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USE OF THE HOOGHOUDT FORMULA FOR DRAIN SPACING CALCULATIONS IN HOMOGENEOUS-ANISOTROPIC SOILS

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ABSTRACT

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Cases of groundwater flow in anisotropic soils can be transformed to equivalent isotropic cases, and then be solved with the Laplace equation. On the basis of this principle, the Hooghoudt formula, developed for isotropic soils, has been used for calculating drain spacings in anisotropic soils. However, the Hooghoudt formula is only an approximate solution of the Laplace equation so that extension to anisotropic soils may not a priori be justified under all conditions. Results obtained with the Hooghoudt formula were compared to those obtained by numerical solutions of the Laplace equation. On the basis of this comparison, it is concluded that the Hooghoudt formula may indeed be used with reasonable confidence for drain spacing calculation in homogeneous-anisotropic soils.

INTRODUCTION

Anisotropy is almost always a soil characteristic to be considered in the design of groundwater drainage systems for salinity control in alluvial plains. Here, wide drain spacings can generally be applied with much of the drainage flow moving through the deeper substrata below the rootzone. Because of the way they were deposited, the particles making up these layers often have a common horizontal orientation of their longest dimensions, referred to as micro-stratification, which makes the hydraulic conductivity of these strata in the horizontal direction (K_h) much higher than that in the vertical direction (K_v). Maasland (1957) showed that anisotropy due to micro-stratification may be treated as homogeneous-anisotropy (meaning that K_h and K_v differ but do not vary within the considered flow region). It can be shown that by a transformation of coordinates the Laplace equation of the isotropic soils can also be made valid for homogeneous-anisotropic soils (by this transformation of coordinates, the anisotropic case is said to be converted into an

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equivalent isotropic case — see Maasland, 1957). This means that solutions of the Laplace equation of steady groundwater drainage problems to parallel drains in isotropic soils can also be applied, after transformation, to the same type of problems in homogeneous-anisotropic soils. In principle, this holds true only for exact solutions of the Laplace equation and therefore would not a priori apply to the drainage formulae used in practice to calculate required drain spacings since, as pointed out by, amongst others, Lovell and Youngs (1984), these formulae are for most real cases only approximate solutions based on a schematization or a particular conception of the flow pattern involved. In this paper it has been investigated which errors are made when, after transformation, the Hooghoudt drainage formula is used for drain spacing calculations in anisotropic soils (as proposed by Boumans, 1979). Drain spacings obtained with the Hooghoudt formula have been compared with those obtained by numerical solutions of the Laplace equation for the same cases.

FLOW EQUATIONS AND TRANSFORMATION RULES

For homogeneous anisotropic soils characterized by $K_x \neq K_z$, where the directions x and z are respectively parallel with and normal to the stratification, the basic equation for groundwater flow in two dimensions is:

$$K_x \frac{\partial^2 h}{\partial x^2} + K_z \frac{\partial^2 h}{\partial z^2} = 0 \tag{1}$$

where h is the hydraulic head at (x, y). Making the substitutions $x_t = x\sqrt{C/K_x}$ and $z_t = z\sqrt{C/K_z}$ (where C is a constant and the suffix t stands for transformation), equation (1) becomes the Laplace equation, the basic equation for groundwater flow in an isotropic soil:

$$\frac{\partial^2 h}{\partial x_t^2} + \frac{\partial^2 h}{\partial z_t^2} = 0 \tag{2}$$

Thus by shrinking or expanding the dimensions in the two principal directions, cases of groundwater flow in an anisotropic soil can be transformed into equivalent cases in an isotropic soil. It can be proved (see e.g. Maasland, 1957) that the hydraulic conductivity of the equivalent isotropic case relates to the real anisotropic hydraulic conductivities as $K_t = \sqrt{K_x K_z}$, which is the geometric mean value of K_x and K_z.

For phreatic flow, where head and vertical dimensions are interrelated, it is convenient to take the constant $C = K_z$. In that case $x_t = x\sqrt{K_z/K_x}$ and $z_t = z$, so that changing the dimensions in horizontal (x) direction only is enough to achieve the desired transformation. The corresponding form of the Laplace equation is:

$$\frac{\partial^2 h}{\partial x_t^2} + \frac{\partial^2 h}{\partial z^2} = 0 \tag{3}$$

To maintain equivalence in the case of a linearly distributed groundwater recharge (q), the latter must also be transformed such that $qx = q_t x_t$ or $q_t = q/\sqrt{K_z/K_x}$.

HOOGHOUDT FORMULA FOR GROUNDWATER DRAINAGE

The case of steady groundwater drainage flow to equidistant parallel drains has been much analysed and has resulted in a number of drainage formulae. Of these, the Hooghoudt drainage formula is widely used in pipe drainage design practice, generally with good results. This formula is here written as:

$$q = \frac{8Kdh}{L^2} + \frac{4Kh^2}{L^2} \tag{4}$$

where q is discharge rate; h watertable height at mid-spacing; d equivalent depth of the impermeable base below the drainage base, $d = f(D, L, u)$; D real depth of the impermeable base below the drainage base; u wet perimeter of the drain; K hydraulic conductivity; and L drain spacing. The Hooghoudt formula is based on separate analyses of horizontal and radial flow, each conceived to be confined to a certain part of the flow region and each solved after some schematization of the conceived flow pattern. Hooghoudt eventually incorporated his findings in the so-called elliptic drainage formula. This formula equals equation (4) except that $d = D$ and may be derived by assuming the drainage flow to the drains to be strictly horizontal Dupuit-Forchheimer flow. Extra resistances due to radial flow in the real case are taken into account in the Hooghoudt formula by using d instead of D, where always $d < D$. Clearly, while founded on the basic laws of groundwater flow, the Hooghoudt formula is far from an exact solution of the Laplace equation. Lovell and Youngs (1984) even maintain that by the introduction of the equivalent depth concept, relationships in the formula have become so unspecific that it should be regarded as an empirical formula. To what extent the structure and underlying approximations of the Hooghoudt formula interact with the transformation used in applying it to anisotropic soils is difficult to predict.

NUMERICAL SOLUTION

Finite element solutions of the Laplace equation, enforcing boundary conditions as in Fig. 1, were obtained for each considered case by applying methods described by Zienkiewics and Cheung (1970). In this approach the flow region is divided into a mesh of small triangular elements. The value of the head h at each mesh point, written as $h = A + Bx + Cz$, is found by determining the minimum of the function:

$$E = \frac{1}{2} \iint \left[K_x \left(\frac{\partial h}{\partial x}\right)^2 + K_z \left(\frac{\partial h}{\partial z}\right)^2 \right] \mathrm{d}x \, \mathrm{d}z \tag{5}$$

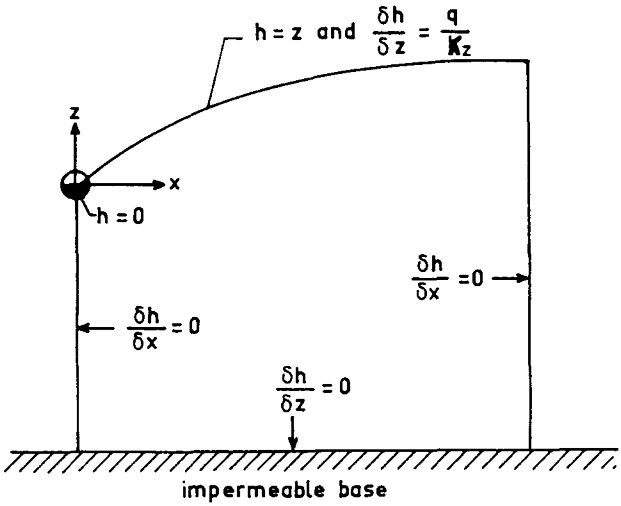

Fig. 1. Flow region of two-dimensional groundwater drainage to parallel drains.

The function E is minimized by subsequently differentiating it with respect to h for all mesh points and setting the resulting equations equal to zero (except for those points in which there is inflow or outflow where they are set equal to the flow rate).

In the case of groundwater drainage design, the drain spacing must be such that the watertable height at mid-spacing attains a pre-determined value for a pre-determined steady recharge of the groundwater. This condition was met by varying the drain spacing until the calculated height matched the required value. First estimates of the drain spacing and watertable position were obtained by solving the Hooghoudt flow model for this purpose, although it is realised that this model does not pretend to give accurate watertable positions.

Use was made of the computer program AFEP (a finite element package) developed by the Department of Mathematics, Technical University, Delft. This program includes a subroutine for covering the flow region with a suitable mesh. The mesh arrangement and density can be varied for different parts of the flow region in order to adapt to convergent or divergent streamline patterns. In the testing stage, different mesh arrangements and densities were compared before deciding on the standard mesh, depicted in Fig. 2, used in all presented results. Drain spacings calculated with this standard mesh were in some cases compared with those obtained with a 3-times denser mesh (in the latter case there are 20 points on the wet entry perimeter of the drain instead of the 10 points used in the standard mesh). Differences were always < 0.5% (see Table 1).

For testing purposes, drain spacings were also calculated both by the numerical and the Hooghoudt method for the following isotropic case: K_h (= K_x) = K_v (= K_z) = 1.5 m/day, D = 5.0 m, h = 1.0 m, q = 0.002 m/day

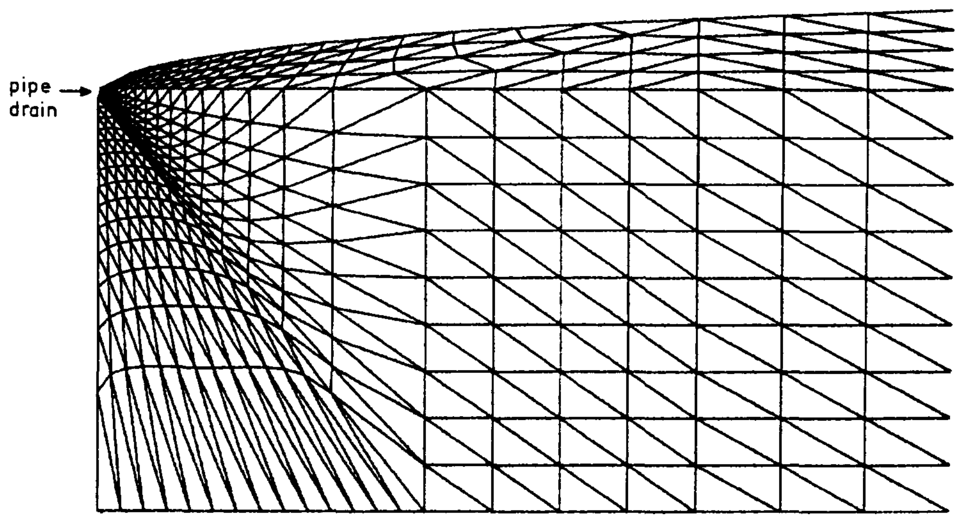

Fig. 2. Mesh used for the numerical solution of the Laplace equation for groundwater drainage to parallel drains.

TABLE 1

Drain spacings calculated by numerical solution and by the Hooghoudt method

Cases	Input parameters[a]						Hooghoudt[b] (m)	Numerical[c] (m)
	D (m)	h (m)	q (m/day)	u (m)	K_h (m/day)	K_v (m/day)		
I	2.5	1	0.002	0.31	1.5	0.06	108	107.3
II	2.5	1	0.002	0.31	1.5	0.12	113	114.2
III	5	1	0.002	0.31	1.5	0.06	118	121.8 (121.2)
IV	5	1	0.002	0.31	1.5	0.12	130	135.8 (135.4)
V	10	1	0.002	0.31	1.5	0.06	125	128.0
VI	10	1	0.002	0.31	1.5	0.12	145	151.4

[a]For explanation of symbols see explanation of symbols in equation (4) in the text.
[b]In all cases the Boumans nomographs were used to solve the Hooghoudt formula.
[c]Results apply to the standard mesh with, between brackets, results for 3-times denser mesh.

and $u = \pi r_0 = 0.31$ m, where $r_0 = 0.10$ m, the drain radius. The calculated drain spacings were nearly equal: numerical method $L = 169$ m and Hooghoudt method $L = 168$ m (in the latter case use was made of the Boumans nomographs, see Wesseling, 1973). This result shows that although the Hooghoudt formula is not an exact Laplace solution, it adequately accounts in this case for the relevant flow characteristics as far as the required drain spacing is concerned. Of course, as Fig. 3 shows, this case conforms closely to the Hooghoudt flow model which assumes the flow to be horizontal in

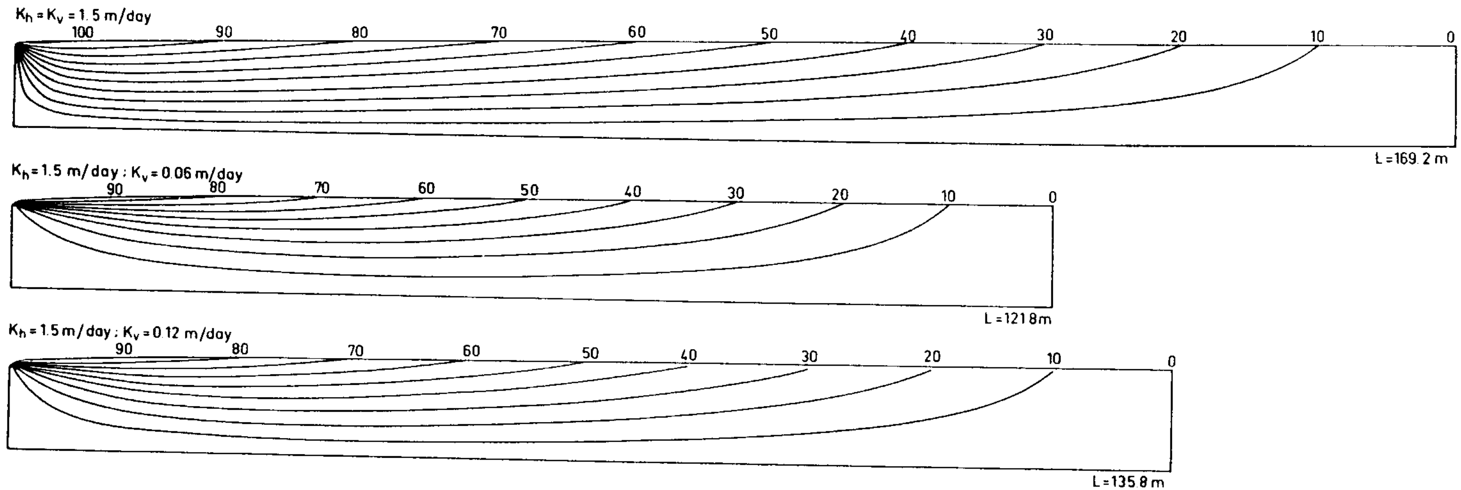

Fig. 3. Comparison of streamline patterns in isotropic and anisotropic soils (cases III and IV of Table 1) compared to the test case.

the region from mid-spacing up to a distance $\frac{1}{2}D\sqrt{2}$ from the drain while the flow is assumed to be radial in the region within distance $\frac{1}{2}D\sqrt{2}$ from the drain.

ANISOTROPIC CASES

Comparison of drain spacings calculated numerically with the finite element method and with the Hooghoudt formula has been made for the six anisotropic cases detailed in Table 1. The Hooghoudt formula was applied after the transformation $x_t = x\sqrt{K_v/K_h}$ and $z_t = z$, which means that all horizontal dimensions were shrunk by a factor $\sqrt{K_v/K_h}$. This applied to the dimensions:

$$L_t = L\sqrt{K_v/K_h}$$

$$u_t = \pi r_0 \tfrac{1}{2} (1 + \sqrt{K_v/K_h}) \qquad \text{(half-circle becoming half-ellipse)}$$

Further:

$$K_t = \sqrt{K_h K_v} \qquad \text{and} \qquad q_t = q/\sqrt{K_v/K_h}$$

The results are presented in Table 1 while examples of streamline patterns, also calculated with the AFEP program, are shown in Fig. 3.

Cases III and IV of Table 1 are comparable to the previously discussed test case, except for the anisotropy. It shows an anisotropy ratio K_h/K_v = 1.5/0.006 = 25 reduces the spacing by some 50 m (L = 168—169 m for the isotropic case vs. L = 118—121 for the anisotropic case) and by some 35—40 m for when the anisotropy ratio is 1.5/0.12 = 12.5. Comparison of the streamline patterns (Fig. 3) show that in the anisotropic cases the bulk of the flow occurs at a shallower depth than in the isotropic case, the more so the higher the anisotropy ratio. In the isotropic case, a better use is made of the full soil depth available for the flow to the drain, resulting in a wider drain spacing. As to be expected, the influence of the anisotropy on the required drain spacing is smaller when the impermeable base occurs at a rather shallow depth (for the cases I and II in Table 1, both with D = 2.5 m, L = 129 m when $K_h = K_v$ = 1.5 m/day, only some 15—20 m wider than for the anisotropic cases) as compared to the case where the impermeable base occurs at greater depth (as e.g. in the cases V and VI where D = 10 m and L = 214 m when $K_h = K_v$ = 1.5 m/day, which is some 60—90 m wider than for the comparable anisotropic cases).

The streamline patterns of Fig. 3 are of course not relevant to the drain spacing calculations with the Hooghoudt formula as this formula was not applied to the real case but to the transformed case obtained by horizontal shrinkage of the flow region by a factor $1/\sqrt{K_v/K_h}$ while simultaneously condensing the recharge by the same factor (these shrinkage/condensing factors are, respectively, 5.00 and 3.57 for anisotropy ratios 25 and 12.5). The applicability of the Hooghoudt formula should be based on the streamline pattern of the transformed case. The example presented in Fig. 4, applying to case III of Table 1, shows that the transformed (equivalent isotropic) flow adheres closely to the classical flow pattern on which the Hooghoudt formula is based and for which situations this formula is known to give reliable drain spacing results. This is confirmed by the closeness of the Hooghoudt results to the numerical results in all the considered cases of Table 1. The latter results may be taken as references ('exact' results) as they are based on the solution of the basic flow equation (eq. 1) of each case. The accuracy of the numerical solution, moreover, was checked by also numerically solving the basic flow equation for the transformed case; in all cases it was found that the relationship $L_t = L\sqrt{K_v/K_h}$ held exactly.

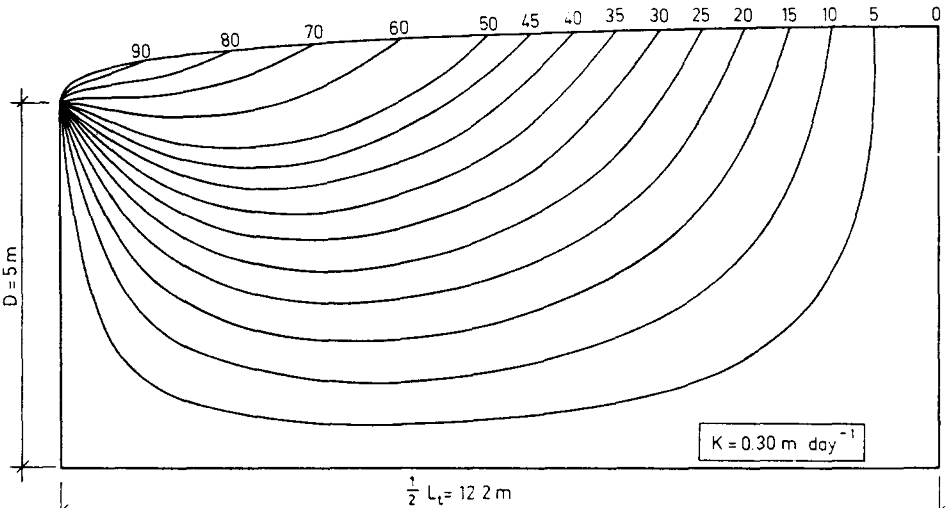

Fig. 4. Streamline pattern for the isotropic equivalent of case III of Table 1 (horizontal and vertical scales are equal).

DISCUSSION

The above results confirm in the first place the validity of the transformation theory: cases of groundwater flow in anisotropic soil may be solved by solving the Laplace equation for the transformed (equivalent isotropic) case. Best results are to be expected when for the latter case the Laplace equation can be solved exactly. For most cases this can readily be done numerically (see above examples) but, as mentioned earlier, few exact analytical solutions are available. Applying drainage formulae like that of Hooghoudt, which are all only approximate solutions of the Laplace equation, to the transformed case can lead to erroneous results as some of the schematizations and simplications underlying these formulae may adversely interact with the applied transformations. The possibility of this occurring depends on the type of schematizations, simplifications and transformations. As stated earlier, for drain spacing calculation problems, transformation in the horizontal direction are most convenient. For a typical case of drainage in an alluvial plain with a substratum having an anisotropy ratio K_h/K_v = 25—30 (Boumans, 1979), horizontal dimensions in the transformed case are shrunk by a factor of 5—5½ as compared to the vertical dimensions which remain unchanged. This may put the transformed case in the range of flow geometries with intermediate L/D values which are less amenable to the Hooghoudt flow schematization. At these intermediate L/D values (the case of Fig. 4 with $L/D \approx 5$ being an example) streamlines are rather curved, meaning that there is considerable vertical flow (neglected in the Hooghoudt formula) while no distinct horizontal and radial flow zones occur. Lovell and Youngs (1984) showed that none of the available drainage formulae are very satisfactory under these conditions. Whether the transformed case ends up in this less favourable L/D range depends on the anisotropy ratio (which determines the shrinkage factor) and on the L/D value of the real case. Given the prevailing conditions in many alluvial plains, most transformed cases may indeed have flow patterns for which the Hooghoudt formula is not at its best. However, even under these less favourable conditions, the Hooghoudt formula is known to give reasonable results for isotropic soils and this would equally apply to the transformed anisotropic cases. The final conclusion therefore is that the Hooghoudt formula can be used with confidence for drain spacing calculation for homogeneous-anisotropic soils.

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