On heat flux estimation from isotherms

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Heat flux $q_w(t)$ estimation using data from the time evolution of isotherms in material depth is considered. The location of isotherms is determined by thermal indicators. The inverse heat conduction problem with spatially distributed data on isotherms is discussed. The results of numerical experiments for different numbers of indicators and different values of input data accuracy are presented. © 1997 by Elsevier Science Inc.

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Introduction

Standard temperature measurements are performed in fixed points in space (where sensors are located). Thermal indicators (TI) can provide spatially distributed data. However, as a rule, TI are not used for the measurement of unsteady heat flux because of the large interval between color change temperatures and (Abramovich Kartavtsev 1978). An attempt to use spatially distributed unsteady data is described in the present paper. An approach to heat flux $q_w(t)$ evaluation using data on TI color change coordinates (in specimen depth) in time dependence is considered. We discuss a situation when TI strips are located depthward from a heated surface (Alekseev 1994, 1996). A heat flux measurement method is described, which can be used in a situation when standard means based on electric measurements are not desirable. Safety conditions or large electromagnetic interference can be reasons for its application. A new type of inverse heat conduction problem (IHCP) arises with consideration of this method.

The heat flux sensor shown in Figure 1 contains strips of temperature-indicating paints located depthward from the heated surface. We discuss the reversible indicators that recover color under cooling (they can be liquid crystals or luminescent temperature indicators) or irreversible ones at the stage of temperature rise. The sensor material should be transparent in the visible range and opaque to infrared rays. It is valid for glasses, for example. It allows us to neglect the radiative heat transfer within the sensor structure and, therefore, simplifies the heat transfer model used. The TI state $C_i(t,x)$ is recorded in time dependence. The state of TI color can be approximated by curves $X_i(t)$, which are the isotherms in two-dimensional (2-D) space, composed of spatial and time coordinates.

Governing equations

The temperature field is nonstationary and spatially one-dimensional (1-D). The heat conduction equation is written with corresponding initial and boundary conditions.

$$C(T)\cdot\rho\frac{\partial T(t,x)}{\partial t}=\lambda(T)\cdot\frac{\delta^2 T(t,x)}{\delta x^2} \tag{1}$$

$$\lambda\frac{\delta T}{\delta x}\bigg|_{x=0}=q_w(t);\qquad \frac{\delta T}{\delta x}\bigg|_{x=L}=0 \tag{2}$$

$$x\epsilon(0,L);\ t\epsilon(0,t_f);\ T(0,x)=T_0(x)$$

The effect of TI thermophysics is neglected. The state of N TI (with temperatures $T_i(i=1\ldots N)$) can be described by the following equation:

$$C_i(t,x)=\int_0^t \delta(T(t,x-T_i)\mathrm{d}t,\ i=1,\ldots,N \tag{3}$$

Here 0 = initial state, 1 = annealed state.

These data can be described by the set of N isotherms corresponding to N indicators color change temperatures

$$X|_{T(t)=T_i}=\tilde{X}_i(t),\qquad i=1\ldots N \tag{4}$$

The values $X_i(t)=\tilde{X}_i(t)+\delta_i(t)$ are known from the experiment. We search such heat flux $q_w(t)$ that reproduces in computation the experimentally measured values $X_i(t)$. We consider the case of the relatively simple shape of $X_i(t)$ corresponding to one impulse of heating (Figure 2). For more complicated events [if $X_i(t)$ is a multi-value function of t, e.g.], the problem is beyond our consideration.

If N is large enough, then we can restore total temperature field $T(t,x)$ and determine $q_w(t)$ by differentiating this field: $q_w(t)=\lambda\cdot\delta T(t,0)/\delta x$. This kind of problem is classified by Alifanov (1994) as a pseudo-inverse one. Such problems are accompanied by fewer numerical difficulties.

If $N\geq 1$, the problem (Equations 1–4) is an inverse heat conduction problem similar to that discussed by Alifanov (1994). The solution region is composed of the boundary value problem zone (from the cold surface to the isotherm and between isotherms) and the Cauchy problem zone (from heated surface to nearest isotherm) (Figure 3). It is the Cauchy problem zone where the problem is ill-posed and can be unstable. If $X_i(t)$ is a good enough function (analytical e.g.) then the problem (Equations 1–4) has a unique solution (Lavrentiev et al. 1980) for only

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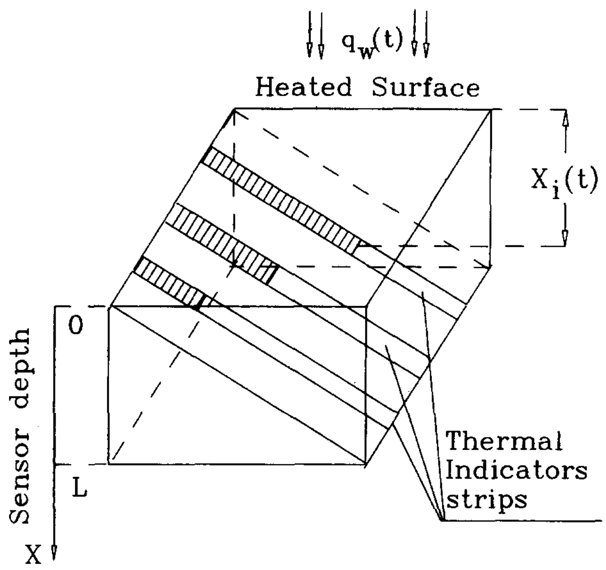

Figure 1 The thermal indicators location sketch

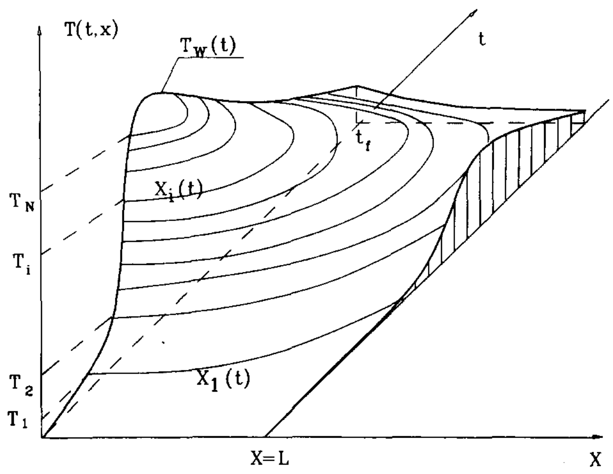

Figure 2 The temperature field and coordinate of color change in time dependence

one isotherm. We observe $X_i^{\exp} = \tilde{X}_i(t) + \delta_i(t)$ that can be neither an analytical nor a continuous function because of an error of experiment $\delta_i(t)$. One subject of this paper is the comparison of stability and precision in dependence on $\delta_i(t)$ and TI number N in computational experiments.

Numerical experiments

The problem (Equations 1–4) is formulated as an optimization one. The heat flux density $q_j = q(t_j)$ should provide a minimum of the mismatch of computed and experimental values of color change coordinates $X_i(t)$.

$$\varepsilon(q_j) = \sum_i \int [X_i^{\exp}(t) - X_i^{\text{comp}}(t)]^2 \, dt \tag{5}$$

Alifanov (1994) offers the conjugated problem solution for calculation of $\Delta\varepsilon/\Delta q_i$ that provide high computational effectiveness. Nevertheless, for the case of a large number of isotherms, this approach is very complicated to program. In the present work the discrepancy gradient $\Delta\varepsilon/\Delta q_i$ was computed in the simplest way by means of a finite-difference approach. The value of $\Delta q_i/q_i$ was varied in the range of 0.01–0.001. The steepest descent method was used as the optimization algorithm. The computations for a sensor having the following characteristics ($\lambda = 0.00074$ kW/(m·s); $C \cdot \rho = 2500$ kJ/m^3; thickness 0.01 m) were performed to confirm the validity of the above method. The heat transfer equation was solved by implicit finite-difference meth-

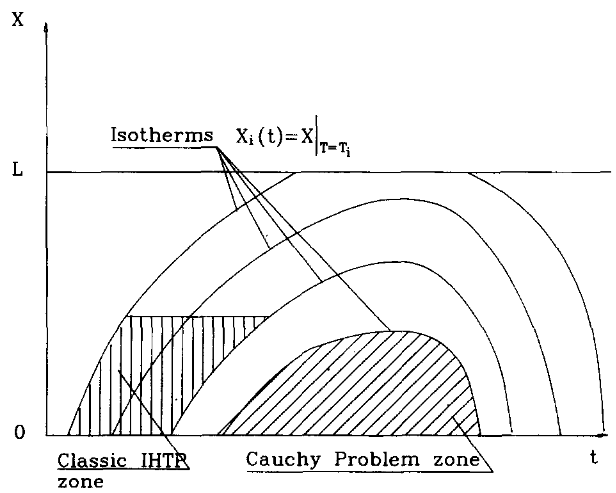

Figure 3 The scheme of different problem zones

Notation

$C(T)$	specific heat
$C_i(t, x)$	state of ith thermal indicator (1:0)
L	thickness of sensor
N	number of thermal indicators
$q_w(t)$	wall heat flux
t	time
$T(t, x)$	temperature
T_0	initial temperature
X	coordinate

Greek

ε	discrepancy of computed and measured data
$\lambda(T)$	thermal conductivity
ρ	density
δ	Dirac's delta function
$\delta_i(t)$	experimental error

Superscripts and subscripts

comp	computed
exp	experimental
i	thermal indicator number
w	wall

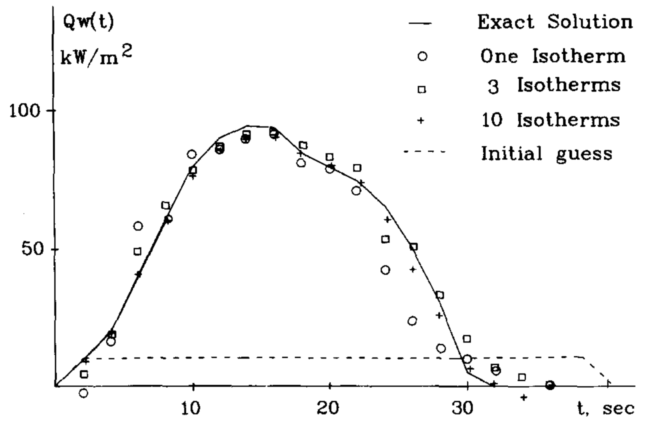

Figure 4 The heat flux estimation using 1, 3, and 10 isotherms with precise data

ods of second-order accuracy in space. The time–space location of isotherms $X_i(t)$ was computed numerically for some heat flux $q_w(t)$ and used as input data. The normally distributed random error with different dispersion was added to input data $X_i(t)$ for experiment error simulation.

The results of computational experiments are presented in Figures 4, 5, 6. The results of heat flux estimation using 1, 3, and 10 isotherms in comparison with the exact solution are shown in Figure 4 for exact input data. A constant heat flux was used as an initial guess. The numerical experiments confirm the feasibility of heat flux evaluation using the isotherms coordinates $X_i(t)$. Figure 5 demonstrates the input data error influence. The heat flux estimation was made using one isotherm for 0% and 5% standard deviation error in input data. An increase of the number of isotherms improves the accuracy. Figure 6 demonstrates that 10 temperature indicators are sufficient for acceptable accuracy with the same input data error level. Solution instability was not found in the computational experiments despite our problem being ill-posed.

Discussion

Direct and inverse heat transfer methods with data on some curves $X(t)$ were discussed by Lavrentiev et al. (1980) and Alifanov (1994). Usually, measurements at a moving boundary are very difficult; here we discuss a natural way of obtaining such

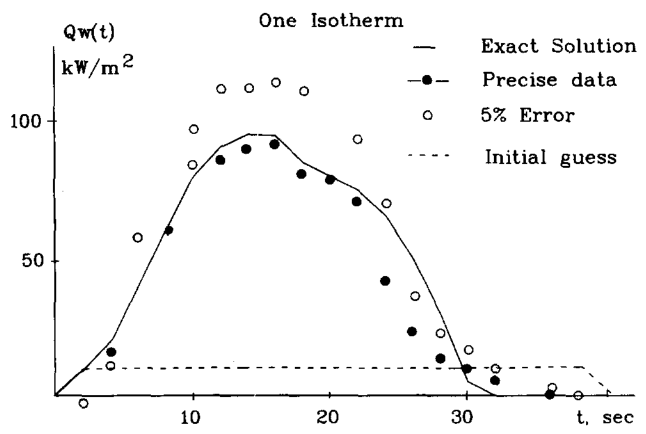

Figure 5 The heat flux estimation using one isotherm for 0 and 5% standard deviation error in input data

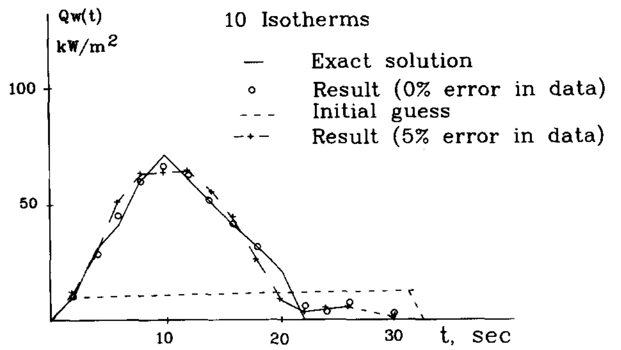

Figure 6 The heat flux estimation using 10 isotherms for 0 and 5% standard deviation error in input data

data. In the standard method of using thermal indicators, we record an indicator color being changed or not as a function of time. It is a piece-wise constant function, and intervals of constant color do not contain any useful information. In the method offered in the present paper, every indicator strip provides more information about the process: continuous functions $X_i(t)$ contain information on heat flux at any moment. The possibility of achieving the sensitivity threshold is far less, because the temperature range is smaller in the sensor depth. The temperature determination error is governed by the color change error ΔT (about 1%). In the standard approach, it is determined by the interval between service temperature of indicators (it provides error of about 10%).

The considered method has the following peculiarities. The sensitivity of isotherms to heat flux depends on the distance from the heated surface to the isotherm and decreases when this distance rises. The signal recorded by this sensor has an integral character in time, and, thus, it is robust to information loss. Direct interpretation of these sensor records is impossible (excluding the stationary case). In the steady case (q_w = CONST), the sensor's conductivity λ can be measured by simple differentiating, which provides an opportunity for sensor calibrating.

Conclusions

(1) The numerical experiments demonstrate the feasibility of heat flux estimation from isotherms.
(2) One isotherm is sufficient for boundary condition estimation.
(3) The increasing of necessary isotherms number improves accuracy. The maximum number is moderate (from 3 to 10) and far less than the number necessary for precise temperature field $T(t, x)$ determination.
(4) The input data $X_i(t)$ accuracy of about 5% was modeled by random number generator and was found to be acceptable.

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