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## The Xinanjiang model applied in China

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### ABSTRACT

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The Xinanjiang model is a rainfall–runoff, distributed, basin model for use in humid and semi-humid regions. The evapotranspiration component is represented by a model of three soil layers. Runoff production occurs on repletion of storage to capacity values which are assumed to be distributed throughout the basin. Prior to 1980, runoff was separated into surface and groundwater components using Horton's concept of infiltration. Subsequently, the concept of hillslope hydrology was introduced with an additional component, interflow, being identified. Runoff concentration to the outflow of each sub-basin is represented by a unit hydrograph or by a lag and route technique. The damping or routing effects of the channel system connecting the sub-basins are represented by Muskingum routing. There are fifteen parameters in all, of which the model is particularly sensitive to six. Optimization of the parameters is achieved with different objective functions according to the nature of each parameter. The model has been widely used in China since 1980, mainly for flood forecasting, though more recently it is also being used for other purposes.

### INTRODUCTION

The Xinanjiang model was developed in 1973 and published in 1980 (Zhao et al., 1980). Its main feature is the concept of runoff formation on repletion of storage, which means that runoff is not produced until the soil moisture content of the aeration zone reaches field capacity, and thereafter runoff equals the rainfall excess without further loss. This hypothesis was first proposed in China in the 1960s, and much subsequent experience supports its validity for humid and semi-humid regions. According to the original formulation, runoff so generated was separated into two components using Horton's concept of a final, constant, infiltration rate. Infiltrated water was assumed to go to the groundwater storage and the remainder to surface, or storm runoff. However, evidence of variability in the final infiltration rate, and

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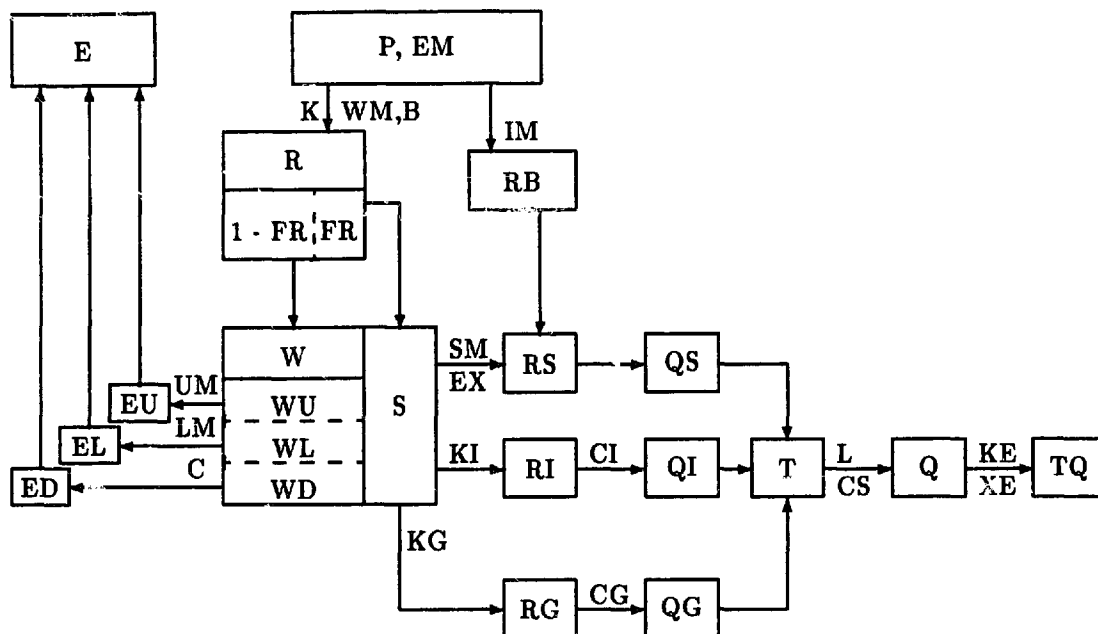


Fig. 1. Flow chart for the Xinanjiang model.

in the unit hydrograph assumed to connect the storm runoff to the discharge from each sub-basin, suggested the necessity of a third component. Guided by the work of Kirkby (1978) an additional component, interflow, was provided in the model in 1980. The modified model is now successfully and widely used in China. It is presented in detail below.

#### THE STRUCTURE OF THE XINANJIANG MODEL

The basin is divided into a set of sub-basins, the outflow hydrograph from each of which is first simulated and then routed down the channels to the main basin outlet. The flow chart is shown in Fig. 1. The inputs to the model are  $P$ , the measured areal mean rainfall depth on the sub-basin and  $EM$ , the measured pan evaporation in the same units. The symbols for inputs, outputs and state variables appear inside the blocks of Fig. 1; those for parameters (constants on each sub-basin) appear outside the corresponding blocks.

The outputs are the discharge,  $Q$ , from each sub-basin,  $TQ$ , the outlet discharge from the whole basin,  $E$ , the actual evapotranspiration from the whole basin, which is the sum of the evapotranspirations from the upper soil layer  $EU$ , the lower soil layer  $EL$ , and the deepest layer  $ED$ .

The state variables are:  $W$  tension water stored at a point in the basin;  $WM$  the corresponding capacity value at a point, varies throughout the sub-basin from zero to a maximum value  $MM$  (a parameter);  $W$  areal mean tension water storage having components  $WU$ ,  $WL$  and  $WD$  in the upper,

lower and deepest layer; the capacity value of  $W$  is  $WM$  (a parameter), equal to the sum of  $UM$ ,  $LM$  and  $DM$ , the capacities of the three layers;  $S'$  free water storage at a point, having a capacity value of  $S'M$ ,  $S$  areal mean free water storage;  $AU$  the ordinate of the point  $x$  in Fig. 2, representing the tension water storage state in the sub-basin;  $BU$  the ordinate of the point  $x$  in Fig. 3, representing the free water storage state;  $R$  runoff from the pervious area having components  $RS$ ,  $RI$  and  $RG$  surface, interflow and groundwater runoff, respectively;  $RB$  runoff from the impervious area  $IM$ ;  $Q$  the discharge from a sub-basin having components  $QS$ ,  $QI$ ,  $QG$ , surface runoff, interflow and ground water, respectively;  $FR$  the (variable) runoff producing area;  $T$  the total sub-basin inflow to the channel network, having components  $TS$ ,  $TI$  and  $TG$ .

The parameters are:  $K$  the ratio of potential evapotranspiration to pan evaporation;  $MM$  the maximum value within the sub-basin of the tension water capacity  $WM$  (related through  $B$  to  $WM$ );  $WM$  the areal mean tension water capacity having components  $UM$ ,  $LM$  and  $DM$  the capacities of the three soil layers;  $C$  a factor, less than unity, by which any remaining potential evaporation is multiplied in application to the deepest soil layer;  $B$  a parameter in the distribution of tension water capacity;  $IM$  the impervious area of the sub-basin;  $SM$  areal mean free water storage capacity;  $MS$  maximum free water storage capacity (related through  $Ex$  to  $SM$ );  $Ex$  a parameter in the distribution of free water storage capacity;  $KI$  a coefficient relating  $RI$ , a contribution to interflow storage;  $KG$  a coefficient relating  $RG$ , a contribution to groundwater storage;  $CI$  the interflow reservoir constant of the sub-basin;  $CG$  the groundwater reservoir constant of the sub-basin;  $L$  the "lag" parameter of the flow concentration within the sub-basin;  $CS$  the "route" parameter of the flow concentration within the sub-basin.

#### EVAPOTRANSPIRATION

Evapotranspiration is related to potential evapotranspiration through a three-layer soil moisture model depending on four parameters  $K$ ,  $UM$ ,  $LM$  and  $C$ . Until the storage  $WU$  of the uppermost layer is exhausted, evaporation occurs at the potential rate, equal to  $K$  times the pan evaporation rate.

$$EU = K \times EM \quad (1)$$

On exhaustion of the upper layer (capacity  $UM$ ) any remaining potential evapotranspiration is applied to the lower layer, but the efficiency is modified by multiplication by the ratio of the actual storage  $WL$  to the capacity storage  $LM$  of that layer.

$$EL = (K \times EM - EU) \times WL/LM \quad (2)$$

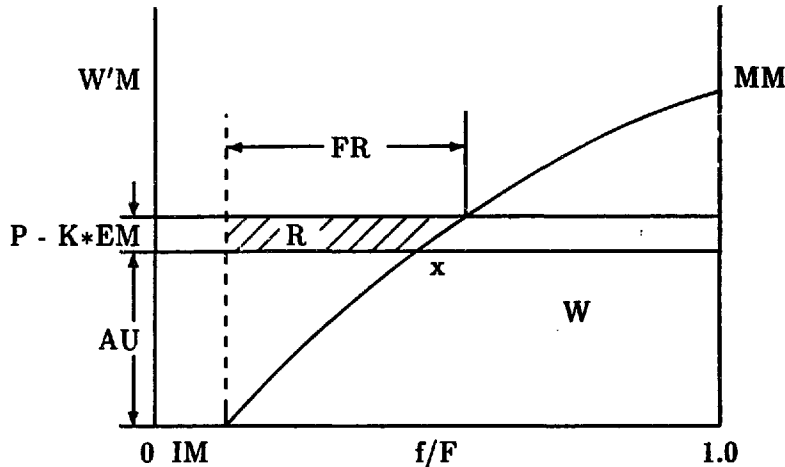


Fig. 2. The distribution of tension water capacity in the sub-basin.

When the lower layer storage  $WL$  is reduced to a proportion  $C$  (a parameter) of  $LM$ , evapotranspiration is assumed to continue, but at a further reduced rate  $ED$  given by

$$ED = C \times (K \times EM - EU) - EL \quad (3)$$

#### RUNOFF PRODUCTION

Runoff production at a point, occurs only on repletion of the tension water storage at that point. To provide for a non-uniform distribution of tension water capacity throughout the sub-basin, a tension water capacity curve (Fig. 2) is introduced.

In Fig. 2,  $f/F$  represents the proportion of the pervious area of the basin whose tension water capacity is less than or equal to the value of the ordinate  $W'M$ . The tension water capacity at a point, ( $W'M$ ) varies from zero to a maximum  $MM$  (a parameter) according to the relationship

$$(1 - f/F) = (1 - W'M/MM)^B \quad (4)$$

where  $B$  is a parameter.

The areal mean tension water capacity,  $WM$ , constitutes an alternative parameter to the maximum value  $MM$ . These are related through the parameter  $B$ . From eqn. (4), by integration, it is easy to show that

$$MM = WM(1 + B)/(1 - IM) \quad (5)$$

The state of the catchment, at any time, is assumed to be represented by a point  $x$  on the curved line of Fig. 2. The area to the right and below the point

$x$  is proportional to the areal mean tension water storage  $W$  (not capacity). This assumption implies that each point in the sub-basin is either at capacity tension (points to the left of  $x$ ) or at a constant tension (points to the right of  $x$ ).

#### RUNOFF GENERATION ON PERVIOUS AREAS

When rainfall exceeds evaporation, the ordinate of Fig. 2 is increased by the excess,  $x$  moves upwards along the curve and runoff is generated proportional to the area shown shaded to the left and above the point  $x$  in Fig. 2.

If  $P - K \times EM + AU$  is less than  $MM$ , then

$$R = P - K \times EM - WM + W + WM \times [1 - (P - K \times EM + AU) / MM]^{1+B} \quad (6)$$

otherwise

$$R = P - K \times EM - WM + W \quad (7)$$

On the other hand, when evaporation exceeds rainfall, the three tension moisture storages are reduced as explained in the previous section and the point  $x$  moves downwards along the curve of Fig. 2 to a level at which the areal mean tension water storage  $W$  (the area to the right and below the point  $x$ ) assumes its appropriate value. It is perhaps worth noting that this implies a redistribution of water within the sub-basin. If, initially, the tension water state of the catchment is represented by the curved line to the left and the horizontal line to the right of  $x$  in Fig. 2, a reduction in tension water storage at all points in the sub-basin would be represented by a constant downward shift of the curved and horizontal lines. Instead however, no reduction is imposed on points to the left and below  $x$  — these points remain at capacity level — and a correspondingly greater reduction is imposed over the remainder of the sub-basin, implying a greater reduction in the position of the horizontal line. This implies a redistribution of soil moisture during the drying period, with water flowing from the more elevated parts of the sub-basin to the lower parts.

#### SEPARATION OF RUNOFF COMPONENTS

The total runoff  $R$ , generated in a wet period in accordance with Fig. 2, must be separated into its three components,  $RS$  surface runoff,  $RG$  the ground water contribution, and  $RI$  a contribution to interflow. To effect this, the concepts of free water storage  $S'$  and free water storage capacity  $S'M$  are used. The latter is assumed to be distributed between zero and a point maximum  $MS$  in a parabolic manner, over  $FR$ , that portion of the sub-basin

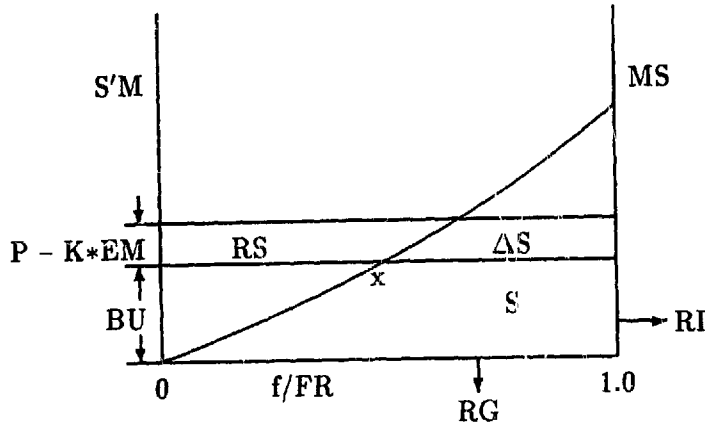


Fig. 3. The separation of runoff components.

which is currently producing runoff (Fig. 3).

$$\left(1 - \frac{f}{FR}\right) = \left(1 - \frac{S'M}{MS}\right)^{Ex} \tag{8}$$

where  $f$  is that portion of the sub-basin area for which the free water storage capacity is less than or equal to  $S'M$  and  $Ex$  is a parameter.

It is also assumed that the current state of free water storage in the sub-basin can be represented by a point (ordinate  $BU$  on the parabola of Fig. 3) implying that the portion of the sub-basin to the left of that point is at capacity storage and to the right the storage is constant, below capacity level.

The areal mean free water storage capacity  $SM$  may be used instead of  $MS$  as a parameter

$$MS = SM(1 + Ex) \tag{9}$$

By integration of  $S'M$  in eqn. (8) and substitution of  $SM$  for  $MS$  from eqn. (9), the equivalent free water storage  $S$  over the runoff producing area  $FR$ , can be found to be

$$1 - \frac{S}{SM} = \left(1 - \frac{BU}{MS}\right)^{1+Ex} \tag{10}$$

The total runoff  $R$ , generated in accordance with Fig. 2, and expressed as the depth  $P - K \times EM$  over the runoff producing area of the sub-basin, is applied by adding  $P - K \times EM$  to  $BU$  in Fig. 3, yielding a contribution  $RS$  to surface runoff.

Algebraically, if  $BU + P - K \times EM < MS$  then

$$RS = (P - K \times EM - SM + S + SM) \times [1 - (P - K \times EM + BU)/MS]^{(1+Ex)} \times FR \tag{11}$$

Otherwise,

$$RS = (P - K \times EM + S - SM) \times FR \quad (12)$$

The remainder of  $R$  becomes an addition,  $\Delta S$ , to the freewater storage  $S$ , which in turn contributes  $RI$  laterally to inflow and  $RG$  vertically to ground water, according to the relations

$$RI = S \times KI \times FR \quad (13)$$

$$RG = S \times KG \times FR \quad (14)$$

where  $KI$  and  $KG$  are parameters.

The surface runoff  $RS$  passes unmodified to the channel system as  $TS$ . The interflow  $RI$  and the ground water  $RG$  are routed through linear reservoirs representing interflow and groundwater storage respectively. Outflows  $TI$  and  $TG$  from these reservoirs, are determined by

$$TI(t) = TI(t-1) \times CI + RI(t) \times (1 - CI) \quad (15)$$

$$TG(t) = TG(t-1) \times CG + RG(t) \times (1 - CG) \quad (16)$$

$TI$  and  $TG$  are added to  $TS$  to become the total sub-basin inflow  $T$  to the channel network.

#### FLOW CONCENTRATION

Within each sub-basin, this is represented by the convolution of  $T$  with an empirical unit hydrograph or by "lag and route" with parameters  $L$  and  $CS$ , to produce  $Q$ , the sub-basin outflow. Flood routing from the sub-basin outlets to the total basin outlet is achieved by applying the Muskingum method to successive sub-reaches (parameters  $KE$  and  $XE$  of Fig. 1).

#### THE PARAMETERS

While all the parameters have clear physical meanings, their determination by measurement in the field is nevertheless impractical and recourse must be had to evaluation by some form of system identification. This is quite a difficult task, made more so by uneven sensitivity in the computed output to changes in the parameter values and dependence among the parameters so that the effect of changing the value of one may be offset by a corresponding change in another. Parameters for changes in which the output is insensitive, can be given fixed values in accordance with experience but dependence results in non-uniqueness and instability in optimized values.

In the model as described, there are 15 parameters for a sub-basin when

using the lag and route method. In addition there are two alternative parameters  $MM$  and  $MS$  related algebraically to some of the fifteen. The 15 parameters may be grouped as follows:

- (1) Evapotranspiration parameters  $K$ ,  $UM$ ,  $LM$ ,  $C$ .
- (2) Runoff production parameters  $WM$ ,  $B$ ,  $IM$ .
- (3) Parameters of runoff separation  $SM$ ,  $Ex$ ,  $KG$ ,  $KI$ .
- (4) Runoff concentration parameters  $CG$ ,  $CS$ ,  $CI$ ,  $L$ .

Generally, the output is more sensitive to the underlined parameters.  $K$  is the ratio of potential evapotranspiration to pan evaporation. The output is particularly sensitive to this parameter which controls the water balance.  $K$  may be looked upon as the product of a number of factors including  $k_1$ , the ratio of pond evaporation to pan evaporation, which can be determined by experiments,  $k_2$ , the ratio of potential evapotranspiration to pond evaporation, usually about 1.3–1.5 in summer and 1.0 in winter,  $k_3$  a coefficient dependent on the elevation of the pan, which converts the measured pan value to the basin mean value. In practice,  $k_1 \times k_2 = 1$ .

$WM$ , the areal mean tension water capacity, is the sum of  $UM$  in the upper layer,  $LM$  in the lower layer and  $DM$  in the deepest layer.  $DM$  is therefore completely dependent on the other three and need not be considered for optimization.  $WM$  is a measure of aridity, which varies from 80 mm in South China to 170 mm in North China. The model operation is generally insensitive to  $WM$ , provided its value is large enough to ensure that the computed areal mean soil moisture content  $W$  does not become negative. Parameters  $UM$  and  $LM$  are determined by experience. Typical values for  $UM$  are from 5 to 20 mm, for deforested to forested areas.  $LM$ , typically 60 to 90 mm, is taken as the range within which it is assumed that evapotranspiration is proportional to soil moisture content.

$B$ , the exponent of the tension water capacity curve, defines the non-uniformity of the surface conditions. Experience indicates that for basins of area less than 10 km<sup>2</sup>,  $B = 0.1$  and for basins measured in thousands of square kilometers,  $B = 0.4$ .

$IM$  is the ratio of the impervious to the total area of the basin. For natural basins in humid regions this is usually negligible, but in semi-humid or more arid regions the impervious area may be a large proportion of the runoff producing area of the basin.

$C$ , the coefficient of deep evapotranspiration, depends on the proportion of the basin area covered by vegetation with deep roots. It varies from 0.18 in South China to 0.08 in North China. The runoff is insensitive to  $C$  in humid areas and humid seasons, but quite sensitive in arid areas and dry seasons. The appropriate value of  $C$  is often dependent upon the sum  $UM + LM$  but as this



is usually kept to a fixed value of approximately 100 mm, the appropriate value of  $C$  can usually be determined.

$SM$  the areal mean of the free water capacity of the surface soil layer, represents the maximum possible deficit of free water storage. Surface runoff is sensitive to the value of this parameter. For thin soils,  $SM$  may be approximately 10 mm, increasing to 50 mm for thick and porous surface soils.

$Ex$ , the exponent of the freewater capacity curve influences the development of the saturated area.  $Ex$  is dependent on  $SM$  but statistical analysis shows that the value of  $Ex$  lies in a small range from 0.5 to 2.0, the best selection being between 1 and 1.5. It may be taken as a constant.

$KG$  and  $KI$  are the outflow coefficients of the freewater storage to groundwater and interflow relationships. Their sum determines the flow rate from freewater storage and their ratio determines the proportion going to interflow and groundwater flow, respectively. In Fig. 3, if the sum  $KG + KI$  is increased and  $SM$  is decreased while keeping the ratio  $KG/KI$  constant, the distribution of the runoff among the three components may be unchanged. Insofar as the operation of the model is sensitive to these parameters, this dependence is undesirable. It may be avoided by fixing the sum  $KG + KI$ . As the recession duration of the upper interflow storage ordinarily lies between 2 and 3 days, we may take  $KG + KI = 0.7-0.8$ .

$CG$ , the recession constant of groundwater storage, usually takes a value between 0.99 and 0.998.  $CI$ , the recession constant of the lower interflow storage, usually takes values between 0.5 and 0.9 in humid regions.  $CS$ , the recession constant in the "lag and route" method for routing through the channel system within each sub-basin, is purely empirical. While this method of routing is not theoretically strong, it is easier to apply than the unit hydrograph.  $L$  is the corresponding "lag", also of empirical value.  $KE$  and  $XE$  are parameters of the Muskingum method which can be determined by hydraulic formulas.

## OPTIMIZATION

The above classification of the parameters in four groups, corresponds to different characteristics in optimization. Those of the first group are most stable and of the fourth group least so. Parameters within the same group tend to be mutually dependent, while those in different groups appear to be relatively independent. The parameter values in the higher groups have little effect on the optimized values in the lower groups. For example, variation of the parameter values of runoff concentration within a reasonable range, will not influence the optimum value of the parameter for runoff separation, and variation of the parameter value of runoff separation will not greatly influence

the optimized value of the parameters of runoff production and evapotranspiration. Therefore, parameters of the lower numbered groups are optimized before those of the higher numbered groups and different objective functions are used for the different groups. For the four groups three objective functions are used:

(1) The water balance error, being the difference between the total computed and the total measured flow.

$$\Delta R = \sum Q - \sum M \quad (17)$$

where  $\sum Q$  is the total computed and  $\sum M$  the total measured flow in the period of calibration.

(2) The relative absolute error, being the sum of the absolute errors of prediction normalized by division by the sum of the measured discharges

$$ABS = \sum \text{abs}(M - Q) / \sum M \quad (18)$$

(3) The absolute logarithmic error, defined by

$$ABS \text{ LOG} = \frac{1}{n} \sum \log \left[ \text{abs} \frac{M - Q}{M} + 1 \right] \quad (19)$$

where  $n$  is the number of ordinates.

The optimization proceeds as follows:

(1) Assume initial values for all the parameters using reasonable values as far as possible.

(2) Optimize  $K$  alone, using daily data and eqn. (17). Normally at least 4 years of data are necessary for calibration. If in the course of running the model, it is observed that  $W$  becomes negative  $WM$  should be increased accordingly. Usually  $UM$  and  $LM$  may be chosen by experience and  $C$  may be adjusted after optimization of  $K$ .

(3) Taking a fixed value (0.7 or 0.8) for the sum of  $KG$  and  $KI$  and fixing  $Ex$  at 1.5, the remaining parameters of the third group  $KG/KI$  and  $SM$  are optimized by a two-dimensional search using eqn. (19). Subsequently, the values of  $CG$  and  $CI$  may be adjusted.

(4) Applying the model to some flood events only, and using very short time intervals with an adjusted value of  $SM$ ,  $L$  and  $CS$  are optimized using eqn. (18). The adjustment to the previously optimized value of  $SM$  is necessary because, due to the unevenness of the distribution of rainfall during a 1-day period,  $SM$  for daily values is always less than the value appropriate to shorter time intervals.

## APPLICATIONS

The efficiency of the Xinanjiang model has been established by long use in China. The model has been applied successfully over a very large area including all of the agricultural, pastoral and forested lands of China except the loess. On an arid watershed, Bird Creek in the United States, the model has performed well as reported from the US National Weather Service River Forecast System offices. On the loess area of China, another model, based on the concept of runoff formation in excess of infiltration would be necessary. Such a model would generate surface runoff only without interflow or ground water.

In China, the Xinanjiang model is used mainly for hydrological forecasting. Many kinds of software have been developed. Many large projects such as Gezhouba, Panjakou, Danjiangkou, Lubuge, Longyangxia and Ertan have involved the use of this model. Many forecasting systems such as Sanmenxia to Huayuenkou on the Yellow River, the middle reach of the Hui River and the Yangtze Gorges have used the model, sometimes with adjustment in real time.

Use of the model has also spread to other fields of application such as water resources estimation, design flood and field drainage, water project programming, hydrological station planning, water quality accounting, etc. The model is being extended and developed to meet with miscellaneous surface conditions such as snow cover, karst, large plains, swamps, etc.

## CONCLUSIONS

Our most important task now is to strengthen the scientific basis of the technique, to prove the model structure, and to find relations between parameter values and natural conditions. Any conceptual model may be considered as a scientific hypothesis to be subjected to scientific examination. High accuracy alone is insufficient for many purposes. The final criterion must be whether the model structure and parameter values accord with the hydrological reality and the natural conditions. By the application of this criterion, the scientific status of modelling will be raised and scientific hydrology stimulated.

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