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DOES THE LAW OF SUPPLY HOLD UNDER UNCERTAINTY?*

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A widely held conjecture of economists is that the quantity supplied by price-taking producers increases with an increase in output price (law of supply). This cornerstone of economics has been proven repeatedly with many techniques under many different sets of assumptions and is found in all textbooks on principles of economics. Most of these proofs assume full certainty of production relationships, single-product technology, and/or risk neutrality. In a world of uncertainty, one of the most crucial aspects of economic behaviour is risk aversion. With risk aversion, diversification is a basic response to economic uncertainty. Thus, models of producer behaviour under uncertainty need to consider multi-output technology where diversification is a possibility.

This paper points out that the widely held 'law of supply' may fail when these generalities (risky multioutput production with risk aversion) are introduced simultaneously where diversification is also affected by a capacity constraint in production. Conditions are developed under which the law fails (holds). The peculiar result occurs because increasing price can increase profit risk more than proportionally while expected profit increases proportionally. This relative increase in risk causes a tendency to shift production capacity into an alternative activity. The first section introduces a model of production under uncertainty. The second section analyses the behaviour of supply in the special case of additive production risk without price risk. The third section shows that similar problems can be encountered with other cases as well. Empirical examples, which demonstrate relevance of the results, are also given.

I. THE PRODUCTION MODEL WITH UNCERTAINTY

Consider a single firm with two production activities and with a joint limitation on production capacity. Suppose, for simplicity, that the output of each technology follows constant (stochastic) returns to scale and that the production function for each activity has one variable input. Specifically, let $q_i = c_i y_i$ and $\pi_i = p_i y_i - w_i x_i$, where q_i is the output of activity i, c_i is plant capacity allocated to activity i, y_i is the output of activity i per unit of plant capacity allocated to it, π_i is (short-run) profit or quasi rent per unit of plant capacity allocated to activity i, x_i is the quantity of the relevant variable input utilised per unit of capacity allocated to activity i, and w_i is the corresponding input price. Both output price and output quantity per unit of plant capacity may be stochastic

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representing producer uncertainty about production and market conditions. Thus, profitability of production activities is uncertain. Specifically, suppose the first two moments of the joint distribution are denoted by

$$\mathrm{E}\begin{pmatrix}\pi_1\\ \pi_2\end{pmatrix} = \begin{pmatrix}\bar{\pi}_1\\ \bar{\pi}_2\end{pmatrix} \quad \mathrm{cov}\begin{pmatrix}\pi_1\\ \pi_2\end{pmatrix} = \begin{pmatrix}\omega_1^2 & \rho\omega_1\omega_2\\ \rho\omega_1\omega_2 & \omega_2^2\end{pmatrix} > 0.$$

The jointness in production is due to a physical capacity constraint which, without loss of generality, is represented as $c_1 + c_2 = 1$ (see Pfouts, 1961, for a similar concept of jointness in production).

Note that this model formulation is quite common particularly in agricultural problems. The capacity variables can represent the share of a farm's land allocated to one crop versus another, and y_i variables can represent yields per unit of land which are random because of weather and crop disease. This formulation has also been used to model general problems of technology choice in production (Stoneman, 1981).

Now assume that the entrepeneur is risk averse with utility function $U(\cdot)$ defined on wealth W ($U' > 0$, $U'' < 0$), where wealth is composed of initial wealth W_0 and current profits π, $W = W_0 + \pi$. Assuming full-capacity utilisation, the decision problem is thus

$$\max_{\substack{0 \leqslant c_1 \leqslant 1\\ x_1, x_2 \geqslant 0}} \mathrm{E}U[W_0 + \pi_2 + c_1(\pi_1 - \pi_2)]. \tag{1}$$

With an internal solution ($0 < c_1 < 1$, $x_i > 0$), first-order conditions are

$$\frac{\partial \mathrm{E}U}{\partial c_1} = \mathrm{E}[(\pi_1 - \pi_2)\, U'] = 0, \tag{2}$$

$$\frac{\partial \mathrm{E}U}{\partial x_1} = \mathrm{E}\left[\frac{\partial \pi_1}{\partial x_1} c_1 U'\right] = 0, \tag{3}$$

$$\frac{\partial \mathrm{E}U}{\partial x_2} = \mathrm{E}\left[\frac{\partial \pi_2}{\partial x_2}(1 - c_1)\, U'\right] = 0. \tag{4}$$

Following Newbery and Stiglitz (1979), consider a first-order Taylor series approximation of U' about expected wealth, $\bar{W} = W_0 + \bar{\pi}_2 + c_1(\bar{\pi}_1 - \bar{\pi}_2)$,

$$U'(W) = \bar{U}' + [e_2 + c_1(e_1 - e_2)]\,\bar{U}'', \tag{5}$$

where $\bar{U}'$ and $\bar{U}''$ are U' and U'' evaluated at expected wealth, respectively, and $e_i = \pi_i - \bar{\pi}_i$ ($i = 1, 2$).[1] Further, let the measure of relative risk aversion be denoted by $\psi(W) = -WU''(W)/U'(W)$ so that at mean wealth

$$\psi = \psi(\bar{W}) = -W\bar{U}''/\bar{U}'.$$

Then using (5) in (2)–(4), first-order conditions are approximated by

$$\frac{1}{\bar{U}'}\frac{\partial \mathrm{E}U}{\partial c_1} = (\bar{\pi}_1 - \bar{\pi}_2) - \frac{\psi}{\bar{W}}[c_1 v + \omega_2(\rho\omega_1 - \omega_2)] = 0, \tag{6}$$

[1] Newbery and Stiglitz (1979) used this technique to examine the welfare effects of price stabilisation under uncertainty. Their results show that some other aspects of supply response under uncertainty can be perverse. Namely, they show that supply response to stabilisation can be in either direction.

$$\frac{1}{\bar{U}'}\frac{\partial \mathrm{E}U}{\partial x_1} = c_1\frac{\partial \bar{\pi}_1}{\partial x_1} - c_1\frac{\psi}{\bar{W}}\left[(1-c_1)\,\mathrm{E}\left(e_2\frac{\partial \pi_1}{\partial x_1}\right) + c_1\mathrm{E}\left(e_1\frac{\partial \pi_1}{\partial x_1}\right)\right] = 0, \tag{7}$$

$$\frac{1}{\bar{U}'}\frac{\partial \mathrm{E}U}{\partial x_2} = (1-c_1)\frac{\delta \bar{\pi}_2}{\partial x_2} - (1-c_1)\frac{\psi}{\bar{W}}\left[(1-c_1)\,\mathrm{E}\left(e_2\frac{\partial \pi_2}{\partial x_2}\right) + c_1\mathrm{E}\left(e_1\frac{\partial \pi_2}{\partial x_2}\right)\right] = 0, \tag{8}$$

where $v = \omega_1^2 + \omega_2^2 - 2\rho\omega_1\omega_2 = \mathrm{var}(\pi_1 - \pi_2)$. Thus, the solution for c_1 can be written as

$$c_1 = \frac{\bar{\pi}_1 - \bar{\pi}_2 + (\psi/\bar{W})\,\omega_2(\rho\omega_1 - \omega_2)}{(\psi/\bar{W})\,v} = \frac{(\bar{\pi}_1 - \bar{\pi}_2)\,\bar{W}}{\psi v} + R, \tag{9}$$

where
$$R = \frac{\omega_2(\omega_2 - \rho\omega_1)}{v}.$$

In each case, the first-order conditions in (6)–(8) equate the marginal mean income effect (the first right-hand terms) to the marginal risk effect (the second right-hand terms) discounted to a certainty equivalent by multiplying by the Arrow–Pratt coefficient of absolute risk aversion $(\psi/\bar{W})$. The numerator of the capacity solution in (9) is the excess of the marginal expected profit $[\partial \mathrm{E}(\pi)/\partial c_1 = \bar{\pi}_1 - \bar{\pi}_2]$ over the certainty equivalent of the marginal variance of profit at zero capacity $[\partial\,\mathrm{var}\,(\pi)/\partial c_1 = \omega_2(\omega_2 - \rho\omega_1)$ at $c_1 = 0]$ where all margins are with respect to capacity allocation, c_1. The resulting optimal capacity allocation is obtained by denominating this excess in terms of the certainty equivalent of the variance of marginal profit $[v = \mathrm{var}\,(\partial\pi/\partial c_1)]$. Alternatively, the optimal capacity allocation can be written as the sum of two ratios as on the right-hand side of (9). The first is the ratio of marginal expected profit to the certainty equivalent of the variance of marginal profit. The second is the ratio of the covariance of marginal profit with profit at zero capacity $[\omega_2(\omega_2 - \rho\omega_1) = \mathrm{cov}\,(\partial\pi/\partial c_1, \pi_2)]$ to the variance of marginal profit. To this extent, the first ratio represents a marginal mean-variance trade-off while the second ratio is a correlation effect that is positive (negative) if correlation is low (high) $[\rho < (>)\ \omega_2/\omega_1]$. In particular, the second term disappears when the second activity is deterministic.

II. THE TWO-OUTPUT CASE OF ADDITIVE PRODUCTION RISK WITH DETERMINISTIC PRICE

To illustrate a common case where the law of supply may fail, suppose production risk is additive, all prices are known at decision-making time, and variable inputs are risk neutral.[1] That is, let $y_i = \bar{y}_i(x_i) + \epsilon_i$, where $\mathrm{E}(\epsilon_i) = 0$ and $\mathrm{var}\,(\epsilon_i) = \sigma_i^2$, so that mean and variance of profit per unit of capacity in each activity are

[1] Input risk neutrality greatly simplifies the mathematical derivation of this paper. Note that most models with stochastic production assume input risk neutrality which is necessary for the existence of dual cost and production relationships independent of risk preferences. Nevertheless, comparing with the results of Just and Zilberman (1983), the same principles demonstrated here clearly govern the general case.

$\bar{\pi}_i = p_i \bar{y}_i - w_i x_i$ and $\omega_i^2 = p_i^2 \sigma_i^2$, respectively. Thus, the first-order conditions in (7) and (8) become

$$\frac{1}{\bar{U}'} \frac{\partial \mathrm{E}U}{\partial x_i} = c_i(p_i \bar{y}_1' - w_i) = 0 \quad (i = 1, 2).$$

Here the assumptions of input risk neutrality and additive production risk lead to the familiar condition that equates the value of marginal product of an input to its price. This leads to an independence of variable input decisions for the capacity allocation choice.

Second-order conditions hold for this case assuming positive and decreasing expected marginal productivity ($\bar{y}_1' > 0$, $\bar{y}_1'' < 0$) if and only if[1]

$$D \equiv \frac{\psi v}{\bar{W}} \left[1 - \frac{\eta (\bar{\pi}_1 - \bar{\pi}_2)^2}{\psi v} \right] > 0, \tag{10}$$

where η is the elasticity of absolute risk aversion,

$$\eta = -[\partial(\psi/\bar{W})/\partial \bar{W}]\, [\bar{W}/(\psi/\bar{W})],$$

which is equal to $1 - \tilde{\eta}$ where $\tilde{\eta}$ is the elasticity of relative risk aversion,

$$\tilde{\eta} = (\partial \psi / \partial \bar{W})\, (\bar{W}/\psi).$$

Note that $\eta = 0$ implies constant absolute risk aversion, $\eta = 1$ implies constant relative risk aversion, $\eta > 0$ implies decreasing absolute risk aversion, and $\eta < 1$ implies increasing relative risk aversion. The condition in (10) is assumed to hold throughout the remainder of this paper.

Next, consider the behaviour of supply. Without loss of generality, consider only the supply of good 1. Since production is uncertain, the law of supply is analysed in terms of the deterministic component of supply. This corresponds to the approach of empirical analysis where coefficients are estimated along expectations of relationships describing supply. Note also that the same qualitative results are obtained if supply is analysed in terms of a given state of nature.

Expected production of good i is

$$\bar{q}_i = \mathrm{E}(q_i) = c_i \bar{y}_i.$$

Thus, the response of expected supply is

$$\frac{d\bar{q}_i}{dp_i} = c_i \bar{y}_i' \frac{dx_i}{dp_i} + \bar{y}_i \frac{dc_i}{dp_i}. \tag{11}$$

The first right-hand term is a productivity effect, and the second term is a capacity effect.

To sign (11), first note from (7) and (8) that

$$\frac{dx_i}{dp_i} = -\frac{\bar{y}_i'}{p_i \bar{y}_i''} = -\frac{w_i}{p_i^2 \bar{y}_i''} > 0, \tag{12}$$

[1] Note that $(1/\bar{U}')\, \partial^2 \mathrm{E}U/\partial c_1^2 = -D$. Second-order conditions in addition to (10) are satisfied since $(1/\bar{U}')\, \partial^2 \mathrm{E}U/\partial x_i^2 = c_i p_i\, \bar{y}_i'' < 0$ $(i = 1, 2)$; $(1/\bar{U})\, \partial^2 \mathrm{E}U/\partial c_1 \partial x_1 = 0$ $(i = 1, 2)$; and $(1/\bar{U}')\, \partial^2 \mathrm{E}U/\partial x_1 \partial x_2 = 0$.

so that the variable input effect in (11) is positive reflecting decreasing marginal productivity. Note further from differentiating (9) totally and using (7) and (8) that

$$\frac{dc_1}{dp_1} = \frac{1}{D}\left\{\left[1 + \frac{\eta c_1(\bar{\pi}_1 - \bar{\pi}_2)}{\bar{W}}\right]\bar{y}_1 - \frac{\psi}{\bar{W}}\left[2c_1\omega_1 - \rho\omega_2(c_2 - c_1)\right]\frac{\partial\omega_1}{\partial p_1}\right\}. \tag{13}$$

Capacity response is, thus, a linear combination of the mean and standard deviation of production per unit of capacity since $\partial\omega_1/\partial p_1 = \sigma_1$. The mean effect is composed of an own-mean effect, $\bar{y}_1/D$, which is constant with constant stochastic returns to scale reflecting that a higher price causes higher marginal revenue and, thus, greater response in capacity allocation. The remainder of the mean effect is a correction for declining absolute risk aversion which is positive since an increase in price increases profit, thus reducing absolute risk aversion and leading to a further tendency to expand c_1. The first component of the second term, $2c_1\psi\omega_1/(D\bar{W})$, is an own-variance effect that reflects a tendency for capacity to respond negatively to price increases because of the associated increase in variance of profits. This tendency is directly proportional to risk aversion, output price, capacity allocated to the activity, and the standard deviation of production per unit of capacity. The remainder of the variance effect is a correlation effect that vanishes with perfect diversification ($c_1 = c_2 = 0{\cdot}5$) and becomes large as capacity is allocated completely to one activity or the other. Higher correlation tends to make capacity more (less) responsive to price if the capacity allocated to the activity in question is high (low) and correlation is positive when overall capacity response is positive.

The conflicting signs of terms in (13) suggest the peculiar possibility that capacity allocation may not respond positively to price. In point of fact, $dc_1/dp_1 < 0$ if

$$\psi > \frac{[\bar{W} + \eta c_1(\bar{\pi}_1 - \bar{\pi}_2)]\,\bar{y}_1}{[2c_1\omega_1 + \rho\omega_2(c_2 - c_1)]\,(\partial\omega_1/\partial p_1)}, \tag{14}$$

or

$$\rho > (<)\,\frac{[\bar{W} + \eta c_1(\bar{\pi}_1 - \bar{\pi}_2)]\,\bar{y}_1 - 2c_1\psi\omega_1(\partial\omega_1/\partial p_1)}{\psi\omega_2(c_2 - c_1)\,(\partial\omega_1/\partial p_1)} \quad \text{as} \quad c_2 > (<)\,c_1, \tag{15}$$

or

$$\eta < \frac{\bar{y}_1 - [2c_1\psi\omega_1 + \rho\psi\omega_2(c_2 - c_1)]\,(\partial\omega_1/\partial p_1)}{c_1\bar{y}_1(\bar{\pi}_2 - \bar{\pi}_1)} \tag{16}$$

or

$$p_1 > \frac{[\bar{W} + \eta c_1(\bar{\pi}_1 - \bar{\pi}_2)]\,\bar{y}_1 - \rho\psi\omega_2(c_2 - c_1)\,(\partial\omega_1/\partial p_1)}{2c_1\psi\sigma_1(\partial\omega_1/\partial p_1)}. \tag{17}$$

These conditions show that, ceteris paribus, capacity response may be negative when risk aversion is sufficiently high, correlation is sufficiently high (positively or negatively), the elasticity of risk aversion is sufficiently low, or the price is sufficiently high. While the plausibility of these cases may not be clear at this point, an example below demonstrates their plausibility clearly.

Next consider the overall expected supply response in (11). Substituting (12) and (13) into (11) obtains

$$\frac{d\bar{q}_1}{dp_1} = -\frac{c_1^* w_1 \bar{y}_1'}{p_1^2 \bar{y}_1''} + \frac{\bar{y}_1}{D}\left\{1 + \frac{\eta c_1(\bar{\pi}_1 - \bar{\pi}_2)}{\bar{W}}\bar{y}_1 - \frac{\psi}{\bar{W}}[2c_1\omega_1 - \rho\omega_2(c_2 - c_1)]\frac{\partial\omega_1}{\partial p_1}\right\}. \quad (18)$$

The first right-hand term of (18) represents the effect of declining marginal productivity, which tends to make the supply curve have positive slope, even though capacity response may be negative. Nevertheless, the first term may not override the second term so that supply response could be negative when one of the conditions in (14)–(17) holds. For example, the first term vanishes as either the marginal productivity or price of the variable input tends to zero. Moreover, the first term vanishes as output price gets high, which is one of the conditions that tends to make capacity response negative [see (15)]. Thus, cases with negative supply elasticity seem to be reasonable.

III. THE GENERAL TWO-OUTPUT CASE OF PRICE AND PRODUCTION RISK

This section considers a general stochastic specification that allows for both price and production risk where both can be of either additive or multiplicative form. To allow this generalisation without unduly complicating the mathematical presentation, the production model is simplified to the case of Leontief technology. As noted above, many agricultural problems fit the framework of this paper. For the deterministic component of such problems, fixed-proportions production functions are often employed (in which case production is not responsive to x_i) so that supply response is reflected simply by the choice of c_1. See, for example, the many programming models that have been used to represent agricultural production (such as Hazell, 1971) or the stochastic theoretical models of technology choice (Feder, 1980). Furthermore, for the special case of the previous section, the law of supply fails only if capacity response is negative and then only if the capacity effect overrides the productivity effect in (11). From the standpoint of qualitative theoretical analysis, a negative capacity effect clearly overrides the productivity effect only if the productivity effect is insignificant ($\bar{y}_1' \to 0$, $w_1 \to 0$ or $p_1/w_1 \to \infty$). Thus, the approach of this section may be viewed as simplifying the model from the outset by employing one of these assumptions.

Suppose both price and production are risky with both additive and multiplicative random components. For example, suppose

$$p_1 = \bar{p}_1 e_p + \epsilon_p, \quad \mathrm{E}(e_p) = 1, \quad \mathrm{E}(\epsilon_p) = 0,$$
$$y_1 = \bar{y}_1 e_y + \epsilon_y, \quad \mathrm{E}(e_y) = 1, \quad \mathrm{E}(\epsilon_y) = 0,$$

where e_p, e_y, ϵ_p, and ϵ_y are all pairwise independent random variables with respective variances, δ_p^e, δ_y^e, δ_p^ϵ, and δ_y^ϵ. Then,

$$\bar{\pi}_1 = \bar{p}_1\bar{y}_1 - w_1 x_1, \quad (19)$$

$$\omega_1^2 = \bar{p}_1^2\bar{y}_1^2(\delta_p^e\delta_y^e + \delta_p^e + \delta_y^e) + \bar{p}_1^2\delta_y^\epsilon(\delta_p^e + 1) + \bar{y}_1^2\delta_p^\epsilon(\delta_y^e + 1) + \delta_p^\epsilon\delta_y^\epsilon. \quad (20)$$

In this context, the decision problem can continue to be represented as in (1) except the x_i variables are no longer choice variables. Thus, the first-order condition is given by (2) and is approximated by (6) with the corresponding solution in (9).

To examine supply in this case, let supply be defined by the relationship of expected output to expected price since price is now also uncertain at production-planning time. Because the x_i variables are no longer decision variables, (11) becomes

$$\frac{d\bar{q}_i}{d\bar{p}_i} = \bar{y}_i \frac{dc_i}{d\bar{p}_i}.$$

Thus, supply response is determined by the response of capacity allocation. Also, capacity allocation response continues to follow (13) (with p_1 replaced by $\bar{p}_1$) since $\partial\bar{\pi}_1/\partial\bar{p}_1 = \bar{y}_1$ from (19). Furthermore, from (20),

$$\frac{\partial\omega_1^2}{\partial\bar{p}_1} = 2\bar{p}_1[\bar{y}_1^2(\delta_p^e\,\delta_y^e + \delta_p^e + \delta_y^e) + \delta_y^\epsilon(\delta_p^e + 1)].$$

Since sign $(\partial\omega_1/\partial\bar{p}_1)$ = sign $(\partial\omega_1^2/\partial\bar{p}_1)$, one finds that $\partial\omega_1/\partial\bar{p}_1 > 0$ if $\delta_p^e > 0$ or $\delta_y^e > 0$ or $\delta_y^\epsilon > 0$. On the other hand, $\partial\omega_1/\partial\bar{p}_1 = 0$ if $\delta_p^\epsilon > 0$ and $\delta_p^e = \delta_y^e = \delta_y^\epsilon = 0$. Thus, the intuition surrounding (13) carries through; and the conditions for negative response in (14)–(16) follow if multiplicative price risk or any production risk exists. Only if all risk arises from additive price disturbances do the second term in (13) and the corresponding possibility of negative supply response vanish. In other words, negatively sloped supply can occur if and only if an increase in average price increases the overall variance of returns per unit of capacity.

IV. THE LAW OF SUPPLY FOR A TWO-TECHNOLOGY FIRM

Consider next the case where both of the firm's production activities produce the same product so that the quantity supplied by the firm is the sum of outputs from allocating capacity between two technologies. For this purpose, let $\bar{p}_1 = \bar{p}_2 = \bar{p}$ represent the common expected output price. Then expected production is

$$\bar{q} = \mathrm{E}(q_1 + q_2) = c_1\bar{y}_1 + c_2\bar{y}_2,$$

and expected supply response, again simplifying to the case of Leontief technology, is

$$\frac{d\bar{q}}{d\bar{p}} = \sum_{i=1}^{2} \bar{y}_i \frac{dc_i}{d\bar{p}}. \tag{21}$$

To further simplify this case, suppose that any price risk is multiplicative so that $p_1 = \bar{p}_1 e_p$, $\mathrm{E}(e_p) = 1$, while $\bar{y}_1 = y_1 e_y + \epsilon_y$, $\mathrm{E}(e_y) = 1$, $\mathrm{E}(\epsilon_y) = 0$ and other assumptions follow Section III. Then, the result in (9) becomes

$$c_1 = \frac{(\bar{\pi}_1 - \bar{\pi}_2)\,\overline{W}}{\psi\bar{p}^2 v^*} + \frac{\tilde{\omega}_2(\tilde{\omega}_2 - p\tilde{\omega}_1)}{v^*}, \tag{22}$$

where $v^* = \tilde{\omega}_1^2 + \tilde{\omega}_2^2 - 2\rho\tilde{\omega}_1\tilde{\omega}_2$ and $\tilde{\omega}_i^2 = \text{var}(e_p y_i)$ $(i = 1, 2)$. Thus, $\tilde{\omega}_i$, v^*, and the entire correlation term is independent of expected price. Therefore,

$$\frac{dc_1}{d\bar{p}} = \frac{1}{\bar{p}D}\left[\bar{p}(\bar{y}_2 - \bar{y}_1) + 2(w_1 x_1 - w_2 x_2) + \frac{\eta c_1(\bar{\pi}_1 - \bar{\pi}_2^2)(\bar{y}_1 - \bar{y}_2)}{\bar{W}}\right]. \tag{23}$$

The last right-hand term in brackets provides a correction for declining absolute risk aversion. The remaining terms reflect the relative importance of price changes on different components of the first right-hand term of the expression for c_1 in (22). The price appears in second order in the variance component in the denominator, in first order in the revenue component of the numerator, and does not appear in the cost component of the numerator. Therefore, an increase sn price reduces the revenue/variance ratio, $[\bar{p}(\bar{y}_1 - \bar{y}_2)]/(\bar{p}^2 v^*)$, but reduces the cost/variance ratio, $(w_2 x_2 - w_1 x_1)/(\bar{p}^2 v^*)$, by an additional order of magnitude. Thus, the variance effect of an increase in price tends to be of overriding importance. The revenue/variance effect is negative if the other activity is less productive (produces less mean output per unit of capacity). Similarly, the cost/variance effect is negative if the other activity entails higher costs (higher variable costs per unit of capacity). Although the declining risk aversion correction is always positive, it can very possibly be overridden since $0 \leqslant \eta \leqslant 1$ and the remainder of the term can be small.

To examine supply response for this case, substitute (23) into (21) and note that $dc_2/d\bar{p} = -dc_1/d\bar{p}$ to obtain

$$\frac{d\bar{q}}{d\bar{p}} = (\bar{y}_1 - \bar{y}_2)\frac{dc_1}{d\bar{p}}. \tag{24}$$

Thus, supply response is negative if production functions follow fixed proportions, absolute risk aversion is constant ($\eta = 0$), and the difference in variable costs per unit of capacity is less than half the difference of mean gross returns per unit of capacity. Since this case is reasonable, particularly in the context of agricultural problems which often assume fixed proportions production, the results show clearly that the law of supply need not hcld. Furthermore, the assumption of constant absolute risk aversion can be easily relaxed in this example when wealth is large relative to expected gross returns (since $\eta \leqslant 1$ with non-decreasing relative risk aversion).

V. AN EXAMPLE

An example can further serve to show that cases which violate the law of supply do not require unreasonably extreme values of parameters. Consider the widely referenced data on rice risk in the Philippines reported by Roumasset (1976, pp. 54 and 55). He reports means and standard deviations of yields, variable costs per unit of land, and output price for production under four technologies. Here we present the comparison of only the two highest-yielding techniques to avoid unnecessary generalisations of the model.[1] The two technologies have mean yields of 80 and 90 cavans per hectare with respective standard deviations of 30 and 35 and respective variable costs per hectare of P410 and P490. The price

[1] Similar results were obtained for about half of all the possible pairwise comparisons.

Table 1

Supply Response with Risk Aversion for a Two-Good Firm

			$\rho = -0.8$				$\rho = 0$				$\rho = 0.8$			
					ν				ν				ν	
ϕ	ψ	$\overline{W}$	c_1	θ	$\eta = 0$	$\eta = 1$	c_1	θ	$\eta = 0$	$\eta = 1$	c_1	θ	$\eta = 0$	$\eta = 1$
0.001	1.0	1000	0.46	0.90	2.42	2.31	0.43	0.84	4.63	4.47	0.16	0.79	57.11	59.44
0.002	1.0	500	0.50	1.76	0.84	0.72	0.50	1.55	1.49	1.30	0.51	1.33	6.99	6.33
0.003	1.0	333	0.52	2.60	0.36	0.25	0.53	2.12	0.64	0.43	0.63	1.64	2.67	1.76
0.004	1.0	250	0.52	3.42	0.14	0.02	0.54	2.56	0.24	0.03	0.69	1.72	1.07	0.06
0.005	1.0	200	0.53	4.20	0.00	−0.11	0.55	2.87	0.01	−0.20	0.72	1.58	0.23	−0.83
0.006	1.0	167	0.53	4.96	−0.08	−0.20	0.55	3.04	−0.14	−0.35	0.75	1.20	−0.29	−1.38
0.007	1.0	143	0.53	5.70	−0.15	−0.27	0.56	3.08	−0.24	−0.46	0.76	0.60	−0.64	−1.75
0.008	1.0	125	0.53	6.40	−0.19	−0.31	0.56	2.99	−0.32	−0.54	*	*	*	*
0.009	1.0	111	0.53	7.08	−0.23	−0.35	0.56	2.76	−0.38	−0.60	*	*	*	*
0.010	1.0	100	0.53	7.73	−0.26	−0.38	0.56	2.40	−0.43	−0.65	*	*	*	*

* Not applicable; maximum certainty equivalent is negative. Definitions: ϕ, absolute risk aversion at expected wealth; ψ, relative risk aversion at expected wealth; $\overline{W}$, expected wealth; c_1, proportion of land allocated to the first output; θ, partial risk aversion at expected wealth; ν, elasticity of supply; η, elasticity of absolute risk aversion; ρ, correlation of profits per unit of capacity of the two outputs.

of output in each case is P16 per cavan. Since Roumasset (1976) did not report a correlation of yields, the calculations here consider correlations ranging from −0·8 to 0·8. Roumasset (1976) also did not estimate risk aversion. Here, we consider a variety of cases under both constant absolute and constant relative risk aversion and where the two technologies generate different crops or the same crop.

Consider first the case with constant absolute risk aversion, $\eta = 0$, where each technology produces a different crop. For each level of absolute risk aversion ($\phi = -\bar{U}''/\bar{U}'$) from 0·001 to 0·010, Table 1 reports the corresponding capacity decision c_1 from (9), the associated supply elasticity,

$$\nu = \frac{dc_1}{dp_1}\frac{p_1}{c_1} = \frac{d\bar{q}_1}{dp_1}\frac{p_1}{\bar{q}_1},$$

where dc_1/dp_1 follows (13), and the measure of partial risk aversion,

$$\theta = -\bar{M}\frac{\bar{U}''}{\bar{U}'} = \bar{M}\phi,$$

where $\bar{M}$ is the certainty equivalent of profit. Binswanger (1980, 1981) has found empirically that the measure of partial risk aversion varies from 0·1 to 10 among farmers as risk aversion varies from slight to extreme. This measure is reported here to verify the plausibility of risk-aversion levels where the law of supply fails. As one can see, none of the partial risk-aversion measures in Table 1 is outside the 0·1 to 10 range; in fact, cases with negative supply elasticity are obtained with partial risk aversion as low as 0·6 in the case with constant absolute risk aversion ($\eta = 0$). Moreover, the negative supply elasticities can get quite large, e.g. −0·64 for $\phi = 0{\cdot}007$ and $\rho = 0{\cdot}8$.

Next, consider the case with constant relative risk aversion ($\eta = 1$). In this case, following the arguments of Arrow (1971), relative risk aversion is assumed to be unity ($\psi = 1$). With levels of wealth from P100 to P1,000, as indicated in Table 1, this corresponds to levels of absolute risk aversion from 0·001 to 0·010 since $\phi = \psi/\bar{W}$. Following (13), the elasticity of supply with $\eta = 1$ is somewhat lower than with $\eta = 0$ for each case in Table 1 since $\bar{\pi}_1 - \bar{\pi}_2 = -80 < 0$. In this case, negative supply response occurs at lower levels of absolute risk aversion and at higher levels of wealth than with constant absolute risk aversion. Also, the negative elasticities reach as high as −1·75.

Now consider the case where both technologies produce crops that enter the same market. In this case, the values of c_1 and θ reported in Table 1 still correspond to the same value of ϕ, ψ, and $\bar{W}$. However, the elasticities must be calculated according to (13) and (24),

$$\nu = \frac{d\bar{q}}{d\bar{p}}\frac{\bar{p}}{\bar{q}} = (\bar{y}_1 - \bar{y}_2)\frac{dc_1}{d\bar{p}}\frac{\bar{p}}{\bar{q}}. \qquad (25)$$

For the case of constant absolute risk aversion, this obtains the peculiar result that $\nu = 0$ for all the cases in Table 1. This occurs because the Roumasset (1976) data just happen to satisfy

$$\bar{p}(\bar{y}_2 - \bar{y}_1) + 2(w_1x_1 - w_2x_2) = 0,$$

so, from (23), $dc_1/d\bar{p} = 0$ if $\eta = 0$. This circumstance also implies, however, that the supply elasticity is negative for any rice price greater than P16 while it is positive for any price less than P16. Thus, whether negative supply elasticity occurs does not depend on the level of absolute risk aversion.

Turning to the case of constant relative risk aversion where both technologies produce crops for the same market, the results are similar. The use of $\eta = 1$ instead of $\eta = 0$ in (23) and (25) only modifies the supply elasticity by a maximum of 0·000007 from the zero levels under constant absolute risk aversion. Thus, negative supply elasticity occurs for all prices only slightly higher than the P16 reported by Roumasset (1976).

VI. CONCLUSION

This paper shows that any price increase which inherently results in increased variability of returns can cause negative supply response. In particular, this occurs with multiplicative price risk and either additive or multiplicative production risk. Because either multiplicative price risk or production risk can cause this problem, eliminating one or the other through providing alternative institutions or contingency markets cannot eliminate the problem. For example, providing an unibased futures market would only reduce the price risk, whereas yield risk can cause negative supply response. Similarly, providing production insurance, such as agricultural crop insurance against yield loss, cannot eliminate the problem if multiplicative price risk persists. Only elimination of all revenue risk other than additive price risk can assure positive supply response. Empirical examples in both cases show these conditions are plausible even in cases with moderate levels of risk aversion. Perhaps econometricians have been too quick to discard empirical results with 'wrong signs' in estimated supply equations!

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