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Dynamics of the interface between streams and groundwater systems in lowland areas, with reference to stream net evolution

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Abstract

Observations reveal a close relationship between groundwater depth and stream net characteristics in the permeable lowlands of the sandy Pleistocene area of the Netherlands. This applies for average conditions as well as for the seasonal expansion and contraction of the network of streams that participate in the drainage process. It is plausible that this observed relation is caused by the close connection between groundwater and surface water in an area where almost all precipitation surplus is discharged as groundwater. Under such conditions a stream system can be considered the outcrop of a groundwater flow system. The hypothesis is that the stream network, as the interface between both systems, must have adapted in response to the discharge capacity that is required to release the precipitation surplus through the continuum of ground waters and surface waters.

A theoretical model of coupled groundwater and surface water drainage systems is proposed that gives the stream net density and channel dimensions as functions of the geological and climatic conditions. The theoretically derived relationships of increasing stream density and decreasing channel size with reducing depth to groundwater and reducing topgraphic slope reflect the observed situation. The model further allows for assessment of the response of a drainage network to a changing environment.

1. Introduction

The study of drainage network evolution in areas of uniform lithology and negligible structural control has been based on both deterministic and stochastic modelling. An example of a deterministic approach to stream network development is the well-known Horton model (Horton, 1945), based on surface erosion. The stochastic models are based on random topology (e.g. Shreve, 1966, 1967). Various

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studies have been based on such concepts as optimal network growth and minimum energy dissipation (e.g. Woldenberg, 1971; Howard, 1990). Rodriguez-Iturbe et al. (1992a, b) have made plausible that a river network organizes itself to comply with the principle of optimal energy expenditure. This can explain the tree-like structure as well as the empirical relations between discharge and channel geometry.

The present study proposes a deterministic approach to stream density and stream size for the specific conditions that prevail in many lowland areas. The characteristics of the areas considered here include: a rather flat topography, shallow groundwater table, pervious subsurface of non-indurated sediments and a moderately humid climate.

Because of the moderate rainfall, the high infiltration capacity and the low relief, unchannelled overland flow is a rare phenomenon in such areas and almost all precipitation surplus percolates to the subsurface to become part of a groundwater drainage system. This groundwater flow eventually reappears at the surface as soon as the subsurface discharge capacity is exceeded by precipitation surplus. At this zone of groundwater exfiltration, a surface drainage network may develop through sapping erosion, caused by the combined process of reduction in the intergranular pressure and subsequent erosion by runoff. The role of subsurface processes in stream network initiation has been previously emphasized by Dunne (1969, 1990) and De Vries (1974, 1976).

The hypothesis is that such a channel system represents an interface between subsurface and surface drainage components, and that the stream system can be considered as a linear outcrop of the groundwater flow system. The aim of the present study is to explore the sensitivity of the characteristics for this interface in terms of stream spacing and channel size, as a function of independent variables, such as subsurface permeability and climate, as well as for the semi-independent variables, like topographic slope and average groundwater depth.

The proposed deterministic model contains the following reasoning: rainfall surplus, combined with groundwater depth and associated storage capacity, determines the groundwater discharge capacity that is needed to prevent ponding and unchannelled surface runoff. This desired groundwater discharge capacity requires, and therefore determines, a unique combination of stream spacing and stream size for given conditions of subsurface permeability and relief. Stream spacing (or stream density) and stream channel size are interdependent because the resistance to subsurface drainage reduces with an increase in stream size and/or an increase in stream density. Stream channel size is in turn controlled by the required stream discharge, which is again related to stream density.

Although a comprehensive deterministic model is not presented here, the present study reveals basic relationships in the continuous system of groundwater and surface water in areas where all precipitation surplus is discharged as groundwater. Such studies can help in evaluating the response of a drainage system to a changing environment.

The first sections describe the observed dynamics for this type of drainage system and indicate how the geological and climatic factors control its dimensional properties. The second part presents a simplified mathematical model to quantify the functional relations. The characteristics of the Netherlands Pleistocene lowland will be used in this study as a reference (Fig.1).

The present study is an extension of part of the author's earlier papers (De Vries, 1974, 1976, 1977), in which the idea of stream evolution by groundwater outcrop erosion was developed and a theoretical model was proposed to quantitatively relate stream spacing to the properties of the associated groundwater flow system. The present study expands on the coupling between the groundwater flow systems and the stream systems through the geometric properties of the stream channel.

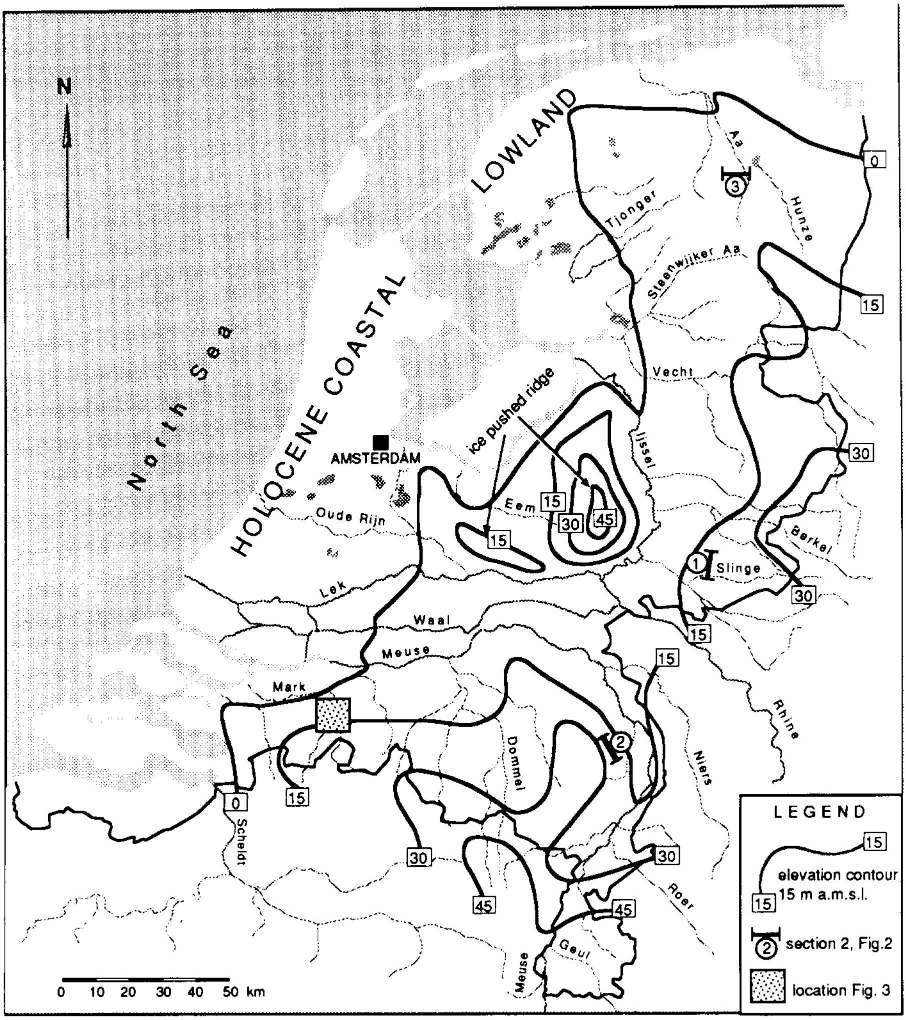

Fig. 1. Topography of the Netherlands Pleistocene sandy lowland showing the primary stream system. The Pleistocene lowland approximately coincides with the area above m.s.l.

2. Drainage system dynamics

2.1. Independent and dependent variables

The hydraulic characteristics that can be adapted by the coupled groundwater–stream system to cope with the drainage requirements in a given geological and topographical situation include stream density, incision depth and channel dimension. These stream properties form the dependent variables.

Subsurface permeability and climate are the controlling independent variables, whereas the topography can be considered as a semi-independent factor that predominantly belongs to an (independent) morphological system of extended time and areal scale. One can distinguish between (1) primary topography that exclusively belongs to the larger scale unit and which among others, comprises the large rivers originating outside the considered area (Fig. 1), and (2) secondary topography that is formed by the local stream system. The secondary topography in flat areas is restrained between narrow boundaries set by the primary system, and can be considered as semi-independent with little flexibility.

The drainage requirement is a function of the (almost) independent climatic situation and storage capacity. The latter is associated with the (almost) independent soil properties and average groundwater depth at the beginning of the wet period, which mainly depends on the primary topography in this flat area (Fig. 2).

In contrast to the average groundwater depth, the seasonal fluctuation of the groundwater table is a dependent variable within an existing drainage system. It regulates the imbalance between recharge and discharge by the combination of storage and hydraulic gradient. This feedback mechanism includes the changing number of drainage channels that participate in the drainage process: a rising groundwater table resulting from an increase in recharge means an increase of storage and an increase of hydraulic gradient, and can subsequently activate an increasing number of lower order streams (Fig. 2).

In this concept, therefore, a drainage system consists of surface and subsurface components which have organized themselves in such a way as to satisfy the drainage requirements in a given geological, geomorphological and climatic situation. This self-regulating process includes the development of a hierarchical stream network which is in equilibrium with conditions on a longer time scale, and which adjusts itself dynamically to the interyear and seasonal meteorological fluctuations by a shift in number and character of streams that participate in the groundwater discharge.

This expanding and contracting drainage network is equivalent to the variable source area concept of Hewlett and Hibbert (1967) that supposes an expanding area of saturated soil to provide a progressively increasing contribution to quick runoff by saturation–excess overland flow or interflow. This process of pseudo-overland flow is considered for our conditions as an extreme situation that occurs only when the rainfall has exceeded a certain statistical criterion, that for the Netherlands means a frequency of about once a year.

A limiting morphological case is that in which no adequate stream system can develop because of a flat or concave topography. This will cause the groundwater

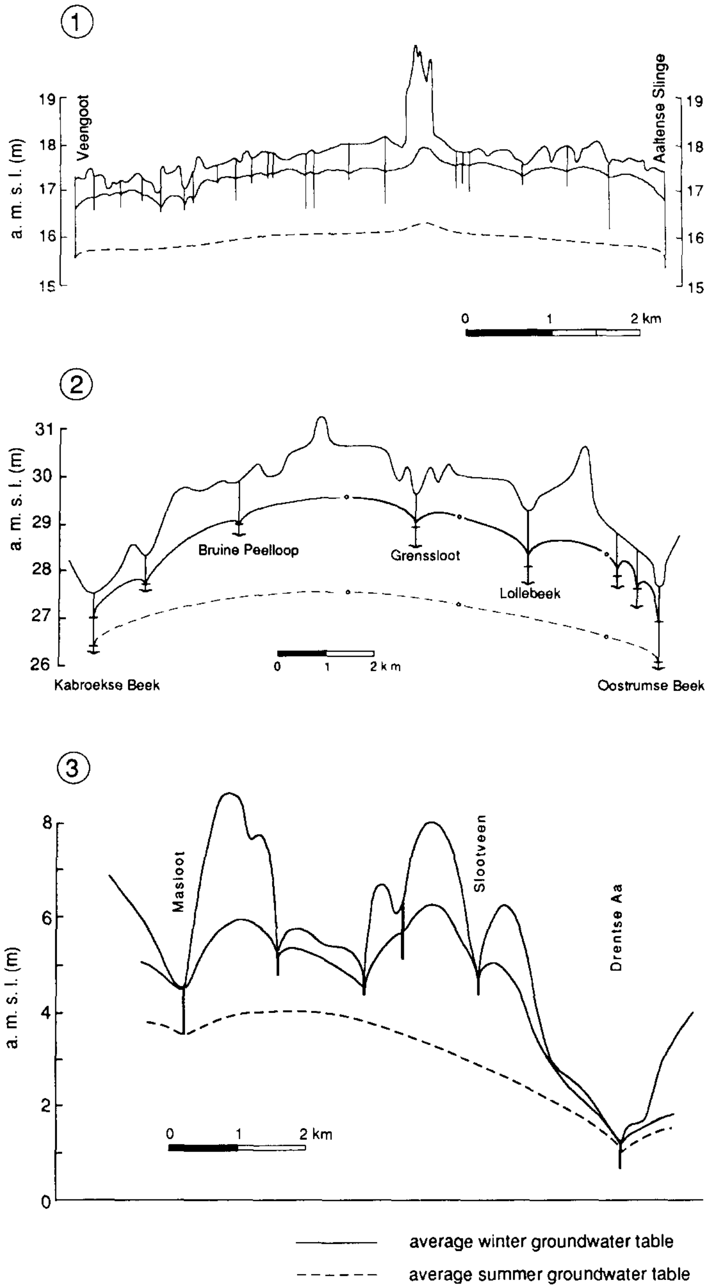

Fig. 2. Hydrologic–topographic sections through some small stream systems in the Netherlands Pleistocene area, showing the relationship between groundwater depth and stream density; for location see Fig. 1. The stream network is essentially natural, with artificial improvement of its discharge capacity by enlargement of the channel cross-sections. The vertical lines indicate position and depth of the drainage channels. (Redrawn from detailed studies by Ernst, 1962 and De Vries, 1974.)

table to be near or at the surface and marshy conditions or peat will develop instead of a stream network. The other end of the topographic scale is represented by the condition that the (primary) topographic slope exceeds the required hydraulic gradient. All groundwater then can be discharged by the existing primary river system, and a deep groundwater table normally prevails. The latter situation is represented in the Netherlands Pleistocene area of the ice-pushed ridges, where topographic slopes of the order of 1% are in contrast to the average terrain slope of 0.04% in the whole Pleistocene lowland region.

2.2. Threshold value or 'design criterion'

The present study presumes that the rainfall events which occur with a frequency of exceedance of 1–5% during the wet season control the stream network evolution. The 1% frequency coincides with the design criterion for artificial drainage of farm land in the Netherlands (De Vries, 1974, p.104), and produces a higher drainage capacity than performed by the original natural drainage system. The 5% frequency of rainfall is thus tentatively applied as 'design criterion' for evolution of the natural system.

It is plausible to consider that the streams of lower order have evolved in equilibrium with the 'design' rainfall frequency. Higher order streams are secondary expressions of the self-organizing system, structured to comply with the network's fractal nature of self-similarity; expressed by Horton's bifurcation law and other principles of drainage network morphology. Non-homogeneities and irregularities within a drainage basin arise if the primary topographic features are of a smaller scale than the considered watershed. This especially applies where the primary topography is of non-fluvial origin, such as the undulating aeolian sands and the ice-pushed ridges in the Netherlands. Other deviations are encountered in the broad stream valleys and on wide and flat divides where shallow groundwater tables cause the development of peat marshes and raised bogs, which can be considered as the creation of an extra storage capacity and an infinite stream density.

2.3. Reference area

The schematization of geomorphology and the generalization of data into a quantitative approach are based on observations from the Netherlands Pleistocene landscape (e.g. De Vries, 1974; Ernst, 1978). This area can be approximately characterized as a Pleistocene fluvial fan with an average thickness for the upper aquifer of 50 m. The predominantly medium-fine to medium-coarse sandy deposits are covered by fine-grained fluvio-aeolian sands and loamy sands from the arctic desert environment of the Weichselian (so-called Coversands), with a thickness of up to 10 m. The whole forms an undulating topography with height differences of up to 3 m. Average regional slope in the area is about 1:2500, sloping towards the north and west where the Pleistocene dips beneath the Holocene fluvio-marine clayey and peaty deposits of the coastal lowland (Fig. 1).

The climate is semihumid with moderate temperatures, an average annual precipitation of 775 mm and an actual evapotranspiration of approximately

500 mm (period 1950–1980). During the summer half-year the precipitation and evaporation more or less counterbalance, so that a precipitation surplus of about 275 mm prevails during the winter period. Infiltration capacity is rarely exceeded because of the favourable entry characteristics of the permeable sandy soil, the flat topography and moderate rainfall intensities. Whenever the infiltration capacity is exceeded, however, temporary retention storage at the surface occurs rather than overland flow. The moderate surface slopes and the small incision depths of the rivers cause the groundwater table to lie within 2 m of the surface. Ice-pushed ridges with a width of several kilometres and a height of tens of metres form exceptional topographic features with deeper groundwater tables.

The natural drainage system is characterized by a network of small streams. The spacing of the first-order streams ranges from about 200 m for areas with a shallow groundwater table (< 0.5 m), to about 2000 m for areas with deeper groundwater tables (> 2 m) (Fig. 2). Fig. 3 shows a typical example of a stream network for an area with a medium groundwater table. The topographic slopes perpendicular to the

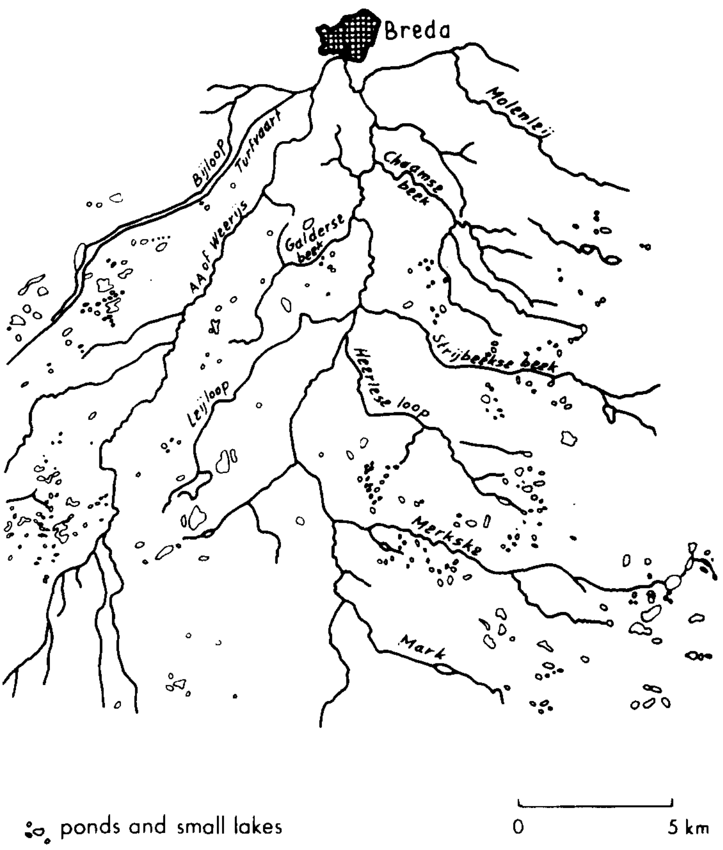

Fig. 3. Example of a stream network in an area with a medium groundwater depth: 20–70 cm for the lower areas; 70–200 cm for the higher areas. (From the topographic map 1:50 000, issued in 1850, before the large-scale artificial drainage works were pursued; for location see Fig. 1.)

stream channels (transversal slopes) are remarkably uniform and close to 1:500 for the smaller as well as for the larger streams. The same applies to the stream gradients (longitudinal slopes), which vary from 1:1500 for the steepest areas to 1:2500 for the most level areas (De Vries, 1974; Ernst, 1978). Horton (1945) suggests that

$$\cos\phi = s_{\mathrm{m}}/s_{\mathrm{t}} \tag{1}$$

where ϕ is the junction angle between tributaries with slope s_{t} and the main stream with slope s_{m}. The above-mentioned ratio of about 0.25 for the longitudinal and transversal slopes in the Netherlands forms a maximum value of the ratio $s_{\mathrm{m}}/s_{\mathrm{t}}$, and this would mean according to Eq. (1) a maximum junction angle of 75°. The observation on the natural stream system in this country indeed shows this relatively high value as an upper limit (see e.g. Fig. 3).

The studied drainage system is tributary to larger streams which have their main drainage basin outside the present area. These larger streams are known to have persisted during the last Pleistocene (Weichselian) period, and are thus associated with the longer time scale of the primary topography. The smaller rivers (maximum spacing about 3000 m) are expressions of the present Holocene climate and have developed in response to this climate and the pre-existing primary topography. It has been argued (De Vries, 1974) that this process evolved as an increase in stream density by headward migrating erosion, beginning at the large Pleistocene stream system.

The following average hydrogeological data for the Pleistocene landscape have been observed (e.g. De Vries, 1974; Ernst, 1978):

(1) transmissivity of the upper part of the Pleistocene fluvial aquifers (T) — 500–2000 m^2 day^{-1};

(2) average permeability of Coversand (K') — 3 m day^{-1}; glacial till and fluvial clayey deposits cause lower permeabilities locally;

(3) average thickness of the Coversand (b') — 5 m.

3. Mathematical model

3.1. Drainage formulae

Groundwater flow to the stream systems can be schematized as a flow towards a parallel set of drains (Fig. 4), and can be described by the formula given by Ernst (1956, 1962)

$$\frac{\Delta h}{U} = \frac{L^2}{8T} + L\Omega \quad (T) \tag{2}$$

where U is the average flux through the phreatic surface per unit area (LT^{-1}), T is the transmissivity of the aquifer ($L^2\,T^{-1}$), Δh is the difference in hydraulic head between the divide and the discharge base (L), L is the stream spacing (L), Ω is the resistance for the radial flow component near the drain ($T\,L^{-1}$), caused by the upward bending and contraction of the flow lines. The radial flow resistance depends on geometry and

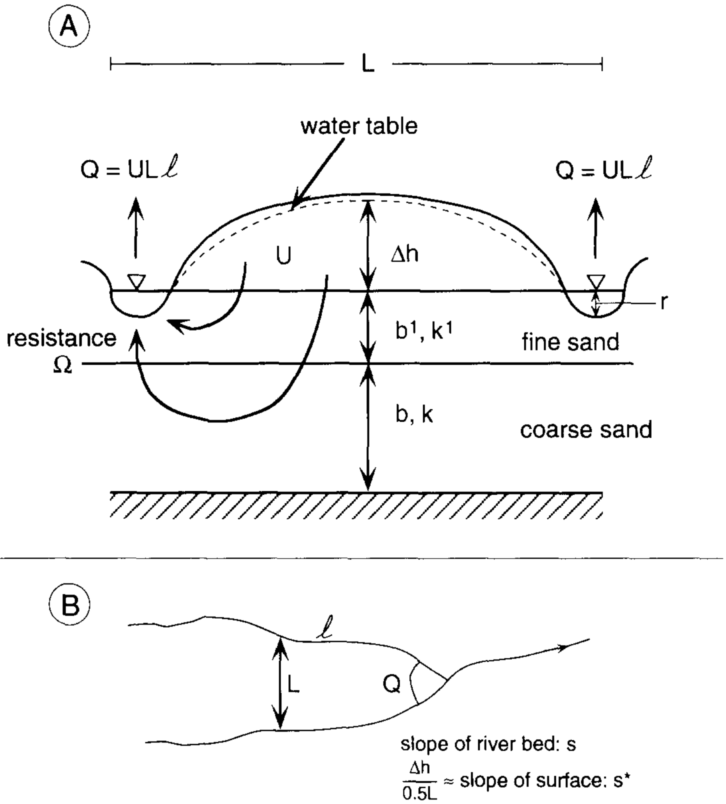

Fig. 4. Conceptualization of the combined groundwater (A) and stream flow model (B).

incision depth of the drainage channel and the ratio between the hydraulic soil properties for the fine Coversand layer and the coarse underlying aquifer material. The ratio of the hydraulic conductivities is about 1:5, whereas the ratio of the thickness for these layers is 1:10. With these figures and modified formulae on radial flow given by Ernst (1962), the following relation can be derived (De Vries, 1974, 1977)

$$\Omega = \frac{1}{\pi K'} \ln \frac{5b'}{B} \quad (TL^{-1}) \tag{3}$$

where K' (LT^{-1}) and b' (L) are the hydraulic conductivity and thickness of the Coversand layer, B is the wetted perimeter of the stream (L). The maximum hydraulic gradient $\Delta h_{\max}/0.5L$ coincides with the surface slope (s^*) perpendicular to the channel (transversal slope).

3.2. Streamflow formulae

Streamflow can be described by the well-known Gauckler–Manning formula

$$Q = k_m A R^{0.67} s^{0.5} \quad (L^3 T^{-1}) \tag{4}$$

where A is the wetted cross-section (L^2); k_m is the roughness coefficient ($L^{0.33}\ T^{-1}$), R is the hydraulic radius $(A/B)(L)$, s is the hydraulic gradient (slope of stream bed for uniform flow), Q is the discharge ($L^3\ T^{-1}$), and B is the wetted perimeter (L).

If we assume for simplicity's sake half-circular channels with radius r, then: $A = 0.5\ \pi\ r^2$ and $B = \pi\ r$, thus $R = 0.5r$.

Substitution of these relations in Eqs. (2)–(4) for the maximum groundwater discharge gives

$$U_{\max} = \frac{0.5s^*}{\dfrac{L}{8T} + \dfrac{1}{\pi K'}\ln\dfrac{5b'}{\pi r}} \quad (LT^{-1}) \tag{5}$$

and

$$Q = k_m r^{2.67} s^{0.5} \quad (L^3T^{-1}) \tag{6}$$

The discharge (Q) at the junction of the stream of the next higher order equals (assuming parallel streams)

$$Q = ULl \tag{7}$$

where l is the stream length. If α is introduced as the ratio l/L and one applies in further considerations the discharge midway between two junctions (Q) as the flow rate controlling the average stream dimensions, then one has

$$\mathrm{Q} = 0.5\alpha UL^2$$

Substitution of Eq. (6) gives

$$U_{\max} = \frac{k_m r^{2.67} s^{0.5}}{0.5\alpha L^2} \quad (LT^{-1}) \tag{8}$$

From observations in the Netherlands Pleistocene area the following average values for the roughness coefficient k_m were obtained (e.g. Ernst, 1978): k_m varies from about 5 $m^{0.33}\ s^{-1}$ for the smallest streams to 40 $m^{0.33}\ s^{-1}$ for the wider streams. α shows an average value of 5 and ranges from 4 for the smaller streams to 12 for the largest streams. This relatively high ratio reflects the pseudo-parallel character of the stream network (see e.g. Fig. 3).

3.3. Required discharge capacity

The discharge capacity required to cope with precipitation excess depends on the combination of rainfall characteristics and storage capacity. Since the rate of increase in the n-day sum of precipition becomes progressively smaller with greater n, the required discharge capacity decreases progressively as the number of days available for the discharge process increases. This number of days is determined by the storage capacity available at the beginning of a rainy period and by the discharge rate when the storage capacity is used up. In other words, the storage capacity and the prevailing discharge rate determine when the n-day sum reaches a critical value.

In order to avoid exceeding groundwater storage capacity, which would result in ponding and marshy conditions, the following condition has to be satisfied

$$t\bar{U} \geqslant ti - S \quad (L) \tag{9}$$

where t is the period length of rainfall, i is the average rainfall rate per time unit, $\bar{U}$ is the average discharge and S is the groundwater storage in the unsaturated zone.

The total storage capacity between depth d and 0 is denoted $S(d)$. The relation between the mean rainfall amount per time-unit that is exceeded with a given probability in some time interval and the length of that time interval, can be expressed by a rainfall intensity–duration–frequency relationship of the type (De Vries, 1974)

$$i = ct^{-m} \tag{10}$$

where i is the mean amount of precipitation per time-unit that is exceeded with a given probability p in a time interval t, c and m are local climatic parameters depending only on p. Substitution of Eq.(10) in Eq.(9) yields

$$\bar{U} = ct^{-m} - St^{-1} \tag{11}$$

The time t_* (critical time) for which $U = U_{\max}$ is obtained by taking

$$\frac{\mathrm{d}\bar{U}}{\mathrm{d}t} = 0$$

If S is assumed to be constant (storage capacity available), this gives

$$-mct_*^{-m-1} + St_*^{-2} = 0$$

or

$$t_*^{-2}(-mct_*^{-m+1} + S) = 0$$

If $t \neq 0$, then

$$t_* = \left(\frac{S}{mc}\right)^{\frac{1}{1-m}} \tag{12}$$

Substitution of Eq. (12) in Eq. (11) yields

$$\bar{U}_{\max} = c(1-m)\left(\frac{S}{mc}\right)^{\frac{-m}{1-m}} \tag{13}$$

or

$$\bar{U}_{\max} = (1-m)i \tag{14}$$

$\bar{U}_{\max}$ is the average groundwater discharge during the rise of the groundwater table that is required to avoid exceeding the available groundwater storage capacity. For simplicity's sake one assumes that the average discharge is half its highest value during the rise of the groundwater table from the depth where discharge starts, to the surface. In the following the stream spacing will be considered as a function of the required discharge U_r at the highest groundwater level. Thus

$$U_r = 2\bar{U}_{max} \tag{15}$$

The storage capacity $S(d)$ (L) is defined as the volume of water that is stored per unit surface area when the groundwater table rises from a given depth d to the surface. It depends on the storage factor μ which is a function of the soil-physical characteristics, the successive cycles of wetting and drying and the depth of the groundwater table. In fact, it is the available porosity in the unsaturated zone that new infiltration can occupy without inducing surface ponding. Its value, therefore, varies with time and location and is difficult to estimate. The relationship between groundwater depth d and storage factor μ obtained from various observations was used to formulate the following approximation (De Vries, 1974)

$$\mu = a(d - z) \tag{16}$$

where a is a constant and z forms the vertical coordinate that is taken positive in a downward direction, so that μ decreases from the surface to the water table.

The storage capacity $S(d)$ between the surface and the groundwater table reads

$$S(d) = \int_0^d \mu \, dz$$

or

$$S(d) = 0.5ad^2 \tag{17}$$

The factor a is close to 0.1 m^{-1} and since the maximum value of μ is approximately 0.25, Eq. (18) is limited to $d < 2.5$ m. The storage capacity $S(d)$ for 0 m $< d <$ 2.5 m can thus be approximated by

$$S(d) = 0.05d^2 \tag{18}$$

with S in metres. The total storage capacity for $d = 2.5$ m is thus 0.3 m, which approximates the total winter precipitation surplus in the Netherlands; Eq. (18) therefore includes any storage event given the present climate. The required discharge is no longer dependent on groundwater depth when $d > 2.5$ m; in this case the storage is really sufficiently large that the precipitation intensity duration relationship is no longer relevant, but annual variation in precipitation excess governs. The discharge required under such conditions reduces eventually to the annual precipitation surplus of about 1 mm day^{-1}, and hence the variation in stream spacing and dimension depend exclusively on the variation in topographic slope and subsurface permeability.

A combination of Eqs. (13), (15) and (18) yields the required discharge U_r as a function of initial groundwater depth d

$$U_r = 2c(1 - m)\left(\frac{0.05d^2}{mc}\right)^{\frac{-m}{1-m}} \tag{19}$$

As discussed previously, it is assumed that the drainage system has developed in equilibrium with, and response to, the winter rainfall events which occur with a frequency of 5% during a winter period. The 1% case will also be considered for comparative purposes, thus complying with the design criterion for the artificial drainage system. The 10% case is also examined, corresponding with conditions with more retention storage. For Eq. (10) the following parameters apply (De Vries, 1974).

Frequency (%)	c (mm day^{-1})	m
1	20	0.38
5	10	0.25
10	6	0.175
20	3	0.05

These figures substituted in Eq. (10) give the average amount of rainfall per time unit that is reached or exceeded during a rainfall event, occurring with indicated percentage of all days within a winter period.

4. Results and discussions

4.1. The theoretically synthesized stream system

The discharge capacity required to prevent ponding is a function of initial groundwater depth and can, for different frequencies of rainfall, be calculated using Eq. (19). Subsequently, the required stream spacing and channel size for given subsurface permeability and topographic slope, can be determined by substitution of U_r, in Eqs. (5) and (8).

Fig. 5 shows the calculated stream spacing, channel radius and discharge capacity determined as a function of initial groundwater depth for the following geological, geomorphological and climatic parameters: transmissivity $T = 1000$ m^2 day^{-1}; permeability of shallow layer $K' = 3$ m day^{-1}; gradient of land surface slope perpendicular to stream channel s^* (transversal slopes) ranging from 1:100 for the smallest streams to 1:500 for the larger streams, with the slope of the stream beds s decreasing from 1:500 to 1:2000 dependent on whether the streams are small or large. These parameters correspond with average conditions for the Netherlands Pleistocene area, except for the topographic slopes s^*, which in reality present lower gradients (about 1:500) and no differentiation according to stream size. This flat and uniform topographic situation probably led to the marshy and inadequate natural drainage condition that prevailed in the area before artificial improvement of the dewatering system had been carried out (Fig. 3). Therefore, the above more plausible topographic pattern was applied to synthesize a well integrated and adequate drainage system. For this theoretical model the ratio s/s^* was kept close to 0.3, so that the stream junction angles according to Eq. (1) approach the upper limit of the observed value of about 70° in the Netherlands, corresponding to the maximum relief conditions in this area.

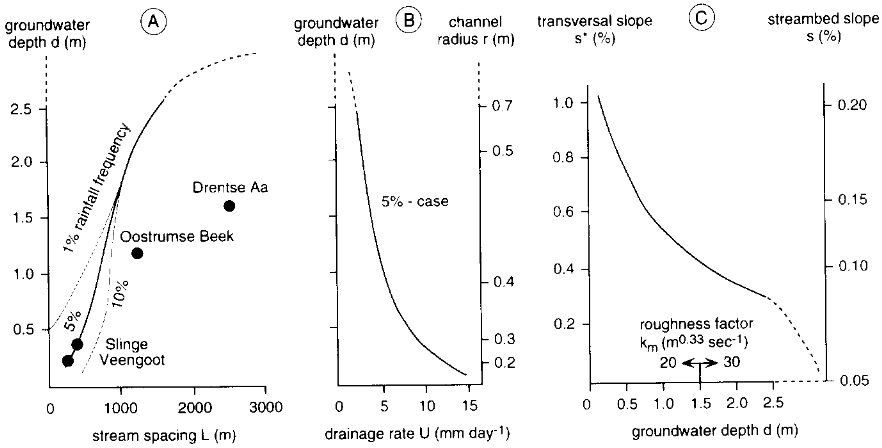

Fig. 5. Stream spacing, channel size and drainage rate as functions of groundwater depth for given terrain slopes and various rainfall-frequency criteria. Calculated from Eqs. (5), (8), and (19) with $T = 1000$ m^2 day^{-1} and $K' = 3$ m day^{-1}. The observed values of d vs. L for the stream systems of Fig. 2 (see also Table 1) are indicated with dots. The deviation from the theoretical curves reflects the low (needed) discharge capacity of the drainage system in the period before the retention storage in marshes was reduced by reclamation works. (The artificial ditch system in the former marshes is not included in the considered stream systems.)

Fig. 5 depicts the results for the assumed topographic slope distribution. For the 5% rainfall-frequency case it shows an increase in stream spacing from 250 to 1500 m, and an increase in channel radius from 0.2 to 0.7 m from areas with an average (wet season) groundwater depth of 0.25 m to areas with an average groundwater depth of 2.5 m. Discharge capacity reduces from 12 mm day^{-1} to 3 mm day^{-1} for the increased groundwater depth. For groundwater depths of greater than 2.5 m the required discharge reduces from the average winter precipitation surplus of 2 mm day^{-1} to the average annual precipitation surplus of 1 mm day^{-1}. Stream spacing then increases from 1700 to 3300 m for the average terrain slopes of the Netherlands Pleistocene landscape, whereas the channel radius comes close to 1 m.

Stream spacing for the 1% case evidently reduces progressively with decreasing depth in comparison with the 5% case. Stream spacing reduces to nil (marshy conditions) in the 1% case for groundwater depths of less than 0.5 m. In contrast, the 10% frequency case allows larger stream spacings which range from 250 to 750 m when groundwater depth increases from 0.1 to 0.5 m.

4.2. The actual stream system in the Netherlands

The transversal topographic slopes s^* in the Netherlands are of the order of 1:500 and thus represent the lower limit of the gradients that were applied in the previous calculations. Such slight slope conditions would, in these calculations for the 5% frequency case and groundwater depths of less than 2 m, result in stream spacings that reduce to zero, i.e. to marshy and peat conditions. It is, therefore, interesting to extrapolate back with the present mathematical model the original drainage capacity

Table 1
Discharge capacity and channel size as calculated for observed stream characteristics

Location	Observed				Calculated	
	L (m)	s^*	s	d (m)	U (mm day^{-1})	r (m)
Veengoot	210	1:550	1:2200	0.3	1.9	0.2
Slingebeek	400	1:550	1:2600	0.4	2.2	0.25
Oostrumse Beek	1000	1:500	1:1750	1.2	2.1	0.6
Drentse Aa	2500	1:500	1:2500	1.5	1.8	1.1

for the natural system, exemplified by the streams depicted in Fig. 2. The results are presented in Table 1 and are indicated in Fig. 5(A).

The extrapolated historical discharge capacity of about 2 mm day^{-1} corresponds rather well with the average winter precipitation surplus. It thus seems that part of the Pleistocene area is too flat for the development of an adequate drainage network. This led to the marshy conditions with high retention storage that prevailed in many parts of the Pleistocene flatlands, especially on flat divides and in depressions. In reality the discharge capacity must have been even less, because the observed (and applied) average winter groundwater tables have been lowered in recent times by improved dewatering through artificial enlargement of the cross-sectional area of the stream channels. Thus, the original drainage capacity might have been close to the average annual precipitation surplus of a marshy area.

The artificial increase in drainage capacity has been accomplished through dewatering of the marshes by ditches and enhancement of the discharge by enlargement and canalization of the existing streams. The artificial deeping of the channels is clear from a comparison of the calculated stream radius (Table 1) with the incision depths, indicated in Fig. 2. Evidently, the groundwater tables depicted in Fig. 2 are somewhat lower than in the past, but they still reflect the original configuration. The artificial ditch system in the former marshes is not included in the determination of the actual stream spacings of Table 1.

4.3. Stream network as an energy dissipating system

From the theoretical concept of optimal energy dissipation and from field observations (e.g. Leopold et al., 1964; Rodriguez-Iturbe et al., 1992a, b), it is plausible that in a drainage network the stream depth is proportional to the square root of discharge, with the proportionality factor being the same everywhere in the network (power-law relation of mass distribution). Fig. 5 reveals that the synthesized stream system quite well accords with this relation for streams with a spacing of greater than 250 m (Table 2). The stream depth (represented by its radius r) is proportional to the discharge during bank-full flow according to the relation

$$r = 0.003Q^{0.5}$$

Another consequence of the theoretical considerations is that the product Qs can be

Table 2
Characteristics of the synthesized stream system depicted in Fig. 5

L (m)	r (m)	U (mm day^{-1})	Area $= \alpha L^2$ (m^2)	$Q = AU$ (m^3) $\times 10^3$	$r/Q^{0.5}$ $\times 10^{-3}$	s %	$Q^{0.5}s$ $\times 10^{-2}$
250	0.25	10	$4L^2$	2.5	5.0	0.2	0.10
500	0.35	8	$6L^2$	12	3.0	0.2	0.23
750	0.4	5	$8L^2$	23	2.7	0.15	0.20
1000	0.5	3.5	$8L^2$	28	3.0	0.1	0.17
1500	0.7	2.5	$10L^2$	56	2.9	0.1	0.24
2000	0.8	2	$10L^2$	80	2.8	0.06	0.17
3000	0.9	1	$12L^2$	108	2.7	0.04	0.13

expected to be constant through the network (power-law relation of energy distribution). The above theoretical network shows values of Qs between 0.0013 and 0.0024. When considering these power-law relations for the distribution of energy and mass, one should realize that the system of streams depicted in Fig. 5 and Table 2 do not form one stream network. They represent the lower order streams of different basins which are each characterized by their own topography and associated groundwater depth. On a larger scale, however, they can be considered as sub-basins of a larger system and, as such, can be expected to comply approximately with the energy concept for the whole basin.

Conditions in the present Netherlands landscape do not fit these relations at all. Ernst (1978) observed that stream depth is proportional to the 0.17 power of the flow. This obviously reflects the artificial (and needed) reduction in flow resistance by stream canalization. This artificial increase in the runoff capacity for the streams was necessary to cope with the enhanced discharge from the reclaimed marshes.

5. Conclusions

(1) Groundwater flow systems and streams form a continuous drainage system in areas with exclusively groundwater discharge. Seasonable groundwater table fluctuations in such systems compensate the inbalance between recharge and discharge by causing a change in storage and by regulating the number of streams that participate in the drainage process.

(2) Considering a stream system to be the outcrop of an associated groundwater flow system enables the development of a theoretical model on the basis of flow formulae that interconnect groundwater flow and stream runoff. This conjunctive model relates geometrical properties of the stream network as dependent variables to the given geological, topographical and climatic conditions.

(3) The theoretically synthesized stream system corresponds to the observed tendency of increased stream spacing with greater average groundwater depths, for shallow groundwater tables (< 2.5 m), within a given topographic situation and given subsurface permeability in the Netherlands Pleistocene area. The same applies to the relationship between increased stream spacing with steeper topographic slope

for groundwater depths exceeding 2.5 m. The groundwater depth of 2.5 m represents the depth that allows phreatic storage of a seasonal precipitation surplus. The figure thus depends on prevailing climate and soil-physical conditions.

(4) The geometrical and hydraulic properties of the synthesized stream network also correspond to characteristics that can be expected from an optimal energy dissipating system. Notably, the proportionality of stream depth and the square root of stream flow are in accordance with theory on the distribution of mass in such systems and with observations elswhere.

(5) Present channel sizes in the Netherlands Pleistocene area (Ernst, 1978) are about twice those calculated. This obviously is due to the artificially improved discharge capacity of the present stream system. Stream depth is proportional to the power 0.17 of stream flow instead of the theoretically determined power 0.5, which can also be explained by a reduced flow resistance through stream canalization.

It may be concluded that the described theoretical model explains rather well the observed close relationships between groundwater depth, topography and stream net properties. This applies to the average stream net conditions and average groundwater depth, as well as to the the dynamic relation between seasonal groundwater table fluctuations and the variable system of streams which participate in groundwater drainage.

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