

# Effect of heat transfer direction on the numerical prediction of beef freezing processes

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## Abstract

Accurate numerical prediction of freezing processes for meats is difficult. One of the reasons is the effect of heat transfer direction during freezing. When meat is being frozen, heat can transfer parallel to or perpendicular to muscle fibres of meats at various rates. In this study the influence of the heat transfer direction on the numerical simulation of the freezing processes was analysed. A computer simulation program using the modified Crank–Nicolson finite difference scheme was developed to predict the temperature profiles during freezing. Two thermal conductivity formulas, the Levy and the modified series equations, were incorporated into the program. Experiments were carried out on beef samples. The comparison between the numerical results and experimental data shows that the program can predict the freezing processes well. © 1999 Elsevier Science Ltd. All rights reserved.

**Keywords:** Beef; Freezing; Freezing time; Heat transfer; Meat; Modelling; Parallel heat transfer; Perpendicular heat transfer

## Notation

$C_c$	equivalent heat capacity (J/kg K)
$F$	function parameter
$h_c$	heat transfer coefficient (W/m <sup>2</sup> K)
$k$	thermal conductivity (W/mK)
$L$	thickness (m)
$t$	time (s)
$T$	temperature (°C)
$r$	intermittent value
$v$	volume fraction
$x$	location (m)
$\rho$	density (kg/m <sup>3</sup> )
$\Delta t$	time step (s)
$\Delta x$	space step (m)

## Subscripts

a	ambient
c	continuous phase
d	dispersed phase
f	fat
$i$	location step
I	ice
$j$	content
$n$	time step
p	protein
sh	sheaths material (epimysium, perimysium, endomysium and ligament)
so	solid
w	water

## 1. Introduction

Freezing and cold storage are common refrigeration processes applied widely in the meat industry (Sun, 1998). The simulation of freezing processes is of importance for optimum design and operation of food freezers. There are a large number of researches applying numerical methods such as finite difference or finite element to predict the freezing time for a variety of foodstuffs under various freezing conditions (Joshi & Tao, 1974; Cleland & Earle, 1977; Abdalla & Singh, 1985; Chau & Gaffney, 1990; Scott & Hedman, 1990; Tocci & Mascheroni, 1994; de Reinick, 1996; Saad & Scott, 1997). Normally the finite difference method is used to calculate the freezing time on regular geometries (Pham, 1985) and the finite element method for irregular geometries (Purwadaria & Heldman, 1982; Cleland, Cleland, Earle & Byrne, 1987). Since the thermal properties of foodstuffs during freezing are temperature dependent and vary with various foodstuffs, a simulation program will predict the freezing time well in some circumstances but not in others (Cleland & Earle, 1984).

There are two aspects which lead to the inaccuracy in predicting freezing time: iteration error and physical approximation (Cleland, Earle & Cleland, 1982). With the increase in computer speed, the iteration error such as numerical error, computer rounding error and

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truncation error can be minimised. Therefore the approximation of thermal properties during freezing will limit the accuracy of the numerical method. From this point the precision of finite difference and finite element calculation would be expected to be identical. Therefore the finite difference method was used in the current study.

As meat freezes, heat will transfer parallel to or perpendicular to the muscle fibres at different rates. Usually research work using numerical methods to predict the freezing time only considered the heat transfer parallel to muscle fibres (Tocci & Mascheroni, 1995a; de Rejnck, 1996) or to samples without muscle fibre direction such as minced samples (Tocci & Mascheroni, 1995b). However the thermal conductivity ( $k$ ) of meat changes with temperature and fibrous direction during freezing, and heat can transfer in the direction that is not only parallel to but also perpendicular to the muscle fibres during freezing. Therefore, in the present study, the effect of heat transfer direction on the numerical prediction of freezing time is examined.

**2. Numerical simulation**

Since during freezing, the thermal conductivity of the sample changes with temperature, the one-dimensional heat transfer equation can be written as follows

$$\rho(T)C_c(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( k(T) \frac{\partial T}{\partial x} \right), \quad t > 0, \quad 0 < x < L. \tag{1}$$

For heat transfer in a slab sample, the initial and boundary conditions are:

$$T(x, 0) = T_i, \quad t = 0, \quad 0 \leq x \leq L, \tag{2}$$

$$-k(T) \frac{\partial T}{\partial x} \Big|_{x=0} = h_c(T_a - T_{x=0}), \quad t > 0, \quad x = 0, \tag{3}$$

$$-k(T) \frac{\partial T}{\partial x} \Big|_{x=L} = h_c(T_{x=L} - T_a), \quad t > 0, \quad x = L. \tag{4}$$

The above equation can only be solved by numerical method (Sun & Deng, 1990b; Sun & Crawford 1993a). The Crank–Nicolson finite difference scheme is an implicit method and widely used in solving heat transfer problems (Sun & Deng, 1988, 1989, 1990a; Sun & Crawford, 1993b). Therefore it is used in this study. The Crank–Nicolson finite different formulation is shown below:

$$\frac{k_-^{n+1/2}}{2\Delta x} T_{i-1}^{n+1} - \left( \frac{k_-^{n+1/2} + k_+^{n+1/2}}{2\Delta x} + ((\rho C_c)_-^{n+1/2} + (\rho C_c)_+^{n+1/2}) \frac{\Delta x}{2\Delta t} \right) T_i^{n+1} + \frac{k_+^{n+1/2}}{2\Delta x} T_{i+1}^{n+1},$$

$$= -\frac{k_-^{n+1/2}}{2\Delta x} T_{i-1}^n + \left( \frac{k_-^{n+1/2} + k_+^{n+1/2}}{2\Delta x} - ((\rho C_c)_-^{n+1/2} + (\rho C_c)_+^{n+1/2}) \frac{\Delta x}{2\Delta t} \right) T_i^n - \frac{k_+^{n+1/2}}{2\Delta x} T_{i+1}^n, \tag{5}$$

where  $k_+$  is evaluated at  $0.5 (T_{i+1} + T_i)$  and  $k_-$  at  $0.5 (T_{i-1} + T_i)$ . The thermal conductivity ( $k$ ) in Eq. (5) should be known. Zhu and Sun have compared various  $k$  equations (Zhu & Sun, 1998) and the appropriate equations are used in the study.

**3. Experiments**

Experiments were designed to freeze lean beef in a cabinet blast freezer Foster Refrigerator, England. T-type thermocouples were connected to a data acquisition system (National Instruments, USA). In each test, two beef samples were placed in the freezer cabinet. One allowed the heat to flow parallel to the muscles fibres and the other perpendicular to its fibril direction (Fig. 1). In order to ensure one-dimensional heat transfer, all the samples were cut into round shapes with 220 mm in diameter and 25 mm in thickness. Because the diameter of the samples is much bigger than its thickness, the heat flows from the round surface of the samples to the core can be ignored. In this case, heat can be assumed to transfer only from the top and bottom surfaces to the centre. The two samples were placed in the freezer cabinet side by side as shown in Fig. 2 so that the freezing conditions could be maintained to be the same for the two samples in the test. All the samples were kept in a refrigerator with a temperature of 10°C overnight for achieving uniform initial temperature of 10°C.

The experiments were carried out using an air velocity of 4 m/s, with average relative humidity of 91%,

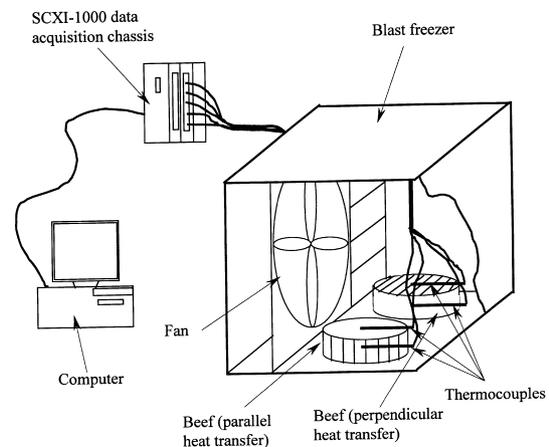


Fig. 1. The cabinet blast freezer and data acquisition system.

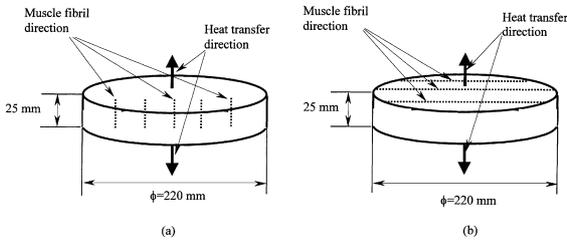


Fig. 2. Beef samples for experiments: (a) parallel heat transfer; (b) perpendicular heat transfer.

and the applied air temperatures were set at  $-15^{\circ}\text{C}$ ,  $-20^{\circ}\text{C}$  and  $-30^{\circ}\text{C}$ , respectively. The moisture content of the beef was determined before the samples were frozen. The applied air temperature was measured using five thermocouples, two inserted in the core, two on the surface of the sample and the last one measuring the temperature of the applied air. Each experiment was repeated five times and temperature profiles were averaged. Table 1 summarises the experimental conditions. The physical properties listed in Table 1 are taken from published literature (Rahman, 1995).

The assessment of the accuracy of the numerical simulation was made by comparing the predicted results with the experimental data, which can be calculated from (Cleland et al., 1987)

Percentage difference

$$= \frac{\text{Predicted time} - \text{Experimental time}}{\text{Experimental time}} \times 100. \quad (6)$$

## 4. Results and discussion

### 4.1. Effect of heat transfer direction

Three sets of experiments under different applied air temperatures (Table 1) were carried out. Fig. 3 shows

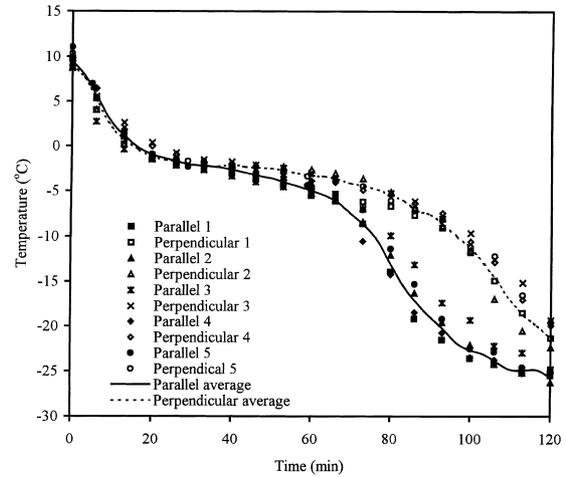


Fig. 3. Results of five replicates for parallel and perpendicular heat transfer at freezing temperature of  $-30^{\circ}\text{C}$ .

the results at the applied air temperature of  $-30^{\circ}\text{C}$ . The results clearly indicate the effect of heat transfer direction. The heat transferred parallel to the muscle fibril direction took place quicker than that perpendicular to the muscle direction. Similar phenomena were also observed for the results at  $-15^{\circ}\text{C}$  and  $-20^{\circ}\text{C}$ .

In the meat industry, manufacturers often encounter longer than expected freezing times (Chadderton & Kemp, 1993). These longer freezing times lead to increasing energy costs and potential losses in product quality. One of the reasons for this error was the inaccuracy in the prediction model used to control the freezing processes. Normally freezing prediction models were developed based on heat transfer without considering the heat transfer direction. The results in Fig. 3 show the necessity of incorporating the heat transfer direction into the numerical simulation in order to improve the accuracy of the prediction.

### 4.2. Simulation of parallel heat transfer

In simulating the parallel heat transfer, a proper thermal conductivity equation should be used. Zhu and Sun (1998) have illustrated that the Levy equation is suitable for calculating the  $k$  value when heat flows parallel to muscle fibers. The Levy model is as follows (Zhu & Sun, 1998)

$$\frac{k}{k_c} = \frac{(1 - 2rF)}{(1 + rF)}, \quad (7)$$

where  $r = (k_c - k_d)/(2k_c + k_d)$  and

$$2F = \frac{2}{s} - 1 + 2v_d - \left[ \left( \frac{2}{s} - 1 + 2v_d \right)^2 - \frac{8v_d}{s} \right]^{1/2},$$

$$s = \frac{(k_d - k_c)^2}{[(k_c + k_d)^2 + k_c(k_d/2)]}.$$

Table 1  
The properties of unfrozen beef and freezing conditions

Experimental parameters			
Initial moisture content	73.0%		
Protein	19.6%		
Fat	11.2%		
Unfrozen thermal conductivity	0.514 W/m K		
Unfrozen specific heat	3.431 kJ/kg K		
Unfrozen density	1076.00 kg/m <sup>3</sup>		
Test No.	1	2	3
Thickness (m)	0.025	0.025	0.025
Applied air temperature (°C)	-30.0	-20.0	-15.0
Initial temperature (°C)	10.0	10.0	10.0
Mean air velocity (m/s)	4.00	4.00	4.00

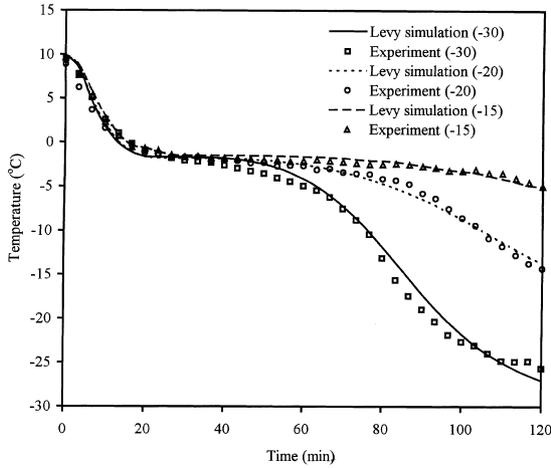


Fig. 4. Comparison of parallel heat transfer experimental data with simulated results using levy model at  $-30^{\circ}\text{C}$ ,  $-20^{\circ}\text{C}$  and  $-15^{\circ}\text{C}$  freezing temperatures.

Fig. 4 compared the simulating results with the experimental results under three applied air temperatures of  $-30^{\circ}\text{C}$ ,  $-20^{\circ}\text{C}$  and  $-15^{\circ}\text{C}$ . The comparison indicates that the mean percentage difference between the data of simulation and the experimental were between 2.25% and 4.58%. Therefore, the Levy equation is suitable for calculating the thermal conductivity of the sample when heat transfers parallel to the sample muscle fibril direction.

4.3. Simulation of perpendicular heat transfer

There is little previous work on simulating the freezing processes for heat flowing perpendicular to muscle fibre of meats. The effect of orientation of heat flowing to the fibrous meats on thermal conductivity

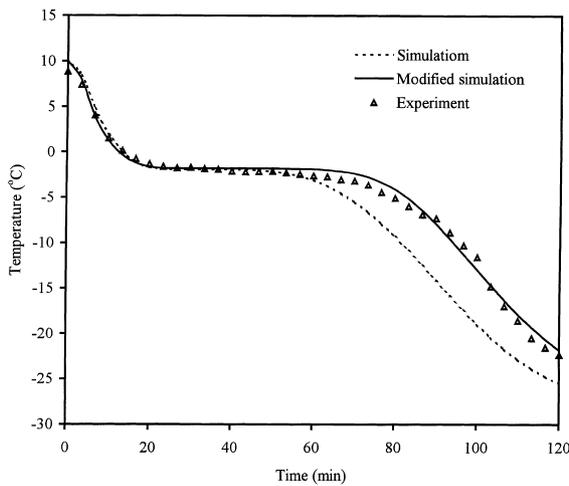


Fig. 5. Comparison of perpendicular heat transfer experimental data with simulated results using series model and modified series model at  $-30^{\circ}\text{C}$  freezing temperature.

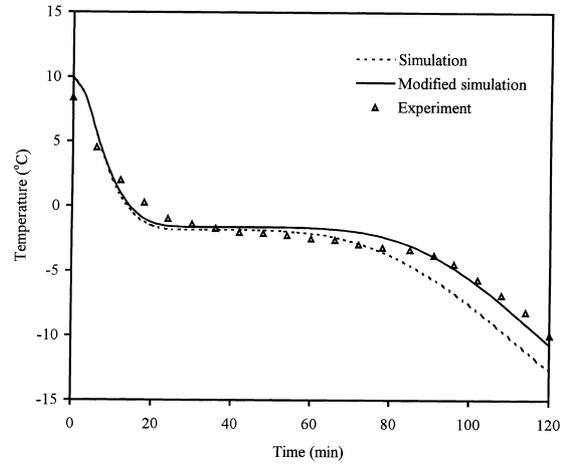


Fig. 6. Comparison of perpendicular heat transfer experimental data with simulated results using series model and modified series model at  $-20^{\circ}\text{C}$  freezing temperature.

was discussed, however, there were considerable discrepancies between the predicted and experimental results (Heldman & Gorby, 1975; Pham & Willix, 1989). For heat flow perpendicular to the sample muscle fibres, the series equation was identified to be a better equation for calculating the thermal conductivity as compared with other five equations (Zhu & Sun, 1998). The series equation is as follows

$$\frac{1}{k} = \sum \frac{v_j}{k_j} \tag{8}$$

The simulation results are shown in dotted lines in Figs. 5–7 as compared with the experimental data. The comparison indicates that using Eq. (8), the predicted results had poor agreement with the experiments. The percentage difference was calculated to be 43.11% for Fig. 5 and the results in Figs. 6 and 7 show similar discrep-

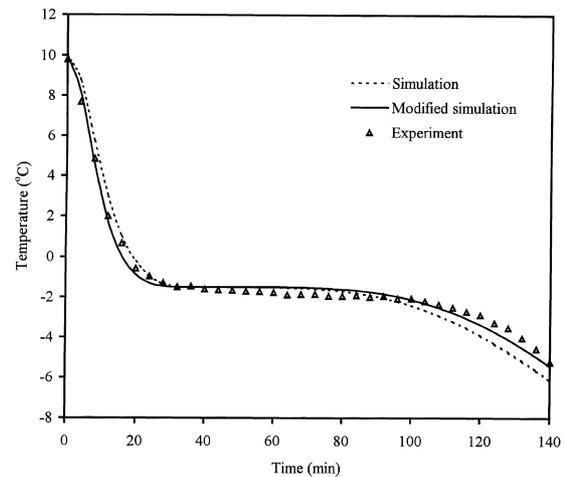


Fig. 7. Comparison of perpendicular heat transfer experimental data with simulated results using series model and modified series model at  $-15^{\circ}\text{C}$  freezing temperature.

ancies. It can also be seen that the experimental freezing rates were lower than the simulated rates, as indicated by Heldman and Gorby (1975) who also used Eq. (8) for calculating the thermal conductivities.

When heat flows perpendicular to the muscle fibres, the heat resistance of the sample components such as epimysium, perimysium, endomysium and ligament sheaths (membrane) should be considered. With considering all these heat resistances, the series equation was modified as follows

$$k = \frac{1}{\sum(v_j/k_j)} = \frac{1}{\left(\frac{v_l}{k_l} + \frac{v_{so}}{k_{so}} + \frac{v_f}{k_f} + \frac{v_p}{k_p} + \frac{v_w}{k_w} + \frac{v_{sh}}{k_{sh}}\right)}. \quad (9)$$

Eq. (9) was then incorporated into Eqs. (1)–(4) and the simulation results are shown in Figs. 5–7 (solid lines), as compared with the experiments. The comparison indicates clearly the improvement in the simulation accuracy when the modified series equation was used. The mean percentage difference in Fig. 5 was reduced to 5.16%.

## 5. Conclusions

Experiments on freezing beef samples were carried out. The results show that the samples freeze at different rates as heat transfers parallel to and perpendicular to the sample muscle fibres. Under the same freezing condition, the heat transferring parallel to the muscle fibril direction took place quicker than that perpendicular to the muscle direction. In order to simulate the freezing processes, a one-dimensional numerical heat transfer model was developed. The model includes the effect of heat transfer direction to the sample muscle fibres. For simulating the heat transfer parallel to the muscle fibres, with the Levy equation for calculating the sample thermal conductivity, the simulated results agree well with the experiments. However, when heat transfers perpendicular to the muscle fibril direction, the series equation modified by the authors should be used to calculate the thermal conductivity, as the simulated results have good agreement with the experimental data.

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