ELSEVIER

Computers and Geotechnics, Vol. 18, No. 4, pp. 245–266, 1996

0266-352X(95)00032-1 0266-352X/96/$15.00 + 0.00

A Simplified Multi-layered Flow Model for use in Landfill Design

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(Received 13 January 1995; revised version received 12 June 1995; accepted 20 June 1995)

ABSTRACT

A simplified analytical flow model, which considers multi-layered soil systems, is developed herein for estimating the steady-state velocity field and the head distribution beneath landfills and in multiple aquitard–aquifer systems. The landfills either can be engineered with a multi-liner and -drainage layer system or can be without any such engineered layers. The application of the method is illustrated with respect to the design of two hypothetical landfills by considering several design options. The effect of landfill size and the potential "shadow effect" is also examined. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

Flow modelling may serve a number of purposes in landfill design. These include calibration of the initial groundwater model, estimating the velocity field for input to contaminant transport models and calculating the head distribution beneath landfills when studying the interaction of the natural hydrogeological system and the engineered system (e.g. liners, drainage layers). If the flow beneath the landfill is essentially one-dimensional and the heads are well-defined at both ends of the pathway, simple hand calculations can be used to estimate the velocity field. Sometimes, the flow system may be too complicated to allow the use of simple one-dimensional hand calculations. For these cases, it is common to perform finite element modelling. At the site selection and preliminary design stages, the limited available data are often insufficient to justify the cost of (in person hours) a detailed numerical analysis; and a simplified analytical method would provide an economical and efficient alternative to the finite element method in these situations.

Rowe and Nadarajah [1] developed simplified flow models, to use in landfill designs, for a system with one or two aquifers. These models cannot be used when there are more than two aquifers. However, the design of landfills may include multiple liner systems with secondary or tertiary leachate collection systems, and the aquitard may have several aquifers within the potential zone of contaminant impact. Thus, for some practical cases there is a necessity for flow modelling in a multi-layered soil system. The present paper provides the development of an analytical solution which can be used to evaluate the velocity field and the head distribution beneath landfills, which are built in multi-aquitard–aquifer systems and in multiple aquitard–aquifer systems.

The application of the method is illustrated by performing flow modelling for two hypothetical landfills, considering several design options. Also the effect of landfill size and the potential "shadow effect" is examined.

PREVIOUS WORK

The simplified solution developed herein is the first of its kind with direct application to the design of landfills. There are a number of different analytical solutions [2–9] which have been developed to analyse the drawdown caused by pumping wells in a leaky multiple-aquifer system. Analytical solutions are also available to analyse the flow in a leaky multiple-aquifer system with a fully or partially penetrating river [9] and with flood areas and reclaimed marsh land [10]. However, while the boundary conditions assumed in these non-landfill solutions varied depending on the flow regime and the geological settings considered, none are suitable for modelling many typical landfill design situations. For example, in existing models the aquifers are generally assumed to be of "infinite extent". In contrast, for landfill problems the drainage layers and liners are finite and the extent is limited by the dimensions of the landfill. At the edge of these layers either the heads or the flows are known.

For typical landfill design calculations, the boundary conditions at the top of the landfill liner are defined by the leachate mound in the landfill. Under normal operating conditions, the mound can be considered to be uniform; with failure to continue to operate the leachate collection system, the leachate mound can be parabolic in shape.

Although the basic equations developed in this paper are similar to those developed by Hemker [3], Mass [9], Yu [10] and Hunt and Curtis [8], the subsequent solution methodology enables the boundary conditions relevant to flow modelling in landfills to be easily implemented in a computer program. Thus the drainage layers are only present beneath the landfill and do not extend below the entire modelled area. To allow for this, the landfill model is divided into three blocks, one beneath the landfill and one on either side, and the blocks are combined by invoking continuity conditions at the boundary between the blocks using an iterative method described later in this paper.

THEORETICAL DEVELOPMENT

Consider the multi-layered system shown in Fig. 1. This system consists of alternating low and high permeability units which may be either man-made (e.g. engineered liners and leachate collection systems) or natural (e.g. aquitards and aquifers). For convenience of the development of the mathematical formulation, the aquitards and aquifers are grouped together as "couplets" with adjacent low and high permeability units being considered together as a "layer". Thus layer 1 (see Fig. 1) consists of the upper low permeability unit and the immediately underlying permeable unit, and so on.

Assumptions

The simplifying assumptions made in the formulation are:

- the fluxes in the low permeability units (aquitards, clay liners) are essentially vertical and the flows in the permeable units (aquifers, leachate collection systems) are essentially horizontal. This is generally a reasonable assumption for typical landfills constructed over relatively thin aquitards when the contrast in the hydraulic conductivities between the permeable units and the low permeability units is more than two orders of magnitude [2];
- there is no significant vertical head drop across a permeable unit;
- the soil layers are saturated, homogenous, isotropic and their boundaries are parallel and nearly horizontal. However, with engineering judgement the model can be approximately used in some situations where the soil boundaries are not parallel;
- the permeable unit at the bottom is underlain by a relatively impermeable boundary;
- the head on the upper aquitard is defined by either a water table in an overlying weathered zone or a surficial sand layer or by a leachate level in a leachate collection system (or waste); and
- flow is in a two-dimensional system (i.e. flow out of the plane is assumed to be zero).

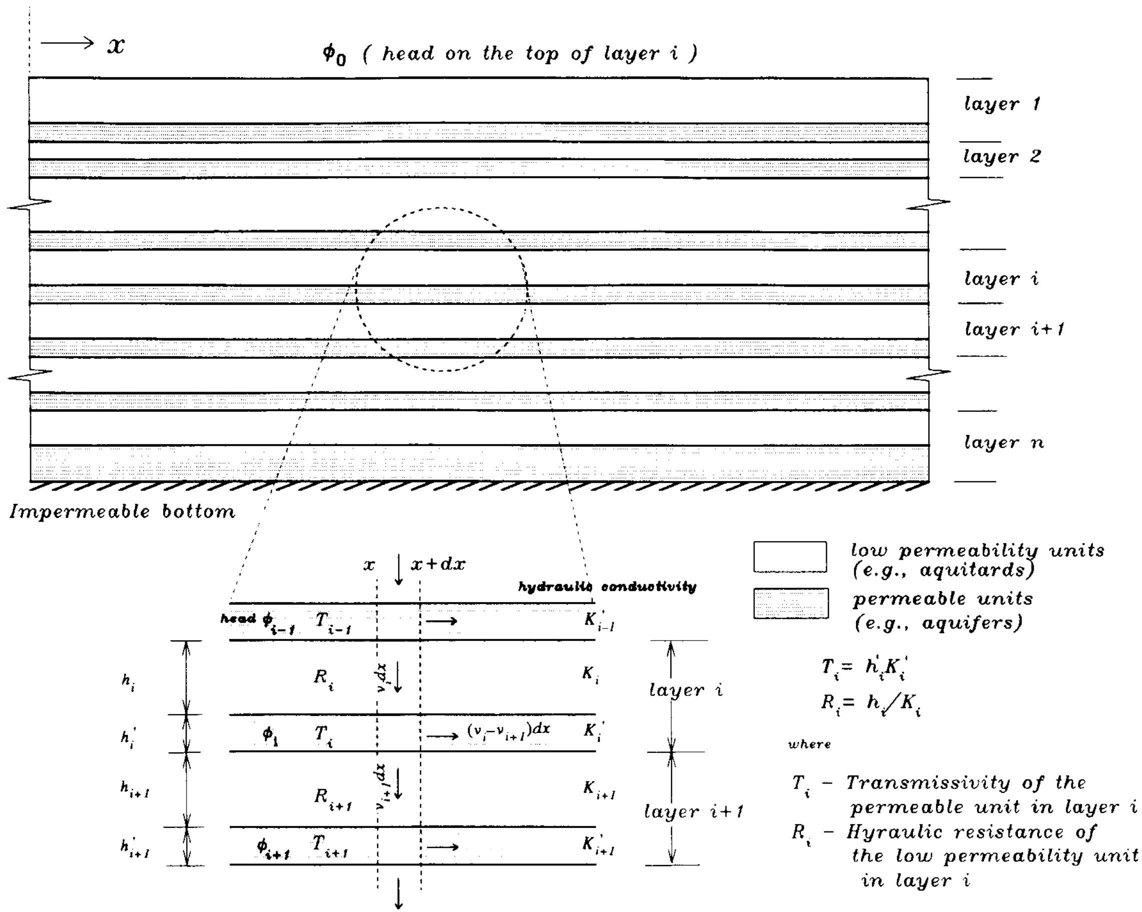


Fig. 1 Schematic of a multi-layer soil system.

Mathematical formulation

Consider the flow through the vertical column of thickness dx in a multi-layered system as shown in Fig. 1. By invoking Darcy's law, the vertical flow through the low permeability unit in layer i is

$$v_i \mathrm{d}x = \frac{K_i}{h_i}(\phi_{i-1} - \phi_i)\mathrm{d}x \tag{1}$$

where $\phi_i = \phi_i(x)$ is the head in the permeable unit of layer i at a distance x; $\phi_{i-1} = \phi_{i-1}(x)$ is the head on top of layer i (e.g. in the permeable unit of layer $i-1$ or at the surface, for $i = 1$); K_i is the hydraulic conductivity of the low permeability unit in layer i; K_i' is the hydraulic conductivity of the permeable unit in layer i; $v_i = v_i(x)$ is the Darcy velocity through the low permeability unit in layer i at a distance x; h_i is the thickness of the low permeable unit in layer i; and h_i' is the thickness of the permeable unit in layer i.

To satisfy the continuity of flow, the difference in vertical flow, i.e. $(v_i - v_{i+1})\mathrm{d}x$, between the low permeability units in layers i and $i + 1$ must be equal to the net change in horizontal flow across the column in the permeable unit of layer i. Using Darcy's law, in the permeable layer and invoking continuity of flow it can be shown that the heads ϕ_{i-1}, ϕ_i and ϕ_{i+1} must satisfy:

$$T_i \frac{\mathrm{d}^2\phi_i}{\mathrm{d}x^2} + \frac{1}{R_i}(\phi_{i-1} - \phi_i) - \frac{1}{R_{i+1}}(\phi_i - \phi_{i+1}) = 0; i = 1, 2, 3....n \tag{2}$$

where $T_i = K_i' h_i'$ is the transmissivity of the permeable unit in layer i and $R_i = h_i/K_i$ is the hydraulic resistance of the low permeable unit in layer i.

By setting $i = 1$ and invoking the Dirichlet boundary condition at the top of the model i, eqn (2) reduces to

$$T_1 \frac{\mathrm{d}^2\phi_1}{\mathrm{d}x^2} + \frac{1}{R_1}(\phi_0 - \phi_1) - \frac{1}{R_2}(\phi_1 - \phi_2) = 0 \tag{3a}$$

where $\phi_0 = \phi_0(x)$, the boundary head on the top of the model at distance x. For the bottom layer (nth), eqn (2) can be written as

$$T_n \frac{\mathrm{d}^2\phi_n}{\mathrm{d}x^2} + \frac{1}{R_n}(\phi_{n-1} - \phi_n) = 0 \tag{3b}$$

Since the bottom (nth) layer is assumed to be underlain by an impermeable layer, $R_{n+1} \rightarrow \infty$.

Equation (2) and eqn (3) can be expressed in matrix form as

$$\boldsymbol{\phi}' = \mathbf{R}\boldsymbol{\phi} + \mathbf{f} \tag{4a}$$

where

$$\mathbf{R}_{n\times n}=\begin{bmatrix} (\frac{1}{R_1T_1}+\frac{1}{R_2T_1}) & -\frac{1}{R_2T_1} & 0 & 0 & \cdot & \cdot & 0 & 0 & 0 & 0 \\ -\frac{1}{R_2T_2} & (\frac{1}{R_2T_2}+\frac{1}{R_3T_2}) & -\frac{1}{R_3T_2} & 0 & \cdot & \cdot & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdot & \cdot & 0 & -\frac{1}{R_{n-1}T_{n-1}} & (\frac{1}{R_{n-1}T_{n-1}}+\frac{1}{R_nT_{n-1}}) & -\frac{1}{R_nT_{n-1}} \\ 0 & 0 & 0 & 0 & \cdot & \cdot & 0 & 0 & -\frac{1}{R_nT_n} & \frac{1}{R_nT_n} \end{bmatrix} \quad (4b)$$

$$\phi'=\frac{d^2\phi}{d^2x}=\left[\phi_1',\phi_2',...,\phi_n'\right]^T \quad (4c)$$

$$\phi=[\phi_1,\phi_2,...,\phi_n]^T \quad (4d)$$

and

$$\mathbf{f}=\left[-\frac{\phi_0}{R_1T_1},0,...,0\right]^T \quad (4e)$$

Solution method

The differential equations system in eqn (4) can be solved using the method of variation of parameters. Firstly, the second-order differential equations in eqn (4) are reduced to a first-order system by the introduction of dependent variables ψ_1, ψ_2,.......ψ_{2n} defined as follows:

$$\psi_1=\phi_1,\psi_2=\phi_1',\psi_3=\phi_2,\psi_4=\phi_2',.......,\psi_{2n-1}=\phi_n,\psi_{2n}=\phi_n' \quad (5)$$

Note that the definitions in eqn (5) yield the following additional relationships

$$\begin{gathered}\psi_1'=\phi_1'=\psi_2,\psi_2'=\phi_1',\psi_3'=\phi_2'=\psi_4,\psi_4'=\phi_2',.......,\\ \psi_{2n-1}=\phi_n'=\psi_{2n},\psi_{2n}'=\phi_n'\end{gathered} \quad (6)$$

Substituting eqn (5) and eqn (6) in eqn (4) gives the non-homogeneous first-order linear system:

$$\boldsymbol{\psi}'(x)=\mathbf{P}\boldsymbol{\psi}(x)+\mathbf{s}(x) \quad (7a)$$

where

$$\boldsymbol{\psi}=[\psi_1,\psi_2,...,\psi_{2n-1},\psi_{2n}]^T=\left[\phi_1,\phi_1',...,\phi_n,\phi_n'\right]^T \quad (7b)$$

$$\mathbf{P}_{2n\times 2n} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \cdot & \cdot & 0 & 0 & 0 & 0 \\ (\frac{1}{R_1T_1}+\frac{1}{R_2T_1}) & 0 & -\frac{1}{R_2T_1} & 0 & 0 & 0 & \cdot & \cdot & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \cdot & \cdot & 0 & 0 & 0 & 0 \\ -\frac{1}{R_2T_2} & 0 & (\frac{1}{R_2T_2}+\frac{1}{R_3T_2}) & 0 & -\frac{1}{R_3T_2} & 0 & \cdot & \cdot & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & -\frac{1}{R_nT_n} & 0 & \frac{1}{R_nT_n} & 0 \end{bmatrix} \tag{7c}$$

and

$$\mathbf{s} = \left[0, -\frac{\phi_0}{R_1T_1}, 0, ..., 0\right]^{\mathrm{T}} \tag{7d}$$

The vector $\psi(x)$ in eqn (7b) consists of the head (e.g. ϕ_i in the permeable unit of layer i) and its derivative (e.g. ϕ_i' in the permeable unit of layer i) in each permeable unit. The derivative term, ϕ_i', for the permeable unit in layer i can be related to the horizontal flow in that unit using Darcy's law:

$$\phi_i' = -\frac{q_i}{T_i} \tag{8}$$

where $q_i = q_i(x)$ is the flow in the permeable unit of layer i at a distance x and T_i is the transmissivity of the permeable unit. Therefore, the vector $\psi(x)$ in eqn (7b) can be formed in terms of head and flow in the permeable units, by substituting the relationship between ϕ' and q_i [i.e. substituting eqn (8) in eqn (7b)]:

$$\psi = [\psi_1, \psi_2, ...\psi_{2n-1}, \psi_{2n}]^{\mathrm{T}} = \left[\phi_1, -\frac{q_1}{T_1}, ..., \phi_n, -\frac{q_n}{T_n}\right]^{\mathrm{T}} \tag{9}$$

The complementary function $\psi_c(x)$ of the homogeneous system

$$\psi'(x) = \mathbf{P}\psi(x) \tag{10}$$

is given in terms of the fundamental matrix $\beta(x)$ as

$$\psi_c(x) = \beta(x)\boldsymbol{\alpha} \tag{11a}$$

where $\boldsymbol{\alpha}$ is a vector of constants, which can be obtained from the known boundary conditions as discussed in the next section.

If the matrix **P** in eqns (7a–d) has a complete set of $2n$ (where n is the number of layers in the soil system) linearly independent eigenvectors ν_1, ν_2,.....ν_{2n} associated with the (not necessarily distinct) eigenvalues λ_1, λ_2,...λ_{2n}, respectively, the corresponding solution vectors of eqn (10) are given by

$$\boldsymbol{\psi}_{c,i}(x) = \boldsymbol{\nu}_i e^{\lambda_i x} \text{ where } i = 1, 2, ...2n. \tag{11b}$$

And the fundamental matrix, $\boldsymbol{\beta}(x)$, of the system in eqn (10) is

$$\boldsymbol{\beta}(x) = \begin{bmatrix} | & | & & | \\ \boldsymbol{\nu}_1 e^{\lambda_1 x} & \boldsymbol{\nu}_2 e^{\lambda_2 x} & \cdot \; \cdot \; \cdot & \boldsymbol{\nu}_{2n} e^{\lambda_{2n} x} \\ | & | & & | \end{bmatrix} \tag{11c}$$

which has the solutions $\boldsymbol{\psi}_{c,1}$, $\boldsymbol{\psi}_{c,2}$,.......$\boldsymbol{\psi}_{c,2n}$ as column vectors.

Then the particular solution of the non-homogeneous system in eqn (7a) is given by

$$\boldsymbol{\psi}_p(x) = \boldsymbol{\beta}(x) \int \boldsymbol{\beta}^{-1}(x)\mathbf{s}(x)\mathrm{d}x \tag{12}$$

If we add this particular solution and the complementary function in eqn (11a), we get the general solution [which contains the head and the flow in each permeable unit, see eqn (9)]:

$$\boldsymbol{\psi}(x) = \boldsymbol{\beta}(x)\boldsymbol{\alpha} + \boldsymbol{\beta}(x) \int \boldsymbol{\beta}^{-1}(x)\mathbf{s}(x)\mathrm{d}x \tag{13}$$

Boundary conditions

At least two boundary conditions (i.e. head or flow) at any location in each permeable unit should be known to evaluate the constant vector $\boldsymbol{\alpha}$ in eqn (13). The boundary conditions on top and at the bottom of the model are already included in the formulation [see eqn (3a) and (3b)]. If the head and the flow in each permeable unit are known at a vertical section, say at a distance $x = x_1$, the vector $\boldsymbol{\alpha}$ can be obtained from eqn (13) as

$$\boldsymbol{\alpha} = \boldsymbol{\beta}^{-1}(x_1)\left\{ \boldsymbol{\psi}(x_1) - \boldsymbol{\beta}(x_1) \int_0^{x_1} \boldsymbol{\beta}^{-1}(x)\mathbf{s}(x)\mathrm{d}x \right\} \tag{14}$$

But, it is not necessary that the head or flow be known at a single vertical section. Often the heads are known in the aquifer at the two ends of the model. If the head is known in each permeable unit at two vertical sections at $x = x_1$ and $x = x_2$, then eqn (13) can be written as:
for $x = x_1$

$$\boldsymbol{\psi}(x_1) = \boldsymbol{\beta}(x_1)\boldsymbol{\alpha} + \boldsymbol{\beta}(x_1) \int_0^{x_1} \boldsymbol{\beta}^{-1}(x)\mathbf{s}(x)\mathrm{d}x \tag{15a}$$

and for $x = x_2$

$$\boldsymbol{\psi}(x_2) = \boldsymbol{\beta}(x_2)\boldsymbol{\alpha} + \boldsymbol{\beta}(x_2) \int_0^{x_2} \boldsymbol{\beta}^{-1}(x)\mathbf{s}(x)\mathrm{d}x \tag{15b}$$

Although the systems in eqn (15a) and eqn (15b) each consist of $2n$ liner equations, only n values of heads or flows in the vectors $\boldsymbol{\psi}(x_1)$ and $\boldsymbol{\psi}(x_2)$ are known. To allow the determination of the $2n$ unknowns in vector $\boldsymbol{\alpha}$, the n equations with known heads or flows in eqn (15a) must be combined with the n equations with known heads or flows in eqn (15b); and the resulting $2n$ liner equations then solved to obtain the vector $\boldsymbol{\alpha}$.

Controlling parameters

The matrix $\mathbf{R}$ in eqn (4b) depends on the transmissivity of the permeable units, T, and the hydraulic resistance of the low permeability units, R. This suggests that the velocity field and the heads in an aquitard–aquifer system and beneath landfills are controlled by

- the hydraulic resistances of the liner(s) (if present) and the aquitard(s); and
- the transmissivities of the drainage layer(s) (if present) and the aquifer(s).

It is obvious from eqn (13) that the other parameters controlling the velocity field and the heads are the boundary conditions.

Verification of the method

The results obtained using the simplified analytical model described in the previous sections were found to be in excellent agreement with those obtained from a finite element analysis for a number of typical hydrogeological settings as described by Nadarajah and Rowe [12].

LANDFILL MODEL

Figure 2 shows an engineered landfill with a multi-layer leachate collection system, built in a multi-layer aquitard–aquifer system. For convenience of presentation, the engineered liners and the drainage layers will be referred to as *engineered layers* and the natural aquitards and aquifers will be referred to as *formation layers*. The bottom aquifer is underlain by an impermeable layer. The heads (or flows) in the permeable layers at the boundaries of the

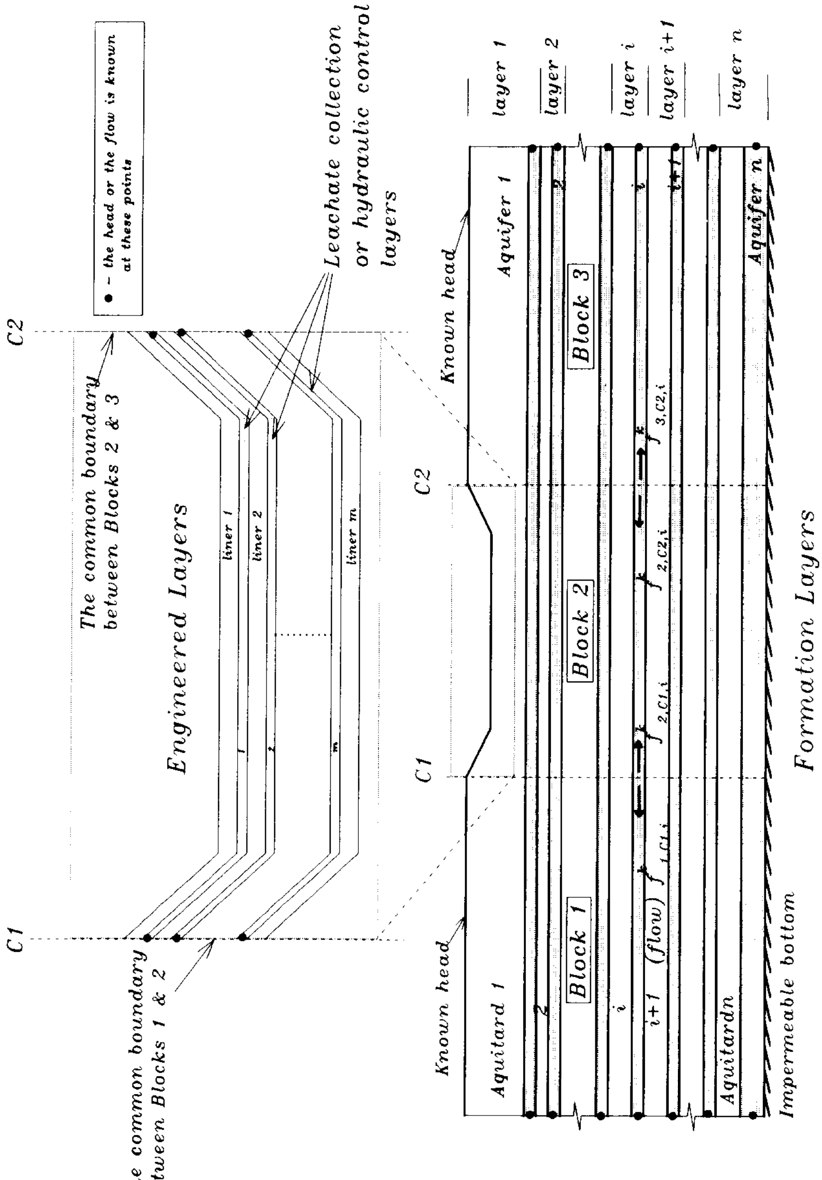


Fig. 2 Schematic of a landfill in a multi-layer system.

model are assumed to be known (i.e. from field data or calculations when using multiple models):

- at (or near) both ends of the landfill in the engineered layers; and
- at two vertical sections in the formation layers, at distances away from the landfill, on both sides of the landfill (see Fig. 2).

Since the engineered layers are not continuous over the entire domain considered in Fig. 2, the theory developed herein requires some modifications. Accordingly, the domain is divided into three blocks so that the soil layers are continuous within each block. The presence of side slopes at both ends of the landfill (see Block 2 in Fig. 2) also violates the assumption made earlier that the soil layers are parallel. As an approximation, the side slopes were replaced by an equivalent head distribution. The heads in the formation layers at the common boundaries (C1 and C2, see Fig. 2) between the blocks are initially assumed and the velocity field and the heads beneath the landfill are determined using the iterative procedure described below:

1 Assume a reasonable head value in each aquifer at the common boundaries C1 and C2.
2 Calculate the flows [using eqn (13)] in the aquifers at the common boundaries in all three blocks.
3 Check whether the calculated flows, in each aquifer, on both sides of the common boundaries are within the chosen convergence tolerance:
   - if within the tolerance, the velocity field and the heads beneath the landfill can be obtained from Block 2 using the current heads in the aquifers at the common boundaries, and the known boundary conditions in the landfill (e.g. the head due to leachate mound) and in the engineered layers;
   - if not, go to the next step.
4 Assign new flow values in the aquifers at the common boundaries, according to the convergence criteria described by Nadarajah and Rowe [12], and calculate the new heads and flows in the aquifers at the common boundaries using Blocks 1 and 3.
5 Using the heads calculated at the common boundaries in Step 4, calculate the flow in the aquifers at the common boundaries using Block 2.
6 Repeat Steps 3–5 until convergence occurs.This procedure was demonstrated to converge rapidly [12].

The procedures described in the previous sections have been coded in a computer program, MULTI, which can be used to determine the velocity field and the head distribution beneath landfills which are built in a multi-aquitard–aquifer system.

The proposed approach can be used as a simplified analytical substitute for the sophisticated numerical groundwater models and is considered suitable for:

- sensitivity studies to assist in the preliminary design of landfills;
- initial calibration of hydrogeological models; and
- estimating velocity inputs for contaminant transport analyses to be performed using finite layer analysis programs (see ref. 11).

APPLICATIONS

Design of landfills

To illustrate the application of the theory, consideration is given to the design of a hypothetical landfill in a three-layer aquitard–aquifer system shown in Fig. 3(a). It is assumed that the terrain generally slopes at a grade of 0.004 from left to right and that a hydrogeological investigation has given the hydraulic conductivity of the various hydrostratigraphic units as well as the heads in the aquifer at two locations well away from the proposed landfill as shown in Fig. 3(a). The upper few meters of aquitard 1 are weathered and fractured, and the water table is 0.8 m below the ground surface.

When designing a landfill, several alternative design options may be considered to establish the level of engineering that is required to meet the regulatory requirements. In this example, 12 different design options were considered, as summarized in Table 1. The table shows the hydraulic conductivity of the liner(s) and the secondary leachate collection system, if any, used for the various design options. In each case the bottom of the primary leachate collection system (PLCS) was taken to be 4 m below the original ground level [see Fig. 3(b) and (c)]. Any engineering in addition to the PLCS (e.g. liners and secondary collection layers) were placed below this by excavating in aquitard 1 [see Fig. 3(c)].

The design option designated by Case 1 [Fig. 3(a)] represents the situation before landfill construction; Case 2 [Fig. 3(b)] considers a landfill with a primary leachate collection system but with no engineered liners; in Cases 3 and 4 the landfill has an engineered primary liner with a hydraulic conductivity of 0.5×10^{-10} and 2×10^{-10} m/s, respectively; in Cases 5 and 6 the landfill has an engineered primary liner and an operating secondary leachate collection system (SLCS) with Cases 5 and 6 differing only in terms of the assumed hydraulic conductivity of the liner; in Cases 7 and 8 the landfill is engineered with both primary and secondary liners and an operating secondary leachate collection system; Cases 9–12 are essentially the same as Cases 5–8, except the secondary leachate collection system is assumed to be clogged in Cases 9–12

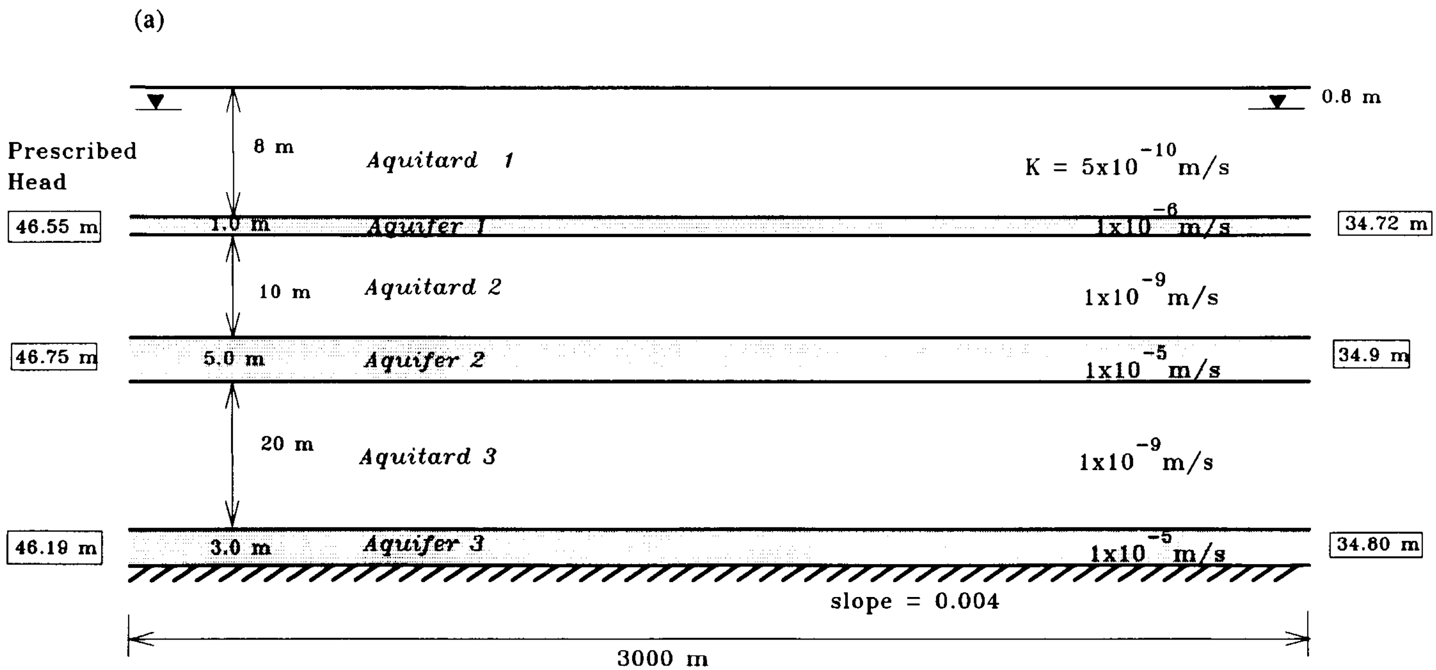


Fig. 3 The configuration for flow modelling. (a) Case 1, no landfill; (b) Case 2, landfill with no engineered layers other than a primary leachate collection system; and (c) landfill with two liners and a SLCS.

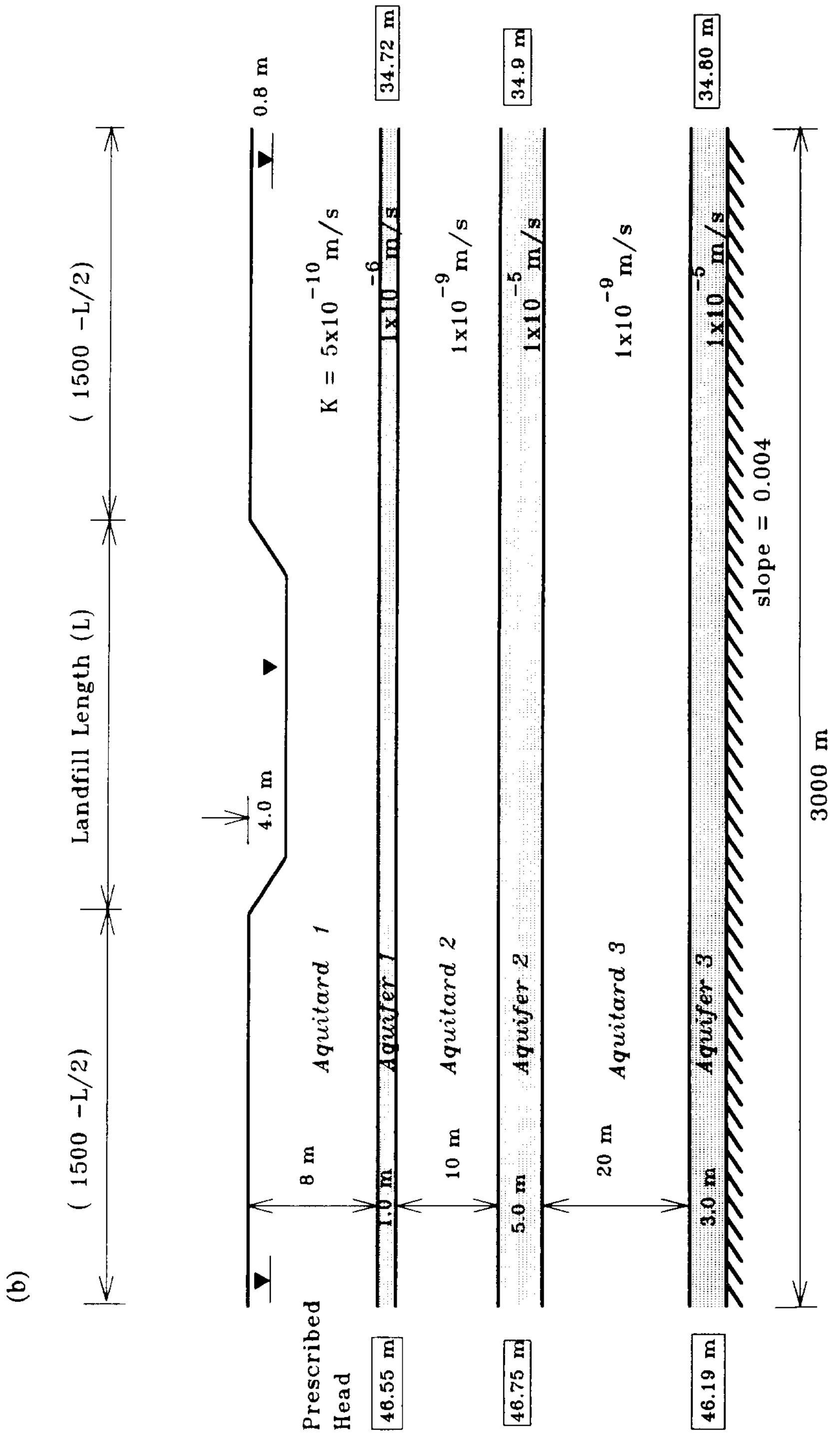

(b)
(1500 -L/2)
Landfill Length (L)
(1500 -L/2)
0.8 m
4.0 m
Prescribed Head
46.55 m
34.72 m
46.75 m
34.9 m
46.19 m
34.80 m
8 m
1.0 m
10 m
5.0 m
20 m
3.0 m
Aquitard 1
Aquifer 1
Aquitard 2
Aquifer 2
Aquitard 3
Aquifer 3
K = 5x10^-10 m/s
1x10^-6 m/s
1x10^-9 m/s
1x10^-5 m/s
1x10^-9 m/s
1x10^-5 m/s
slope = 0.004
3000 m

Fig. 3b

(c)

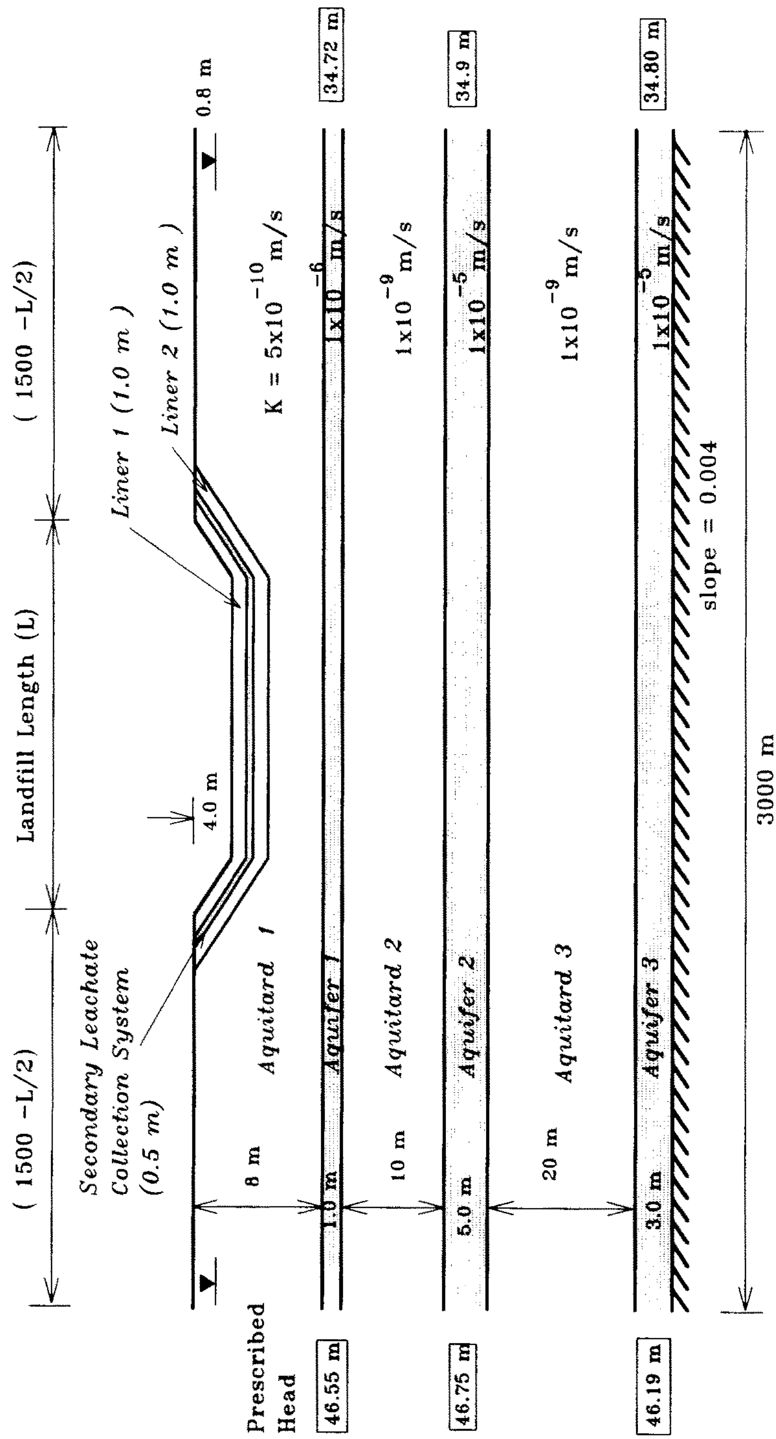

(1500 −L/2)
Landfill Length (L)
(1500 −L/2)
Secondary Leachate Collection System (0.5 m)
Liner 1 (1.0 m)
Liner 2 (1.0 m)
0.8 m
4.0 m
K = 5x10^{-10} m/s
1x10^{-6} m/s
1x10^{-9} m/s
1x10^{-5} m/s
1x10^{-9} m/s
1x10^{-5} m/s
Aquitard 1
Aquifer 1
Aquitard 2
Aquifer 2
Aquitard 3
Aquifer 3
8 m
1.0 m
10 m
5.0 m
20 m
3.0 m
slope = 0.004
3000 m
Prescribed Head
46.55 m
46.75 m
46.19 m
34.72 m
34.9 m
34.80 m

Fig. 3c

TABLE 1
Engineering options considered in the flow modelling

Case	*Hydraulic conductivity (m/s)*			*Remarks*
	Liner 1	*Liner 2*	*SLCS*	
1	—	—	—	no landfill
2	—	—	—	landfill with no engineered layers other than PLCS
3	0.5×10^{-10}	—	—	landfill with a liner
4	2.0×10^{-10}	—	—	landfill with a liner
5	0.5×10^{-10}	—	1.0	landfill with a liner and a SLCS
6	2.0×10^{-10}	—	1.0	landfill with a liner and a SLCS
7	0.5×10^{-10}	0.5×10^{-10}	1.0	landfill with two liners and a SLCS
8	2.0×10^{-10}	2.0×10^{-10}	1.0	landfill with two liners and a SLCS
9	0.5×10^{-10}	—	1×10^{-6}†	landfill with a liner and a clogged SLCS
10	2.0×10^{-10}	—	1×10^{-6}†	landfill with a liner and a clogged SLCS
11	0.5×10^{-10}	0.5×10^{-10}	1×10^{-6}†	landfill with two liners and a clogged SLCS
12	2.0×10^{-10}	2.0×10^{-10}	1×10^{-6}†	landfill with two liners and a clogged SLCS

Notes: (1) SLCS = secondary leachate collection system; (2) †indicates that the SLCS is clogged.

with a hydraulic conductivity that has dropped to 10^{-6} m/s. In all the design options, the thicknesses of the liner and the secondary leachate collection system were taken as 1.0 and 0.5 m, respectively.

Flow modelling was performed using the theory presented herein (as coded in program MULTI) for two landfill lengths, 200 and 1000 m, to study the effect of landfill size on the velocity field. For both landfill sizes, the following two different landfill operating conditions were considered:

- Normal operating conditions in which the leachate mound in the landfill is taken as 0.3 m. Only Cases 2–8 are included in this category. Since clogging of the secondary collection system is not likely during this period, Cases 9–12 are excluded.
- Complete failure of the primary leachate collection system, in which the leachate mounds parabolically between the two extreme ends of the landfill. The average mounding will be proportional to the length of the landfill, and it is estimated to be 6.9 and 34.5 m for the 200 and 1000 m landfills, respectively, for an infiltration rate of 0.15 m/a and a hydraulic conductivity of the waste of 1×10^{-6} m/s. Thus, under complete failure of the primary leachate collection system, a larger leachate head would act on the liner for the larger landfill than for the smaller landfill and therefore the downward velocities can be expected to be higher for the larger landfill under this failure condition.

Normal operating conditions

A key consideration in hydrogeological investigations is the understanding of the groundwater flow direction, the natural variability in the water table and in the potentiometric surface in any underlying aquifers, and the flow in these aquifers. Landfills have been designed based on this information; however, it is important to recognize that the construction of a landfill may change these conditions. This is particularly true when the landfill is of large areal extent as will be demonstrated in the following.

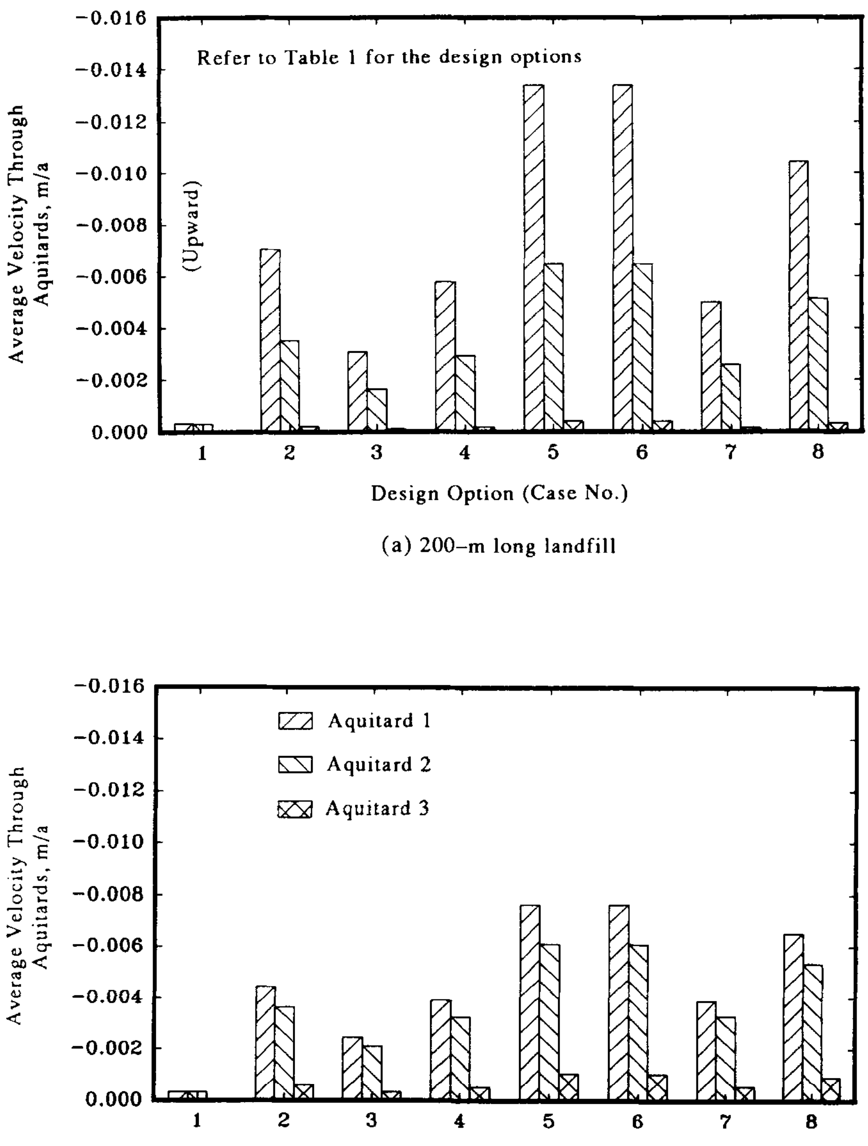
In this example, the landfills are being designed as a natural "hydraulic trap". Here the landfill base contours have been placed below the potentiometric surface of the underlying aquifer and water is being extracted from the aquifer to provide the inward flow necessary to have a "hydraulic trap" wherein the inward flow of water is intended to resist the outward migration of contaminant. However, the extraction of water from the aquifer can lead to a significant drop in water levels in the aquifer, known as the "shadow effect" [13], and this can reduce the effectiveness of the "hydraulic trap".

The calculations show that the flows in the aquifers would be altered by the construction of the landfill. Both landfills were predicted to cause a drop in head in the upper aquifer which ranged from a maximum of about 3 m for the 200 m long landfill to about 4 m for the 1000 m long landfill (Cases 5 and 6). This resulted in inward flow toward the landfill from all directions for the design options considered. The inward flow would prevent advective transport away from the landfill in the top aquifer, but there would likely be an impact on adjacent well users drawing water from aquifer 1. Even in the second aquifer the head drop due to the shadow effect was up to 1.5 m although this did not result in a change in flow direction.

Figure 4(a) and (b) shows the average Darcy velocities through the aquitards for 200 and 1000 m landfills, respectively, for various design options considered under normal operating conditions. Since the landfill is located in a discharge zone, the flows in the aquitards are upward [see Fig. 4(a) and (b)], creating a natural "hydraulic trap" for all the design options considered. The effect of landfill size on the estimated velocities is evident from Fig. 4(a) and (b):

- The velocity in the top aquitard for the 200 m long landfill is about twice as high as that in the 1000 m long landfill. This is because the larger landfill casts more of a "hydraulic shadow" over the aquifer resulting in a greater drop in head in the aquifer and hence a lesser "hydraulic trap". Thus, the larger landfill would allow more diffusive transport of the contaminant to the underlying aquifer.

- The velocities in aquitards 2 and 3 are less affected by the size of the landfill but are affected by the design of the barrier system as can be seen by comparing the results for the different design options.



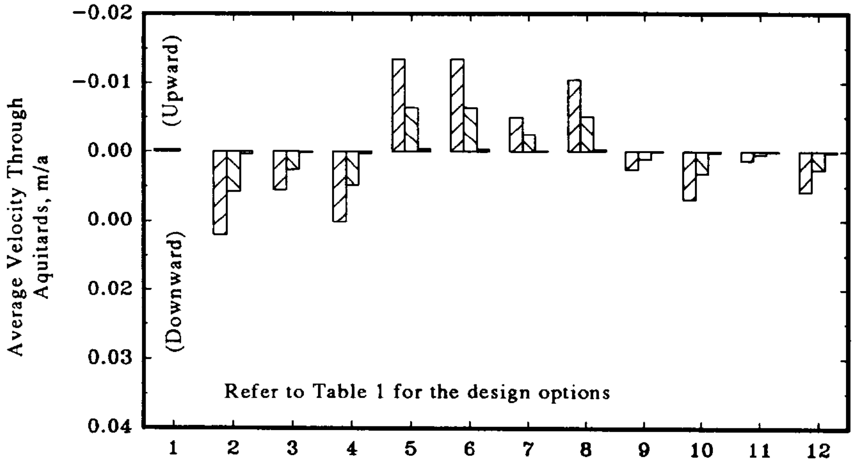
(a) 200-m long landfill

(b) 1000-m long landfill

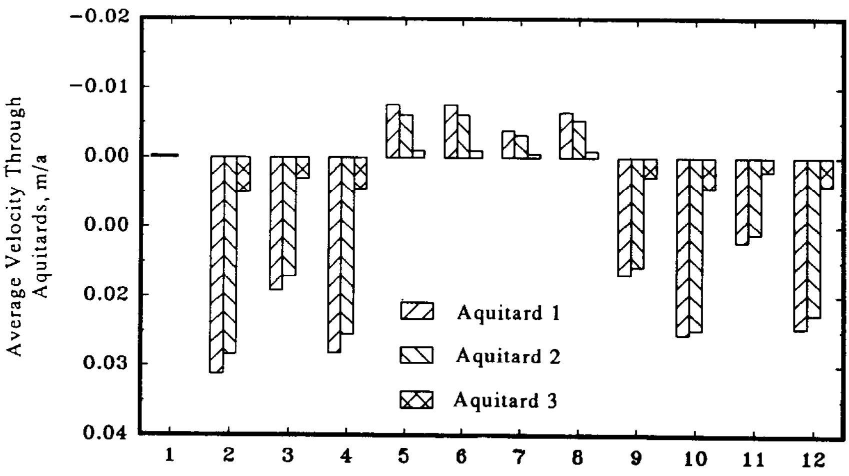
Fig. 4 Average Darcy velocity in the aquitards (normal operating conditions).

Complete failure of the primary leachate collection system

Figure 5(a) and (b) shows the average Darcy velocities through the aquitards for 200 and 1000 m landfills, respectively, for the various design options



(a) 200-m long landfill



(b) 1000-m long landfill

Fig. 5 Average Darcy velocity in the aquitards (under complete failure of the primary leachate collection system).

considered under the complete failure condition of the primary leachate collection system. For Cases 5–8, which involve an operational secondary leachate collection system, the flows in the aquitards are upward. Thus, while SLCS is functioning, the "hydraulic trap" is maintained since the SLCS effectively decouples the leachate mound in the landfill from the natural system. Once the SLCS clogs, it ceases to decouple the engineered and the natural system, destroying the hydraulic trap as is evident from Fig. 5 for Cases 9 and 12. Under the clogged condition, the flow is downward to all aquifers implying that there is a potential for contamination by downward advection of contaminants. The velocities in Cases 2–4, where there is no secondary leachate collection system, are also downward and higher than those for the clogged secondary leachate collection system, Cases 9–12.

The effect of landfill size under complete failure of the primary leachate collection system is also evident from Fig. 5(a) and (b):

No SLCS (Cases 2–4) and clogged SLCS (Cases 9–12)

- For the 1000 m long landfill, the velocity in the top aquitard is about 3 times higher than that for the 200 m landfill. Under complete failure of the primary leachate collection system a larger leachate head acts on the liner for the large landfill, than for the smaller landfill and hence the velocity will be higher for the larger landfill, resulting in more advective transport of the contaminants to the underlying aquifers.
- The velocity is nearly the same in aquitards 1 and 2 for the larger landfill indicating that in this case the majority of flow is to the second aquifer. For the smaller landfill the Darcy velocity in the second aquitard is about half that in the first aquitard indicating that only about half the flow from the landfill is down to the second aquifer.
- The Darcy velocity in the bottom aquitard is very small for the smaller landfill but is significant for the larger landfill.

With SLCS (Cases 5–8)

- The Darcy velocity in the top aquitard (i.e. aquitard 1) for the 200 m long landfill is about twice as high as that in the 1000 m long landfill. This suggests that the larger landfill creates a lesser "hydraulic trap" and therefore would allow more diffusive transport of the contaminant to the underlying aquifer as previously discussed.
- The Darcy velocities in aquitards 2 and 3 are only moderately affected by the size of the landfill.

Effect of landfill size

It is evident from the foregoing discussions (based on flow modelling) that the size of the landfill is a factor affecting the velocity field beneath landfills.

For landfills built with a natural "hydraulic trap", the upward flow into the landfill is greater for small landfills under normal operating conditions or under complete failure of the primary leachate collection system where the SLCS is still functioning. Under the failure condition, with no SLCS or with a clogged SLCS, the downward velocity is larger for the larger landfill. It is therefore likely that the larger the landfill, the more contaminant transport can be anticipated under both normal operating conditions and complete failure of the primary leachate collection system.

SUMMARY AND CONCLUSIONS

A simplified analytical method for estimating the steady-state velocity field and the head distribution beneath landfills and in multi-aquitard–aquifer systems has been presented. The method is applicable to landfills that are built in a multi-aquitard–aquifer system and engineered with multi-layered compacted clay liners and leachate collection systems (or hydraulic control layers). This technique is considered suitable for

- sensitivity studies to assist in the preliminary design of landfills;
- initial calibration of hydrogeological models; and
- estimating velocity inputs for contaminant transport analyses.

Using the equations presented, the velocity field and the head distribution in aquitard–aquifer systems and beneath the landfills are shown to be controlled by

- the hydraulic resistance of the liner(s) and the aquitards; and
- the transmissivity of the engineered drainage layer(s) and the aquifers.

The application of the method was illustrated by performing flow modelling for two hypothetical (200 and 1000 m long) landfills, considering several design options. It has been observed that for landfills built with a natural "hydraulic trap", the upward flow into the landfill is greater for small landfills under normal operating conditions or under complete failure of the primary leachate collection system where the SLCS is still functioning. But, under the failure condition, with no SLCS or with a clogged SLCS, the downward velocity is larger for the larger landfill. For the hydrogeological setting examined, it is likely that the larger the landfill, the greater will be the potential for more contaminant transport under both normal operating conditions and with complete failure of the primary leachate collection system.

The study also examined the potential shadow effects for various design options. It is shown that there can be a significant drop in water level due to the operation of a hydraulic trap and this "shadow effect" is more likely to be significant for larger landfills.

ACKNOWLEDGEMENTS

This paper was prepared with funding from the National Science and Engineering Research Council of Canada under Grant A1007.

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