A SIMPLE MODEL FOR PREDICTING THE EFFECTS OF WINTER LEACHING OF RESIDUAL NITRATE ON THE NITROGEN FERTILIZER NEED OF SPRING CROPS

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Summary

A model is described for predicting the amounts of nitrate residues remaining in the soil in the autumn and the extent to which these are lost by leaching during the winter. It is used to predict the effects of winter rainfall on the growth of spring crops. The model is based on the assumption that any nitrate which remains in the effective rooting zone of the spring crop after the winter will be equally available and any nitrate leached below it will be totally unavailable. The losses of nitrate were estimated from the excess rainfall and the water holding capacity of the soil.

The model was tested against the results of published N response experiments in the Netherlands and the UK. It predicted the differential effects of winter rainfall with reasonable accuracy, but tended to overestimate the spring nitrate contents observed in the Dutch experiments and underestimate those for the English data. These deviations appeared to be associated with errors in the estimation of the amounts of nitrate remaining in the soil in the autumn.

With certain reservations, the model would appear to provide helpful advice for adjusting spring fertilizer dressings for the effects of differences in winter rainfall.

Introduction

A LARGE proportion of the residual nitrate (from soil and fertilizer) which remains in the soil after harvest in the autumn is lost by leaching during the winter. The size of these losses depends on the extent of winter rainfall, so the amounts of residual nitrate which are retained in the spring can be extremely variable. Results have shown that these effects can contribute substantially to variations in the response of crops to N fertilizer (Lehr and Veen, 1952; Boyd *et al.*, 1957; van der Paauw, 1962, 1963*a* and *b*, 1968, 1972; Eagle, 1971; Lidgate, 1978).

Attempts have been made to use the results of such expeiments as the basis of empirical methods for adjusting spring N dressings for differences in winter rainfall (van der Paauw, 1959; Eagle, 1971). However, these methods are limited to the relatively few crops grown on the small number of soils for which suitable response data is available. An alternative approach, based on soil analysis in which samples are taken to a depth of a metre, has also been proposed for winter wheat (Jungk and Wehrmann, 1978). Although this method may prove to be more widely applicable (provided the depth of sampling for each crop is known), it may not always be convenient to make the necessary measurements in the period

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immediately before fertilizer application. There is, therefore, a need for a simple method for rapidly predicting the influence of winter rainfall on the N fertilizer needs of all spring crops.

Previous publications have described a simple practical procedure for calculating the extent to which yields can be effected by nitrate leaching during the growing season (Burns, 1980*a*, *b*). This was based on the assumption that nitrate was uniformly available within the effective rooting zone of the crop, and totally unavailable below it. The object of this paper is to describe a procedure for estimating the amount of residual nitrate remaining in the soil in the autumn and to show how the above model can be adapted to predict the extent of leaching during the winter. The new model has been deliberately framed in a simple form so that it may be used for making on-site predictions of the influence of winter rainfall on the N fertilizer needs of individual crops. The validity of the model is tested against the results of published experiments in both The Netherlands and the UK.

Methods

The method comprises four separate steps:

1. Calculation of an effective rooting depth typical of the spring crop at harvest.
2. Estimation of the size of the nitrate residues remaining in the soil in the previous autumn from tabulated data for different cropping systems.
3. Calculation of the amount of residual nitrate displaced from the rooting zone during the previous winter, assuming that it was uniformly distributed within the soil at the start.
4. Calculation of the amount of residual N which remains available to the spring crop from the difference between the amount of residual nitrate present in the autumn and that leached during the winter.

Details of these steps are given below.

1. *Effective rooting depth*

An average value of effective rooting depth for each mature spring crop was calculated from typical values of its aerial dry weight and population density and the diameter of its roots using a simple equation derived from a previous study of root distribution data for vegetable and cereal crops (Burns, 1980*a*). The values were assumed to correspond to the depth at which nitrate becomes unavailable to each crop.

2. *Residues of nitrate present in the autumn*

Estimates of the average amounts of inorganic N remaining in the soil in the autumn were selected from a table of data (see Table 2) for soils of different N index, defined on the basis of recent cropping history (M.A.F.F., 1973). These data were calculated from the size of the N dressings applied to the previous crop and the amount of N released from the organic matter. Adjustments were made for the N recovered by this crop, but no account was taken of any losses of nitrate by leaching or denitrification during its growth.

Net release of inorganic N depends on the rates of mineralization and immobilization in the soil. Although recent work has suggested that organic matter should be considered as containing 'pools' of different organic material each with its own decay characteristics (Jenkinson and Raynor, 1977), there is much evidence which suggests that its decomposition often loosely approximates to a first order reaction (Jenkinson, 1965; Kolenbrander, 1975), in which the average half life of the native organic matter is relatively constant at *ca* 20 to 25 years (Monteith *et al.*, 1964; Jenkinson, 1965). If the organic matter is of reasonably uniform composition, the net release of inorganic N should mirror the decline in organic matter and also follow first order kinetics:

$$\frac{dN_{org}}{dt} = -k\,N_{org} \qquad (1)$$

where N_{org} is the organic N content of the soil (kg ha^{-1}), t is the time (years) and k is the rate constant for the net release of inorganic N (year^{-1}). Integrating Equation 1 gives:

$$N = N^{*}_{org}(1 - e^{-kt}) \qquad (2)$$

where N is the net amount of inorganic N released (kg ha^{-1}) and N^{*}_{org} is the initial value of N_{org}.

Calculations show that N^{*}_{org} for the top metre of a typical index 0 soil at Wellesbourne is *ca* 6000 kg N ha^{-1}. If it is assumed that $k = 0.0347$ year^{-1} (corresponding to an organic N half life of 20 years), and that only 75 per cent of the N mineralized per annum is released before harvest, then according to Equation 2 the maximum amount of mineralized N available to crops is 153 kg N ha^{-1}. In general, only about half of the available N will be recovered (Cooke, 1977), suggesting that an average uptake of 76 kg N ha^{-1} could be expected for this soil. This prediction is in good agreement with average data for the uptake of N from unfertilized plots by a number of vegetable crops (Greenwood, personal communication).

Although soils of higher N index release more N than those of index 0, this does not appear to be directly associated with a higher organic N content. The differences appear to be related more to differences in the type of organic matter present (resulting from their various cropping histories), and the influence of this on the net rate of release of inorganic N. Nevertheless, these soils behave as though they have a higher *effective* organic N content than those of index 0, where the effective organic N content may be defined as the amount of organic N of equivalent form to that at Wellesbourne which must be postulated to be present to produce the amount of inorganic N observed. By assuming different effective organic N contents for soils of different N index, calculations of their net rates of mineralization can be made using the same value of k.

As a result of advisory experience, it has been found that, as a general rule, an index 4 soil declines to one of index 0 *ca* 12 years after ploughing out good quality grassland (M.A.F.F. 1973). On this basis it can be calculated from Equation 2 that an index 4 soil must have an effective organic N content of 9094 kg N ha^{-1} in the top metre if it is to decline to 6000 kg N ha^{-1} in 12 years and to half its value after 20 years. Similar

TABLE 1

Estimated effective organic N contents and amounts of inorganic N available to crops in the top metre of different soils

N index	*Time taken for soil to decline to N index 0 (year)**	*Effective organic N content (kg ha^{-1})*	*N mineralized during crop growth (kg ha^{-1})*	*Average N fertilizer appl. (kg ha^{-1})**		*Total N available to crop (kg ha^{-1})*	
				Cereals	*Other crops*	*Cereals*	*Other crops*
0	0	6000	153	88	163	241	316
1	4	6892	176	63	157	239	333
2	7	7647	195	38	119	233	314
3	10	8485	217	13	100	230	317
4	12	9094	233	6	63	239	296
Mean	—	—	—	—	—	236.4	315.2

*Based on advisory experience (M.A.F.F., 1973).

TABLE 2

Data for the estimation of autumn N contents of the top metre of soil

N index for current year	*Recent cropping history*	*Conditions in the previous year*				
		N Index	*Total N available to crop (kg ha^{-1})*	*N remaining at harvest (kg ha^{-1})*	*N mineralized after harvest (kg ha^{-1})*	*Autumn N (kg ha^{-1})*
0	cereals (>2 years)	0	241	121	51	172
0	cereals (2 years)	1	239	119	59	178
1	cereals (1 year)	2	233	117	65	182
2	other arable crops	2	314	157	65	222
3	other arable crops	3	317	159	72	231
4	other arable crops	4	296	148	77	225

calculations may also be made to determine the effective organic N contents of soils of intermediate index, as shown in Table 1.

The maximum amount of mineralized N available to crops from each of these soils was calculated from Equation 2 (again assuming that only 75 per cent of the N mineralized per annum is released before harvest) and the values given in Table 1. Typical average dressings of N fertilizer for cereals and various other arable crops (M.A.F.F. 1973) are also given in this Table. The sum of the corresponding amounts of mineralized and fertilizer N represents an average estimate of the available N in the soil when no residues of nitrate remain from the previous year. The data show that these totals are approximately constant within each of the crop groups, which suggests that N fertilization for a given crop raises the N status of all soils to the same level irrespective of their N index. This conclusion agrees with the findings of Greenwood *et al.*, (1974), who showed that the variability in N response of crops between sites was reduced by half when this assumption was made.

These data were used to estimate the amounts of residual nitrate remaining in the top metre of soil in the autumn in Table 2. The N indices for the current year and a summary of recent cropping history are given in the first two columns; the remainder of the Table refers to conditions in the previous year. The calculations were made assuming the N index did not change over the two years, except when there was a previous history of 1 or 2 years of cereals.

The amounts of soil plus fertilizer N available to the previous crop were taken from Table 1. These data assume that no residues remain from the year before last, so their values are likely to be underestimated. However, any errors introduced by this omission are likely to have only a small effect in the current year (Fisher, 1925; van der Paauw, 1972), and will tend to be counterbalanced by any losses of nitrate (*e.g.* by denitrification). The amounts of N remaining in the soil at harvest were calculated assuming that all crops recover 50 per cent of this total available N (Cooke, 1977). This figure is also in good agreement with average data of Widdowson *et al.*, (1967), Greenwood *et al.*, (1974) and Johnston (1976) for the recovery of fertilizer N by various non-leguminous crops. Autumn N contents were calculated from the sum of the N remaining at harvest and the amount of N mineralized subsequently (which was assumed to correspond to 25 per cent of the total annual N release). No other changes in N level were assumed to occur before the onset of winter leaching, by which time all of the residual N was considered to be in the nitrate form. Following the data of Needham (1976) and Shaw (1975), all of this nitrate was assumed to be uniformly distributed within the soil profile.

The data in Table 2 fall into two distinct groups, depending on the nature of the last crop. Where cereals were grown, residual nitrate contents of the top metre average 178 kg N ha^{-1}, whereas the corresponding amounts after other arable crops are 226 kg N ha^{-1}.

3. *Nitrate displacement from the root zone*

The amount of nitrate displaced from the potential rooting zone of the spring crop during the previous winter was calculated using a simple chromatographic equation which assumes that nitrate is uniformly distributed within the soil when leaching occurs (Burns, 1976):

$$L \approx \frac{Ah}{100}\left(\frac{P}{P+\theta_m}\right)^{h/2} \tag{3}$$

where h is the effective rooting depth in cm (Section 1), A is the amount of residual nitrate $-$N present in the top metre of soil in the autumn in kg ha^{-1} (Section 2), P is the accumulated drainage in cm (*i.e.* the total amount of rain or irrigation water percolating through the soil) and θ_m is the volumetric field capacity of the soil (cm^3 cm^{-3}). The derivation of values of P and θ_m have been given previously (Burns, 1980*b*).

4. *Nitrate remaining in the spring*

This was considered to correspond to the difference between the autumn N content and the amount of residual nitrate lost by leaching during the winter. It was assumed that all of these spring residues were equally available to the spring crop no matter how they were distributed within the rooting zone.

Tests of the method

Residual nitrate present in the autumn

Estimated amounts of residual nitrate remaining in the top 60 cm of soil in the autumn are compared with corresponding experimental data for a number of sites between 1973 and 1975 in Table 3. The measured values were calculated from analytical data (Needham, 1976), assuming a bulk density of 1.5 g cm^{-3}. The estimated values were taken from Table 2 after scaling for the shallower depth of sampling in this survey.

The results show that the estimates were generally higher than the average amounts of autumn-nitrate found in these soils. However, since most of the estimates fell within the range of experimental data observed (which varied considerably from site to site), it is clear that they are of the correct order of magnitude.

Residual nitrate present in the spring

Fertilizer residues

Tests of the model were made by comparison with the experimental data of van der Paauw (1963*a*). The experiments were designed to measure the effects of fertilizer residues on the growth of spring-sown crops at seven sites between 1955 and 1962 and at one site between 1955 and 1960 in the Netherlands. Potatoes, wheat or rye, and oats were grown at each site in rotation. The residual effects were expressed as the ratio of the increase in yield from the residues of a previous dressing to the increase in yield when the same amount of fertilizer was applied directly to the current crop. By defining residual effect in this way, van der Paauw was able to eliminate the influence of nitrate released from the organic matter.

The corresponding predicted residual effects (N_{Rp}) for each crop were calculated by the equation:

$$N_{Rp} = h(1 - u_F)(1 - 100L/Ah)$$

TABLE 3

Comparison of estimated and measured autumn nitrate levels for soils of N index 0 and 2

N index	*Previous crop*	*Soil*	*Number of data*	*Measured autumn nitrate (kg N ha^{-1})* range of data*	*Measured autumn nitrate (kg N ha^{-1})* mean*	*Estimated autumn nitrate (kg N ha^{-1})**
0	Cereals	sandy	27	28–99	61	105
		clay	27	42–75	63	
		silty	12	60–106	89	
2	Potatoes	sandy clay loam silty clay loam	12	53–241	110	133
2	Oil seed rape	clay loam	3	51–112	74	

*For the top of 60 cm of soil.

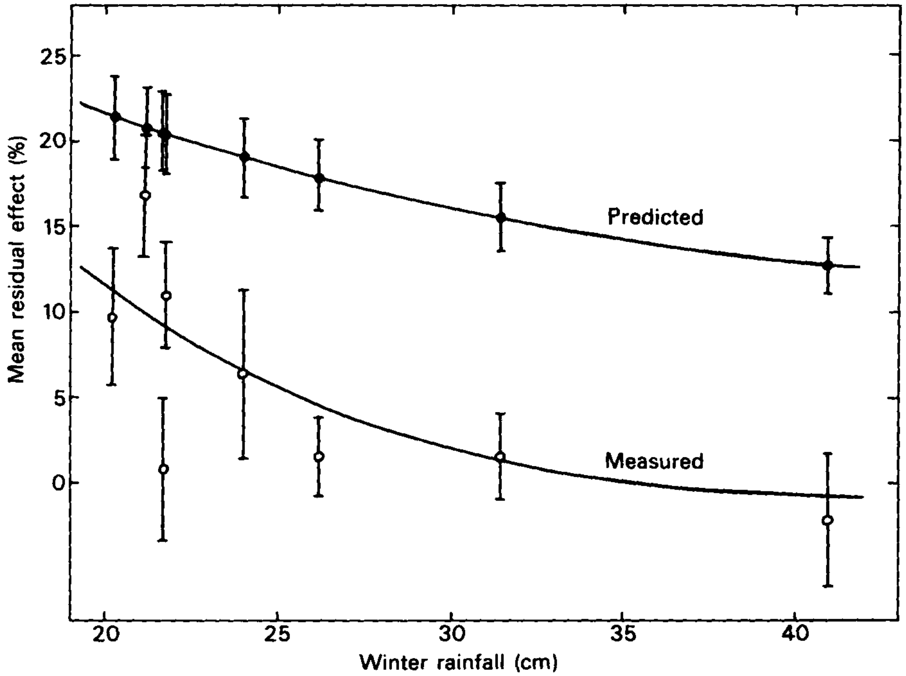

FIG. 1. Changes in mean residual effects with winter rainfall (November to March) in 8 rotation experiments in the Netherlands between 1955–62; (● predicted residual effects; ○ measured residual effects). The bars represent the standard errors of the means.

where u_F is the fraction of fertilizer N recovered by the previous crop, and $100L/Ah$ was estimated from Equation 3. u_F was assumed to be 0.5 (Cooke, 1977). This equation assumes that the residues of fertilizer were uniformly distributed within the top metre of soil in the previous autumn in the same way as the combined effects of soil and fertilizer (Needham, 1976; Shaw, 1975). Mean predicted residual effects for each year (together with the corresponding standard errors) were calculated from this equation, assuming that all crops were grown at all sites in each of the years.

The influence of winter rainfall on the estimated and observed residual effects is compared in Fig. 1. Although the predictions consistently overestimated the experimental data, the difference in slope between the two curves was fairly small (especially after wet winters), indicating that the predicted differential effect of winter rainfall on nitrate leaching was broadly correct. This suggests that much of the difference between the predicted and measured effects was caused by errors in the estimation of the amount and distribution of autumn nitrate in the soil, and by other losses of nitrate (*e.g.* by denitrification) which may have occurred during the winter. Nevertheless, despite these effects, statistical analysis showed that the predicted effects were significantly correlated with the experimental data at the 5 per cent level and that the slope of the corresponding regression line was not significantly different from unity.

Residues of nitrate from both soil and fertilizer

The model was tested against data from 4 separate experiments in the Netherlands in which residual effects were measured from changes in yield of different plant parts observed after the amount of winter rainfall intercepted by the soil had been reduced by the intermittent use of covers or supplemented by irrigation (van der Paauw, 1962, 1968). The results of a similar experiment in which spring rainfall was varied before sowing an autumn crop (van der Paauw, 1968) were also included. Some variation in measured data was observed depending on the plant fraction used for assessing the yield increases.

The estimated amount of residual nitrate remaining in the effective rooting zone of each crop was calculated by the model at the end of the period during which the different rainfall treatments were imposed (*i.e.* in the autumn or summer respectively). Nitrate contents of the soil at the start were derived from information on previous cropping (Table 2). Where this information was not available, it was assumed that the rotation: potatoes, wheat or rye, and oats used in similar experiments (van der Paauw, 1963*a*) was followed.

A comparison of the predicted differences in N residues between treatments and the corresponding mean differences in measured residual

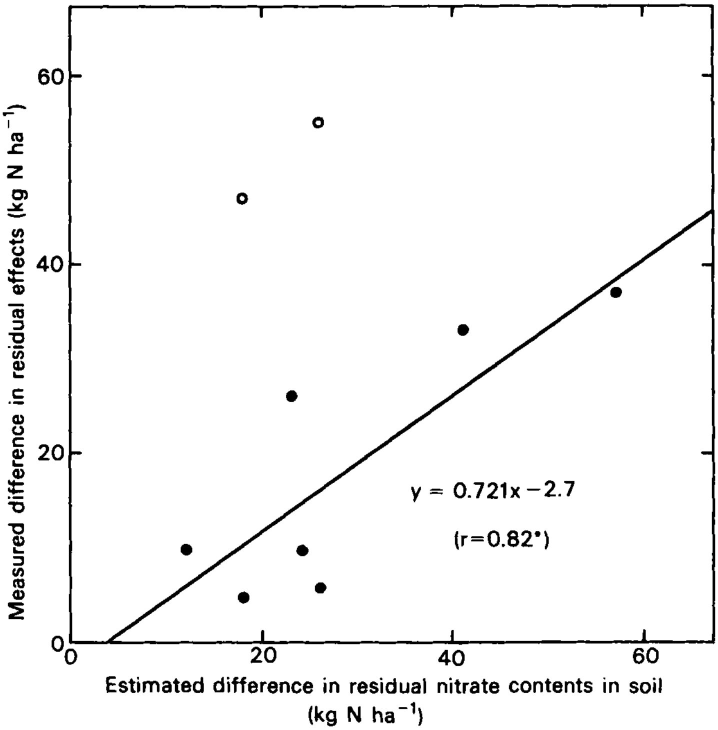

FIG. 2. Plot of the measured difference in residual effects *vs* the estimated difference in spring nitrate residues after intermittent covering or irrigation of the plots during the winter in the Netherlands; (○ with or ● without FYM application in the previous autumn).

effects is illustrated in Fig. 2. This shows that the predicted differences were in reasonable agreement with the observed data for most soils, except where dressings of FYM had been made shortly before the start of the experiment. The contribution of FYM to the nitrate content of these soils was calculated assuming that 10 t FYM was equivalent to 15 kg N for an average application (M.A.F.F., 1973), and under the conditions of these experiments this would appear to be a gross underestimation. When the results of the FYM treatments were ignored, the predictions were found to be significantly correlated with the measured data at the 5 per cent level and the regression line was not significantly different from the line of perfect agreement. This provides further evidence that the model gave satisfactory estimates of the differences in spring N residues in Dutch conditions, when no FYM had been applied.

The predictions of the model were also examined using N response data for the second winter wheat crop grown after a two-year ley (Lidgate, 1978). The experiments, which were conducted in a sandy loam soil overlying a clay subsoil at Jealott's Hill between 1969 and 1977, showed that more N was normally required to maintain yields after a wet winter than after a dry one. Optimum N dressings were estimated from Lidgate's data by fitting two straight lines through the data for average yield versus N level following the method of Boyd *et al.* (1970).

Estimates of the spring N contents were made from the losses of nitrate

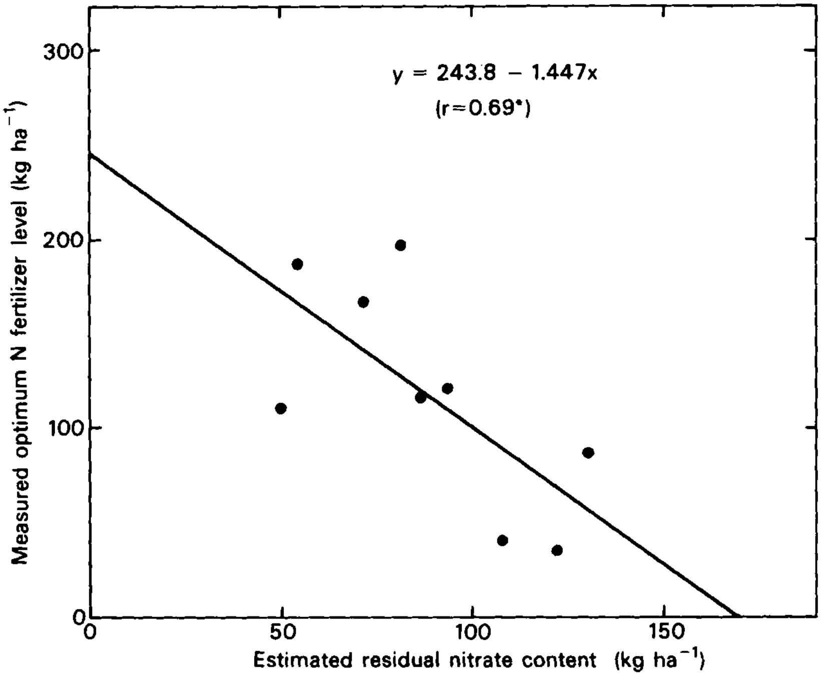

FIG. 3. Relationship between the estimated contents of residual nitrate in spring and the measured optimum N fertilizer dressings for 9 winter wheat crops in the U.K. between 1969–77.

between September and March for each of the years using the model. The amount of nitrate present in the autumn was derived from Table 2 assuming a N index of 3 for this soil.

The predictions are plotted against the measured optimum N fertilizer dressings in Fig. 3. It can be shown (see Appendix) that the data should fall on the line:

$$N_F = k_1 - \frac{u_R}{u_F} N_R \tag{4}$$

where N_F is the optimum amount of N fertilizer, N_R is the estimated amount of residual N, u_F and u_R are the fractions of fertilizer and residual N recovered by the crop, and k_1 is approximately constant. If the soil and fertilizer N were equally utilized (*i.e.* $u_R = u_F$) the graph should produce a line of slope -1.0. The actual slope in Fig. 3 is -1.447, which suggests that either N_R and N_F were not used in equal amounts, or that the residual nitrate contents were consistently underestimated (by *ca* 31 per cent) causing an increase in the gradient of the line.

Similar deductions were made when the grain yields from the unfertilized treatments (Y_o) in Lidgate's experiments were plotted against the predicted amounts of spring N (Fig. 4). It has been shown in the

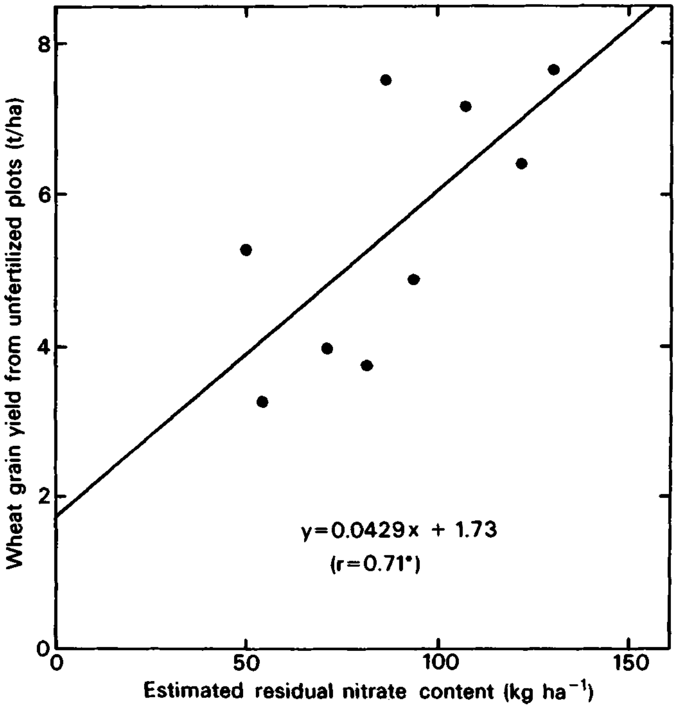

FIG. 4. Relationship between the estimated contents of residual nitrate in spring and the measured wheat grain yields from unfertilized plots in 9 experiments in the U.K. between 1969–77.

Appendix that these data should fall on the line:

$$Y_o = \frac{u_R}{F_N} N_R + k_2 \tag{5}$$

where F_N is the fractional N content of the grain and k_2 is approximately constant. The slope of the regression line was 42.9 kg grain/kg soil N at 85 per cent dry matter. This suggests that 73 per cent of the residual N would have to be recovered in the grain to give the 1.7 per cent N content normally observed at this dry matter percentage (Lidgate, 1978; Greenwood, personal communication), when no fertilizer was applied. Alternatively, the same N content could have been obtained with a residual N recovery of 50 per cent (Cooke, 1977) if all of the spring nitrate levels had been underestimated by 31 per cent.

Although it is not clear from the results which of these two explanations is more likely to be correct, both probably contributed to the results observed. Differences in the distributions of residual and fertilizer N within the soil profile mean that N_R and N_F will probably be available to a crop at different stages of growth, and it is clearly an approximation to assume that both sources of N will always be recovered in equal proportions by harvest. In addition, N_R was almost certainly underestimated at least to some extent for these experiments. Both Equations 4 and 5 assume that there was no uptake of N by the winter wheat crop before the onset of leaching, but in practice this is an exaggeration. Any residual nitrate recovered in the autumn will cause both A and L (in Equation 3) to be overestimated and result in an underestimation of all spring N contents.

Discussion

The results of these tests show that although the model appeared to predict the differential effects of winter rainfall with reasonable accuracy, the corresponding estimates of spring residues were more prone to error. Thus, it would appear that the major limitation to the accuracy of the method is associated with the estimation of the amount and distribution of nitrate present in the soil at the start of leaching. This is confirmed by the conclusions from the small number of tests in which predicted and measured autumn residues were compared directly.

The importance of accurate estimation of nitrate contents of the soil when leaching occurs is emphasised by tests of the sensitivity of the model to errors in each of the main input parameters. The tests were made by differentiating Equation 3 with respect to each:

$$\frac{dL}{L} = \frac{dA}{A} = \left(1 + \ln\left(\frac{100L}{Ah}\right)\right)\frac{dh}{h} = \left(\frac{\theta_m h}{2(P + \theta_m)}\right)\frac{dP}{P} = -\left(\frac{\theta_m h}{2(P + \theta_m)}\right)\frac{d\theta_m}{\theta_m}$$

since $f = 100L/Ah$ where f represents the fraction of nitrate lost by leaching, and since $\theta_m h/(2P + 2\theta_m) \simeq \theta_m h/2P \simeq -\ln f$ when $P \gg \theta_m$ (Burns, 1975), these equations can be rewritten as:

$$\frac{dL}{L} = \frac{dA}{A} = (1 + \ln f)\frac{dh}{h} \simeq (-\ln f)\frac{dP}{P} \simeq (\ln f)\frac{d\theta_m}{\theta_m}$$

These show that $|dL/L|$ is less than both $|dP/P|$ and $|d\theta_m/\theta_m|$ when

$0.368 < f < 2.718$ and is less than $|dh/h|$ when $0.135 < f < 1.0$, but is always equal to $|dA/A|$. This means that for all practical purposes the accuracy of L is influenced more by errors in A than by equivalent errors in P, θ_m or h. Errors in these parameters only become dominant when leaching losses are small and relatively unimportant. Thus, provided there are no gross errors in the estimation of P, θ_m or h, the overall accuracy of the model is likely to depend on the precision with which the autumn nitrate content of the soil can be estimated.

Although the autumn nitrate contents were derived from a combination of the average effects of mineralization and uptake, and the average dressings of N fertilizer applied to the previous crop, the values were not tailored to individual crop or soil conditions. Deviations may also have resulted when losses of N occurred during the previous summer, or when a substantial proportion of the mineralized N remained in the ammomium form. In addition, other errors may have been introduced when the combined effects of all these processes caused deviations from the uniform distribution of autumn nitrate assumed by the model, or when significant denitrification occurred during the winter. However, in view of the complexities of these effects, it is doubtful if more accurate determination of residual nitrate contents could be made without using a much more sophisticated (and far less practical) model.

By comparison, it has been shown that accurate estimates of the N fertilizer needs of winter wheat crops can be made by soil analysis, provided the samples are taken to the correct depth in the spring (Jungk and Wehrmann, 1978). In practice, however, problems may arise in obtaining the results in time to adjust spring N dressings. Since the model appears to predict the differential effects of winter rainfall with reasonable accuracy, these difficulties could be overcome by taking the soil samples in the autumn and using the results as an input to the model for predicting spring nitrate contents. This would have the advantage that the sampling would be made at a quiet time of the year and at the same time considerably reduce the risk of error from the use of the model.

However, despite all reservations, the model still produced satisfactory predictions about the influence of different amounts of winter rainfall on spring nitrate levels and could, therefore, be used to provide helpful advice for adjusting spring fertilizer dressings for differences in winter leaching.

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Appendix

Derivation of Equation 4

The total amount of N taken up by a mature crop (U_N) may be defined by the equation:

$$U_N = u_F N_F + u_M N_M + u_R N_R \tag{I}$$

where N_F is the amount of spring-applied fertilizer, N_M is the amount of N mineralized by harvest and N_R is the amount of residual nitrate (from soil and fertilizer) left over from the previous crop, and u_F, u_M and u_R are the corresponding frections of N recovered by the current crop. Rearranging this equation gives:

$$N_F = \frac{(U_N - u_M N_M)}{u_F} - \frac{u_R}{u_F} N_R \tag{II}$$

Provided no losses of N occur after the winter and no other factors limit growth, then U_N should be approximately constant for all wheat crops grown in different years whenever maximum yields are attained. Assuming N_M is unlikely to vary greatly for the same soil from year to year, equation (II) can be rewritten as:

$$N_F = k_1 - \frac{u_R}{u_F} N_R$$

where N_F becomes the optimum amount of fertilizer and k_l is approximately constant. This equation is identical to equation 4.

Derivation of Equation 5

When $N_F = 0$, Equation I simplifies to:

$$U_N = u_R N_R + u_M N_M \tag{III}$$

U_N may also be defined by the equation:

$$U_N = F_N \cdot Y_o \tag{IV}$$

Where Y_o is the yield of grain from these unfertilized treatments and F_N is its fractional N content. Combining Equation III and IV and rearranging gives:

$$Y_o = \frac{u_R}{F_N} N_R + \frac{u_M N_M}{F_N} \tag{V}$$

Since year-to-year variations in both N_M and F_N are likely to be small for the same soil, Equation V can be rewritten as:

$$Y_o = \frac{u_R}{F_N} N_R + k_2$$

where k_2 is approximately constant. This equation is identical to Equation 5.

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