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Development of a Mathematical Model for the Simulation of the Biodegradation of Organic Substrates in a High-Solids Anaerobic Digestion Process

Masoud Kayhanian, George Tchobanoglous

Department of Civil and Environmental Engineering, University of California at Davis, Davis, California 95616, USA

& Joan Mata-Alvarez*

Department of Chemical Engineering, University of Barcelona, Martí i Franquès 1, pta 6, E-08034 Barcelona, Spain

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Abstract: A kinetic model is developed to simulate the biodegradation of the organic fraction of municipal solid waste in a high-solids complete-mix continuous-flow anaerobic digestion process. A mass balance correction factor (f) is used to account for (i) the water that is incorporated into the production of biogas, and (ii) the water vapor present in the biogas. With the inclusion of the factor f, expressions are developed to determine the substrate removal efficiency based on first-order and Monod kinetic models. Simulations using first-order and Monod kinetic models are carried out to illustrate the importance of the f factor on the substrate mass removal and microorganism concentration in the high-solids anaerobic digestion process. The effect of the value of f on kinetic constants, and the error involved in considering the high-solids process as a low-solids process are discussed. Finally, experimental results from a pilot-scale high-solids complete-mix batch-fed anaerobic digestion process are used to estimate the first-order rate constant for the biodegradable fraction of municipal solid waste.

Key words: dry digestion, kinetic, municipal solid waste, biomethanization.

NOTATION

a	Dimensionless constant defined by eqn (40)
A	Dimensionless lumped constant defined by eqn (37)
b	Dimensionless constant defined by eqn (41)
BOF	Biodegradable organic function
BVS	Biodegradable volatile solids
f	Overall mass balance correction factor accounting for stoichiometric and evaporation water (dimensionless)
f'	Stoichiometric correction factor accounting for water (dimensionless)
HRT	Hydraulic retention time (days)
k	First-order rate contant (day^{-1})
k_d	Microorganism rate decay contant (day^{-1})
K_s	Saturation constant (kg substrate per kg wet mass)
M	Total reactor wet mass (kg)
M_w	Average reactor wet mass for batch-fed reactor (kg)
MRT_e	Effluent mass retention time (days)
MRT_i	Influent mass retention time (days)
MSW	Municipal solid waste

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* To whom correspondence should be addressed.

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P	Total pressure (kPa)
p_v	Water vapor pressure (kPa)
Q	Inlet or outlet volumetric flow rate ($dm^3\ day^{-1}$)
r_d	Decay rate of microorganisms (kg microorganism per kg wet mass per day)
r_g	Growth rate of microorganisms (kg microorganism per kg wet mass per day)
r_{ng}	Net growth rate of microorganisms (kg microorganism per kg wet mass per day)
r_{su}	Reaction rate per reactor unit mass (kg BVS per kg wet mass per day)
S	Substrate concentration ($kg\ kg^{-1}$)
S_e	Effluent substrate concentration (kg BVS per kg wet mass)
$S_{e\,pred}$	Predicted effluent substrate concentration (kg BVS per kg wet mass)
S_i	Influent substrate concentration (kg BVS per kg wet mass)
t	Time (days)
T	Temperature (°C)
TS	Total solids ($kg\ kg^{-1}$ wet mass)
V	Reactor volume (dm^3)
V_b	Wet biogas volumetric flow rate ($m^3\ day^{-1}$)
V'_b	Dry biogas volumetric, flow rate ($m^3\ day^{-1}$)
VS	Volatile solids ($kg\ kg^{-1}$ wet mass)
W_b	Wet biogas mass flow rate ($kg\ day^{-1}$)
W'_b	Dry biogas mass flow rate ($kg\ day^{-1}$)
W_e	Total wet effluent mass flow rate ($kg\ day^{-1}$)
W_{ew}	Evaporated water mass flow rate of ($kg\ day^{-1}$)
W_i	Total wet influent mass flow rate ($kg\ day^{-1}$)
X_e	Microorganism concentration at the reactor outlet (kg microorganism per kg wet mass)
X_i	Microorganism concentration at the reactor inlet (kg microorganism per kg wet mass)
ρ	Density ($kg\ m^{-3}$)
μ	Specific microorganism growth rate (kg microorganism per kg wet mass per day)
μ_m	Maximum specific microorganism growth rate constant (kg microorganism per kg wet mass per day)

1 INTRODUCTION

Digestion of organic substrates at higher total solids (TS) concentrations is advantageous when considering the anaerobic digestion of the biodegradable organic fraction of municipal solid waste. Digestion at TS concentrations of between 23 and 32% is classified as high-solids anaerobic digestion (also as dry fermentation).[1,2] An early study of the high-solids biodegradation of sewage sludge was conducted by Schulze.[3] More recent studies on high-solids anaerobic digestion processes, in the United States, focused primarily on municipal solid waste.[4–6] Other significant contributions to this subject have been made by several European groups.[7–9] All published work to date is related to system design, operational and performance characteristics. A mathematical model that can address an overall mass balance in a high-solids anaerobic digestion process is not available in the literature. Recently, an attempt was made by Richards *et al.*[10] to analyze the kinetics of a high-solids digester that partially satisfies the overall mass balance as described below.

In general, the following Buswell and Muller[11] stoichiometric relationship is used to approximate the removal of substrate and to predict the biogas volume and composition.

$$\begin{aligned} C_aH_bO_cN_d &+ (4a-b-2c+3d)/4H_2O \\ &\rightarrow (4a-b+2c+3d)/8CO_2 \\ &\quad + (4a+b-2c-3d)/8CH_4 \\ &\quad + dNH_3 \end{aligned} \tag{1}$$

Equation (1) can be modified to include bicarbonate as a product, as given below.[10]

$$\begin{aligned} C_nH_aO_bN_c &+ (n-0{\cdot}25a-0{\cdot}5b+1{\cdot}75c)H_2O \\ &\rightarrow (0{\cdot}5n+0{\cdot}125a-0{\cdot}25b-0{\cdot}375c)CH_4 \\ &\quad + (0{\cdot}5n-0{\cdot}125a+0{\cdot}25b-0{\cdot}625c)CO_2 \\ &\quad + cNH_4^+ + cHCO_3^-. \end{aligned} \tag{2}$$

In eqn (2) it is assumed that the ammonia released is retained in solution and is balanced by bicarbonate derived from product carbon dioxide.

When using either eqn (1) or eqn (2), it is important to note that the mass of gas formed is greater than the mass of the substrate converted in the process. Because of the water requirement (see the stoichiometry of reactions), the actual substrate mass removed is equal to a fraction of the measured dry biogas mass (a small fraction comes also from water). Additionally, the gas produced from an anaerobic digestion process is normally saturated with water vapor. To develop an accurate mass balance, the quantity of gas produced, measured at a given temperature and pressure at the reactor outlet, must be converted to the corresponding quantity of dry gas at standard temperature and pressure. When dealing with the high-solids anaerobic digestion process, it is critical to take into account (i) the water that is incorporated into the production of biogas, and (ii) the water vapor present in the biogas. Both elements influence the mass balance which has not been addressed in the literature.

The present study was undertaken to advance the following specific objectives:

1. to develop a mathematical model for the substrate removal efficiency and the growth rate of microorganisms in a high-solids complete-mix continuous-flow anaerobic digestion process, using a generalized mass balance correction factor;

2. to simulate the high-solids complete-mix continuous-flow anaerobic digestion process using first-order and Monod kinetic model; and
3. to predict the first-order kinetic constant for the biodegradable organic fraction of MSW (BOF/MSW) from field data.

2 GENERAL STEPS USED TO DEVELOP THE MATHEMATICAL MODEL

Steps followed to develop a generalized mass balance for mathematical expressions include (i) unit convention, (ii) water uptake, (iii) water vapor in digester gas, (iv) generalized mass balance correction factor, and (v) mass retention time versus hydraulic retention time.

2.1 Unit convention

In a conventional low-solids anaerobic digester the kinetic relationships, based on a mass or volume basis, are essentially the same, as the density of the active reactor media is close to unity. The concentrations and rates in these systems are reported using the units g dm^{-3} or g dm^{-3} time^{-1}. For digesters with high-solids concentrations, these relationships do not hold true, as the density of the material in the reactor is variable.[10] In addition, in a high-solids complete-mix anaerobic digester, there is a significant difference between the influent and effluent quantities, based on mass. Using a mass basis for process analysis is more reliable for research evaluations, as the reactor mass is not affected by substrate characteristics and gas production rate. For the mathematical models developed in this paper, mass concentrations, mass flow rates, and mass growth (or decay) rates are expressed as kg kg^{-1}, kg time^{-1}, and kg kg^{-1} time^{-1}, respectively.

2.2 Water uptake in anaerobic digestion

Water uptake during the production of biogas must be considered in the development of a substrate mass balance for a high-solids anaerobic digestion process. As indicated in the introduction, because of the water requirement, the actual biodegradable substrate mass removed is some fraction, f', of the measured mass of dry biogas. The value of the stoichiometric correction factor f' can be estimated from the chemical composition of the biodegradable portion of the organic waste. The biodegradable fraction of an organic waste is defined as the fraction of the waste that can be converted to biogas and other end products under optimum conditions in a period of 30–60 days.[5] Biodegradable volatile solids (BVS) is a better measure of the substrate than the use of volatile solids (VS), especially when dealing with the organic fraction of MSW which contains a significant amount of lignin.[4,12] Determination of BVS is discussed in detail in Chandler *et al.*[13] The chemical composition of the mixed organic fraction of MSW, and the corresponding value of the biodegradable fraction used as a feedstock in the pilot-scale studies, is reported in Table 1. Using the data given in Table 1, and assuming that the composition of BOF/MSW is the same as that for the total organic waste, the empirical formula for the BOF/MSW is found to be $C_{43}H_{68}O_{29}N$. Using this empirical formula, and eqn (2), the corresponding value of f' for this waste is about 0·82. The value of f' normally ranges from 0·7 to 1, and depends largely on the chemical characteristics of the organic substrate (e.g. $f' = 1$ for glucose).

2.3 Water vapor in digester gas

The gas produced from the high-solids anaerobic digestion process is normally saturated with water vapor. The quantity of gas produced, measured at a given temperature and pressure at the reactor outlet, must be converted to the corresponding quantity of dry gas at standard temperature and pressure conditions. The mass of water vapor present in the digester gas can be estimated using the perfect gas law.

2.4 Generalized mass balance correction factor

An overall mass balance for the high-solids complete-mix continuous-flow anaerobic digestion process depicted in Fig. 1, assuming steady-state conditions ($dM/dt = 0$), will yield:

$$W_i = W_e + W_b \tag{3}$$

where W_i and W_e are the influent and effluent mass flow rates, respectively, and W_b is the wet biogas mass flow

TABLE 1
Representative Composition Data for the Organic Fraction of Municipal Solid Waste used as a Substrate for the Pilot-Scale High-Solids Complete-Mix Batch-Fed Anaerobic Digestion Process[a]

Item	*Symbol*	*Units*	*Value*
Ultimate analysis			
Carbon	C	% TS	45·66
Nitrogen	N	% TS	1·23
Hydrogen	H	% TS	5·99
Oxygen	O	% TS	40·59
Sulfur	S	% TS	0.25
Chlorine	Cl	% TS	0·28
Residue		% TS	6·10
Biodegradability			
Lignin content	LC	% VS	4·75
Biodegradable fraction[b]	BF	% VS	69·7

[a] From Reference 5. Note: additional waste characterization data are available in this reference.
[b] BF = 0·83 − (0·028) × LC (see Ref. 13).

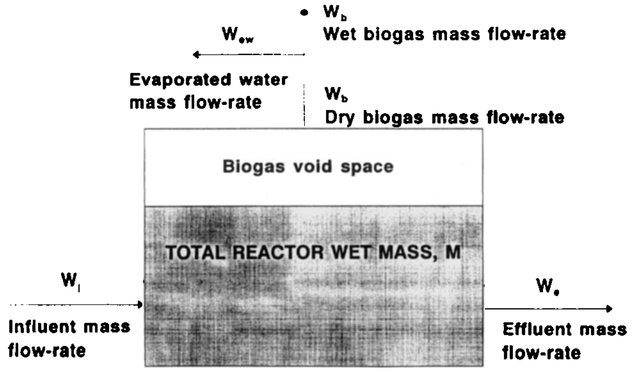

Fig. 1. Mass balance diagram for a high-solids complete-mix continuous-flow anaerobic digestion process at steady state condition.

rate. The wet biogas mass flow rate is equal to the dry biogas mass flow rate (W'_b) plus the mass flow rate of the water vapor in the biogas (W_{ew}):

$$W_b = W'_b + W_{ew} \tag{4}$$

In eqn (7), W'_b is equal to the difference between the substrate influent and effluent mass flow rates, corrected by the stoichiometric factor f'. The value of W'_b can be expressed in a more general form as follows:

$$W'_b = (W_i S_i - W_e S_e)/f' \tag{5}$$

where S_i and S_e are the influent and effluent substrate mass concentrations. An alternative, and more practical expression, for eqn. (5) in which the wet biogas production is used directly is:

$$W_b = (W_i S_i - W_e S_e)/f \tag{6}$$

where f is a mass balance correction factor, that takes into account both the stoichiometric factor f' and the evaporated water, W_{ew}. From eqns (4), (5), and (6) it can be shown that f is equal to:

$$f = (f' W'_b)/(W'_b + W_{ew}) \tag{7}$$

From eqn (7), it is clear that the generalized mass balance correction factor is a fraction of the stoichiometric factor f'. The variation in this fraction is associated with moisture content of biogas.

2.5 Mass retention time versus hydraulic retention time

Conventionally, the hydraulic retention time (HRT) is defined as the reactor volume divided by the volumetric input rate. The mathematical expression of HRT is:

$$HRT = V/Q \tag{8}$$

where V is the reactor volume and Q the influent (or effluent) volumetric flow-rate. In low-solids liquid digestion systems, it is generally assumed that the rate of inflow equal the rate of outflow. The assumption that the inflow is equal to outflow is not correct for the high-solids anaerobic digestion process because of the significant removal of mass in the form of biogas. Thus, a mass based definition of retention time is more appropriate for a high-solids anaerobic digestion process. The

TABLE 2
Typical Values for the Operating Variables for a Pilot-Scale High-Solids Complete-Mix Batch-Fed Anaerobic Digestion Process using BOF/MSW as a Substrate[a]

Item	*Symbol*	*Unit*	*Operating condition*	
			1	*2*
Average reactor wet mass	M_w	kg	1890	1890
Influent mass flow rate	W_i	kg day^{-1}	93	63·2
Influent total solids concentration	TS	kg kg^{-1} inflow wet mass	0·29	0·29
Influent volatile solids concentration	VS	kg kg^{-1} inflow wet mass	0·23	0·27
Influent substrate (BVS) concentration	S_i	kg BVS per kg inflow wet mass[b]	0·129	0·19
Effluent mass flow rate	W_e	kg day^{-1}	80·1	49·3
Wet biogas volumetric flow rate at 328 K and 101·3 kP	V_b	m^3 day^{-1}	12·356	13·264
Wet biogas mass flow rate	W_b at 328 K and 101·3 kPa	kg day^{-1}	12·9[c]	13·9[c]
Influent mass retention time	MRT_i	days	20·3	29·9
Effluent mass retention time	MRT_e	days	23·6	38·3

[a] Data are from Ref. 5. The operating data are based on the digestion of the organic fraction of municipal solids waste as reported in Table 1.
[b] BVS, biodegradable volatile solids (see Table 1).
[c] Computed value by difference ($W_b = W_i - W_e$).

mass retention time referred to the inflow stream (MRT_i) is defined as:

$$MRT_i = M/W_i \tag{9}$$

where M is the total reactor wet mass and W_i is the total wet influent mass flow rate, which includes substrate, dilution water, and other components. An alternative definition based on the effluent mass flow rate, W_e, is:

$$MRT_e = M/W_e \tag{10}$$

where MRT_e is defined as the effluent mass retention time. Because MRT_e and MRT_i are substantially different, conventional mass balances are not reliable in high-solids systems.

From eqns (9) and (10), the following relationship between both the influent and effluent mass retention times can be obtained:

$$MRT_e/MRT_i = W_i/W_e \tag{11}$$

Using eqns (3) and (6), it is now possible to write:

$$W_i/W_e = (f - S_e)/(f - S_i) \tag{12}$$

Substituting eqn (12) into eqn (11), the relationship between MRT_i and MRT_e as a function of substrate concentrations and the generalized mass balance correction factor is:

$$MRT_e/MRT_i = (f - S_e)/(f - S_i) \tag{13}$$

From eqn (13), it is clear that only in the case of no biodegradation can the value of MRT_i equal the value of MRT_e. With biological activity, the ratio MRT_e/MRT_i will always be greater than one. Using the digester operating characteristics reported in Table 2 and assuming (i) an organic substrate with the chemical characteristics shown in Table 1, (ii) a complete biodegradation of the BVS ($S_e = 0$), and (iii) an f value of 0·75, the ratio of MRT_e/MRT_i for operating conditions 1 and 2 based on eqn (13) is approximately 1·2 and 1·34, respectively. The computed values would be the maximum ratios expected for a high-solids anaerobic digestion process operating with this substrate.

3 MATHEMATICAL MODEL USING FIRST-ORDER KINETICS

A substrate mass balance for an anaerobic digestion process, carried out in a complete-mix and continuous-flow reactor, is expressed as follows:

$$(dS_e/dt)M = W_i S_i - W_e S_e - r_{su} M \tag{14}$$

where r_{su}, the reaction rate per unit mass, will depend on the reactor substrate concentration in accordance with the kinetic expressions used to describe the system. Considering eqn (6), the conversion term is related to the biogas production through:

$$f W_b = r_{su} M \tag{15}$$

Using eqns (3) and (15) and assuming steady state ($dS_e/dt = 0$, eqn (14) is transformed into:

$$W_i(S_i - S_e) = r_{su} M(1 - S_e/f) \tag{16}$$

In low-solids systems, it is assumed that $W_i = W_e$, so that eqn (16) becomes:

$$W_i(S_i - S_e) = r_{su} M \tag{17}$$

Comparing eqns (16) and (17), it can be seen that substrate mass balance in high-solids anaerobic digestion processes differs from conventional substrate mass balances for low-solids processes term by $(1 - S_e/f)$.

Dividing both sides of eqn (16) by W_i and applying eqn (9) yields the following relationship:

$$(S_i - S_e)/MRT_i = r_{su}(1 - S_e/f) \tag{18}$$

In a similar manner, the following equation can be obtained.

$$(S_i - S_e)/MRT_e = r_{su}(1 - S_i/f) \tag{19}$$

The basic relationship for first-order kinetic model is:

$$r_{su} = kS_e \tag{20}$$

Substituting this rate expression into eqn (18) and rearranging terms, the following quadratic relationship is obtained:

$$(kMRT_i/f)S_e^2 - (1 + kMRT_i)S_e + S_i = 0 \tag{21}$$

Solving for S_e yields:

$$S_e = \{1 + kMRT_i - [(1 + kMRT_i)^2 - 4S_i kMRT_i/f]^{1/2}\}/(2kMRT_i/f) \tag{22}$$

It should be noted that the corresponding expression for S_e for a low-solids anaerobic digestion process would be as follows:

$$S_e = S_i/(1 + kMRT_i) \tag{23}$$

Similarly, substituting kS_e for r_{su} into eqn (18) yields:

$$(S_i - S_e)/MRT_e = kS_e(1 - S_i/f) \quad (24)$$

Expression (24) can be used to estimate the first-order kinetic coefficient, k, using the value obtained from eqn (22) and other given parameters.

4 MATHEMATICAL MODEL USING MONOD KINETICS

Most of the kinetic models for continuous biological treatment systems are based on the Monod equation. Lawrence and McCarty[14] were among the first researchers to apply the Monod equation to model the growth rate of bacteria in anaerobic digesters. The classic Monod expression relates the specific growth rate with substrate concentration according to the following relationship:

$$\mu = \mu_m S/(K_s + S) \quad (25)$$

where μ is the specific growth rate, whereas μ_m and K_s are the constants of the model (see Notation for units).

If the microorganism growth rate r_g is defined by $r_g = \mu X$, where μ is the specific growth rate and X is the cell concentration, the resulting expression for the growth rate in a complete-mix reactor, such as shown in Fig. 1 is:

$$r_g = \mu_m S_e X_e/(K_s + S_e) \quad (26)$$

Substrate utilization and growth rate are linked by the cell yield coefficient Y:

$$r_g = Yr_{su} \quad (27)$$

Thus, for Monod type kinetics:

$$r_{su} = kS_e X_e/(K_s + S_e) \quad (28)$$

where $k = \mu_m/Y$, is the maximum rate of substrate utilization.

If the rate of endogenous decay, expressed as:

$$r_d = -k_d X_e \quad (29)$$

is taken into account, the overall net growth rate is given by:

$$r_{ng} = Yr_{su} - k_d X_e \quad (30)$$

A microorganism mass balance for a high-solids complete-mix continuous-flow anaerobic digestion process, is as follows:

$$(\mathrm{d}X_e/\mathrm{d}t)M = W_i X_i - W_e X_e + r_{ng} M \quad (31)$$

where X_i and X_e are the microorganism mass concentrations at the digester inlet and outlet, respectively. Assuming that X_i is relatively small compared with X_e and steady-state conditions ($\mathrm{d}X_e/\mathrm{d}t = 0$), eqn (31) can be combined with eqn (10) and written in the following form:

$$X_e/MRT_e = r_{ng} \quad (32)$$

If eqn (28) and (30) are combined with the microorganism balance, eqn (32), then:

$$1/MRT_e = \mu_m S_e/(K_s + S_e) - k_d \quad (33)$$

Rearranging terms, an expression for S_e can be found:

$$S_e = K_s(1 + k_d MRT_e)/(\mu_m MRT_e + k_d MRT_e - 1) \quad (34)$$

To develop an expression for S_e as a function of MRT_i and the generalized mass correction factor, the relationship between MRT_i and MRT_e given in eqn (13) can be used. Using eqn (13), eqn (34) can be transformed into the following quadratic equation:

$$(k_d + \mu_m)S_e^2 - [\mu_m f + k_d(K_s + f) - (f - S_i)/MRT_i]_e + K_s[(f - S_i)/MRT_i + k_d f] = 0 \quad (35)$$

which can be solved for S_e. Assuming that the digestion process is operated in a region where the decay rate can be ignored.

$$S_e^2 - (f - A)S_e + K_s A = 0 \quad (36)$$

where A is a lumped constant, given by:

$$A = (f - S_i)/(\mu_m MRT_i) \quad (37)$$

Solving eqn (36) for S_e yields:

$$S_e = \{(f - A) - [(f - A)^2 - 4K_s A]^{1/2}\}/2 \quad (38)$$

An expression for microorganism concentration may be obtained for the Monod kinetic model. Substituting eqn (28) in the substrate mass balance (eqn (18)) and rearranging terms, the following relationship is obtained:

$$X_e = [Y(K_s + S_e)(S_i - S_e)]/[MRT_i(1 - S_e/f)\mu_m S_e] \quad (39)$$

which, when combined with eqn (38), can be used to obtain an expression for X_e as a function of the kinetic constants and the operating parameter MRT_i. It should be noted that the relationship between constants, $k = \mu_m/Y$ has also been used to develop eqn (39).

The substrate and microorganism mass balances developed above are used subsequently to simulate a high-solids anaerobic digestion process.

5 PROCESS SIMULATION

Mathematical modes developed for the first-order and Monod kinetics were used to simulate the substrate removal efficiency and the growth rate of microorganisms in a high-solids anaerobic digestion process. The importance of a mass balance correction factor for both simulation processes is examined and its significance in comparison to a low-solids anaerobic digestion process is discussed. It is important to note that most input data used are obtained from a long-term pilot study.[4,5]

5.1 Simulation of substrate removal efficiency using first-order kinetics

As given by eqn (23), substrate removal is only dependent on the retention time and the first-order kinetic coefficient for low-solids process. In a high-solids system, the substrate removal not only depends on retention time and first-order kinetic constant, it also depends upon the absolute value of the initial substrate concentration S_i and on the overall mass balance factor f.

The removal of biodegradable volatile solids (BVS) has been computed using eqns (22) and (23) to study the behavior of a high-solids complete-mix anaerobic digestion process using the parameters f' and f and to compare the results with the low-solids process. As a basis for comparison, the operating values for condition 2, as presented in Table 2, have been used. The influent substrate concentration used for operating condition 2 is 0·19 kg BVS per kg inflow wet mass. Profiles of substrate removal as a function of the term $kMRT_i$ for different values of f ranging from 0·7 to 1 for operating condition 2 are presented in Fig. 2. As shown in Fig. 2, the substrate removal percentage increases in the values of f and the product $kMRT_i$. To achieve removals greater than 90%, the value of the term $kMRT_i$ must be greater than 6·2 for $f = 1{\cdot}0$ and greater than 9·5 for $f = 0{\cdot}7$. Consequently, for a given value of k, assuming a value of $f = 1$, if, for instance, the real value is 0·7, the increase in the MRT_i value would be roughly 50%. This increase in the MRT_i value would cause a significant error on the computation of the reactor size.

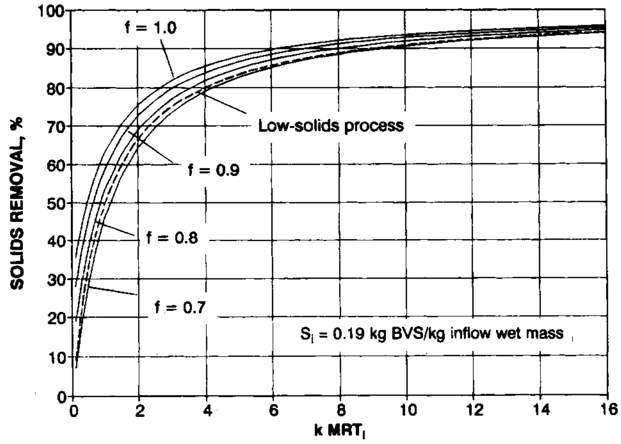

Fig. 2. Substrate (BVS) removal profiles for a high-solids complete-mix continuous-flow anaerobic digestion process, following a first-order model for different values of the mass balance correction factor f. The dotted line corresponds to the profile obtained with the hypothesis of a low-solid process.

The influence of S_i on substrate removal is limited. For example, with values of S_i ranging from 0·1 to 0·2 kg BVS per kg wet mass, the corresponding removal percentages range from 90·15 to 90·03. Clearly, the variation is absorbed completely by the experimental error of measurement.

5.2 Simulation of substrate removal efficiency using Monod kinetics

Assuming the same conditions used in the simulation with the first-order model, with the same basic biodegradable substrate concentration values ($S_i = 0{\cdot}19$ kg BVS per kg inflow wet mass, see operating condition 2 in Table 2), a simulation of the substrate removal efficiency can be carried out using Monod kinetics. To facilitate the generalization of the results, two dimensionless parameters, a and b, have been used. They are defined as follows:

$$a = MRT_i \mu_m \tag{40}$$

and

$$b = S_i/K_s \tag{41}$$

Parameter a is a dimensionless residence time: good conditions for growth are achieved with increasing values of a, which can either be obtained with increasing values of μ_m or with increasing values of the MRT_i. Parameter b is a dimensionless concentration term, characteristic of the microorganism growth rate conditions. Large values of b favoring the rate of growth, are obtained either from small values of the kinetic constant K_s or from large initial concentrations, S_i.

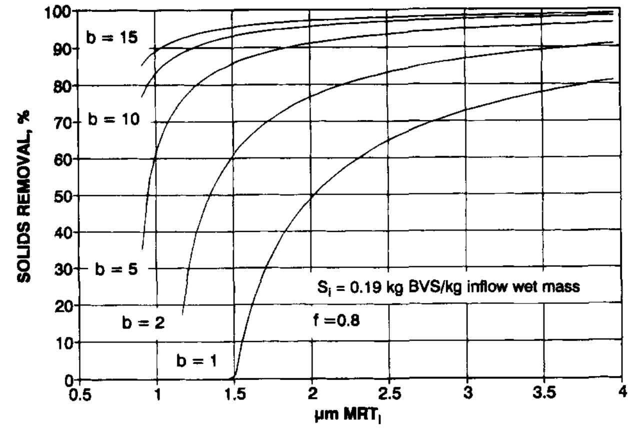

Fig. 3. Substrate (BVS) removal profiles for a high-solids complete-mix continuous-flow anaerobic digestion process, following a Monod model for different values of parameter *b*.

As before, the solids removal rate is the parameter chosen to measure the response of the system. In the first case, a basic value of $f = 0{\cdot}8$ has been set and the effect of parameters *a* and *b* is assessed. A plot of the substrate (BVS) removal percentage as a function of parameter a ($\mu_m MRT_i$), for a range of *b* values from 1 to 15 is given in Fig. 3. As can be seen, substrate removal is dependent on parameter *b*, especially at small values of *a*. For instance, for a 90% removal and for a given set of kinetic constants K_s and μ_m, if S_i is decreased five times, so that *b* moves from 10 to 2, the corresponding value of the product $\mu_m MRT_i$ will increase from about 1·2 to about 3·7, which means that the MRT_i will have to increase by about 300%.

The effect of the factor *f* on substrate removal for a basic value of $b = 5$ is reported in Fig. 4. As can be seen, the effect of *f* on a Monod model for biodegradation is similar to that in the first-order model. However, a large difference will occur if it is assumed wrongly that the process behaves as a low-solids process. The equation for this latter case would be

$$S_e = K_s/(\mu_m MRT_i - 1) \qquad (42)$$

and, using parameters *a* and *b*:

$$S_e = (S_i/b)/(a - 1) \qquad (43)$$

The performance profile for a low-solids system using the same values of S_i and *b* is presented in Fig. 4. As can be seen, there is a large difference, especially with the profiles at lower *f* values, which are the most probable. For instance, assuming a value of $f = 0{\cdot}8$ and a substrate removal efficiency of 90% the necessary value of $\mu_m MRT_i$ predicted for the high-solids process is nearly 50% lower than the value predicted by a conventional low-solids approach.

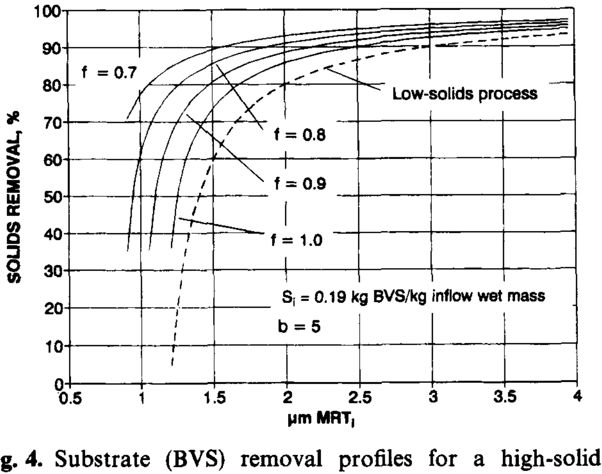

Fig. 4. Substrate (BVS) removal profiles for a high-solids complete-mix continuous-flow anaerobic digestion process, following a Monod model for different values of the mass balance correction factor *f*. The dotted line corresponds to the profile obtained with the hypothesis of a low-solid process.

5.3 Simulation of microorganism concentration using Monod kinetics

To simulate the effect on microorganism concentration, eqn (39) is transformed using the dimensionless parameters defined in eqns (40) and (41) to obtain:

$$X_e = [Y(1/b + S_e/S_i)(S_i - S_e)]/[a(1 - S_e/f)S_e/S_i] \qquad (44)$$

The value of X_e is plotted as a function of a ($\mu_m MRT_i$) in Figs 5 and 6. As can be seen in eqn (44), the microorganisms, concentration is proportional to the yield coefficient *Y*. Both figures were constructed using a typical value of $Y = 0{\cdot}05$ for an anaerobic digestion process.[15] Figure 6 also includes a profile for a low-solids process. The expression of X_e for a low-solids process derived from eqns (17) and (28), is similar to eqn (44), but with the term $(1 - S_e/f)$ absent in the denomi-

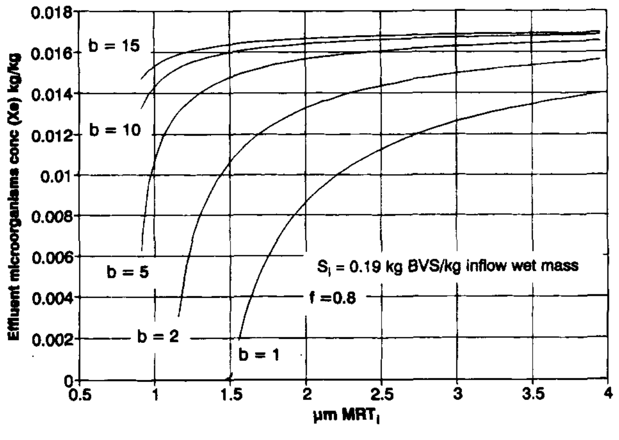

Fig. 5. Effluent microorganism concentration (X_e) profiles for a high-solids complete-mix continuous-flow anaerobic digestion process, following a Monod model for different values of parameter *b*.

nator. The corresponding value for S_e is given by eqn (43). As can be seen, there is a complete parallelism between Figs 3 and 5 on one hand and Figs 4 and 6 on the other. It is clear that the selection of a correct mass balance factor f for microorganism concentration, is essential when the Monod kinetic model is used.

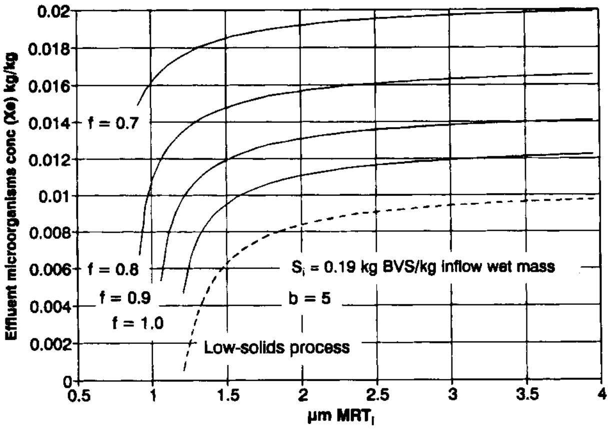

Fig. 6. Effluent microorganism concentration (X_e) profiles for a high-solids complete-mix continuous-flow anaerobic digestion process, following a Monod model for different values of the mass balance correction factor f. The dotted line corresponds to the profile obtained with the hypothesis of a low-solid process.

6 ESTIMATION OF FIRST-ORDER REMOVAL RATE CONSTANT FOR THE BIODEGRADABLE ORGANIC FRACTION OF MSW

Long-term experimental results for two operating conditions for a high-solids complete-mix batch-fed anaerobic digestion process used to digest the BOF/MSW (see Table 2) were used to estimate the first-order removal rate constant and to test the first-order kinetic model

TABLE 3

Estimated and Predicted First-Order Removal Rate Constants for a Pilot-Scale High-Solids Complete-Mix Batch-Fed Anaerobic Digestion Process using BOF/MSW as a Substrate

Item	*Symbol*	*Unit*	*Operating condition*	
			1	*2*
Total influent mass flow rate	W_i	kg day^{-1}	93	63·2
Total effluent mass flow rate	W_e	kg day^{-1}	80·1	49·3
Influent substrate (BVS) concentration	S_i	kg kg^{-1}	0·129[b]	0·19[b]
Wet biogas volumetric flow rate at 378 K and 101·3 kPa	V_b	m^3 day^{-1}	12·356	13·264
Biogas composition				
Carbon dioxide	CO_2	%	49	48
Methane	CH_4	%	51	52
Dry biogas volumetric flow rate at STP at 273 and 101·3 kPa	V'_b	m^3 day^{-1}	8·64[c]	9·28[c]
Biogas density at 273 K and 101·3 kPa	ρ	kg m^{-3}	1·346	1·346
Dry biogas mass flow rate	W'_b	kg day^{-1}	11·64[d]	12·49[d]
Stoichiometric correction factor	f'		0·82[e]	0·82[e]
Effluent substrate concentration	S_e	kg kg^{-1}	0·028[f]	0·03[f]
Evaporated water mass flow rate	W_{ew}	kg day^{-1}	1·29[g]	1·39[g]
Mass balance correction factor	f		0·74[h]	0·74[h]
First-order rate constant	k	day^{-1}	0·185[i]	0·187[i]
Predicted effluent substrate concentration	$S_{e\,pred}$	kg kg^{-1}	0·028[j]	0·031[j]

[a] The operating conditions 1 and 2 reported are based on an MRT_i value of 20·3 and 29·9 days respectively.

[b] BVS fraction of organic waste based on lignin content and the results of long-term batch studies.

[c] The volumetric flow rate of dry biogas at standard temperature and pressure is computed using the following expression: $V'_b(STP) = V_b(273/273 + T)[(P - p_v)/P]$

[d] $W'_b = V'_b \times \rho$

[e] Estimated from empirical formula ($C_{43}H_{68}O_{31}N$) using eqn (2).

[f] Computed from field measurements using eqn (5).

[g] Computed using the expression: $n = p_v V/RT$ (The value of W_{ew} can be also obtained by subtraction ($W_b - W'_b$)).

[h] Computed using eqn (7).

[i] Computed using eqn (24).

[j] Computed using eqn (22).

developed in this paper. The data and the estimated k values are presented in Table 3. The equations derived from a complete-mix continuous-flow process and the average reactor wet mass (M_w) were used to estimate the k values. The estimated k values, based on dry biogas, for the two operating conditions are essentially the same, with a mean value of 0·186 day^{-1}. The corresponding value based on VS is about 0·13 day^{-1}. Similar results for the first-order rate constant for the organic fraction of MSW based on VS have been reported.[16] The estimated k values, along with the other experimental results given in Table 3, can also be used to predict the effluent substrate concentration. As reported in Table 3, the predicted S_e values for both operating conditions are similar to the experimental values.

7 CONCLUSIONS

The conclusions that can be drawn from this study are:

(1) For the high-solids anaerobic digestion process, mass units should be used to define retention times instead of volumetric units. Moreover, an accurate mass balance must account for the water that is incorporated into the production of biogas, and the water vapor present in the biogas. These two elements can be combined and used as a generalized mass balance factor f. If an available kinetic model is used in a high-solids system without the inclusion of factor f, a significant error for the prediction of substrate removal will result.

(2) When first-order and Monod kinetics are used for a generalized mass balance, it is important to estimate the value of the mass balance factor f correctly, to avoid significant errors in estimating substrate removal, microorganism concentration and the required reactor volume. The reactor volume estimation error increases with increased substrate removal. The influence of influent substrate concentration, S_i, on the performance of a high-solids process, using a first-order kinetic model, is negligible, whereas the kinetic constant plays a major role on the system performance. With the Monod model, the kinetic constant and initial substrate concentration are both significant, influencing process design and performance.

(3) The model developed can be used to predict the first-order kinetic constant. The first-order kinetic constant computed for the BOF/MSW, using two average influent mass retention times of about 29·9 and 20·3 days were found to be essentially the same, with a mean value of 0·186 day^{-1}. Compared with a low-solids process, the first-order kinetic constant value is about 5–10 times high in a high-solids process for similar substrate.

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REFERENCES

1. Wuijick, W. J. & Jewell, W. J., Dry anaerobic fermentation. *Biotechnol. Bioeng. Symp.*, **10** (1980) 43–65.
2. Jewell, W. J., Dell'Orto, S., Fanfoni, K. J., Fast, S., Jackson, D. & Cabrick, D. J., *Dry fermentation of agricultural wastes.* Annual Report No. XB-0-9308-1-7. Cornell University, Ithaca, New York, 1981.
3. Schulze, K. L., Sludge digestion and methane fermentation. Part 1, Increased solids concentration. *Sewage and Industrial Wastes*, **30** (1958) 28–33.
4. Kayhanian, M., Lindenauer, K., Hardy, S. & Tchobanoglous, G., Two-stage process combines anaerobic and aerobic methods. *Biocycle*, **32** (1991) 48–53.
5. Kayhanian, M. & Tchobanoglous, G., Innovative two-stage anaerobic process for the recovery of energy and compost from the organic fraction of municipal solid waste (MSW). *Water Sci. Tech.*, **27** (2) (1993) 133–43.
6. Rivard, J. C., Himmel, M. E., Vinzant, T. B., Adney, W. S., Wyman, C. E. & Krohmann, K., *Anaerobic high solids fermentation of processed municipal solid wastes for the production of methane.* Biotechnology Research Branch, Solar Energy Research Institute, Golden, Colorado 80401, 1987.
7. De Baere, L., Verdonck, O. & Verstraete, W., High rate dry anaerobic composting process for the organic fraction of solid wastes. *Biotechnol. Bioeng. Symp.*, **15** (1985) 321–32.
8. The Valorga Process, Waste recovery as a source of methane and fertilizer. In *2nd Annual Int. Symp. On Ind. Resource Management*, February 17–20, Philadelphia, USA, 1985, pp. 234–44.
9. Cecchi, F. & Mata-Alvarez, J., Biodegradation of solid waste by anaerobic digestion: a developing technology in Europe. In *Biotechnology and Biodegradation*, eds D. Kamely, A. Chackrabardy & G. S. Omenn. Portfolio Publ. Comp., 1990, pp. 1–27.
10. Richards, K. B., Cummings, R. J., White, E. T. & Jewell, W. J., Methods for kinetic analysis of methane fermentation in high solids biomass digesters. *Biomass and Bioenergy*, **1**(2) (1991) 65–73.
11. Buswell, A. M. & Muller, H. F., Mechanics of methane fermentation. *Journal of Industrial Engineering Chemistry*, **44** (3) (1952) 550–9.
12. Mata-Alvarez, J., Cecchi, F., Pavan, P. & Fazzini, G., Performances of digesters treating the organic fraction of municipal solid waste differently sorted. *Biological Wastes*, **33** (1990) 181–99.
13. Chandler, J. A., Jewell, W. J., Gossett, J. M., Vansoset, P. J. & Robertson, J. B., Predicting methane fermentation biodegradability. *Biotechnology and Bioengineering Symp.*, **10** (1980) 93–107.
14. Lawrence, A. W. & McCarty, P. L., Kinetic of methane fermentation in anaerobic treatment. *Journal WPCF*, **41** (1969) R1–R17.
15. Mata-Alvarez, J. & Cecchi, F., A review of kinetic models applied to the anaerobic biodegradation of complex

organic matter. Kinetics of the biomethanization of the organic fraction of municipal solid waste. In *Biotechnology and Biodegradation*, eds D. Kamely, A. Chackrabardy, & G. S. Omenn. Portfolio Publ. Comp, 1990, pp. 1123–63.

16. Owens, J. M. & Chynoweth, D. P., Biochemical methane potential of MSW components. In *Proceedings of the International Symposium on Anaerobic digestion of Solid Waste*, eds F. Cecchi, J. Mata-Alvarez & F. G. Pohland. Venice, Italy, April 14–17, 1992, pp. 29–42.