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Thermal Dispersion–Radiation Effects on Non-Darcy Natural Convection in a Fluid Saturated Porous Medium

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Abstract. The effects of thermal dispersion and thermal radiation on the non-Darcy natural convection over a vertical flat plate in a fluid saturated porous medium are studied. Forchheimer extension is considered in the flow equations. The coefficient of thermal diffusivity has been assumed to be the sum of molecular diffusivity and the dispersion thermal diffusivity due to mechanical dispersion. Rosseland approximation is used to describe the radiative heat flux in the energy equation. Similarity solution for the transformed governing equations is obtained. Numerical results for the details of the velocity and temperature profiles which are shown on graphs have been presented. The combined effect of thermal dispersion and thermal radiation, for the two cases Darcy and non-Darcy porous medium, on the heat transfer rate which are entered in tables is discussed.

Key words: radiation, thermal dispersion, natural convection.

Nomenclature

A	constant.
C	empirical constant.
C_T	temperature difference.
d	pore diameter.
f	nondimensional stream function.
g	gravitational constant.
K	permeability of the porous medium.
k_d	dispersion thermal conductivity.
k_e	effective thermal conductivity.
Nu_x	local Nusselt number.
p	pressure.
q	local heat flux.
q^r	radiative heat flux.
R	radiation parameter.
Ra_x	Rayleigh number.
T	temperature.
u, v	velocity components in the x and y directions.
x, y	cartesian coordinates.
α	molecular thermal diffusivity.
α_d	dispersion diffusivity.

α_x, α_y thermal diffusion coefficients in x and y directions, respectively.
β thermal expansion coefficient.
γ mechanical dispersion coefficient.
δ_T boundary layer thickness evaluated at $\theta(\eta) = 0.001$.
η similarity parameter.
ψ dimensional stream function.
θ nondimensional temperature.
ρ fluid density.
μ viscosity.
ν fluid kinematic viscosity.
σ Stefan–Boltzman constant.
χ the mean absorption coefficient.

Subscripts
w evaluated on the wall.
∞ evaluated at the outer edge of the boundary layer.

1. Introduction

Recently, the study of convection boundary layer flow in porous media has received considerable interest, because of its wide applicability in energy, such as geothermal energy technology, petroleum recovery, filtration processes, packed bed reactors and underground disposal of chemical and nuclear waste. Cheng [1] presented a comprehensive review about heat transfer in geothermal systems. Plumb and Huenefeld [2] and Nakayman *et al.* [3] used the Forchheimer extension to study the non-Darcy natural convection from the vertical wall. Study of the thermal dispersion effects becoms, prevalent in the porous media flow region. Fried and Combarnous [4] proposed a linear function to express the thermal dispersion. Also, a linear dispersion model taking the porosity of the porous medium into account is used for free convection in a horizontal layer heated from below was introduced by Georgiadis and Catton [5]. Cheng [6] and Plumb [7] gave another model for flow and heat transfer in porous media by taking thermal dispersion effects into consideration. An analysis of thermal dispersion effect on vertical plate natural convection in porous media is presented by Hong and Tien [8]. Lai and Kulacki [9] investigated thermal dispersion effect on non-Darcy convection from horizontal surface in saturated porous media. Effects of thermal dispersion and lateral mass flux on non-Darcy natural convection over a vertical flat plate in a fluid saturated porous medium was studied by Murthy and Singh [10].

At high temperatures thermal radiation can significantly affect the heat transfer and the temperature distribution in the boundary layer flow of participating fluid. Gorla [11], and Gorla and Pop [12] has investigated the effects of radiation on mixed convection flow over vertical cylinders. Ali *et al.* [13] investigated natural convection–radiation interaction in boundary layer flow over a horizontal surface. Ibrahiem and Hady [14] studied mixed convection–radiation interaction in boundary layer flow over a horizontal surface. Forced convection–radiation interaction heat transfer in boundary-layer over a flat plate submersed in a porous medium was analyzed by Mansour [15]. The present investigation is devoted to study the combined effect of radiation and thermal dispersion on Forchheimer natural convection over a vertical flat plate in a fluid saturated porous medium. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The wall temperature distribution is assumed to be uniform.

2. Analysis

Consider the non-Darcy natural convection–radiation flow and heat transfer over a semi infinite vertical surface in a fluid saturated porous medium. The governing equations for this problem are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u + \frac{C\sqrt{K}}{\nu} uq = -\frac{K}{\mu}\left(\frac{\partial p}{\partial x} + \rho g\right), \tag{2}$$

$$v + \frac{C\sqrt{K}}{\nu} vq = -\frac{K}{\mu}\left(\frac{\partial p}{\partial y}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial x}\left(\alpha_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\alpha_y \frac{\partial T}{\partial y}\right) - \frac{1}{(\rho_\infty C_\mathrm{p})_\mathrm{f}}\frac{\partial q^\mathrm{r}}{\partial y}, \tag{4}$$

$$\rho = \rho_\infty[1 - \beta(T - T_\infty)], \tag{5}$$

where $q^2 = u^2 + v^2$ along with the boundary conditions

$$\begin{aligned} y = 0: &\qquad v = 0, \quad T_\mathrm{w} = \text{const.}, \\ y \to \infty: &\qquad u = 0, \quad T \to T_\infty, \end{aligned} \tag{6}$$

where u and v are the velocity components in the x and y directions, respectively, $(\rho_\infty C_\mathrm{p})_\mathrm{f}$ is the product of density and specific heat of the fluid, k_e is the effective thermal conductivity of the saturated porous medium. p is the pressure, T is the temperature, K is the permeability constant, C is an empirical constant, β is the thermal expansion coefficient, μ is the viscosity of the fluid, ρ is the density, and g is the acceleration due to gravity, α_x, α_y are the components of the thermal diffusivity in x and y directions, respectively. The quantity q^r on the right-hand side of Equation (4) represents the radiative heat flux in the y direction. The radiative heat flux term is simplified by the Rosseland approximation (cf. Sparrow and Cess [16]) and is as follows:

$$q^\mathrm{r} = -\frac{4\sigma}{3\chi}\frac{\partial T^4}{\partial y}, \tag{7}$$

where σ and χ are the Stefan–Boltzmann constant and the mean absorption coefficient.

The radiative heat flux in the x direction is considered negligible in comparison with that in the y direction (Sparrow and Cess [16]). The normal component of the velocity near the boundary is small compared with the other component of the velocity and the derivatives of any quantity in the normal direction are large compared with derivatives of the quantity in the direction of the wall. Under these assumptions, the Equations (1)–(5) become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{8}$$

$$u + \frac{C\sqrt{K}}{\nu} u|u| = -\frac{K}{\mu}\left(\frac{\partial p}{\partial x} + \rho g\right), \tag{9}$$

$$\frac{\partial p}{\partial y} = 0, \tag{10}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha_y \frac{\partial T}{\partial y}\right) - \frac{1}{(\rho_\infty C_\mathrm{p})_\mathrm{f}}\frac{\partial q^\mathrm{r}}{\partial y}. \tag{11}$$

The quantity α_y is variable and is defined as the sum of molecular thermal diffusivity α and dispersion thermal diffusivity α_d. Plumb [7] has been expressed for the dispersion thermal diffusivity as $\alpha_\mathrm{d} = \gamma|u|d$, where γ is the mechanical dispersion coefficient whose value depends on the experiments and d is the pore diameter.

Having invoked the Boussinesq approximations, with substituting Equation (5) into Equations (9) and (10), eliminating the pressure and the velocity components u and v can be written in terms of stream function ψ as: $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, we obtain

$$\frac{\partial^2\psi}{\partial y^2} + \frac{C\sqrt{K}}{\nu}\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right)^2 = \frac{Kg\beta}{\mu}\frac{\partial T}{\partial y}, \tag{12}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left[(\alpha + \alpha_\mathrm{d})\frac{\partial T}{\partial y}\right] - \frac{1}{(\rho_\infty C_\mathrm{p})_\mathrm{f}}\frac{\partial q^\mathrm{r}}{\partial y}. \tag{13}$$

Introduce the similarity variable and similarity profiles:

$$\eta = \mathrm{Ra}_x^{1/2}\frac{y}{x}, \quad f(\eta) = \frac{\psi}{\alpha \mathrm{Ra}_x^{1/2}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_\mathrm{w} - T_\infty}, \tag{14}$$

where Ra_x is the modified Rayleigh number, $\mathrm{Ra}_x = Kg\beta(T_\mathrm{w} - T_\infty)x/\alpha\nu$.

The problem statement then becomes

$$f'' + 2F_0\mathrm{Ra}_\mathrm{d}\, f'f'' - \theta' = 0, \tag{15}$$

$$\theta'' + \tfrac{1}{2} f\theta' + \gamma \mathrm{Ra_d}(f'\theta')' + \tfrac{4}{3} R[(C_T + \theta)^3\theta']' = 0. \tag{16}$$

The boundary conditions become

$$f(0) = 0, \quad \theta(0) = 1, \quad f'(\infty) = \theta(\infty) = 0 \tag{17}$$

where the parameter $F_0 = C\sqrt{K}\alpha/\upsilon d$ represents the structural and thermophysical properties of the porous medium, the radiation parameter is defined by $R = 4\sigma(T_\mathrm{w} - T_\infty)^3/\chi k_\mathrm{e}$, $C_T = T_\infty/(T_\mathrm{w} - T_\infty)$ is the temperature difference, and $\mathrm{Ra_d} = Kg\beta(T_\mathrm{w} - T_\infty)d/\alpha\nu$ is the pore diameter dependent Rayleigh number which describes the relative intensity of the buoyancy force, such that d is the pore diameter.

It is noteworthy that $F_0 = 0$ corresponds to the Darcian free convection and $\gamma = 0$ represents the case where the thermal dispersion effect is neglected. Also, $R = 0$ corresponds the case where the thermal radiation effect is neglected.

From the definition of the stream function, the velocity components become

$$u = \frac{\alpha}{x}\mathrm{Ra}_x\, f', \quad v = -\frac{\alpha}{2x}\mathrm{Ra}_x^{1/2}(f - \eta f').$$

The local heat transfer rate which is the primary interest of the study is given by

$$q_\mathrm{w} = -k_\mathrm{e}\frac{\partial T}{\partial y}\bigg|_{y=0} - \frac{4\sigma}{3\chi}\frac{\partial T^4}{\partial y}\bigg|_{y=0} = -(k + k_\mathrm{d})\frac{\partial T}{\partial y}\bigg|_{y=0} - \frac{4\sigma}{3\chi}\frac{\partial T^4}{\partial y}\bigg|_{y=0}, \tag{18}$$

where k_e is the effective thermal conductivity of the porous medium which is the sum of the molecular thermal conductivity k and the dispersion thermal conductivity k_d. Together with the definition of the local Nusselt number,

$$\mathrm{Nu}_x = \left(\frac{q_\mathrm{w}}{T_\mathrm{w} - T_\infty}\right)\frac{x}{k_\mathrm{e}}, \tag{19}$$

one can write

$$\mathrm{Nu}_x\,\mathrm{Ra}_x^{-1/2} = -[1 + Ds\, f'(0) + \tfrac{4}{3}R(\theta(0) + C_T)^3]\theta'(0) \tag{20}$$

where $Ds = \gamma \mathrm{Ra_d}$ is the dispersion parameter.

3. Results and Discussion

In order to get the physical insight, the system of ordinary differential equations (15)–(16) along with the boundary conditions (17), are integrated numerically by means of the fourth-order Runge–Kutta method with shooting technique. The step size $\Delta\eta = 0.05$ is used while obtaining the numerical solution with $\eta_\mathrm{max} = 10$ and five-decimal accuracy as the criterion for convergence. Numerical computations are carried out for $0 \leqslant F_0 \leqslant 0.5$ and $0 \leqslant R \leqslant 7$, $0 \leqslant C_T \leqslant 1$, $1 \leqslant \mathrm{Ra_d} \leqslant 7$ and $\gamma = 0.0, 0.3$.

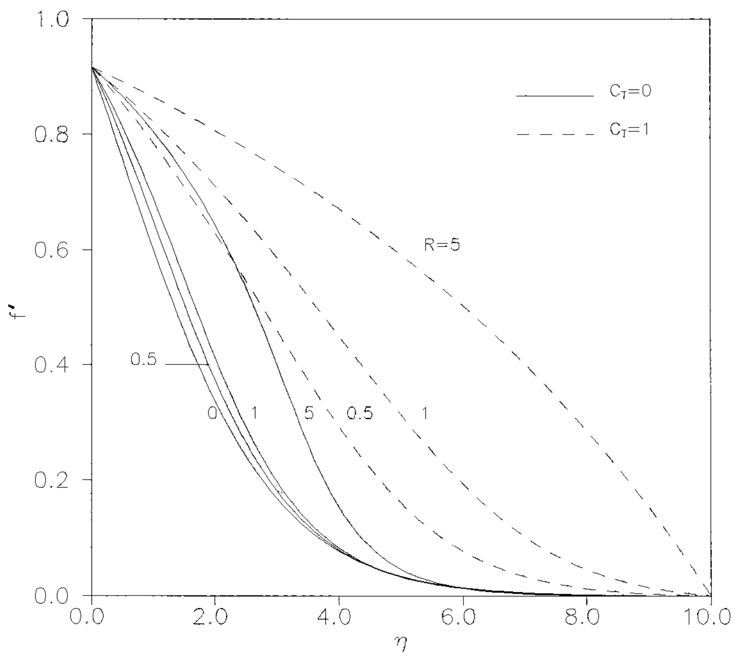

Figure 1. Variation of velocity profiles with similarity variable η, for varying R and C_T, at $\mathrm{Ra_d} = 1.0$, $F_0 = 0.1$ and $\gamma = 0.3$.

Figures 1 and 2 illustrate the velocity and temperature fields, respectively, for different values of R and C_T with fixed values of F_0, $\mathrm{Ra_d}$ and γ. It can be seen from Figures 1 and 2 that the velocity and temperature profiles thicken as the radiation parameter and the temperature difference increase. Figures 3 and 4 show the velocity and temperature profiles, respectively, for different values of F_0 and γ with fixed values of $\mathrm{Ra_d}$, R, and C_T. It can be seen from Figures 3 and 4 that both velocity and temperature profiles thicken as the mechanical dispersion coefficient increases. Also, it is noteworthy that an increase in the parameter F_0 reduces the velocity profiles while it enhances the temperature profiles. Figure 5 shows the velocity profiles for varying $\mathrm{Ra_d}$ and F_0, with fixed γ, R and C_T. From Figure 5 we observe that, for the Darcy case ($F_0 = 0$), the velocity profiles thicken as the parameter $\mathrm{Ra_d}$ increases. It is interesting to note that, for the non-Darcy case ($F_0 > 0$), the velocity profiles thin near the wall, while it thicken far from the wall as the parameter $\mathrm{Ra_d}$ increases. Also, we observe from Figure 6 that due to an increase in $\mathrm{Ra_d}$ there is a thickening in the temperature profiles.

The value of the similarity variable at which $\theta(\eta)$ becomes equal to 0.001 is noted as the boundary layer thickness (Murthy and Singh [10]). Variation of the boundary layer thickness δ_T as a function of the radiation parameter R in the non-Darcy ($F_0 = 0.1$, $\mathrm{Ra_d} = 1.0$) porous medium is plotted in Figure 7 for varying values of the temperature difference C_T and the dispersion parameter γ.

Numerical results of the Nusselt number for varying values of R, C_T, γ and F_0 and with $\mathrm{Ra_d} = 1$ are presented in Table I. It is obvious that, an increase in

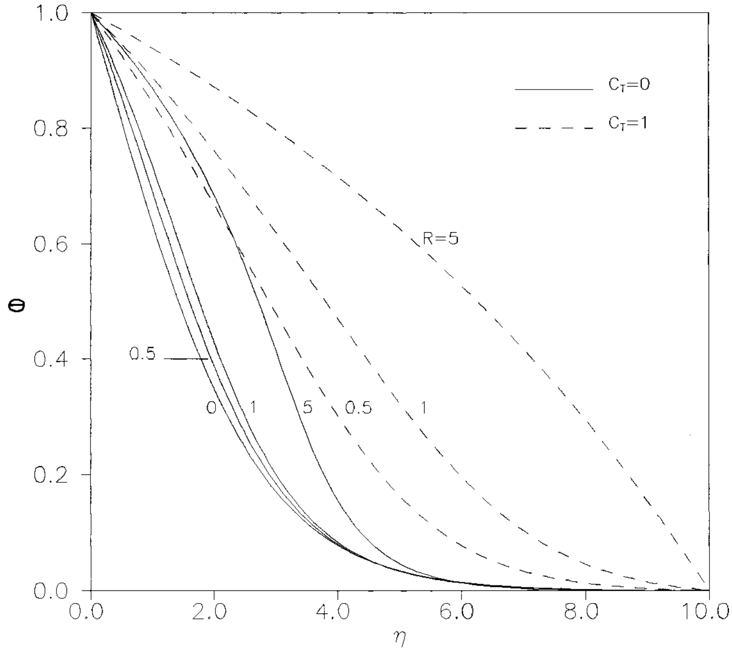

Figure 2. Variation of temperature profiles with similarity variable η, for varying R and C_T, at $\mathrm{Ra_d} = 1.0$, $F_0 = 0.1$ and $\gamma = 0.3$.

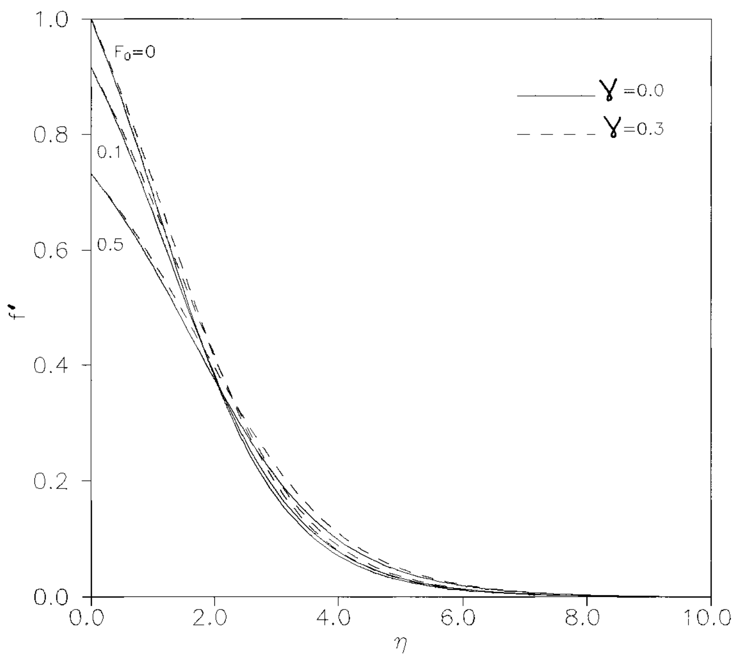

Figure 3. Variation of velocity profiles with similarity variable η, for varying F_0 and γ at $\mathrm{Ra_d} = 1.0$, $R = 5.0$, and $C_T = 0.2$.

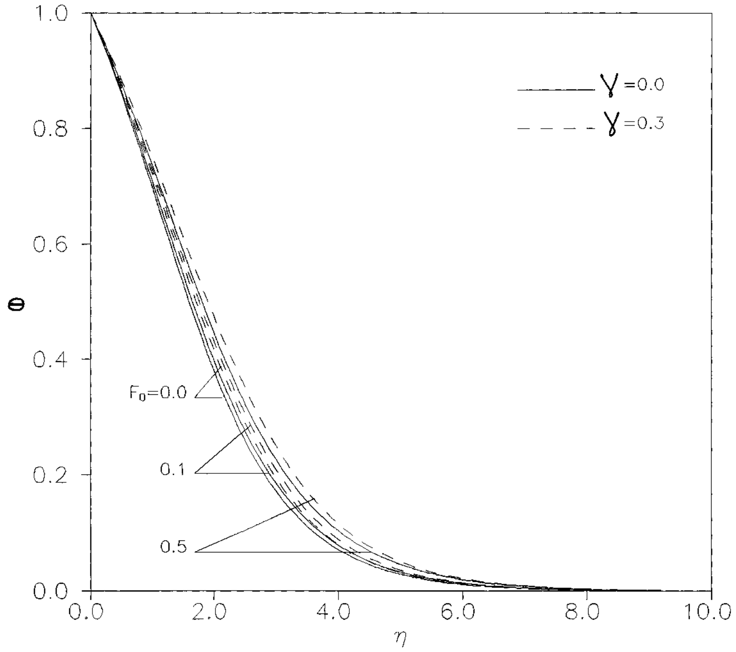

Figure 4. Variation of temperature profiles with similarity variable η, for varying F_0 and γ at $\mathrm{Ra_d} = 1.0$, $R = 5.0$, and $C_T = 0.2$.

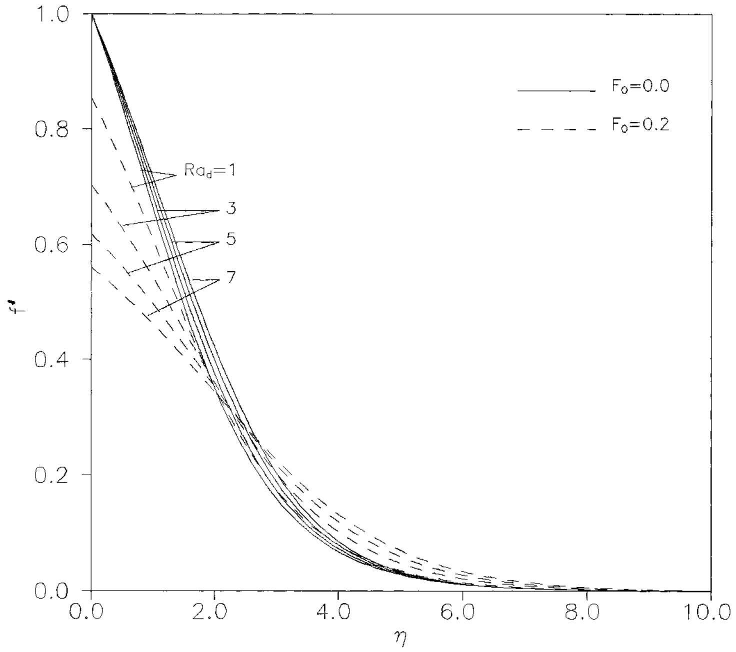

Figure 5. Variation of velocity profiles with similarity variable η, for varying $\mathrm{Ra_d}$ and F_0, at $\gamma = 0.1$, $R = 0.5$ and $C_T = 0.01$.

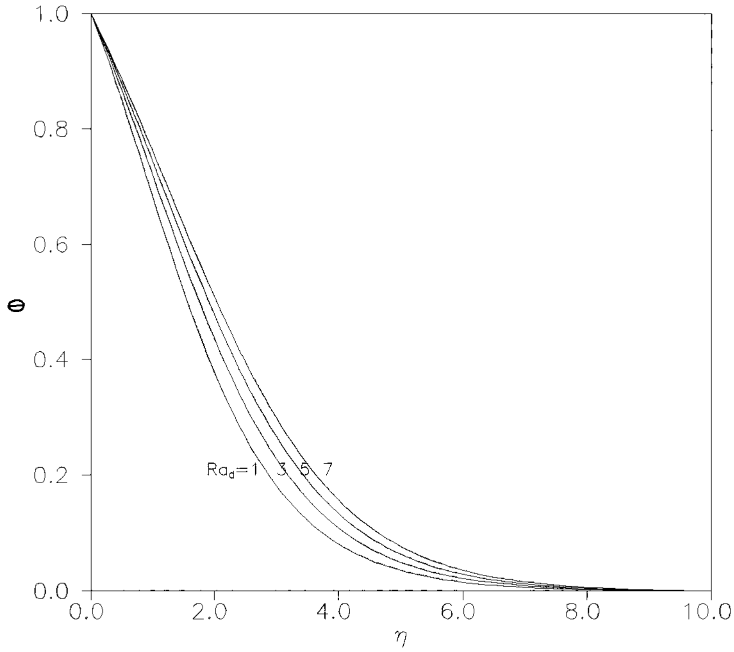

Figure 6. Variation of temperature profiles with similarity variable η, for varying $\mathrm{Ra_d}$, at $\gamma = 0.1$, $R = 0.5$, $F_0 = 0.2$ and $C_T = 0.01$.

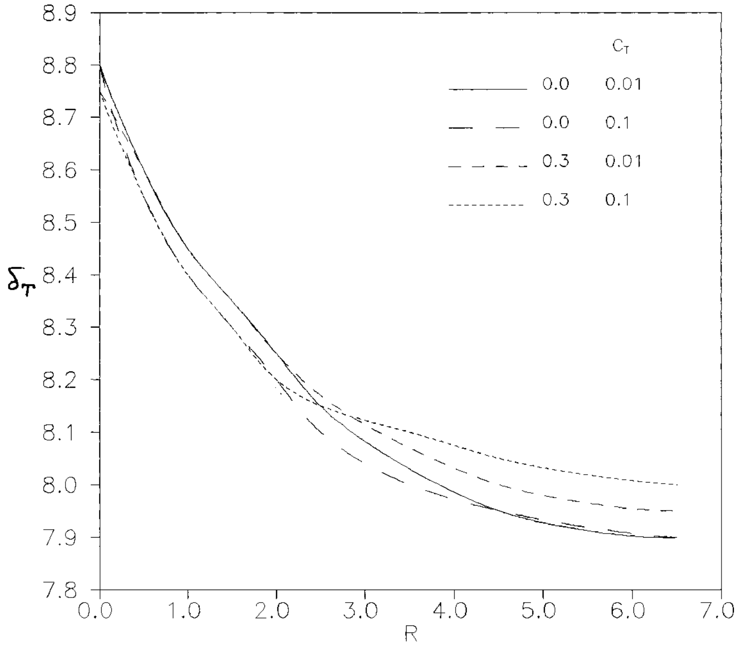

Figure 7. Variation of boundary layer thickness δ_T as a function of radiation parameter R in the non-Darcy ($F_0 = 0.1, \mathrm{Ra_d} = 1.0$) porous medium for varying values of temperature difference and dispersion parameter.

Table I. Values of $\mathrm{Nu}_x/\mathrm{Ra}_x^{1/2}$ for selected values of γ, R, C_T and F_0 with $\mathrm{Ra_d} = 1$

		$\gamma = 0.0$			$\gamma = 0.3$		
R	C_T	$F_0 = 0.0$	$F_0 = 0.1$	$F_0 = 0.5$	$F_0 = 0.0$	$F_0 = 0.1$	$F_0 = 0.5$
0.0	–	0.44390	0.42969	0.39314	0.48983	0.47107	0.42442
0.5	0.0	0.50931	0.49245	0.44947	0.54961	0.52877	0.47693
	0.2	0.56269	0.54384	0.49589	0.59936	0.57689	0.52089
	1.0	0.94906	0.91563	0.83643	0.97130	0.93715	0.85166
1.0	0.0	0.56669	0.54754	0.49898	0.60309	0.58037	0.52381
	0.2	0.65961	0.63704	0.57994	0.69112	0.66545	0.60143
	1.0	1.27141	1.22972	1.12617	1.28850	1.24524	1.13831
5.0	0.0	0.89918	0.86714	0.78686	0.92259	0.88828	0.80290
	2.0	1.17293	1.13120	1.02654	1.19097	1.14748	1.03891
	1.0	3.25379	3.20437	3.10475	3.26825	3.21802	3.17684
7.0	0.0	1.02515	0.98819	0.89618	1.04579	1.00695	0.91032
	0.2	1.35838	1.30982	1.28175	1.37400	1.32393	1.30042
	1.0	4.24058	4.19233	4.14758	4.25523	4.20616	4.16549

Table II. Values of $\mathrm{Nu}_x/\mathrm{Ra}_x^{1/2}$ for selected values of $\mathrm{Ra_d}$, γ and F_0 with $R = 0.5$ and $C_T = 0.01$

	$\gamma = 0.0$		$\gamma = 0.1$	
$\mathrm{Ra_d}$	$F_0 = 0.0$	$F_0 = 0.2$	$F_0 = 0.0$	$F_0 = 0.2$
1	0.51154	0.48102	0.52526	0.49234
3	0.51154	0.44380	0.55167	0.46977
5	0.51154	0.41978	0.57686	0.45574
7	0.51154	0.39881	0.60099	0.44570

the values of the parameters R, C_T and γ enhances the heat transfer rate. Also, it is clear that, the values of the Nusselt number is reduced as the parameter F_0 increases. In Table II, the Nusselt number results are presented for varying $\mathrm{Ra_d}$, for Darcy and non-Darcy cases, and considering or neglecting the thermal dispersion effects, with fixing R and C_T. It is clear that, with neglecting the thermal dispersion effect ($\gamma = 0$), there is no variation in the heat transfer rate with varying $\mathrm{Ra_d}$ for the Darcy case ($F_0 = 0$), while an increase in $\mathrm{Ra_d}$ enhances it for the non-Darcy case ($F_0 > 0$). Also, from the same table we note that, for the non-Darcy case ($F_0 > 0$), the increase in the value of $\mathrm{Ra_d}$ reduces the heat transfer rate.

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