynolds number $\mathrm{Re} = 2Ua/\nu$ where ν is the kinematic viscosity.

Since this paper by Langmuir there have been several theoretical and empirical expressions derived for the collection efficiency of a fibre due to diffusion, e.g. Stairmand (1950), Davies (1952), Ranz (1952), Friedlander (1957), Natanson (1957), Stechkina (1957), Torgeson (1958), Pich (1966), Kirsch *et al.* (1968), Yeh *et al.* (1974) and Yeh (1972).

The results which are accepted as being, mathematically, correct, at very large values of the Peclet number, are those of Stechkina and Natanson which are

$$\varepsilon = 2.92\kappa^{-1/3}\mathrm{Pe}^{-2/3} + 0.624\mathrm{Pe}^{-1}, \qquad \mathrm{Re} \ll 1, \tag{3}$$

$$\varepsilon = 2.26\mathrm{Pe}^{-1/2} \qquad , \qquad \mathrm{Re} \gg 1. \tag{4}$$

In practice the Peclet number is sometimes very large and much greater than the Reynolds number.

In this paper the effects of diffusion on the collection efficiency at finite values of the Reynolds number is investigated but with $\mathrm{Re} \ll \mathrm{Pe}$ and $\mathrm{Pe} \gg 1$. In this regime of parameters interception is important and the combined effects of diffusion, interception and inertial deposition is now being investigated. The general theory developed here is applicable to a real filter where the packing density must be included but only the numerical results are given here for the case of an isolated cylinder.

FLUID FLOW EQUATIONS

Consider the steady two-dimensional flow of an incompressible Newtonian fluid normal to the axis of a circular cylinder. The flow is governed by the non-dimensional Navier–Stokes equations

$$\nabla^2\omega = \frac{\mathrm{Re}}{2r}\left(\frac{\partial\psi}{\partial\theta}\frac{\partial\omega}{\partial r} - \frac{\partial\psi}{\partial r}\frac{\partial\omega}{\partial\theta}\right), \tag{5}$$

$$\nabla^2\psi = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\psi = \omega, \tag{6}$$

where (r, θ) are polar coordinates, $\theta = 0$ being in the direction of the flow. The radial distance, r, stream function, ψ, and vorticity, ω, have been non-dimensionalised with respect to a, Ua and U/a respectively. The non-dimensional radial and transverse velocities of the fluid, u and v, are given by

$$u = \frac{1}{r}\frac{\partial\psi}{\partial\theta}, \quad v = -\frac{\partial\psi}{\partial r}. \tag{7}$$

As is usual with this type of problem where a boundary condition has to be applied at large distances the radial distance is transformed by $r = e^{\xi}$ and then equations (5) and (6) become

$$\frac{\partial^2\omega}{\partial\xi^2} + \frac{\partial^2\omega}{\partial\theta^2} - \frac{\mathrm{Re}}{2}\left(\frac{\partial\psi}{\partial\theta}\frac{\partial\omega}{\partial\xi} - \frac{\partial\psi}{\partial\xi}\frac{\partial\omega}{\partial\theta}\right) \doteq 0, \tag{8}$$

$$\frac{\partial^2\psi}{\partial\xi^2} + \frac{\partial^2\psi}{\partial\theta^2} = \omega e^{2\xi}. \tag{9}$$

Assuming the flow to be symmetrical about the axes $\theta = 0$ and $\theta = \pi$, then the boundary conditions to be satisfied are

$$\left.\begin{aligned} &\psi = \frac{\partial\psi}{\partial\xi} = 0 \qquad \text{on } \xi = 0, \\ &\psi = \omega = 0 \qquad \text{on } \theta = 0 \text{ and } \pi, \\ &\frac{\partial\psi}{\partial\xi} \to e^{\xi}\sin\theta, \quad \frac{\partial\psi}{\partial\theta} \to e^{\xi}\cos\theta \quad \text{as} \quad \xi \to \infty. \end{aligned}\right\} \tag{10}$$

A grid system is set up in the region $0 \le \xi \le \xi_m$ and $0 \le \theta \le \pi$. Constant angular and radial mesh sizes $k = \pi/N$ and $h = \xi_m/M$ are used where M and N are integers. A typical mesh point of the grid system is shown in Fig. 1 using the Allen and Southwell (1953) notation. The line $\xi = \xi_m$ is taken as an outer boundary on which approximations to the conditions at infinity may be assumed to hold. The numerical method consists of replacing the partial differential equations (8) and (9) by finite difference approximations on the grid, i.e.

$$\psi_1 + \frac{h^2}{k^2}\psi_2 + \psi_3 + \frac{h^2}{k^2}\psi_4 - 2\left(1 + \frac{h^2}{k^2}\right)\psi_0 = h^2\omega_0 e^{2\xi_0} \tag{11}$$

$$(1 + h\lambda_0)\omega_1 + \frac{h^2}{k^2}(1 + k\mu_0)\omega_2 + (1 - h\lambda_0)\omega_3 + \frac{h^2}{k^2}(1 - k\mu_0)\omega_4 - 2\left(1 + \frac{h^2}{k^2}\right)\omega_0 = 0, \tag{12}$$

where

$$\lambda(\xi, \theta) = -\frac{1}{4}R\frac{\partial\psi}{\partial\theta}, \quad \mu(\xi, \theta) = \frac{1}{4}R\frac{\partial\psi}{\partial\xi}. \tag{13}$$

One of the difficulties in solving equation (8) is that the vorticity, ω, at $\xi = \xi_m$ is unknown although to a first approximation one could take $\omega = 0$ there. In this paper a gradient-type condition for ω is used on the assumption that the flow for $\xi \ge \xi_m$ is governed by Oseen's linearised equations, the solution for which is

$$\omega(\xi, \theta) \sim G(\theta)\chi^{-1/2}\exp\{(\cos\theta - 1)\}, \quad \xi \gg 1, \tag{14a}$$

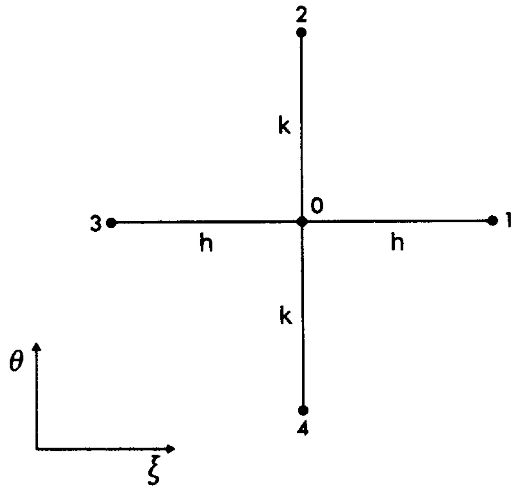

Fig. 1. Notation for the grid points.

where $\chi = \frac{1}{4}\mathrm{Re}^{\xi}$ and $G(\theta)$ is an unknown function of θ. Result (14a) cannot be employed near the cylinder as the Oseen approximation is violated in this region, at finite values of the Reynolds number. Assuming result (14a) to hold for $\xi \geq \xi_m$ we obtain, in a similar way to Dennis, Hudson and Smith (1968), that

$$\omega(\xi, \theta) = \omega(\xi_m, \theta) \exp\{(\chi - \chi_m)(\cos\theta - 1) - \tfrac{1}{2}(\xi - \xi_m)\}, \tag{14b}$$

where χ_m is the value of χ at $\xi = \xi_m$. In particular, if we put $\xi = \xi_m + h$ in expression (14b) we obtain $\omega(\xi_m + h, \theta)$ in terms of $\omega(\xi_m, \theta)$ which can be used, in a similar manner to a gradient-type boundary condition, to eliminate ω_1 from equation (12) whenever the point 0 is situated on $\xi = \xi_m$.

The condition for ω on $\xi = 0$ depends on the solution of (11). Retaining second order accuracy we obtain

$$\omega_0 = \left(\psi_1 - \frac{h^2}{6}\omega_1\right) \bigg/ \left\{\frac{h^2}{3}(1 + h)\right\}. \tag{15}$$

The difficulty with solving iteratively the finite difference equations (12) is that the associated matrix may fail to be diagonally dominant at large values of the Reynolds number, although only in a limited region of the computation. Diagonal dominance is a sufficient condition for the convergence of the Gauss–Seidel or successive over relaxation iterative procedures and the procedures may fail to converge for matrices which are not diagonally dominant. Thus previous investigators have had to use small relaxation parameters which may be as low as 0.05 at Re = 100, see Hamielec and Raal (1969). The matrices associated with the finite difference equations obtained by using forward or backward differencing of the first order derivatives depending on the direction of the flow give rise to matrices which are diagonally dominant. The big disadvantage is that the finite difference equations are first order accurate only.

Allen and Southwell (1953), Dennis (1960) and Dennis, Ingham and Cook (1979) developed a method of representing the two dimensional Navier–Stokes equations, in cartesian form, in finite difference form in which the associated matrices are diagonally dominant and further the truncation error is second order accurate. Here we shall extend the method as described by Dennis, Ingham and Cook to deal with the cylindrical geometry present in this problem.

Equation (8) can be written

$$\frac{\partial^2 \omega}{\partial \xi^2} + \frac{\partial^2 \omega}{\partial \theta^2} + 2\lambda \frac{\partial \omega}{\partial \xi} + 2\mu \frac{\partial \omega}{\partial \theta} = 0, \tag{16}$$

and the difficulty in using the standard central difference numerical scheme is the presence of the first derivative terms. Therefore we split equation (16) up into two equations, one involving the ξ derivatives and the other the θ derivatives only, i.e.

$$\frac{\partial^2 \omega}{\partial \xi^2} + 2\lambda \frac{\partial \omega}{\partial \xi} = A(\xi, \theta), \tag{17}$$

$$\frac{\partial^2 \omega}{\partial \theta^2} + 2\mu \frac{\partial \omega}{\partial \theta} = -A(\xi, \theta), \tag{18}$$

where A is an unknown function of ξ and θ. The offending first derivative terms in equation (17) is transformed locally for $\theta = \theta_0$ in $\xi_0 - h \le \xi \le \xi_0 + h$ by the substitution

$$\left.\begin{aligned} \omega &= F \exp\{-s(\mu, \theta_0)\}, \\ \text{where} \quad s(\xi, \theta_0) &= \int_{\xi_0}^{\xi} \lambda(\xi, \theta_0) \mathrm{d}\xi. \end{aligned}\right\} \tag{19}$$

Equation (18) is transformed locally for $\xi = \xi_0$ in $\theta_0 - k \le \theta \le \theta_0 + k$ by

$$\begin{aligned} \omega &= G \exp\{-t(\xi_0, \theta)\}, \\ \text{where} \quad t(\xi_0, \theta) &= \int_{\theta_0}^{\theta} \mu(\xi_0, \theta) \mathrm{d}\theta. \end{aligned} \tag{20}$$

Substitution of expressions (19) and (20) into the equations (17) and (18) respectively, eliminating A, expanding the exponentials in a Taylor expansion near the point 0 and replacing second order derivatives with central derivatives, gives

$$\begin{aligned} &\omega_1 \left[1 + \lambda_0 h + \underline{\frac{\lambda_0^2 h^2}{2}}\right] + \frac{h^2}{k^2}\omega_2 \left[1 + \mu_0 k + \underline{\frac{\mu_0^2 k^2}{2}}\right] \\ &+ \omega_3 \left[1 - \lambda_0 h + \underline{\frac{\lambda_0^2 h^2}{2}}\right] + \frac{h^2}{k^2}\omega_4 \left[1 - \mu_0 k + \underline{\frac{\mu_0^2 k^2}{2}}\right] \\ &- 2\omega_0 \left[\left(1 + \frac{h^2}{k^2}\right) + \underline{\tfrac{1}{2}(\lambda_0^2 h^2 + \mu_0^2 k^2)}\right] = 0 \end{aligned} \tag{21}$$

see Dennis, Ingham and Cook for further details.

It is seen that the conventional central difference equations (12) can be obtained by setting equal to zero the terms underlined in expression (21). It is very easy to verify that the matrix associated with the finite difference equation (21) is always diagonally dominant since the sum of the first four coefficients in equation (21) is equal in magnitude to the coefficient of ω_0. There is no obvious way of deciding if the finite difference equation in the form (21), with or without the terms underlined, is the more accurate in general as both schemes are second order accurate. The main advantages of the method described here is that it is second order accurate and the associated matrix is diagonally dominant. The finite difference equations (11) and (21) are now solved iteratively by the point by point Gauss Seidel method. The procedure is straightforward and hence no further details will be given here.

Computations were performed for $R = 0.5$, 1, 5, 7, 10, 20, 40, 70 and 100 with various values of h, k and ξ_m. In all cases the iterative procedure was convergent without the need to use a relaxation parameter. Grid sizes of $h = k = \pi/20$, $\pi/30$, $\pi/40$, $\pi/50$ and $\pi/60$ were used and the results presented here are those obtained by using h^2—extrapolation although these results are indistinguishable from those obtained when using $h = k = \pi/60$. The position of ξ_m was varied but in general $\xi_m = \pi$ was found to be satisfactory because of the application of the boundary condition (14) on ω. The variation of the surface vorticity on the cylinder is

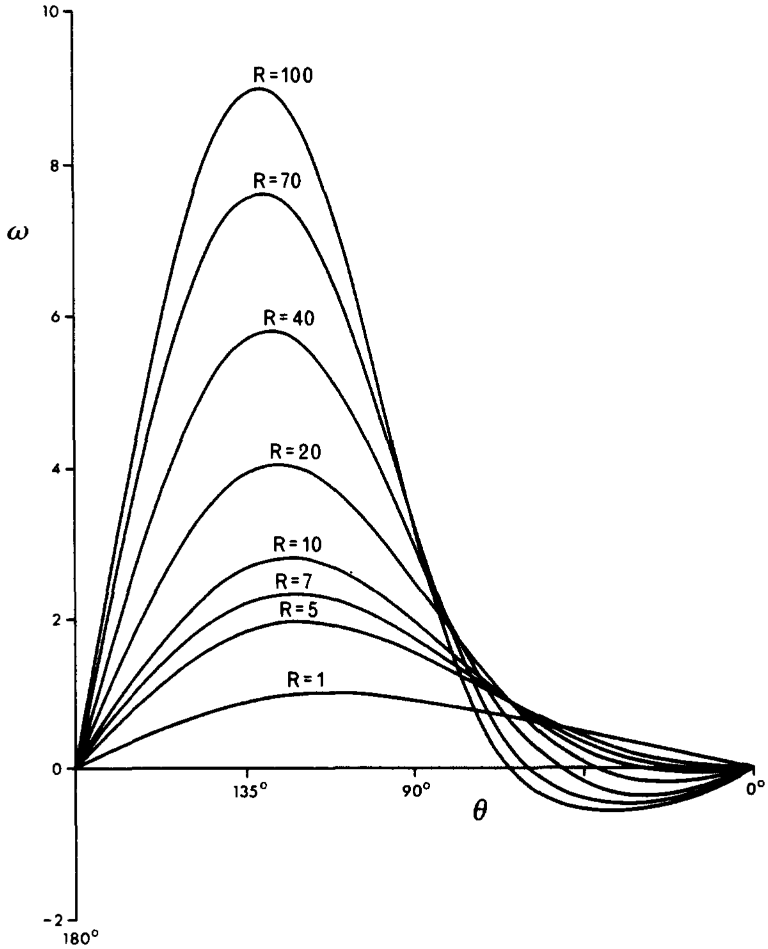

Fig. 2. Vorticity distribution over the surface of the cylinder.

shown in Fig. 2. When the Reynolds number exceeds about 7 it is seen that the flow separates at the rear of the cylinder.

DIFFUSION EQUATION

The non dimensional equation for the particle concentration in (ξ, θ) coordinates can be written

$$u_p\frac{\partial c}{\partial \xi} + v_p\frac{\partial c}{\partial \theta} = \frac{2}{\mathrm{Pe}}e^{-\xi}\left[\frac{\partial^2 c}{\partial \xi^2} + \frac{\partial^2 c}{\partial \theta^2}\right], \tag{22}$$

where c, u_p and v_p are the non dimensional concentration, radial and transverse particle velocities, being non-dimensionalised with respect to the uniform concentration c_0 at large distances and the undisturbed fluid velocity. If the Peclet number is very large, as is usually the case in practice, then the $\frac{\partial^2 c}{\partial \theta^2}$ term in equation (22) can be neglected. Also, for fine particles u_p and v_p can be replaced by the fluid velocities u and v and hence the differential equation (22) reduces to

$$u\frac{\partial c}{\partial \xi} + v\frac{\partial c}{\partial \theta} = \frac{2}{\mathrm{Pe}}e^{-\xi}\frac{\partial^2 c}{\partial \xi^2}. \tag{23}$$

In the case under investigation here, i.e. $\mathrm{Pe} \gg \mathrm{Re}$ and $\mathrm{Pe} \gg 1$, a diffusion boundary layer exists very close to the boundary of the cylinder. In this diffusion boundary layer the vorticity is effectively a constant across it, although it varies with θ. Hence within the diffusion boundary layer we have, approximately,

$$\left.\begin{aligned} \psi &= \tfrac{1}{2}\xi^2 B(\theta), \\ \text{where} \qquad & \\ B(\theta) &= \omega(0,\theta). \end{aligned}\right\} \tag{24}$$

The function $B(\theta)$ has to be determined numerically at intermediate values of the Reynolds number although at small values of the Reynolds number it can be obtained analytically and is given by Lamb (1932) and used by Natanson (1957).

On changing the independent variables to ψ, θ equation (23) becomes

$$\frac{\partial c}{\partial \theta} = -\frac{2(2B(\theta))^{1/2}}{\text{Pe}}\frac{\partial}{\partial \psi}\left(\psi^{1/2}\frac{\partial c}{\partial \psi}\right), \tag{25}$$

and introducing the independent variable

$$\phi = \frac{2\sqrt{2}}{\text{Pe}}\int_{\theta}^{\theta_1}(B(\theta))^{1/2}\,\mathrm{d}\theta, \tag{26}$$

then the equation (25) becomes

$$\frac{\partial c}{\partial \phi} = \frac{\partial}{\partial \psi}\left(\psi^{1/2}\frac{\partial c}{\partial \psi}\right). \tag{27}$$

The solution of equation (27), subject to the boundary conditions that $c = 0$ at $\psi = 0$ (on the surface of the cylinder) and $c = 1$ as $\psi \to \infty$ (far away from the cylinder), is given in Levich (1952) and is

$$c = \frac{4}{\Gamma(\frac{1}{3})}\left(\frac{4}{9}\right)^{1/3}\int_0^z \exp\left(-\frac{4}{9}z^3\right)\mathrm{d}z, \tag{28}$$

where $z = \psi^{1/2}/\phi^{1/3}$.

The collection efficiency is given by

$$\varepsilon = \frac{2}{\text{Pe}}\int_0^{\pi}\left(\frac{\partial c}{\partial \xi}\right)_{\xi=0}\mathrm{d}\theta, \tag{29}$$

which gives, on using the result (28), that

$$\varepsilon = 0.8546\left[\int_0^{\pi}\frac{[B(\theta)]^{1/2}\,d\theta}{\{\int_{\pi}^{\theta}[B(\theta)]^{1/2}\,\mathrm{d}\theta\}^{1/3}}\right]\text{Pe}^{-2/3}. \tag{30}$$

At a given value of the Reynolds number the collection efficiency can be evaluated using the result (30) given the vorticity distribution on the surface of the cylinder which has already been determined and is shown in Fig. 2. The difficulty arises for flows which separate. As seen in Fig. 2 no separation occurs for Re $\lesssim 7$ and hence ε can be determined using result (30). For Re $\gtrsim 7$ there is a recirculating region behind the cylinder and the flow picture is shown, schematically, in Fig. 3. At large values of the Peclet number equation (22) gives that the concentration is a constant along a streamline except near regions of rapid change. Thus in the bulk of the flow the streamlines originate at large distances from the cylinder and therefore the concentration is uniform, c_0, there. Near the boundary of the cylinder a thin diffusion layer, of thickness $\text{Pe}^{-1/3}$ exists. In the recirculating region behind the cylinder the concentration on a streamline will again be constant and in fact probably the same constant on all the closed streamlines. The value of the concentration, on these closed streamlines, however, is unknown. It is most probable that the concentration is zero on the closed streamlines and hence the values of ε would be given by

$$\varepsilon = 0.8546\left[\int_{\theta_s}^{\pi}\frac{[B(\theta)]^{1/2}\,\mathrm{d}\theta}{[\int_{\pi}^{\theta}[B(\theta)]^{1/2}\mathrm{d}\theta]^{1/3}}\right]\text{Pe}^{-2/3} = \varepsilon_1\,\text{Pe}^{-2/3}, \text{ say,} \tag{31}$$

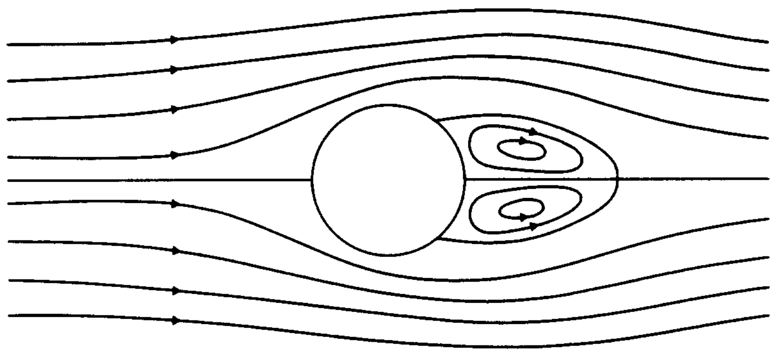

Fig. 3. Schematic flow picture for Re $\gtrsim$ 7.

where θ_s is the angle at which separation occurs. It is possible that the concentration on the recirculating steamlines is that of the mainstream and then ε would be given by

$$\left.\begin{aligned} \varepsilon &= \varepsilon_1 \mathrm{Pe}^{-2/3} + 0.8546 \left[\int_0^{\theta_s} \frac{[-B(\theta)]^{1/2}\,\mathrm{d}\theta}{[\int_0^{\theta}[-B(\theta)]^{1/2}\,\mathrm{d}\theta]^{1/3}} \right] \mathrm{Pe}^{-2/3} \\ \varepsilon &= (\varepsilon_1 + \varepsilon_2)\mathrm{Pe}^{-2/3}, \text{ say.} \end{aligned}\right\} \quad (32)$$

RESULTS AND CONCLUSIONS

Figure 4 shows the variation of the collection efficiency with the Reynolds number when the Peclet number is very large for flow past an isolated circular cylinder. At Re $= \frac{1}{2}$ the numerically obtained value of ε, using result (30), is in excellent agreement with Natanson's analytical expression (3). Up to Re $= 7$ expression (30) has been used to determine ε whereas for Re > 7 both expressions (31) and (32) have been used to evaluate ε_1 and ε_2.

It is seen from Fig. 4 that for the range of Reynolds number under consideration here that

$$\varepsilon_1 \sim \mathrm{Re}^{1/11}. \quad (33a)$$

This increase in ε_1 with Reynolds number is at a much slower rate than that suggested by Johnstone and Roberts (1949) and Ranz (1951), namely

$$\varepsilon \sim \mathrm{Re}^{1/6}. \quad (33b)$$

However, it is observed from expression (30) that $\varepsilon \sim B^{1/3}$ and from Fig. 2 that up to the maximum value of the vorticity $B \sim R^{1/2}$. Hence the contribution to ε from $\theta = \pi$ to $\theta \sim \frac{3}{4}\pi$ varies as $R^{1/6}$ but in the region $\theta_s < \theta < \frac{\pi}{2}$ there is a greater contribution to ε at the smaller Reynolds numbers than at the larger Reynolds numbers. Hence it is not surprising that the value of ε increases more slowly with increasing Reynolds number, in the range $1 \leq \mathrm{Re} \leq 100$, than that suggested in the empirical result (33b).

The results presented in this paper are only valid for Pe $\gg 1$ and Re $\ll$ Pe and hence as the Reynolds number increases we do not expect the inviscid solution (3) as obtained by Natanson to be approached. In fact the results presented in expression (4) will never be achieved from solving a viscous flow problem because the flow always separates at very large values of the Reynolds numbers and hence the potential solution for the flow past a circular cylinder is not valid near the surface of the cylinder.

The results presented in expression (30) are valid for any viscous flow provided Pe $\gg 1$ and Re $\ll$ Pe. Hence for real filters the fluid mechanics problem has to be solved first. Having obtained the vorticity distribution on each fibre the total collection efficiency may be calculated using expression (30).

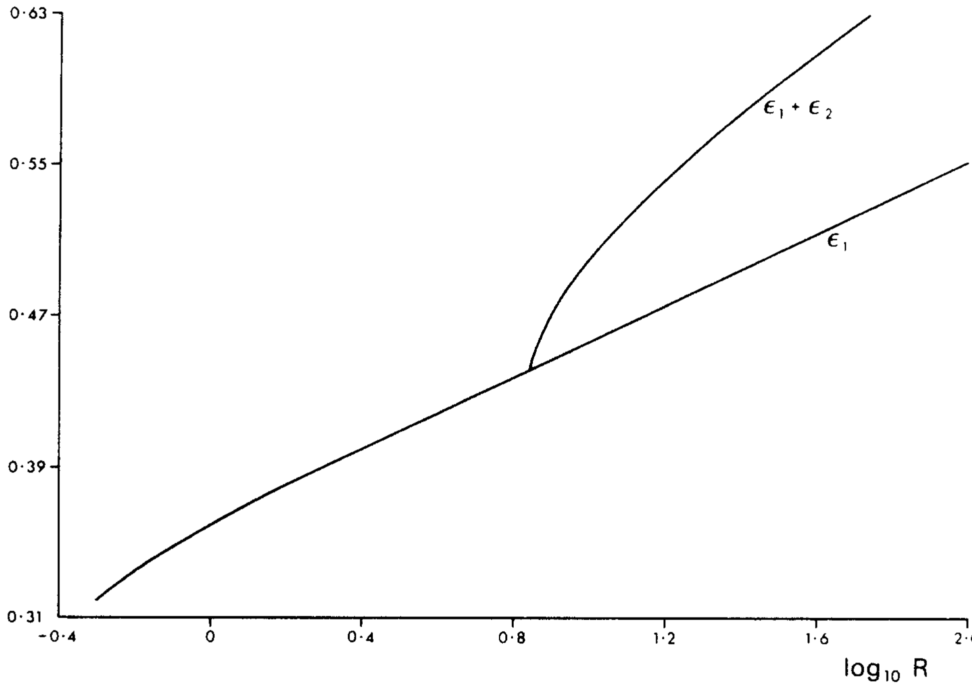

Fig. 4. Variation of the diffusion collection efficiency of an isolated fibre at various values of the Reynolds number.

In real filters fibres are orientated, more or less, parallel to the basal planes of the filters. However, the orientations of the fibre sizes are usually not uniform. A rigorous theoretical study of the real situation seems very difficult due to the complicated nature of the geometry and therefore a simplified model must be used. According to Kirsch and Fuchs (1967), the "fan model" resembles the real filter closer than either the "staggered-array model" or "cell model". A rigorous theoretical study of the fan model is very complicated as it is a three dimensional model. Yeh and Liu (1974) and Yeh (1972) have investigated the staggered-array model based on the solution of the Navier–Stokes equations for Reynolds numbers up to 30. They compared their results with the theoretical studies of Kuwabara (1959) and Happel (1959) which are based on a cell model with an arbitrarily prescribed boundary condition on the outer boundary of the cell and an approximation to the Navier–Stokes equations, the slow flow equation, which is valid only for Reynolds numbers approaching zero. At Re $= 0.1$ the results of Kuwabara, Happel and Yeh and Liu are in very good agreement but at Re $= 30$ the results obtained by the cell and staggered array models do not agree.

Hence for real filters the vorticity distribution on the surface of each fibre has to be determined. This is most easily done by assuming either the staggered-array or cell model. The collection efficiency is then determined from expression (30).

Emi *et al.* (1980) are investigating the diffusion collecting efficiency of fibres for $1 \le \text{Re} \le 100$ and $1 \le \text{Pe} \le 10^6$. They have also considered the effects of the packing of the fibres by using a modified Kuwabara–Happel cell model for the flow. From this flow model the present results could be extended to include the effects of the fibre packing. Further the results to be given by Emi *et al.*, at finite but large values of the Peclet number, will indicate the value towards which the concentration is tending on the closed streamlines as the Peclet number becomes very large.

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