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A method for estimating unsaturated hydraulic properties of vertically heterogeneous soils from transient capillary pressure profiles

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Abstract

The hydraulic properties of soil are represented by the relationship between the volumetric water content θ and the soil capillary pressure ψ and the relationship between the hydraulic conductivity K and ψ. A method to estimate the hydraulic properties for each horizontal layer of a soil profile was developed by combining the instantaneous profile method to compute $K(\psi)$ values and Mualem's model to derive the K–ψ relationship from the soil water retention characteristic. With this method, three parameters that are contained both in the θ–ψ and K–ψ models are optimized by an iterative procedure using transient capillary pressure profiles during water redistribution. Two hypothetical soil columns were used to test the parameter estimation procedure, and it was shown that the proposed method can be applied to soil profiles which have homogeneous or heterogeneous hydraulic properties with depth.

Keywords: Unsaturated soil water flow; Water retention curve; Hydraulic conductivity; Parameter estimation; Soil heterogeneity

1. Introduction

Modeling unsaturated water flow in soils requires knowledge of the hydraulic properties of the soils. The hydraulic properties of unsaturated soil are represented by the relationship between the volumetric water content θ and the soil capillary pressure ψ (the water retention characteristic) and the relationship between the unsaturated hydraulic conductivity K and ψ. The conventional steady state methods to determine these properties are laborious and time consuming. Hence many soil scientists and hydrologists have proposed various kinds of methods to determine the θ–ψ relationship and/or the K–ψ relationship by analyzing transient data associated with unsaturated soil water flow.

Watson (1966) proposed the instantaneous profile method to determine the K–ψ relationship from transient data obtained by drainage experiments in an initially saturated sand column. Vachaud (1967) and Rogers and Klute (1971) applied the instantaneous profile method to laboratory experiments and Rose et al. (1965) used it to determine the K–ψ relationship of soils in situ. The instantaneous profile method can provide K–ψ relationships quicker than the steady state methods. However, it is still laborious because it requires measurements of transient profiles of both water content and capillary pressure (Watson, 1966; Rogers and Klute, 1971), or measurements of tran-

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sient profiles of either water content or capillary pressure and the θ–ψ relationship of each soil layer (Rose et al., 1965; Vachaud, 1967).

Many mathematical expressions that employ a small number of parameters have been proposed to describe the θ–ψ and K–ψ relationships. With these θ–ψ and K–ψ models, the problem of determining the hydraulic properties of soil becomes a problem of estimating values of initially unknown parameters. Libardi et al. (1980) and Jones and Wagenet (1984) proposed some simplified methods to estimate the parameters of the K–ψ model from transient soil water content profiles. In their studies, the hydraulic conductivity was assumed to be expressed by an exponential function of soil water content. Ahuja et al. (1980, 1988) assumed piecemeal power functions for the K–ψ model and piecemeal linear and logarithmic functions for the θ–ψ model, and proposed a simplified method to determine the parameters of these models from transient capillary pressure profiles. These simplified methods produced good results for some soils in situ. However, special functional forms of the θ–ψ and K–ψ models seem acceptable only for specific media. Using the evaporation method proposed by Wind (1968), Tamari et al. (1993) and Wendroth et al. (1993) determined the parameters of the θ–ψ model suggested by van Genuchten (1980). Changes of total water content and capillary pressures at some depths of soil columns during evaporation were used to estimate the parameters of the θ–ψ model, while the K–ψ relationships were determined using the instantaneous profile methodology. The evaporation method is applicable only to small soil samples which have uniform hydraulic properties with depth.

In recent years, the inverse method to determine the soil hydraulic properties proposed by Zachmann et al. (1981) has received interest. In the inverse method, the unsaturated soil water flow equation is solved numerically using the estimated initial parameter values of the θ–ψ and K–ψ models. Solution of the flow equation is repeated with improved parameter estimates until the computed results are identical to the measured cumulative out flow or water content profiles or capillary pressure profiles. Kool et al. (1985) and Parker et al. (1985) applied the inverse method to one-step out flow data and obtained good results. Russo (1988) and Russo et al. (1991) examined the problem of model selection associated with parameter estimation procedures based on the inverse method. They considered three different widely used models for soil hydraulic properties: the van Genuchten (1980) and Brooks and Corey (1964) models, and a combination of K–ψ model of Gardner (1958) and the θ–ψ model of Russo (1988). Using the θ–ψ model, which can express hysteretic phenomena, Kool and Parker (1988) estimated the hydraulic properties of soil from one-dimensional, transient infiltration and redistribution events. Dane and Hruska (1983) applied the inverse method to field data. They determined in situ hydraulic properties from water content profiles during drainage measured in a field lysimeter. In all these studies soil uniformity was assumed, which is unrealistic for undisturbed field soils.

Another problem related to the inverse method is solution uniqueness. Toorman et al. (1992) numerically evaluated solution uniqueness using response surfaces and concluded that cumulative one-step out flow data should be combined with capillary pressure data in order to improve the sensitivity of parameter estimation. Similar results were also obtained from laboratory experiments (e.g. van Dam et al., 1992; Eching and Hopmans, 1993). In the inverse method, initial and boundary conditions are required to solve the flow equation numerically. It is very difficult to assume these conditions appropriately for in situ experiments. In particular, the boundary condition at the bottom of the soil profile is not generally known.

Ross (1993) and Ross and Parlange (1994) recently suggested an inverse method for estimating soil hydraulic properties from a water content profile after a known drainage time from near saturated condition. This method was applied to soil profiles which have heterogeneous hydraulic properties with depth, and good results were obtained. However, relatively simple functional forms should be assumed for the θ–ψ and K–ψ models in this method. These hydraulic models do not have sufficient flexibility.

This study proposes a method to estimate the hydraulic properties of soil from transient soil capillary pressure profiles during water redistribution. The method is developed by combining the instantaneous profile method to compute the K–ψ relationship and the Mualem (1976a) model to derive the

K–ψ relationship from soil water retention characteristic. In this method, the θ–ψ relationship is expressed by the water retention model developed by Kosugi (1994) from the van Genuchten (1980) model. This model exhibits great flexibility for determining retention curves of various soils. An iterative procedure is used for parameter optimization. With this method, the θ–ψ and K–ψ relationships are estimated simultaneously for each horizontal layer of a soil profile. The method requires measurements of transient soil capillary pressure profiles during drainage, while no boundary condition at the bottom of the soil profile is necessary. In this study, the method is tested using numerically generated data sets for two hypothetical soil columns, one of which has uniform hydraulic properties with depth and one of which has heterogeneous hydraulic properties with depth.

2. Theory

2.1. Method to compute $K(\psi)$ values from transient capillary pressure profiles

The one-dimensional, vertical flow equation for soil water (the Richards' equation) can be written as

$$C(\psi)\frac{\partial\psi}{\partial t}=\frac{\partial}{\partial z}\left\{K(\psi)\left(\frac{\partial\psi}{\partial z}+1\right)\right\} \quad (1)$$

where $C(\psi)=\mathrm{d}\theta/\mathrm{d}\psi$ is the water capacity function, $K(\psi)$ is the hydraulic conductivity at a certain capillary pressure ψ, t is time, and z is the vertical distance taken positive upward. With the soil divided into layers (elements) along the z-axis as shown in Fig. 1, the finite difference form of Eq. (1) for an interior node ($i=2$ to $n-1$) is

$$\Delta z\frac{\overline{C}_{i-1}+\overline{C}_i}{2}\frac{\psi_i^{t+\Delta t}-\psi_i^t}{\Delta t}=\overline{K}_i\left(\frac{\overline{\psi}_{i+1}-\overline{\psi}_i}{\Delta z}+1\right)-\overline{K}_{i-1}\left(\frac{\overline{\psi}_i-\overline{\psi}_{i-1}}{\Delta z}+1\right) \quad (2)$$

where Δz and Δt are the distances between the vertical nodes and the time steps, respectively, and $\overline{\psi}$ is the node capillary pressure when $t=t+\Delta t/2$ (i.e. $\overline{\psi}=\psi^{t+\Delta t/2}=(\psi^t+\psi^{t+\Delta t})/2$). $\overline{C}_i$ and $\overline{K}_i$ correspond to the element water capacity and the element hydraulic conductivity, respectively, when

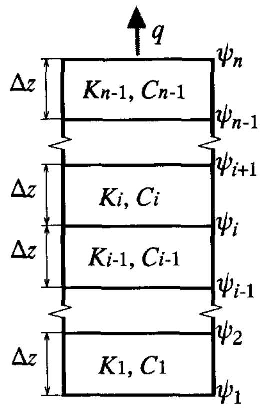

Fig. 1. Elements and nodes for computing $K(\psi)$ values from transient capillary pressure profiles.

$\psi=(\overline{\psi}_i+\overline{\psi}_{i+1})/2$. For the node at the soil surface, the finite difference form of Eq. (1) is

$$\Delta z\frac{\overline{C}_{n-1}}{2}\frac{\psi_n^{t+\Delta t}-\psi_n^t}{\Delta t}=-q-\overline{K}_{n-1}\left(\frac{\overline{\psi}_n-\overline{\psi}_{n-1}}{\Delta z}+1\right) \quad (3)$$

where q is the water flux supplied at the soil surface when $t=t+\Delta t/2$.

When the θ–ψ relationship of each soil layer in Fig. 1 is known a priori, the $K(\psi)$ values of each soil layer are computed by Eq. (2) or Eq. (3) from measured changes of ψ_i ($i=1, 2, \cdots, n$) with time during drainage (that is, $q=0$). The value of $\overline{K}_{n-1}$ of each time step is obtained from Eq. (3) using the value of $\overline{C}_{n-1}$, which is derived by differentiating the θ–ψ relationship of the $(n-1)$th soil layer. The $\overline{K}_{n-2}$ value of each time step is then derived by substituting the $\overline{K}_{n-1}$ value of the same time step into Eq. (2) with $i=n-1$ using the θ–ψ relationships of the $(n-1)$th and $(n-2)$th layers. Repeating this procedure in the lower layers one by one ($i=n-2$ to 2), the $K(\psi)$ value of each soil layer of each time step is obtained. It should be noted that the flux at the bottom of the soil profile is not needed in the $K(\psi)$ determination, because $\overline{K}_1$ is obtained from Eq. (2) with $i=2$ by using the measured ψ_1, ψ_2, and ψ_3 values.

2.2. Functional K–ψ relationship derived from soil water retention model

A soil water retention model described by the following function performs fairly well for retention data sets of various soils (Kosugi, 1994; Kosugi and Fukushima, 1994):

$$S_e = \left\{1 + m\left(\frac{\psi_c - \psi}{\psi_c - \psi_0}\right)^{1/1-m}\right\}^{-m} \quad \psi < \psi_c \tag{4}$$
$$S_e = 1 \quad \psi \geq \psi_c$$

where ψ_c, ψ_0, and m are estimated parameters and the effective saturation S_e is defined as

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{5}$$

where θ_s and θ_r are the saturated and residual water contents, respectively. Here, θ_r is defined as the water content at which ψ is infinitely small and K is assumed to be zero. However, this condition is only met when $\theta_r = 0$. Therefore, θ_r is commonly regarded as an empirical parameter. Three parameters ψ_c, ψ_0, and m have physical meanings for the θ–ψ curve. The parameter ψ_c is the bubbling pressure (or the air entry value of ψ) and ψ_0 is the capillary pressure at the inflection point on the θ–ψ curve. The parameter m ($0 < m < 1$) is dimensionless and related to the width of the soil pore size distribution. The value of S_e at the inflection point is derived by substituting $\psi = \psi_0$ into Eq. (4):

$$S_e(\psi_0) = (1 + m)^{-m} \tag{6}$$

Consequently, m determines the value of $S_e(\psi_0)$. Considering that m can only take on values between 0 and 1, $S_e(\psi_0)$ has a restriction of $0.5 < S_e(\psi_0) < 1$. It should be noted that Eq. (4) is identical to the retention model proposed by van Genuchten (1980) when ψ_c is equal to 0.

The Mualem (1976a) model for predicting the unsaturated hydraulic conductivity from soil water retention curve is written in the form

$$K(S_e) = K_s S_e^{1/2} \left\{ \int_0^{S_e} \frac{dS_e}{\psi(S_e)} \Big/ \int_0^1 \frac{dS_e}{\psi(S_e)} \right\}^2 \tag{7}$$

where K_s is the saturated hydraulic conductivity. Substituting Eq. (4) into Eq. (7) and integrating yields the functional relationship between K and ψ.

In this study, the unsaturated soil hydraulic properties are assumed to be described by Eqs. (4) and (7). Thus, the problem of how to determine the θ–ψ and K–ψ relationships is converted into the problem of how to determine the values of the six parameters, θ_s, θ_r, ψ_c, ψ_0, m, and K_s.

2.3. Parameter estimation procedure

Combining the method to compute $K(\psi)$ values from transient capillary pressure profiles (Eqs. (2) and (3)) and the soil hydraulic models expressed by Eqs. (4) and (7), the θ–ψ and K–ψ relationships of each soil layer are determined simultaneously by the following method. With this method, the three parameters ψ_c, ψ_0, and m that are contained both in the θ–ψ model expressed by Eq. (4) and the K–ψ model expressed by Eqs. (4) and (7) can be optimized for each soil layer. Parameters θ_s and θ_r in the θ–ψ model and K_s in the K–ψ model have to be determined independently. In practice, θ_s and K_s can be measured relatively easily, and θ_r should be guessed from measurement of water content at a low capillary pressure.

The parameter estimation procedure begins at the surface soil layer (the $(n-1)$th layer in Fig. 1) and is repeated in each lower layer one by one. An iterative procedure for estimating the parameters of the $(i-1)$th soil layer ($i = n, n-1, \cdots, 2$) is summarized in Fig. 2.

1. One starts with an initial guess for the three parameters ψ_{cR}, ψ_{0R}, and m_R of the objective $(i-1)$th layer. Here, ψ_{cR}, ψ_{0R}, and m_R represent the parameters of the retention model expressed by Eq. (4). The subscripts R stand for the parame-

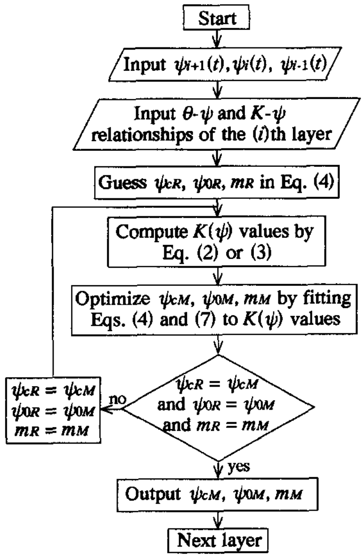

Fig. 2. Iterative procedure for estimating the hydraulic properties of the $(i-1)$th soil layer.

ters that are used to solve the Richards' equation. The guess may not be so difficult because, as mentioned before, ψ_c, ψ_0, and m all have physical significance. This is one of the reasons the retention model expressed by Eq. (4) is used in this study.

2. The $K(\psi)$ value of the $(i-1)$th soil layer is computed by Eq. (2) using the profile of ψ at each time step, the θ–ψ and K–ψ relationships of the upper (i)th soil layer, and the θ–ψ relationship of the objective $(i-1)$th layer expressed by Eq. (4) using the initial parameter set (ψ_{cR}, ψ_{0R}, m_R). In the case of the surface $(n-1)$th layer, the $K(\psi)$ value is computed by Eq. (3) using $q=0$ instead of using the K–ψ relationship of the upper soil layer.
3. The computed $K(\psi)$ values by Eq. (2) or Eq. (3) are then compared with the functional K–ψ relationship generated by Eqs. (4) and (7). Using a non-linear least squares optimization procedure based on Marquardt's maximum neighborhood method (Marquardt, 1963), ψ_{cM}, ψ_{0M}, and m_M are optimized to fit the functional K–ψ relationship to the computed $K(\psi)$ values. Here, ψ_{cM}, ψ_{0M}, and m_M represent the fitted parameters in Eqs. (4) and (7). The subscripts M stand for the parameters that are used in Mualem's model.
4. When ψ_{cR}, ψ_{0R}, and m_R are guessed correctly, ψ_{cM}, ψ_{0M}, and m_M are equal to ψ_{cR}, ψ_{0R}, and m_R, respectively. Conversely, the initially guessed parameter set (ψ_{cR}, ψ_{0R}, m_R) can be assumed to be incorrect when $\psi_{cR} \neq \psi_{cM}$ or $\psi_{0R} \neq \psi_{0M}$ or $m_R \neq m_M$. In this case, the fitted parameter set (ψ_{cM}, ψ_{0M}, m_M) is used as a new parameter set (ψ_{cR}, ψ_{0R}, m_R) to compute the $K(\psi)$ values by Eq. (2) or Eq. (3).

This procedure is repeated until the agreement between the parameter sets (ψ_{cR}, ψ_{0R}, m_R) and (ψ_{cM}, ψ_{0M}, m_M) is satisfactory. After the iterative procedure is finished for the $(i-1)$th soil layer, the parameters ψ_c, ψ_0, and m of the $(i-2)$th layer are estimated using the determined θ–ψ and K–ψ relationships of the $(i-1)$th layer.

3. Numerically simulated experiments

3.1. Hypothetical soil columns

In order to examine the parameter estimation procedure, two hypothetical soil columns were studied. Each column was 70 cm in depth and divided into seven layers (the 7th, 6th, $\cdots$, 1st layers from the soil surface) with eight equally spaced nodes, as shown in Fig. 3. One soil column had uniform hydraulic properties with depth (the uniform soil column) and the other had heterogeneous hydraulic properties with depth (the stratified soil column).

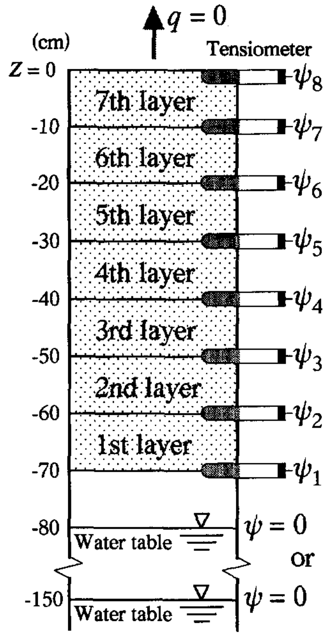

Fig. 3. Schematic of simulated soil profile with location of tensiometers and imposed boundary conditions, showing the numbering system of soil layers and capillary pressures. The hydraulic properties of soil below the 1st layer are the same as those of the uniform soil column (material A in Table 1).

All soil layers of the uniform soil column (material A) had the same hydraulic properties expressed by Eqs. (4) and (7) using the parameters summarized in Table 1. These parameters were derived from measured hydraulic properties of Plainfield sand (Black et al., 1969) (Mualem's soil index 4147; Mualem, 1976b). The values of K_s and θ_s were taken from Mualem's catalog. Parameters θ_r, ψ_c, ψ_0, and m

Table 1
Assumed hydraulic parameters in Eqs. (4) and (7) for materials A and B

Material	K_s (cm s^{-1})	θ_s	θ_r	ψ_c (cm)	ψ_0 (cm)	m	S_e (ψ_0)
A	0.00312	0.307	0.057	−14.1	−22.7	0.517	0.806
B	0.01000	0.307	0.057	−10.1	−12.7	0.417	0.865

Values of $S_e(\psi_0)$ were derived by substituting m into Eq. (6).

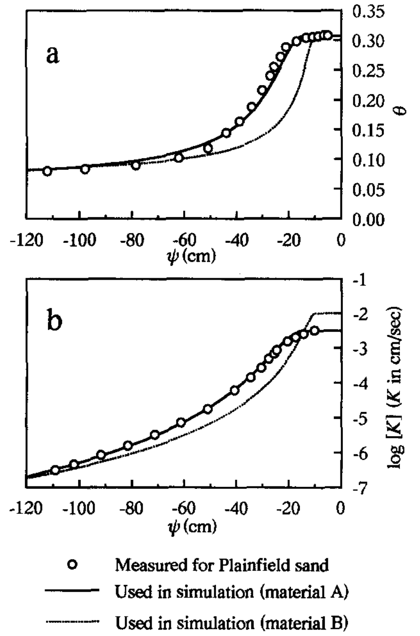

Fig. 4. (a) $\theta-\psi$ curves and (b) $K-\psi$ curves used in numerically simulated experiments. Circles indicate measured $\theta(\psi)$ and $K(\psi)$ values for Plainfield sand (Black et al., 1969).

were generated by fitting Eqs. (4) and (7) to the measured θ–ψ and K–ψ curves using a non-linear least square optimization procedure based on the RETC code developed by van Genuchten et al. (1991). A numerical integration program based on Clenshaw–Curtis type integration formula was used to derive the functional K–ψ relationship from Eqs. (4) and (7). Fig. 4(a) and (b) show measured and fitted θ–ψ and K–ψ curves, respectively. One can see that Eqs. (4) and (7) produced acceptable fits.

Five of the soil layers of the stratified soil column had the same hydraulic properties as every layer of the uniform soil column (material A), while ψ_c, ψ_0, m, and K_s of the 6th and 4th layers were different (material B in Table 1). Considering the greater ψ_c and ψ_0 values, the K_s value of material B was assumed to be about three times greater than that of material A. The smaller m value of material B resulted in a greater $S_e(\psi_0)$ value. The θ–ψ and K–ψ curves of material B are shown with dotted lines in Fig. 4(a) and (b), respectively.

3.2. Simulating drainage experiment

Assuming the experimental apparatus shown in Fig. 3, drainage processes from the two hypothetical soil columns were simulated to provide transient capillary pressure profiles which were subsequently used for the parameter estimation procedure. Eq. (1) was solved using capillary pressure as a dependent variable and a Crank–Nicholson finite difference scheme with equally spaced nodes (element lengths were 2 cm) and a variable time step (between 5 and 60 s).

For drainage simulations, a constant water table condition ($\psi = 0$ cm) was imposed at 80 cm below the soil surface. The hydraulic properties of soil below the 1st layer were the same as those of the uniform soil column (material A in Table 1). The initial condition was hydraulic equilibrium under a constant rainfall of 50 mm h^{-1} ($q = -1.4 \times 10^{-3}$ cm s^{-1}). The uniform soil column was simulated for 30 h under a zero-flux condition ($q = 0$ cm s^{-1}) at the soil surface, while the stratified soil column was simulated for 60 h. Cumulative errors in the mass balance at the end of calculations were 0.28 and 0.31% for the uniform and stratified soil columns, respectively. The numerical simulations provided the changes of capillary pressure heads (ψ_8, ψ_7, $\cdots$, ψ_1) at eight depths (0, 10, $\cdots$, 70 cm below the soil surface) during drainage, which were subsequently used for the parameter estimation procedure. Fig. 5(a) and (b) show the changes of capillary pressure heads for $0 \le t \le 30$ h computed for the uniform and stratified soil columns, respectively. The changes of ψ_5, ψ_6, ψ_7, and ψ_8 of the stratified soil column were more gradual than those of the uniform column because of the smaller K values of the 4th and 6th soil layers of the stratified column. The value of pressure head at the soil surface (ψ_8) of the uniform soil column at the end of calculation ($t = 30$ h) was -76.2 cm, while ψ_8 of the stratified soil column at the end of calculation ($t = 60$ h) was -77.7 cm.

In drainage processes in field, the shallow water table condition (80 cm below the surface) and the equilibrium initial condition are generally unrealistic. Another numerical experiment was made for the stratified soil column under a constant water table condition ($\psi = 0$ cm) at 150 cm below the soil surface (Fig. 3). The soil column was firstly drained for 24 h from the equilibrium condition under a constant rainfall of 1 mm h^{-1}. Then a condition of constant rainfall of 20 mm h^{-1} ($q = -5.6 \times 10^{-4}$ cm s^{-1}) was imposed for 1.7 h to simulate an

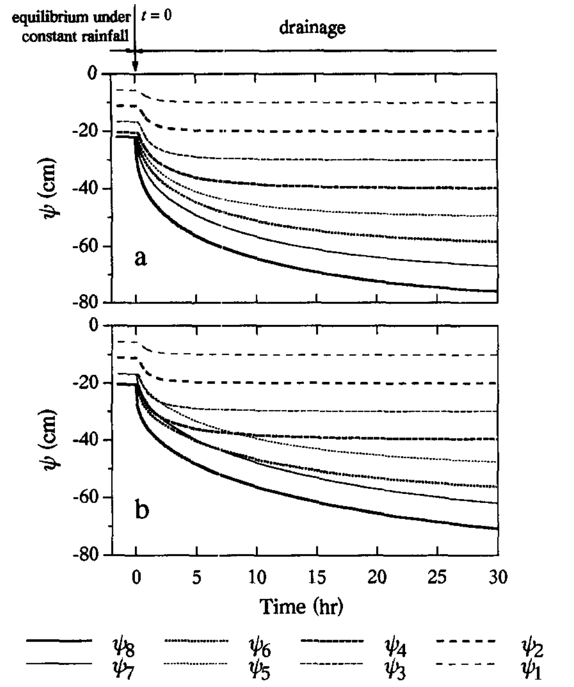

Fig. 5. Simulated transient capillary pressure profiles for (a) the uniform soil column and (b) the stratified soil column under equilibrium initial condition and shallow water table condition (80 cm below soil surface).

infiltration process. Using this non-equilibrium condition as the initial condition (i.e. $t = 0$), the second drainage process was simulated for 48 h under a zero-flux condition at the soil surface. Fig. 6 shows the computed changes of capillary pressure heads

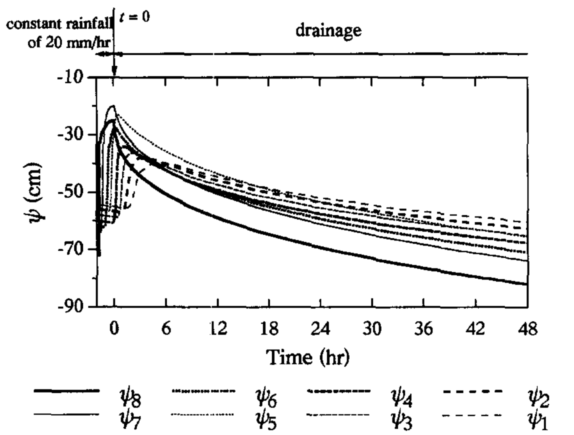

Fig. 6. Simulated transient capillary pressure profiles for the stratified soil column under non-equilibrium initial condition and deep water table condition (150 cm below soil surface).

during the second drainage process, which were subsequently used for the parameter estimation procedure. Values of ψ_8, ψ_7, ψ_6, and ψ_5 were between -30 and -20 cm at the beginning of drainage ($t = 0$ h), while ψ_3, ψ_2, and ψ_1 had not increased at all during the same time. The value of ψ_8 at the end of calculation ($t = 48$ h) was -82.0 cm.

4. Results and discussion

4.1. Uniform soil column

Parameter estimation runs were carried out for the uniform soil column using the simulated change of capillary pressure heads shown in Fig. 5(a). The value of Δz in Eqs. (2) and (3) was 10 cm and the minimal value of Δt was 1 min. Eqs. (2) and (3) did not provide a reasonable K value when the change of capillary pressure from t to $t + \Delta t$ was too small. Hence, automatically varying time steps were employed to satisfy the constraint that the change of capillary pressure head at the (i)th node from t to $t + \Delta t$ must be greater than 0.5 cm. The iterative optimization procedure was continued until the relative change in each parameter became $< 1\%$. The values of θ_s, θ_r, and K_s of every soil layer were kept constant at their true values shown in Table 1.

Fig. 7(a) and (b) show the updating processes of the θ–ψ and K–ψ curves, respectively, of the 7th soil layer using the initial parameter values of $\psi_{cR} = -10$ cm, $\psi_{0R} = -15$ cm, and $m_R = 0.8$. Simulated ψ_8 and ψ_7 values in Fig. 5(a) were used for the parameter estimation. The number of iterations necessary to reach convergence was 11. Values of $K(\psi)$ computed by using the instantaneous profile method (Eq. (3)) with the initial parameter set (presented by squares in Fig. 7(b)) were much smaller than the actual $K(\psi)$ values. The optimized parameter set of (ψ_{cM}, ψ_{0M}, m_M) for these $K(\psi)$ values was (-14.2, -20.6, 0.674). The second iteration produced $K(\psi)$ values which were closer to the actual values (presented by triangles), and the optimized parameter set was (-20.8, -26.0, 0.548). After the 11th iteration, the estimated θ–ψ and K–ψ curves were fairly similar to the actual curves. Final parameter values were (-16.5, -24.8, 0.514), which were about the same as the actual parameter values. Specifically, the estimated curves had slightly greater θ and K values than the actual curves.

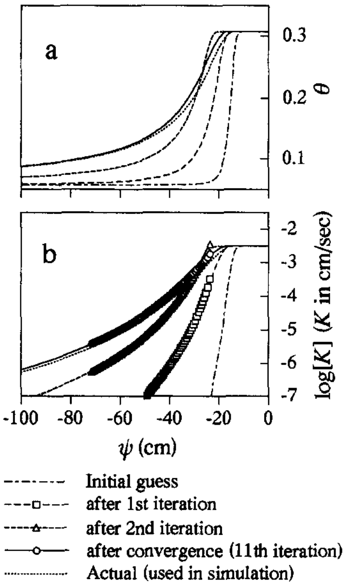

Fig. 7. Updating processes of (a) $\theta-\psi$ curve and (b) $K-\psi$ curve for the 7th layer of the uniform soil column. Squares, triangles, and circles represent $K(\psi)$ values computed by using the instantaneous profile method (Eq. (3)) in the 1st, 2nd, and 11th iterations, respectively.

In order to analyze the effect of initial parameter values on the solution convergence, 20 different initial parameter sets of $(\psi_{cR}, \psi_{0R}, m_R)$ were chosen for the parameter estimation procedure for the 7th soil layer. Initial ψ_{cR} value was 0, -10, -30, or -50 cm. With an initial ψ_{cR} value smaller than -71 cm, all $K(\psi)$ values computed by Eq. (3) were zero. This was because C_7 in Eq. (3) was equal to zero when $\psi \geq \psi_{cR}$. Therefore, the initial ψ_{cR} value for the surface layer must be greater than the smallest observed capillary pressure value. Initial ψ_{0R} values were from -5 to -100 cm which were chosen to satisfy the constraint that $\psi_{0R} < \psi_{cR}$. Initial m_R value was either 0.2 or 0.8. Table 2 shows the average values and the standard deviations of predicted parameters in the 20 cases. One can see predicted parameters are similar to actual parameters and the standard deviations are small. This indicates that initial parameter estimates do not influence the results of the parameter estimation procedure. Specifically, standard deviations of parameter ψ_c and m are relatively larger than that of ψ_0 considering the absolute value of each parameter. However, the differences between the maximal and minimal predicted parameter values were 1.0 cm and 0.009 for ψ_c and m, respectively, which were not significant.

Parameter estimation runs were carried out for the 6th soil layer using the same initial parameter sets as those used for the 7th layer. Here, the θ–ψ and K–ψ relationships of the 7th layer were expressed by Eqs. (4) and (7) using the average values of the predicted parameters in Table 2. Again, good agreement be-

Table 2
Average values and standard deviations of the final parameters of each soil layer predicted using 20 different initial parameter sets

Layer	Material	ψ_c (cm)	ψ_0 (cm)	m
Uniform soil column				
7th	A	−16.1 (3.1E−1)	−24.8 (3.9E−2)	0.518 (3.1E−3)
6th	A	−14.8 (5.1E−3)	−23.3 (4.2E−3)	0.514 (6.8E−5)
5th	A	−14.6 (1.5E−3)	−23.0 (1.1E−3)	0.511 (2.2E−5)
4th	A	−14.2 (1.8E−3)	−22.9 (1.1E−3)	0.516 (1.7E−5)
3rd	A	−13.7 (1.4E−3)	−22.8 (8.0E−4)	0.528 (4.7E−5)
2nd	A	−13.0 (2.2E−3)	−22.9 (2.8E−3)	0.561 (1.6E−4)
1st	A	−13.0 (1.3E−2)	−21.8 (3.3E−1)	0.264 (4.0E−2)
Stratified soil column				
7th	A	−13.1 (2.0E−1)	−24.3 (2.7E−2)	0.542 (1.9E−3)
6th	B	−11.2 (1.8E−2)	−13.4 (1.1E−2)	0.398 (1.5E−4)
5th	A	−12.9 (5.1E−3)	−22.4 (3.3E−3)	0.522 (5.5E−5)
4th	B	−10.0 (1.9E−3)	−13.3 (5.3E−4)	0.445 (3.9E−5)
3rd	A	−13.7 (2.3E−3)	−22.0 (1.4E−3)	0.503 (8.3E−5)
2nd	A	−11.5 (3.0E−3)	−23.2 (2.0E−3)	0.592 (1.6E−4)
1st	A	0.0 (4.1E−6)	−25.8 (1.8E−1)	0.830 (1.7E−3)

Numbers in parentheses represent the standard deviations.

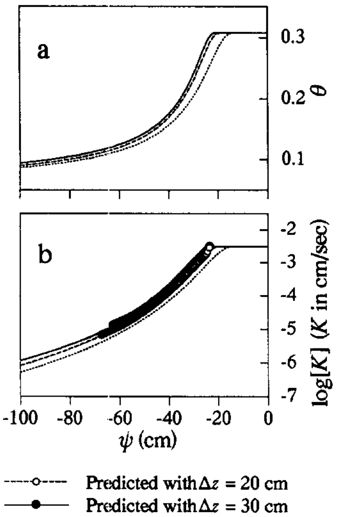

Fig. 8. Predicted (a) $\theta-\psi$ and (b) $K-\psi$ curves of the surface layer of the uniform soil column using a simulated tensiometer distance (Δz) of 20 and 30 cm. Circles on the $K-\psi$ curves indicate $K(\psi)$ values computed by using the instantaneous profile method (Eq. (3)) in the final iteration.

tween predicted and actual parameter sets was obtained in all cases. The standard deviation of every parameter for 6th layer was much smaller than that for the 7th layer (Table 2). The average values of predicted parameters were used for the parameter estimation runs for the 5th soil layer. The parameter estimation procedure was applied to the lower layers one by one in this way. Table 2 indicates that the predicted parameters were similar to the actual parameters in all layers except the 1st layer. The predicted m value of the 1st layer was about a half of the actual value. This is because the experimental data spanned a narrow range of capillary pressures for the lowest soil layer, as shown in Fig. 5(a).

The distance between the tensiometers (i.e. the finite difference step Δz in Eqs. (2) and (3)) can affect the parameter estimation. Fig. 8 shows the results of parameter estimation runs for the surface layer of the uniform soil column using Δz values of 20 cm and 30 cm instead of 10 cm. Simulated ψ_8 and ψ_6 values in Fig. 5(a) were used for the parameter estimation in the case of $\Delta z = 20$ cm, and ψ_8 and ψ_5 values were used in the case of $\Delta z = 30$ cm. The initial parameter values of $\psi_{cR} = -10$ cm, $\psi_{0R} = -15$ cm, and $m_R = 0.8$ were used for both estimation runs. In both runs, predicted $\theta-\psi$ curves had larger θ values than the actual curve and predicted $K-\psi$ curves had larger K values than the actual curve. In comparison with the case of $\Delta z = 10$ cm shown in Fig. 7, it is clear that as Δz becomes larger the difference between the predicted and actual values becomes larger. As a result, the shorter distance between tensiometers (10 cm) was required for accurate parameter estimations.

4.2. Stratified soil column

Parameter estimation runs were carried out for every layer of the stratified soil column using the simulated change of capillary pressure heads shown in Fig. 5(b) and the same initial parameter sets as those used for the uniform soil column. The details of the estimation procedure were as described in the preceding section.

The average values and the standard deviations of the predicted parameters of each soil layer are summarized in Table 2. The predicted parameters were similar to the actual parameters in all layers above the 2nd layer. The predicted m value of the 2nd layer was about 0.07 greater than the actual value. In the 1st layer, m was predicted to be about 0.3 greater than the actual value and ψ_c was predicted to be zero. Standard deviations of the predicted parameters of each soil layer were small. This indicates the parameter estimation procedure did not depend on the initial parameter values.

Fig. 9 shows the predicted and actual $\theta-\psi$ and $K-\psi$ curves of each soil layer using initial parameter values of $\psi_{cR} = -10$ cm, $\psi_{0R} = -15$ cm, and $m_R = 0.8$. The figure clearly shows that the parameter estimation procedure succeeded in reproducing hydraulic properties of the soil layers above the 2nd layer. This result indicates that the procedure can be successfully applied to soil profiles which have heterogeneous hydraulic properties with depth. In case of the 2nd layer, $K(\psi)$ values computed by using the instantaneous profile method (Eq. (2)) span narrow ranges of ψ. The predicted $\theta-\psi$ and $K-\psi$ curves produce good agreements with the actual curves in the range of $\psi > -30$ cm where $K(\psi)$ values computed by Eq. (2) span. The predicted $\theta-\psi$ and $K-\psi$ curves have smaller θ and K values, respectively, than the actual curves when $\psi < -30$ cm. This indicates that the parameter estimation procedure does not work well when experimental data span narrow ranges of capillary pressure. In the case of the 1st layer, the experimental data spanned narrower ranges of ψ than in the case of the 2nd layer (see Fig. 5(b)) and the parameter estimation procedure produced the poorer result.

The time interval between the tensiometer readings can also affect the results of the parameter estimation procedure. Fig. 10(a) and (b) show results of parameter estimation runs using the minimal Δt values of 12 min and 24 min, respectively, instead of 1 min. Values of $\psi_{cR} = -10$ cm, $\psi_{0R} = -15$ cm, and $m_R = 0.8$ were used as the initial parameter set for each estimation run. As the minimal Δt value becomes large, number of $K(\psi)$ values near saturation computed by Eq. (2) or Eq. (3) decreases. This is because the changes of capillary pressures with

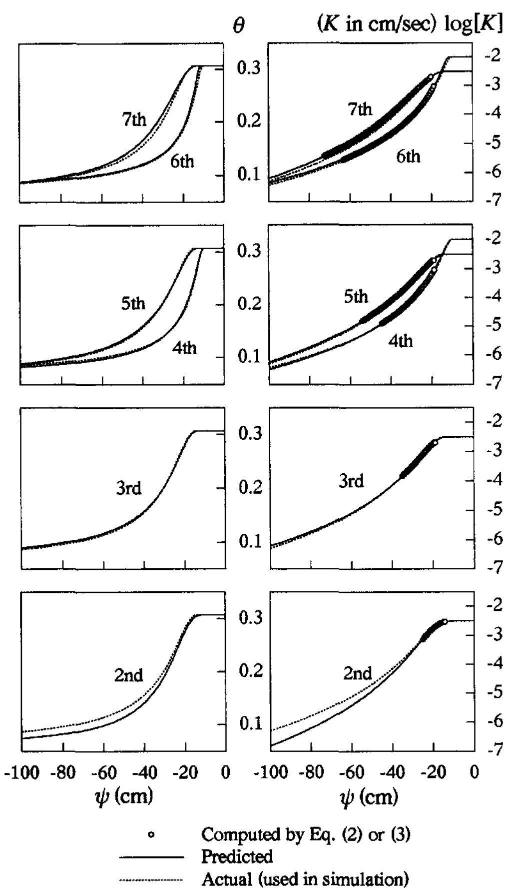

Fig. 9. Predicted and actual $\theta-\psi$ and $K-\psi$ curves of each layer of the stratified soil column. Circles on the $K-\psi$ curves indicate $K(\psi)$ values computed by using the instantaneous profile method (Eq. (2) or Eq. (3)) in the final iteration.

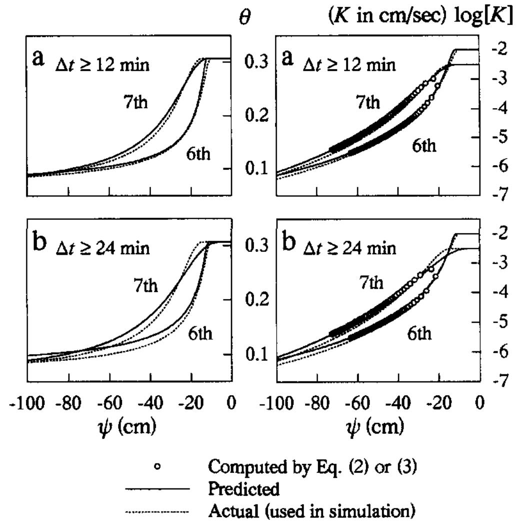

Fig. 10. Predicted $\theta-\psi$ and $K-\psi$ curves of the 7th and 6th layers of the stratified soil column using minimal Δt values of (a) 12 and (b) 24 min. Circles on the $K-\psi$ curves are as defined in Fig. 9.

time were rapid near saturation as shown in Fig. 5(b). With the restrictions of $\Delta t \geq 12$ min, the parameter estimation procedure produced good results both for the 7th and 6th layers (Fig. 10(a)). Noted that the parameter estimation procedure succeeded in reproducing hydraulic properties of other layers above the 2nd layer which are not shown in the figure. With the restriction of $\Delta t \geq 24$ minutes, predicted θ–ψ and K–ψ curves still showed acceptable fits to the actual curves (Fig. 10(b)). Specifically, the predicted θ–ψ and K–ψ curves of the 7th layer had smaller θ and K values, respectively, near saturation. The predicted θ–ψ curve of the 6th layer had greater θ values than the actual curve. Noted that the parameter estimation procedure produced similar results both for the 5th and 4th layers. As a result, the proposed method worked well using larger time intervals (12 and 24 min) between the tensiometer readings. However, the shorter time interval (1 or 12 min) was required for more accurate estimations.

4.3. Sensitivity to experimental error

To investigate the sensitivity of the parameter estimation procedure to the experimental error, a normally distributed measurement error term was added to the simulated capillary pressures for the stratified soil column shown in Fig. 5(b). Fig. 11

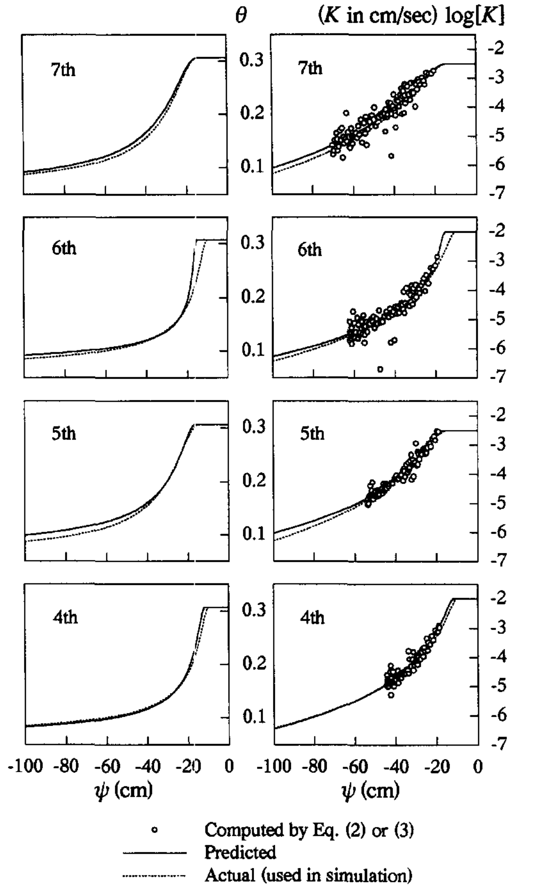

Fig. 11. Predicted $\theta-\psi$ and $K-\psi$ curves of each layer of the stratified soil column when standard deviation of normally distributed measurement error is 0.25 cm. Circles on the $K-\psi$ curves are as defined in Fig. 9.

shows results of the parameter estimation runs using initial parameter values of $\psi_{cR} = -10$ cm, $\psi_{0R} = -15$ cm, and $m_R = 0.8$ when the error standard deviation was taken to be 0.25 cm. Values of $K(\psi)$ computed by Eq. (2) or Eq. (3) were scattered along the actual $K-\psi$ curves. Predicted $\theta-\psi$ and $K-\psi$ curves of each layer produced acceptable fits with the actual curves. In comparison with the results when there was no experimental error (Fig. 9), the predictions were slightly poorer, especially near saturation of the 6th layer and in the range of $\psi < -40$ cm of the 5th layer.

Results of parameter estimation runs carried out using the same initial parameter sets (20 cases) as those used for the uniform soil column are summarized in Table 3. Average values of the predicted parameters of the 7th and 4th layers were similar to the actual values. Values of m predicted for the 6th and 5th layers were about 0.1 smaller than the actual values, and the ψ_c value of the 6th layer was predicted to be about 5 cm smaller than the actual value. The small values of standard deviations of the predicted parameters indicate the parameter estimation procedure did not depend on the initial parameter values for these layers. The parameter estimation procedure did not work well for the 3rd, 2nd, and 1st layers where the experimental data spanned relatively narrow ranges of capillary pressure.

The parameter estimation runs were also carried out for the 7th soil layer taking the error standard deviations to be 0.5 and 1.0 cm. Fig. 12 shows the results using initial parameter values of $\psi_{cR} = -10$ cm, $\psi_{0R} = -15$ cm, and $m_R = 0.8$. When the error standard deviation was 0.5 cm (Fig. 12(a)), the predicted $\theta-\psi$ and $K-\psi$ curves had greater θ and K values, respectively, than the actual curves. When the error standard deviation was 1.0 cm (Fig. 12(b)), computed $K(\psi)$ values by Eq. (3) were scattered in a wide range and the predicted $\theta-\psi$ and $K-\psi$ curves were different from the actual curves.

Table 3
Average values and standard deviations of the final parameters predicted for the stratified soil column when a normally distributed measurement error term is added to the simulated capillary pressures

Layer	Material	ψ_c (cm)	ψ_0 (cm)	m
7th	A	−14.6 (1.4E − 1)	−23.4 (3.0E − 2)	0.506 (1.8E − 3)
6th	B	−15.4 (1.4E − 2)	−16.1 (1.1E − 2)	0.321 (2.8E − 4)
5th	A	−16.4 (9.3E − 3)	−21.4 (1.9E − 3)	0.432 (2.2E − 4)
4th	B	−11.9 (2.2E − 2)	−14.2 (7.7E − 3)	0.418 (5.4E − 4)
3rd	A	−18.1 (7.3E − 3)	−19.7 (3.6E − 3)	0.244 (6.9E − 4)
2nd	A	−15.4 (3.9E − 1)	−15.9 (3.5E − 1)	0.050 (3.7E − 5)
1st	A	−6.8 (1.0E − 0)	−15.7 (1.3E − 1)	0.900 (1.2E − 6)

Numbers in parentheses represent the standard deviations. The error standard deviation is 0.25 cm.

4.4. Estimation under non-equilibrium initial condition

The parameter estimation procedure was tested under the non-equilibrium initial condition using the simulated change of capillary pressure heads shown in Fig. 6. As the ground water table was set to be 80 cm deeper than the bottom of the stratified soil column, the effect of the constant ground water level condition was not significant for these simulated ψ values. The details of the estimation procedure were as described in the preceding section.

Fig. 13 shows the predicted and actual θ–ψ and K–ψ curves of each soil layer using initial parameter values of $\psi_{cR} = -10$ cm, $\psi_{0R} = -15$ cm, and $m_R = 0.8$. The predicted θ–ψ and K–ψ curves of the 7th and 5th layers produced results as good as the predicted curves shown in Fig. 9. This clearly shows the parameter estimation procedure can be successfully used under non-equilibrium initial conditions. Values of $K(\psi)$ computed by Eq. (2) for the 6th layer were plotted on the actual K–ψ curve, and the predicted θ–ψ and K–ψ curves produced acceptable fits. Specifically, the predicted curves had slightly greater θ and K values near saturation. While $K(\psi)$ values computed by Eq. (2) for the 4th layer were plotted on the actual K–ψ curve, the differences between the predicted and the actual θ–ψ and K–ψ curves near saturation were great for this layer. Though the parameter estimation procedure produced acceptable results for the 3rd and 2nd layers, there were slight differences between the predicted and the actual θ–ψ and K–ψ curves near saturation. Because the constant rainfall of 20 mm h^{-1} was continued only for 1.7 h, maximal values of capillary pressure heads simulated in this numerical experiment were smaller than those in other experiments (see Figs. 5 and 6). It seems reasonable to say that the lack of observed ψ data near saturation is what caused the differences between the predicted and the actual θ–ψ and K–ψ curves near saturation for these layers. As a result, no equilibrium initial condition is necessary, but measurements of change of

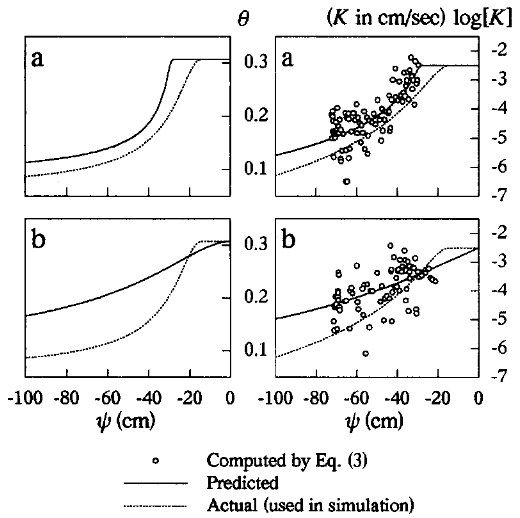

Fig. 12. Predicted $\theta-\psi$ and $K-\psi$ curves of the 7th layer of the stratified soil column when standard deviations of normally distributed measurement error are (a) 0.5 and (b) 1.0 cm. Circles on the $K-\psi$ curves are as defined in Fig. 8.

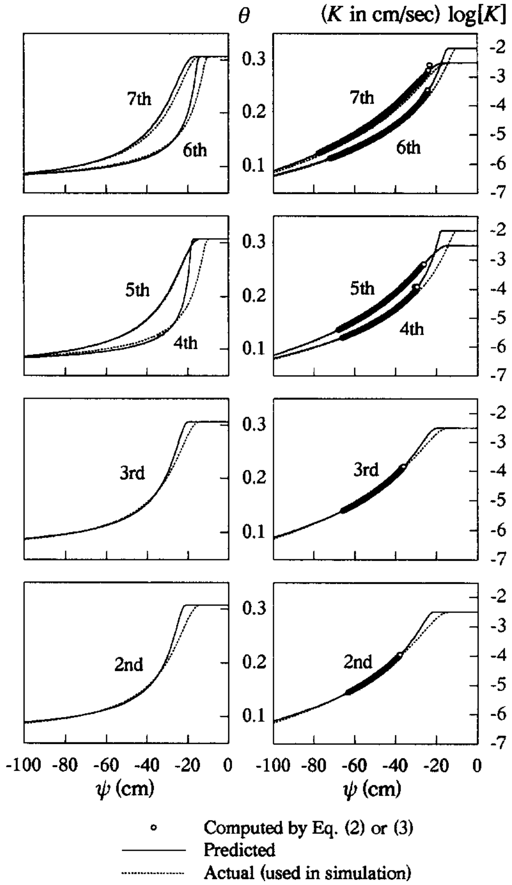

Fig. 13. Predicted $\theta-\psi$ and $K-\psi$ curves of each layer of the stratified soil column using the change of capillary pressures generated under non-equilibrium initial condition. Circles on the $K-\psi$ curves are as defined in Fig. 9.

capillary pressures near saturation are required when applying the parameter estimation procedure to drainage experiments.

5. Summary and conclusions

This study proposed a method to predict the relationship between the volumetric water content θ and the capillary pressure ψ and the relationship between the unsaturated hydraulic conductivity K and ψ simultaneously for each horizontal layer of a soil profile. The method was developed by combining the instantaneous profile method to compute the $K(\psi)$ values and the Mualem (1976a) model to derive the K–ψ relationship from soil water retention characteristic. In this method, the θ–ψ relationship was assumed to be expressed by the soil water retention model suggested by Kosugi (1994). Soil hydraulic properties were then characterized by the following six parameters: the saturated hydraulic conductivity K_s, the saturated water content θ_s, the residual water content θ_r, the bubbling pressure ψ_c, the capillary pressure at the inflection point (ψ_0) on the θ–ψ curve, and the dimensionless parameter m which determines the effective saturation at the inflection point. In the proposed method, the values of ψ_c, ψ_0, and m were optimized for each horizontal soil layer by the iterative procedure from transient soil capillary pressure profiles during drainage. The values of K_s, θ_s, and θ_r had to be determined independently.

The method was examined for two hypothetical soil columns, one of which had uniform hydraulic properties with depth (the uniform soil column) and one of which had heterogeneous hydraulic properties with depth (the stratified soil column). Each soil column was 70 cm in depth and divided into seven layers of the same thickness. Data sets of transient capillary pressure profiles were generated by numerical drainage experiments.

The results showed that the parameter estimation procedure produced good results both for the uniform and stratified soil columns using the vertical distance between capillary pressure measurements of 10 cm and the time interval between tensiometer readings of 1 min. Twenty different initial parameter sets were chosen to examine the solution convergence, and it was shown that initial parameter estimates do not influence the results of the parameter estimation.

When the vertical distance between capillary pressure measurements was 20 or 30 cm, predicted θ–ψ and K–ψ curves for the uniform soil column had larger θ and K values than the actual curves. With the time interval between the tensiometer readings of 12 or 24 min, the parameter estimation procedure produced acceptable results for the stratified soil column. However, the shorter time interval of 1 or 12 min was required for more accurate estimations.

In order to investigate the sensitivity of the parameter estimation procedure to the experimental error, a normally distributed measurement error term was added to the simulated capillary pressures. When the error standard deviation was 0.25 cm, the predicted θ–ψ and K–ψ curves for the stratified soil column produced acceptable fits with the actual curves. When the error standard deviation was 1.0 cm, the parameter estimation procedure did not work well. Testing the parameter estimation procedure using the change of capillary pressures generated under less ideal conditions, it was shown that the proposed method requires no equilibrium initial condition but measurements of change of capillary pressures near saturation.

In summary, we conclude that the proposed method can be applied to soil profiles which have heterogeneous hydraulic properties with depth as well as to soil profiles which have uniform hydraulic properties. The method requires measurements of transient soil capillary pressure profiles during water redistribution, while no boundary condition at the bottom of the soil profile is needed. This method should be examined in the future research using data sets obtained by both laboratory and in situ experiments.

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