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Flow Properties of Stirred Yogurt: Calculation of the Pressure Drop for a Thixotropic Fluid

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ABSTRACT

A new equation, which enables the prediction of the mean value of the friction factor during the flow of a thixotropic fluid in a horizontal rectilinear cylindrical pipe, was obtained using a method proposed by Kemblowski & Petera. According to the obtained equation, the pressure drop is a function of three dimensionless numbers: the generalized Reynolds number Re standing for the ratio of inertial to viscous forces for an Ostwaldian fluid, a modified memory effects number De and a 'structural' number Se which is correlated to the maximum breakdown of structure of the fluid. This relation is only valid for laminar flows and is based on several hypotheses described in the present article. The proposed rheological model is based on a structural approach, featuring a rheological state equation describing shear stress, and a structural decay equation. The fluid is stirred yogurt and its structural parameter λ follows a second order kinetic equation. Experimental validation of the friction factor formula showed good agreement. © 1998 Elsevier Science Limited. All rights reserved.

NOTATION

A	Function defined in eqn (36)
B	Constant defined in eqn (31)

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C	Rate of structural decay, s^{-1}
D	Pipe diameter, m
De	Modified Deborah number defined in eqn (43)
$f/2$	Friction factor
$\dot{\gamma}$	Shear rate, s^{-1}
$\dot{\gamma}_w$	Wall shear rate, s^{-1}
k_e	Consistency factor at equilibrium, Pa s^{n_e}
k_i	Initial consistency factor, Pa s^{n_i}
L	Pipe length, m
λ	Structural parameter
λ_e	Structural parameter at equilibrium
λ_i	Initial structural parameter
μ	Apparent viscosity, Pa s
μ_e	Apparent viscosity at equilibrium, Pa s
n_e	Flow behaviour index at equilibrium
n_i	Initial flow behaviour index
p	Structural decay order
P	Pressure, Pa
P_i	Pressure at the pipe entrance, Pa
Q	Flow rate, $m^3 h^{-1}$
ρ	Fluid density, kg m^{-3}
R	Pipe radius, m
R_i	Internal radius of the single gap concentric cylinder, mm
R_e	External radius of the single gap concentric cylinder, mm
r	Radial distance in the pipe, m
Re	Generalized Reynolds number
Se	Structural number defined in eqn (42)
t	Time, s
τ	Shear stress, Pa
τ_e	Shear stress at equilibrium, Pa
τ_i	Initial shear stress, Pa
Um	Mean linear velocity in the pipe, m s^{-1}
Uz	Local axial velocity in the pipe, m s^{-1}
z	Axial coordinate, m

INTRODUCTION

Our interest is to focus on the study of the flow properties of stirred yogurt, a dairy product whose rheological properties are highly time-dependent. Like other cultured dairy products, stirred yogurt has a delicate protein gel structure that develops during fermentation. The microstructural changes that occur during shearing are decisive in stirred yogurt processing when transporting through tubes. Three factors will influence its apparent viscosity excluding its chemical composition: temperature, shear rate and time. The behaviour of stirred yogurt will depend upon its thermal and mechanical history. Here we are concerned with a thixotropic fluid which follows the definition given by Barnes *et al.* (1989): a material is thixotropic when there is a 'decrease [in time]...of viscosity under constant shear stress or shear rate, followed by a gradual recovery when the stress or shear is removed'.

Quantitative relations were formulated between shear stress, shear rate, temperature and time for different kinds of foodstuffs (mayonnaise, bechamel sauce, white egg, soft cheeses, buttermilk, salad dressing sauce, stirred yogurt, ι-carrageenan). Rheological data are presented and show that thixotropic behaviour (Barnes, 1997) can be fully characterized (Tung *et al.*, 1970; Holdsworth, 1971; Tiu & Boger, 1974; De Kee *et al.*, 1983; Shoemaker & Figoni, 1983, 1984; Massager-Roig *et al.*, 1984; Ford & Steffe, 1986; Paredes *et al.*, 1988; Martinez-Padilla & Hardy, 1989; Ramaswamy & Basak, 1991, 1992; Benezech & Maingonnat, 1993; Butler & McNulty, 1995; Baravian & Quemada, 1996).

Several studies have evaluated the flow properties of stirred yogurt summarized by Benezech & Maingonnat (1994). Using different kinds of rheological measurement [up and down shearing procedures, constant shear rate (or shear stress) experiments], empirical formulae and afterwards models based on structural concept appeared to be valid and widely used to quantify thixotropy. One of the most successful phenomenological concept of thixotropy was given by Cheng & Evans (1965), who generalized results of earlier work by Moore (1959), and introduced a structural parameter λ which accounts for the details of structure and for the transient kinetics changes under shear.

The aim of this work was to develop a simple analytical solution to calculate the mean friction factor of a thixotropic fluid flowing through a horizontal cylindrical pipe. Previous studies which are connected closely with this important engineering problem are often based on hypotheses leading to a numerical resolution (Cheng, 1973; Cawkwell & Charles, 1987; Billingham & Fergusson, 1993; Escudier & Presti, 1996). Kemblowski and Petera (1981) proposed a method to obtain a formula, resumed by Sestak (1988) and Toure (1995), which allows the calculation of the pressure drop due to friction in the case of isothermal flow in a circular tube. As far as we are concerned, we consider only the stabilized steady flow. Kemblowski and Petera (1981) used a rheological model presented in two papers (Kemblowski & Petera, 1979, 1980), Sestak (1988) used another thixotropic model to describe the same flow properties while Toure (1995) worked on a third model including a yield stress. Both of them arrive at the same mathematical expression (Kemblowski & Petera, 1981; Sestak, 1988) of the friction factor but with different Re, Se and De equations.

In this paper we follow the hypotheses used by Kemblowski and Petera (1981) in order to calculate the pressure drop of a thixotropic fluid which has a time dependency behaviour different from that described by these authors. In the remainder of the article, 'duct' or 'pipe' refers to a rectilinear horizontal cylindrical duct.

EXPERIMENTAL INVESTIGATION

The experimental study is concerned with the determination of dynamical magnitudes (flow, and pressure drop) and rheological data (Fig. 1).

A progressing cavity pump (PCM) was used to decant stirred yogurt from the tank at a constant flow rate. Its impact on yogurt is not described. However, one of the interests of this work is that a product sample is taken on line between the pump and the test pipe and therefore we know the initial state (λ_i) of the fluid coming out of the pump. A piece of tube (2 m) allows to reach steady flow just before the first manometer. A viewfinder was set in order to check whether the pipe

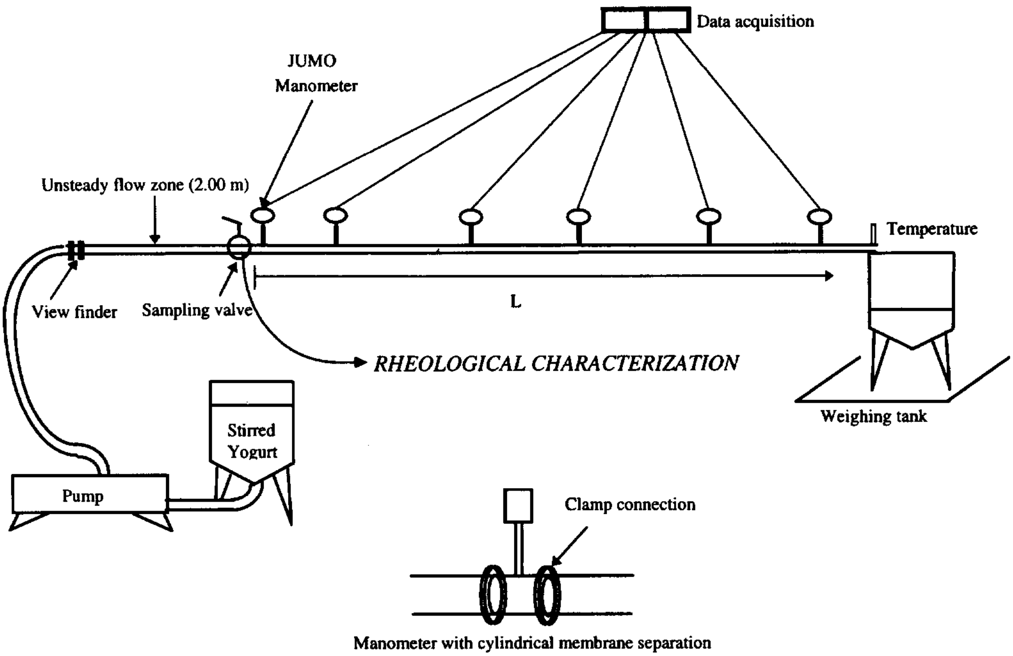

Fig. 1. Pilot plant.

was completely full or not. Clamp connections were used to provide as little disruption in the pipe as possible; experiments were made with a maximum continuous flow. Data acquisition (pressures) was made using Labview.

The stirred yogurt was sampled at the entrance of the duct and sheared between a single gap concentric cylinder DIN145 ($R_i/R_e = 1.084$, $R_e - R_i = 1.9$ mm) of a controlled speed rotating viscometer: Rheomat 260 (Contraves, France). The sample volume was approximately 100 ml. Temperature was controlled by a circulating oil bath (Huber). All measurements were made as soon as the samples were taken and the whole experiment took place at a state value (20 ± 0.2°C). Sampling was made carefully and we assume that it altered as little as possible the viscosity of the product. A complete rheological characterization (detailed in Section 3) was made to determine the parameters of the thixotropic model.

All rheological measurements took place in the 10 min following the beginning of the tank emptying because differences in texture between the top and the bottom of the tank were identified. We have evaluated the reproducibility of the measurement on the same batch of product by replicating the rheological tests three times and taking an average. The maximum coefficient of variation for replicates was 4.9% with an average value at 3%.

During the characterization, the same stirred yogurt issued from the processing plant was pumped through the duct, which can be dismantled (variable length L, diameter $D = 0.023$ m) where six manometers (Jumo, model 4AP-30 with membrane separator 4MDT) measure the pressure drop. These manometers, which can be moved to different locations along the pipe, were calibrated in place in the set up and were checked with a model Ostwaldian fluid (these results are not published here). The flow rate was determined by collecting the fluid in a weighing tank and the mass of the sample obtained in a given time was derived.

MODELLING, RESULTS AND DISCUSSION

Rheological model

On the basis of the theoretical results and other applied studies (Cheng, 1987; Chavan *et al.*, 1975), the following rheological model was suggested:

— a rheological state equation describing shear stress:

$$\tau(t,\dot{\gamma}) = \lambda(t,\dot{\gamma}).\tau_i(\dot{\gamma}) = \lambda(t,\dot{\gamma}).k_i\dot{\gamma}^{n_i} \quad (1)$$

— a structural decay equation [developed by Petrellis & Flumerfelt (1973)]:

$$\frac{d\lambda}{dt} = -C.(\lambda(t,\dot{\gamma}) - \lambda_e)^p \text{ when } \lambda > \lambda_e \quad (2a)$$

$$\frac{d\lambda}{dt} = 0 \text{ when } \lambda \leq \lambda_e \quad (2b)$$

where C is a rate of structural decay which stands for the speed of structural breakdown, λ_e being the equilibrium state which is taken independent of shear rate and p, the order of the structural breakdown. The stirred yogurt studied exhibits a partially irreversible structural breakdown under constant shear rate (even at low shear rate values), as stated by eqn (2b). The influence of time on the structural parameter is represented by the differential eqn (2a) including only structural breakdown phenomena. The initial condition is introduced corresponding to the initial value of the structural parameter λ_i which is equal to one for zero shear time (eqn (3)).

$$\lambda(t=0,\dot{\gamma}) = \lambda_i = \frac{k_i\dot{\gamma}^{n_i}}{\tau(t=0,\dot{\gamma})} = \frac{k_i\dot{\gamma}^{n_i}}{\tau_i(\dot{\gamma})} = 1 \quad (3)$$

At zero shear time, stirred yogurt is shear-thinning and follows a power law model: $\tau_i(\dot{\gamma}) = k_i\dot{\gamma}^{n_i}$. In order to simplify our considerations, we have assumed that the material had no yield stress. The structural parameter λ ranges from the initial value λ_i for zero shear time to an equilibrium value λ_e $(0 < \lambda_e < 1)$ when $t \to \infty$.

Equations (2a) and (2b) can easily be integrated at constant shear rate in order to obtain:

$$\lambda = \lambda_e + [(\lambda_i - \lambda_e)^{1-p} - (1-p).C.t]^{1/1-p} \quad (4)$$

Equation (4) combined with eqn (1) gives:

$$\tau = \lambda_e.k_i\dot{\gamma}^{n_i} + [(\lambda_i.k_i\dot{\gamma}^{n_i} - \lambda_e.k_i\dot{\gamma}^{n_i})^{1-p} - (1-p).C.t.(k_i\dot{\gamma}^{n_i})^{1-p}]^{1/1-p} \quad (5)$$

which can be simplified:

$$\tau = k_e\dot{\gamma}^{n_e} + [(k_i\dot{\gamma}^{n_i} - k_e\dot{\gamma}^{n_e})^{1-p} - (1-p).C.t.(k_i\dot{\gamma}^{n_i})^{1-p}]^{1/1-p} \quad (6)$$

From eqn (6) we can derive:

$$\tau = \tau_e + [(\tau_i - \tau_e)^{1-p} - (1-p).C.t.(\tau_i)^{1-p}]^{1/1-p} \quad (7)$$

For one shear rate, a mathematical expression

$$y = a + [(b)^{1-d} - (1-d).c.x]^{1/1-d} \tag{8}$$

is fitted on rheological data using the standard least squares minimization procedure, with constrains ($a > 0$, $b > 0$, $c > 0$ and $d > 0$). The order of the breakdown process p was identified equal to two, which is in agreement with the literature (Benezech & Maingonnat, 1993). eqn (4) becomes:

$$\lambda(t) = \lambda_e + \left(\frac{1}{\lambda_i - \lambda_e} + C.t \right)^{-1} \tag{9}$$

Three parameters are deduced: τ_i, τ_e and C. Repeating this procedure for each shear rate (a new sample of product is taken at each shear rate), the dependence of τ_i, τ_e and C with shear rate is obtained. The zero-time shear stress τ_i and the equilibrium shear stress τ_e follow power-laws (Fig. 2). The four parameters k_i, n_i, k_e, n_e are calculated. Assuming a constant value of C independent of shear rate, eqn (2a) can then be integrated analytically even under conditions of varying shear rate. This approximation is needed in the theoretical calculation of the friction factor. This is the reason why we considered C as independent of shear rate. λ_e is calculated by dividing τ_e by τ_i and is similarly independent of shear rate. Averages for C and λ_e are determined (Fig. 2).

Four different shear rate levels, each one for 5 min, were enough to obtain the model parameters. With four shear rates, shear rate dependence can be modeled and 5 min is sufficient to reach a 'plateau' (where viscosity decreases slowly enough) in order to evaluate the equilibrium viscosity. The rheological model was applied successfully to stirred yogurt as illustrated in Fig. 3. The model is determined for the range of shear rates 45–110 s^{-1}. We notice a good agreement between the model and the rheology of stirred yogurt.

Friction factor for a thixotropic fluid: theoretical calculation

Our aim is the demonstration of the effect of thixotropy on the flow of fluids on an industrial scale in a pipe.

Let us consider an isothermal steady-state laminar flow of a fluid described by the above rheological eqns (1) and (9). In the following text, we take for granted that the entrance of the pipe is considered as the $z = 0$ axial coordinate (Fig. 1). The proposed calculation [inspired by the hypotheses developed by Kemblowski & Petera (1981)] allows to calculate the pressure drop of a completely characterized thixotropic fluid with a simple equation.

Hypotheses

The fluid entering the pipe has a uniform structure, characterized by the structural parameter λ_i. We assume that there is no slip at the wall, i.e. the velocity of the fluid is equal to zero at the wall. As described in Fig. 1, an adequately long zone (2 m) allows the fluid to reach its steady-state flow. Therefore, the boundary layer is totally developed and we may assume that the fluid effectively enters the pipe with an established velocity profile which depends on the rheological properties and the

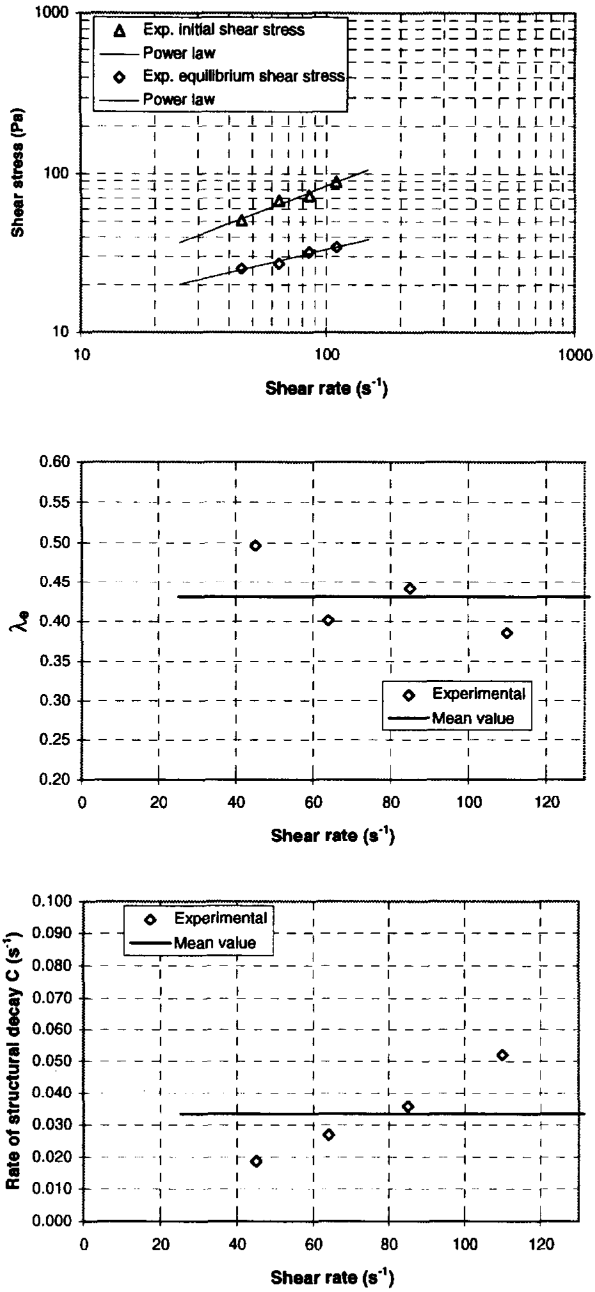

Fig. 2. Shear rate dependence of τ_i,τ_e, λ_e and C.

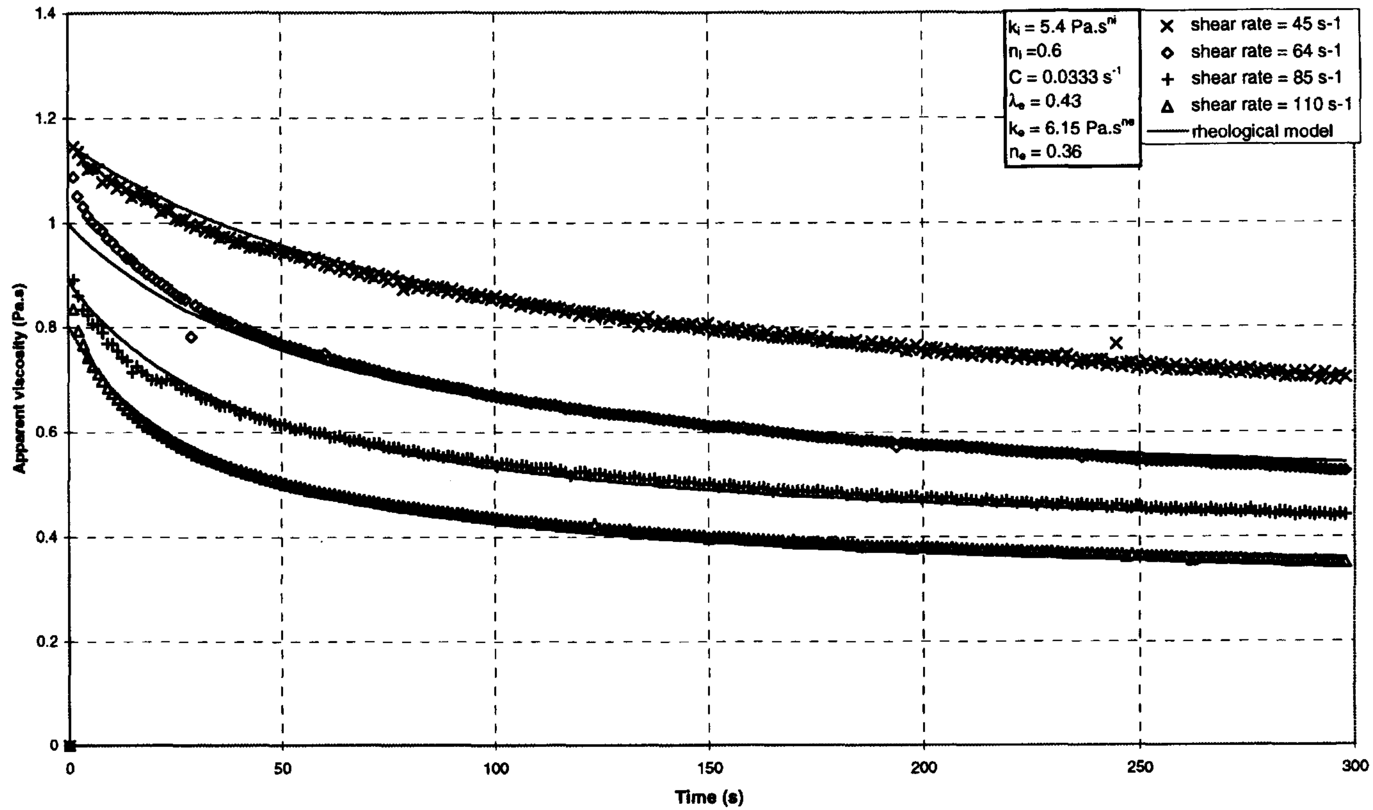

Fig. 3. Rheological model defined by eqns (1), (2a) and (2b) and experimental acquisition on stirred yogurt.

entrance conditions (flow rate). Samples (Fig. 1) are taken at this critical point of the pipe.

Velocity profiles
Let us consider the velocity distribution at a cross-section near the pipe entrance, where it is already fully developed, but where the structural parameter is still approximately equal to λ_i = constant, i.e. a few millimeters after the sampling valve.

According to Midoux (1985), the local axial velocity profile in the previous cross-section considered, for an Ostwaldian fluid (time-independent shear thinning) with a consistency factor k_i and a flow behaviour index n_i which corresponds to the fully structured fluid, is given by:

$$U_{z,i}(r)=\left(\frac{3n_i+1}{n_i+1}\right).Um.\left[1-\left(\frac{r}{R}\right)^{1_i+1/n_i}\right] \tag{10}$$

where

$$Um=\left(\frac{n_i.R}{3.n_i+1}\right).\left[\left(-\frac{\delta P}{\delta z}\right)_i.\frac{R}{2k_i}.\right]^{1/n_i} \tag{11}$$

and

$$-\left(\frac{\delta P}{\delta z}\right)_i$$

axial pressure gradient at the pipe entrance.

In the case of a long pipe ($z\rightarrow\infty$), the structural parameter of a thixotropic fluid changes from λ_i to the equilibrium value λ_e. The local shear stress at a cross-section distant from the pipe entrance is given by the following formula:

$$\tau_e=\lambda_e.k_i\dot{\gamma}^{n_i}=k_e.\dot{\gamma}^{n_e} \tag{12}$$

Thus the local axial velocity is equal to:

$$U_{z,e}(r)=\left(\frac{3n_e+1}{n_e+1}\right).Um.\left[1-\left(\frac{r}{R}\right)^{1_e+1/n_e}\right] \tag{13}$$

where

$$Um=\left(\frac{n_e.R}{3.n_e+1}\right).\left[\left(-\frac{\delta P}{\delta z}\right)_e.\frac{R}{2k_e}.\right]^{1/n_e} \tag{14}$$

and

$$-\left(\frac{\delta P}{\delta z}\right)_e$$

axial pressure gradient at the pipe as z approaches infinity.

Because the Ostwaldian parameters are unequal between the entrance and at a distance from the pipe entrance, eqn (10) and eqn (13) describe, at the same mean linear velocity Um, different radial distributions of the local velocity. This evolution of the velocity profile along the duct gives a radial component to the velocity field of the fluid. The radial component arises from the fact that the fluid must move perpendicularly to the axial direction because the velocity profile is changing.

$$U(z,r) = U(r) + U(z) \tag{15}$$

The flow field is studied here at very low Re (inertia is negligible) and therefore it is assimilated to laminar flow. However, even in this case, some authors do not define this flow as laminar because changes in velocity profile exist, leading to flow instabilities (Toure et al., 1994). In the pipe, the velocity profile changes from an initial profile typical of a power law fluid with a consistency factor k_i and a flow behaviour index n_i to a profile typical of a power law fluid with a consistency factor k_e and a flow behaviour index n_e. (k_i; n_i) being different from (k_e; n_e), the velocity profile changes significantly: it tends to be more flat, the maximum velocity at the center of the duct getting lower when the fluid is less structured in our case. Fig. 4 shows these velocity profiles calculated in the case of an experiment on stirred yogurt.

Let us now consider a steady-state distribution of the structural parameter λ in the pipe. Considering experimental lengths are quite small (13.70 m), the equilibrium state of the thixotropic fluid has not been reached. Thus, the velocity profile $U(z,r)$ changes slightly. Taking into account that the radial velocity component can be neglected in eqn (15), and eqn (2a) becomes:

$$\frac{d\lambda}{dt} = \frac{dz}{dt} \cdot \frac{d\lambda}{dz} = Uz.\frac{d\lambda}{dz} = -C.(\lambda(r,z) - \lambda_e)^2 \tag{16}$$

eqn (16) is valid at any constant value of the radial distance $r \neq R$. It is an ordinary differential equation with the initial condition:

$$z = 0 \rightarrow \lambda(r,0) = \lambda_i \tag{17}$$

The integration of eqn (16) leads to:

$$\forall r \neq R$$

$$\lambda(r,z) = \lambda_e + \frac{[\lambda_i - \lambda_e].Uz(r)}{[C.z.(\lambda_i - \lambda_e) + Uz(r)]} \tag{18}$$

and for $r = R$ ($Uz = 0$):

$$\lambda(R,z) = \lambda_e \tag{19}$$

Multiplying eqn (18) by $k_i\dot{\gamma}^{n_i}$ and remembering that $\tau_e(r) = \lambda_e.k_i\dot{\gamma}^{n_i}$, $\tau_i(r) = \lambda_i.k_i\dot{\gamma}^{n_i}$ and $\tau(r,z) = \lambda(r,z).k_i\dot{\gamma}^{n_i}$ we obtain for $r \neq R$:

$$\tau(r,z)=\tau_e(r)+\frac{[\tau_i(r)-\tau_e(r)].Uz(r)}{[C.z.(\lambda_i-\lambda_e)+Uz(r)]} \tag{20}$$

At the entrance, the structural parameter $\lambda_i = 1$ on the whole cross-section. Thus eqn (20) becomes:

$$\tau(r,z)=\tau_e(r)+\frac{[\tau_i(r)-\tau_e(r)].Uz(r)}{[C.z.(1-\lambda_e)+Uz(r)]} \tag{21}$$

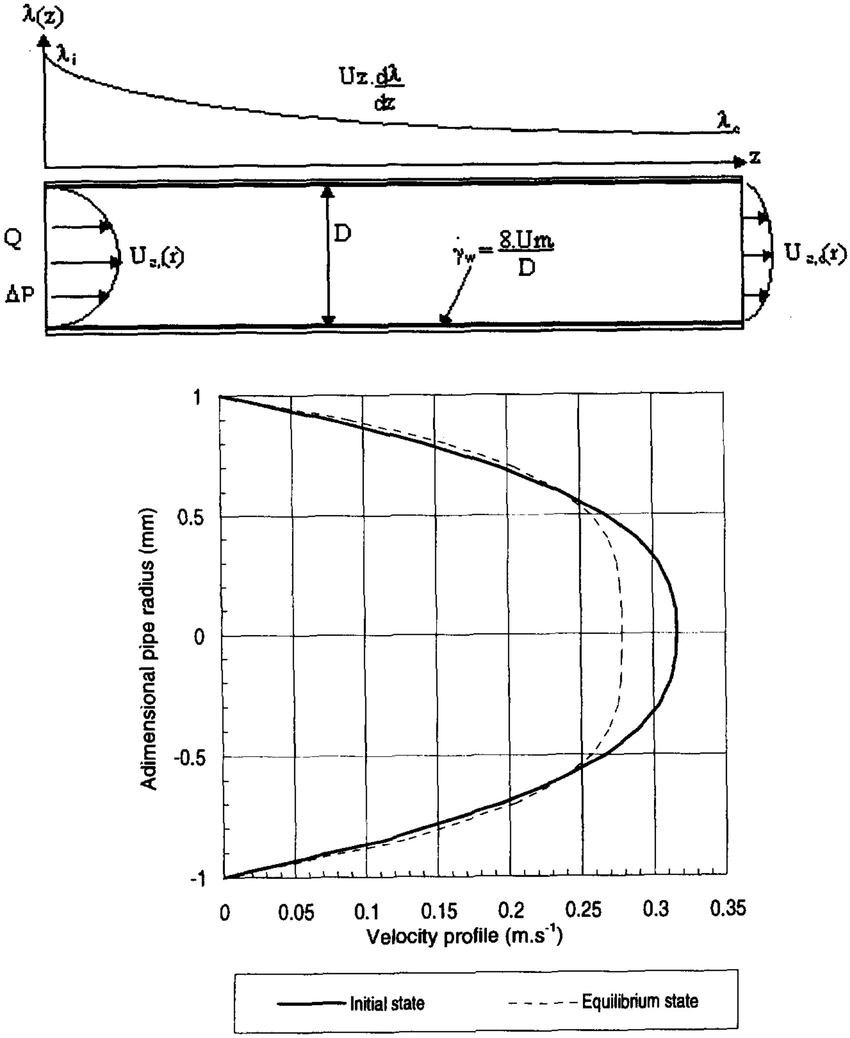

Fig. 4. Evolution of the velocity profile in the pipe for the thixotropic fluid described in Fig. 3.

The shear stress τ is linked to the local pressure drop by relations (22) and (23) whatever the fluid providing it is flowing through a cylindrical duct in a laminar way.

$$\text{at the entrance } \tau_i(r) = \left(-\frac{\delta P}{\delta z}\right)_i . \frac{r}{2} \quad (22)$$

$$\text{For } z \to \infty \ \tau_e(r) = \left(-\frac{\delta P}{\delta z}\right)_e . \frac{r}{2} \quad (23)$$

The pressure gradients are determined on the basis of eqn (11) and eqn (14). According to eqn (22), the shear stress varies linearly with the radial distance r. Therefore, we may approximate eqn (21) by assuming a linear dependence (resulting from the classical Taylor development):

$$\tau(r,z) = \tau(0,z) + \left(\frac{\delta \tau}{\delta r}\right)(0,z).r \text{ and } \tau(0,z) = 0 \quad (24)$$

hence

$$\frac{\tau(r,z)}{r} = \left(\frac{\delta \tau}{\delta r}\right)(0,z) \quad (25)$$

Introducing (21), (22) and (23) into eqn (25), we obtain:

$$\tau(r,z) = \left(-\frac{\delta P}{\delta z}\right)_e . \frac{r}{2} + \frac{\left[\left(-\frac{\delta P}{\delta z}\right)_i - \left(-\frac{\delta P}{\delta z}\right)_e\right].Uz(0).\frac{r}{2}}{[C.z.(1-\lambda_e) + Uz(0)]} \quad (26)$$

$$\text{with } Uz(0) = \left[\frac{3n_i + 1}{n_i + 1}\right].Um \quad (27)$$

Shear stress calculation
Kemblowski & Petera (1981) assumed that:

$$\tau = [A(z) + B].\frac{r}{R} \quad (28)$$

when $A(z) = 0$, eqn (28) describes the flow of a fluid with no memory. Therefore, the function $A(z)$ has to fulfil the following conditions:

$$A(z) \to 0 \text{ for } z \to \infty \tag{29}$$

$$A(z) \to A \text{ for } z \to 0 \tag{30}$$

In this case, according to eqns (28) and (26):

$$B = \left(-\frac{\delta P}{\delta z} \right)_{\mathrm{e}} . \frac{R}{2} \tag{31}$$

Then, the axial pressure gradient is constant and the radial pressure gradient is equal to zero:

$$\left(-\frac{\delta P}{\delta r} \right) = 0 \tag{32}$$

and

$$\tau_{\mathrm{e}}(r) = \left(-\frac{\delta P}{\delta z} \right)_{\mathrm{e}} . \frac{r}{2} \tag{33}$$

For $z \to \infty$, the fluid becomes time-independent. Therefore, conditions (29), (31) and (33) are fulfilled for the thixotropic fluid flowing at a constant velocity Um.

On the other hand, for $z \to 0$, in order to get the relation:

$$\tau_{\mathrm{i}}(r) = \left(-\frac{\delta P}{\delta z} \right)_{\mathrm{i}} . \frac{r}{2} \tag{34}$$

we have to assume that:

$$A = A(0) = \left[\left(-\frac{\delta P}{\delta z} \right)_{\mathrm{e}} - \left(-\frac{\delta P}{\delta z} \right)_{\mathrm{i}} \right] . \frac{R}{2} \tag{35}$$

Let us consider such a function $A(z)$ which fulfils conditions (29) and (30). The function $A(z)$ determines the rate of change of the pressure gradient in the axial direction and depends on the structural parameter λ. Reordering the terms of eqn (26), we derive the function $A(z)$:

$$A(z) = \left(\frac{R}{2} \right) . \frac{\left[\left(-\frac{\delta P}{\delta z} \right)_{\mathrm{i}} - \left(-\frac{\delta P}{\delta z} \right)_{\mathrm{e}} \right] . Uz(0)}{[C.(1 - \lambda_{\mathrm{e}})z + Uz(0)]} \tag{36}$$

Pressure gradients

In the preceding calculation, Kemblowski & Petera (1981) neglected the dependence of the local pressure with the radial direction (see eqn (32)) and they assumed that the shear stress was approximately equal to:

$$\tau(r,z)=\left(-\frac{\delta P}{\delta z}\right).\frac{r}{2} \tag{37}$$

Comparing eqn (37) with eqn (26), we can write:

$$\left(-\frac{\delta P}{\delta z}\right)=\left(-\frac{\delta P}{\delta z}\right)_{\mathrm{e}}+\frac{\left[\left(-\frac{\delta P}{\delta z}\right)_{\mathrm{i}}-\left(-\frac{\delta P}{\delta z}\right)_{\mathrm{e}}\right].Uz(0)}{[C.(1-\lambda_{\mathrm{e}})\dot{z}+Uz(0)]} \tag{38}$$

Integrating eqn (38) with the initial condition $P(z=0)=P_{\mathrm{i}}$, one obtains:

$$P=Pi-\left(-\frac{\delta P}{\delta z}\right)_{\mathrm{e}}.z-\left[\left(-\frac{\delta P}{\delta z}\right)_{\mathrm{i}}-\left(-\frac{\delta P}{\delta z}\right)_{\mathrm{e}}\right].\frac{\ln\left[\frac{C.(n_{\mathrm{i}}+1).(1-\lambda_{\mathrm{e}})z}{(3n_{\mathrm{i}}+1).Um}+1\right]}{\left[\frac{C.(n_{\mathrm{i}}+1).(1-\lambda_{\mathrm{e}})}{(3n_{\mathrm{i}}+1).Um}\right]} \tag{39}$$

Equation (39) describes the dependence of pressure on axial coordinate z of the cylindrical duct. By substituting L for z in eqn (39), the pressure drop for a pipe of length L can be calculated:

$$P=Pi-\left(-\frac{\delta P}{\delta z}\right)_{\mathrm{e}}.L-\left[\left(-\frac{\delta P}{\delta z}\right)_{\mathrm{i}}-\left(-\frac{\delta P}{\delta z}\right)_{\mathrm{e}}\right].\frac{\ln\left[\frac{C.(n_{\mathrm{i}}+1).(1-\lambda_{\mathrm{e}})L}{(3n_{\mathrm{i}}+1).Um}+1\right]}{\left[\frac{C.(n_{\mathrm{i}}+1).(1-\lambda_{\mathrm{e}})}{(3n_{\mathrm{i}}+1).Um}\right]} \tag{40}$$

In order to have a simpler equation and to find out a general form for the mean thixotropic friction factor, we can introduce three dimensionless numbers.

We may first introduce a dimensionless number, Se, defined as the ratio of the axial pressure gradients at the pipe entrance and at the axial distance $z\to\infty$ where rheological equilibrium of the fluid occurs.

$$\mathrm{Se}=\frac{\left(\frac{\delta P}{\delta z}\right)_{\mathrm{i}}}{\left(\frac{\delta P}{\delta z}\right)_{\mathrm{e}}}=\frac{\left[\left(\frac{3n_{\mathrm{i}}+1}{n_{\mathrm{i}}}\right).\frac{Um}{R}\right]^{n_{\mathrm{i}}}.\frac{2k_{\mathrm{i}}}{R}}{\left[\left(\frac{3n_{\mathrm{e}}+1}{n_{\mathrm{e}}}\right).\frac{Um}{R}\right]^{n_{\mathrm{e}}}.\frac{2k_{\mathrm{e}}}{R}} \tag{41}$$

We can write it:

$$\mathrm{Se} = \frac{\left(\dfrac{\delta P}{\delta z}\right)_i}{\left(\dfrac{\delta P}{\delta z}\right)_e} = \left[\frac{\left(\dfrac{3n_i+1}{4n_i}\right)^{n_i}}{\left(\dfrac{3n_e+1}{4n_e}\right)^{n_e}}.\left(\frac{8Um}{D}\right)^{n_i-n_e}\right].\frac{k_i}{k_e} \tag{42}$$

where

$$\frac{8Um}{D}$$

is the wall shear rate. The dimensionless number Se was called the 'structural number' by Kemblowski & Petera (1981) and by Sestak (1988).

We can consider another dimensionless number, De:

$$\mathrm{De} = \frac{(3n_i+1).Um}{C.L.(n_i+1).(1-\lambda_e)} \tag{43}$$

The dimensionless number De can be assimilated to a 'modified Deborah number' (Kemblowski & Petera, 1981). It may be interpreted as 'a ratio of the characteristic time of the fluid to the minimum residence time of the liquid particle in the tube or as a dimensionless reciprocal tube length' (Sestak, 1988). Introducing eqn (42) and eqn (43) into eqn (40) and rearranging, one obtains:

$$\frac{\Delta P.D}{4\rho.Um^2.L} = f/2 = \frac{8}{\mathrm{Re}}.\left[1-(1-\mathrm{Se}).\mathrm{De}.\ln\left(1+\frac{1}{\mathrm{De}}\right)\right] \tag{44}$$

with

$$\mathrm{Re} = \left[\frac{4n_e}{(3n_e+1)}\right]^{n_e}\frac{Um^{(2-n_e)}.D^{n_e}.\rho}{8^{(n_e-1)}.k_e} \tag{45}$$

where Re is the well known generalized Reynolds number (Midoux, 1985).

RESULTS AND DISCUSSION

Using eqn (44) one may determine the mean value of the friction factor $f/2$ as follows:

For complete rheological characterization, stirred yogurt can be described by eqns (1) and (9) with six parameters (k_i, n_i, k_e, n_e, C, λ_e) evaluated experimentally as described above. The structural parameter λ_i at the pipe entrance is uniform in the whole section and equal to one. The density ρ of the fluid was found to be $1032\,\mathrm{kg\,m^{-3}}$. The experimental flow rate Q allows to calculate the mean velocity Um and the wall shear rate $\dot{\gamma}_w$. Knowing the diameter D and the length L of the pipe, one can determine the values Re, De and Se according to eqns (42), (43) and (45). One may now calculate the mean friction factor $f/2$ and deduce the pressure drop for any length L of the duct.

A comparison of the theoretical predictions with experimental data is shown in Fig. 5 (same yogurt as that illustrated in Figs 2 and 3) at constant flow rate ($0.271\,\mathrm{m^3\,h^{-1}}$).

On the graphical representation of eqn (44), pressure drop of the thixotropic fluid varies from an initial pressure drop ΔP_i of a time-independent power-law model (which would have kept all along the pipe its initial structure $\lambda_i = 1$) to an equilibrium pressure drop ΔP_e. The slope of the time-dependent pressure drop decreases as the fluid travels through the pipe. The wall shear-rate and residence time induces a decrease of the structural parameter λ of the fluid, and thus a decrease of the apparent viscosity. The fluid, defined by an initial power-law model, evolves to a final power-law model (corresponding to its equilibrium state) as it flows through the tube. This phenomenon is illustrated by Fig. 5 which shows the variation of the pressure drop with length L of the pipe.

Considering our fluid, we always have $\mathrm{Se} \geq 1$. This means that during the flow in the pipe, either the fluid has the same apparent viscosity or a breakdown of the structure occurs. In all cases, the structure cannot rebuild when flowing through the pipe because of eqn (2b).

If De increases (corresponding to higher memory effects), there is an increase in the deviation of the value $f/2$ from 8/Re (Fig. 6).

For liquids exhibiting a rapid structural breakdown, long residence time in the circular tube, or for conditions in which the structural state of the incoming liquid is close to equilibrium, the effect of thixotropy becomes negligible. Pressure drop will rapidly reach its equilibrium state.

When $\mathrm{Se} = 1$, eqn (44) reduces to $f/2 = 8/\mathrm{Re}$, i.e. to the well-known relation for fluids with no memory. In that case, the fluid is completely broken down at the entrance of the pipe or it is not a thixotropic fluid.

We notice that according to eqn (44), for a given pair of values of Re and De, the value of friction factor $f/2$ increases when Se increases.

When $L = 0$, expression (44) reduces to $f/2 = 8.\mathrm{Se}/\mathrm{Re}$ because

$$\lim_{\mathrm{De}\to\infty} \mathrm{De}.\ln\left(1+\frac{1}{\mathrm{De}}\right)=1$$

The results of pressure drop may be exploited in order to predict the destructuring or level of viscosity drop when flowing through a cylindrical duct. The procedure calculation is not established yet but one can see the relationship between viscosity and pressure drop in eqn (44). Even if the flow of thixotropic fluid in a pipe is not viscosimetric at all because of the radial motions, the pipe behaves from an engineering point of view as a Poiseuille's rheometer for thixotropic fluids.

For the experimental work, we changed the flow rate (between 0.07 and $0.75\,\mathrm{m^3\,h^{-1}}$) corresponding to Re values (eqn (45)) varying over the range 1–100

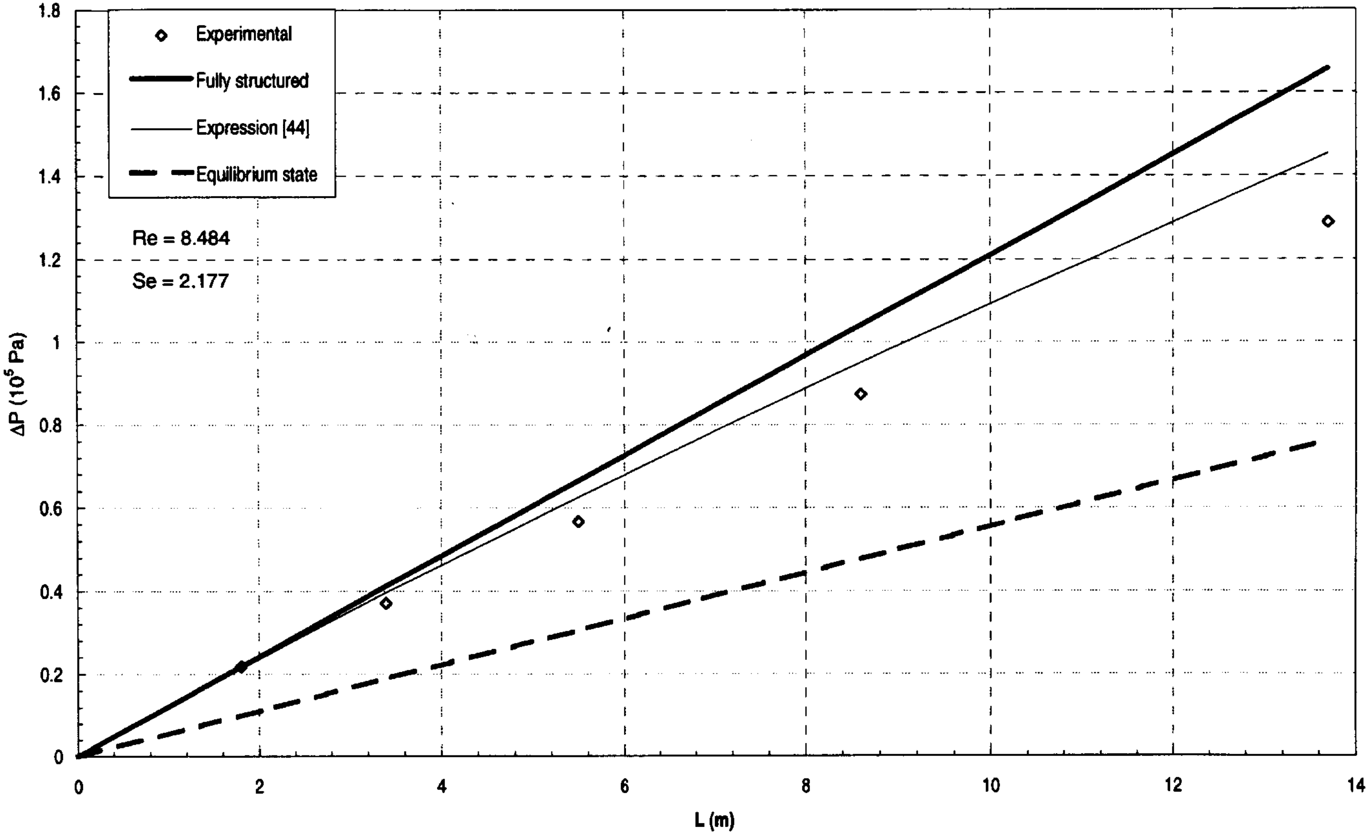

Fig. 5. Pressure drop for stirred yogurt flowing in laminar steady flow through a horizontal cylindrical rectilinear (rheological parameters, see Fig. 3).

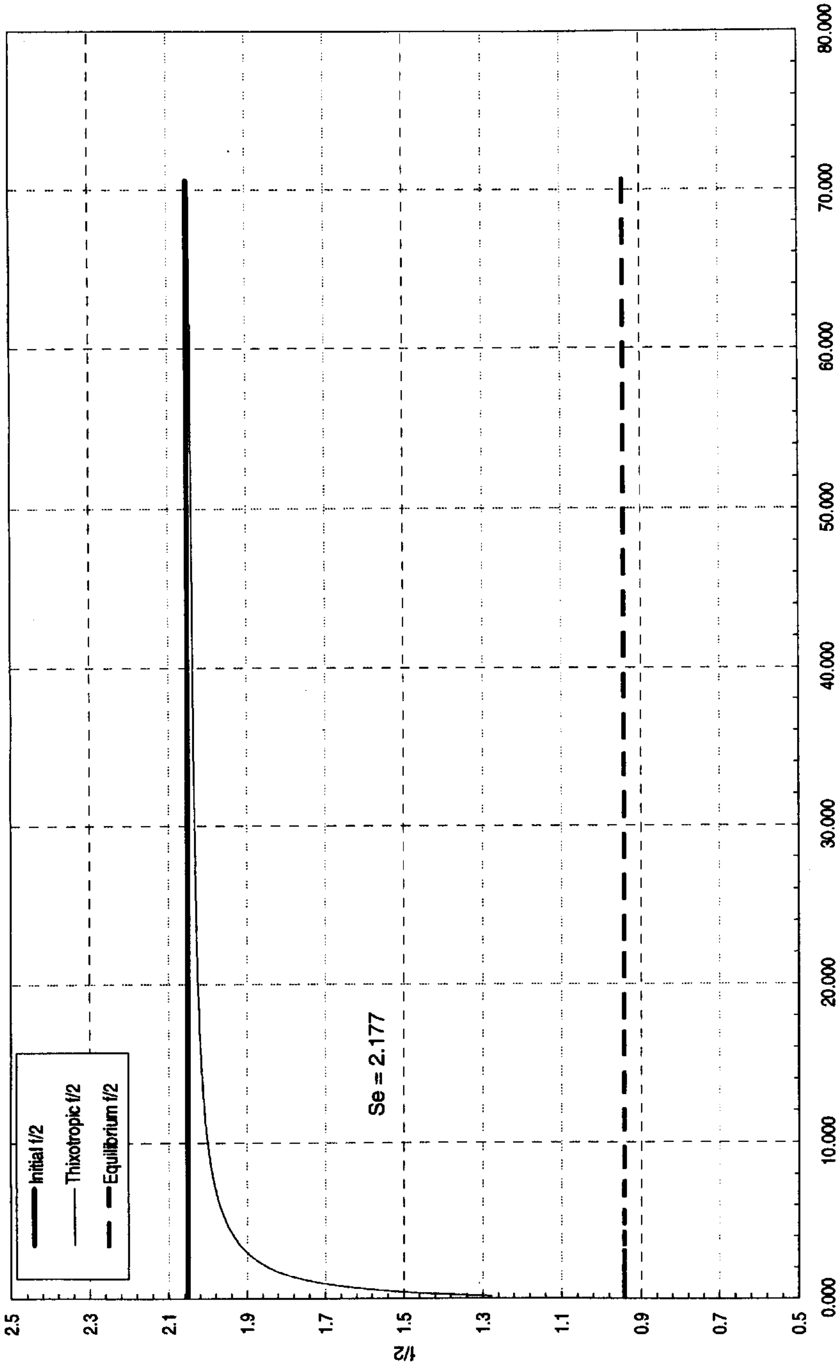

Fig. 6. Theoretical friction factor *f*/2 for steady flow of stirred yogurt in a pipe (case described in Fig. 3).

and to different values of Se ($1.1 < \text{Se} < 3.5$). The yogurt has a uniform structural state in the tank. Because of the impact of the pump on the structure parameter λ, rheological parameters (k_i, n_i, C, λ_e, k_e, n_e) determined at the entrance of the test pipe were different in each experiment.

Figure 7 presents the results obtained with stirred yogurt at different flow rates. The model overestimates the real pressure drop. If sampling altered the stirred yogurt, eqn (44) would underestimate experimental pressure drops. One explanation is that the radial component of the structural parameter is neglected in eqn (16). Another explanation is that average values of C and λ_e are considered for calculation despite their variation with shear rate. The difference is however tolerable. For lower values of De (i.e. longer L), data seem to be situated further from the model whereas experiments in the higher De region (i.e. shorter L) support the validity of the approximate expression (44). The higher Se, the more time-dependent the fluid and the less accurate the pressure drop model.

Through the results presented, we can assert that the radial dependence of the structural parameter is negligible with respect to the axial variation for short pieces of pipe. The existence of instabilities in flow would have little influence on pressure drop compared to those of a stable laminar flow. This means that the hypotheses used to calculate the resulting relation (44) are valid.

Few figures were given by Kemblowski & Petera (1981) and Sestak (1988) except Se values. Kemblowski and Petera studied flow with two paints that were not very time-dependent (Se = 0.98) whereas Sestak (1988) studied a high consistency thixotropic plaster (Se = 1.77). We note that in both results presented, the equations proposed overestimated the experimental pressure drop obtained. Kemblowski & Petera (1981) used the mean value of the rate of structural decay determined for the range of shear rates to calculate the pressure drop. Sestak (1988) assumed the radial dependence of the structural parameter to be negligible with respect to the axial variation and integrated the momentum equation in the direction of the tube centerline. However, the equilibrium value of the structural parameter λ_e depended on the instantaneous wall shear rate. Toure (1995) showed that the friction factor formula integrating the yield stress could correctly predict the pressure drop of an aqueous carboxymethylcellulose gel in steady flow (within the limit of 10% confidence levels).

CONCLUSION

Through this analysis, we can see the complexity of flow of a thixotropic fluid even in a simple geometry. Experimentation at a semi-industrial scale proved that simple calculation for prediction of thixotropic flow was possible.

More work concerning this structural approach is to be carried out to enforce the prediction of pressure drop. Longer pipes will be tested to validate the accuracy of the model when distribution time increases. The influence of radial variations of the structural parameter upon the accuracy of eqn (44) will also be investigated by other means.

This simple case enables us to develop procedures of calculation to predict flow behaviour in various types of equipment used for processing and transportation of thixotropic fluids (vertical rectilinear pipes, bends, valves, etc).

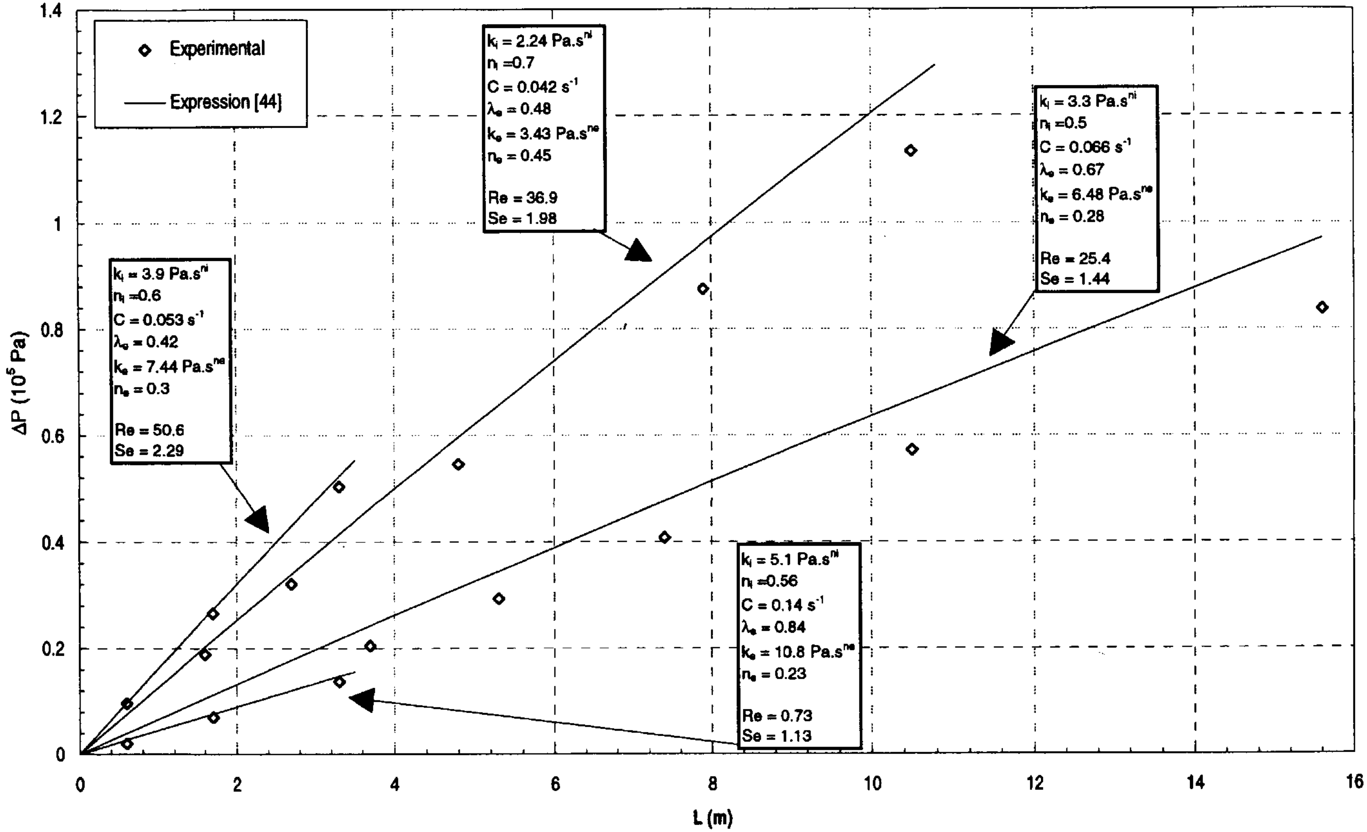

Fig. 7. Several experimentations with stirred yogurt: different rheological parameters and different process conditions tested on the pilot plant (length $3.30 < L < 15.60$ m).

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