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# Three-phase foam equation of state

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**Abstract**—A three-phase foam equation of state has been developed to complete a previous equation defined for aqueous foams by other authors. An original compression system has been used to define a pressure-specific volume relationship taking into account the presence of a co-volume in the three-phase foams. The results showed that three-phase foams are governed by an equation depending on pressure, temperature, foam specific volume, foam specific incompressible volume and moles of gas. This relation, $P(V_s - V_{si}) = N_g RT$, will be, in addition way, a function of the solid content and the expansion ratio of the foam. Also, it has been found that the theoretical model of the three-phase foam equation coincides well with the experimental results, with an error less than 5%. © 1998 Published by Elsevier Science Ltd

*Keywords*: Foams; suspensions; equation of state; modelization; compressibility.

## INTRODUCTION

Due to the fact that they could be desirable or not in some industrial processes, there is a particular interest in the study of foams, loaded or not (Bikerman, 1973; Antonini *et al.* 1985, 1994, 1995; Kundu and Peterson, 1986; Wilson, 1989; Antonini and Perotin, 1994; Hazi *et al.* 1995; Mbama Gaporaud *et al.* 1997). Although, the studies of the equation of state of foams have almost concerned aqueous foams, the few reports on three-phase foams present results with different factors which cannot be obtained easily.

The equation of state of foams defined by previous authors (Derjaguin, 1953; Ross, 1969; Morrison and Ross, 1983) joins six state variables (volume, pressure, moles of gas, temperature, surface area of liquid and surface tension). In the case of three-phase foams, some of these variables seem to be difficult in application. Furthermore, this equation is, at best, adapted to foams whose volume fraction of the gas, $\Phi$, is close to the unity. Princen (1988) proposed an equation taking into account the volume fraction of the gas, but as in Morrison and Ross, it is not easy to use this equation for three-phase foams.

Three-phase foams consist of a pulverized solid component, air and a liquid fraction. The latter is mainly water, with addition of an organic surfactant additive. Considering the above remarks, an original compression system has been developed in order to define an equation of state taking into account the presence of a co-volume due to the liquid and solid contents. Some results are reported and discussed, especially the influence of the solid content on different parameters of the equation of state. Also, a mathematical model of the three-phase foam equation of state is defined and a comparison between the theoretical and experimental results are presented.

## DESCRIPTION OF THE PLANT AND EXPERIMENTAL PROCEDURE

The experimental facility used in this work consists of a three-phase foam preparation and formulation device and a compression system (Fig. 1).

The experimental preparation of three-phase foams is made in a 5 l steel batch vessel. First, a small quantity of water is introduced into the batch vessel, with a chemical foaming agent to a concentration of about 2%, which corresponds to the critical micellar concentration (CMC) of this surfactant in water. An aqueous foam (that is to say, from water and the foaming agent) is then generated by vigorous shaking. The shaking is maintained for about 5 min in order to improve the aqueous foam quality. Next, the shaking speed is slowed down and the solid is gradually incorporated into the aqueous foam, until the desired concentration is achieved. An air admission and a pressure indicator allows pressurization of the foam up to 300 kPa.

The foam compression facility is an original device made up of a hydraulic screw jack sliding inside a corrected cylinder, the whole being fixed to a metallic frame. The diameter of the cylinder is 0.04 m and its length is 0.6 m. Toric joints ensure the tightness of the system up to pressures higher than 1600 kPa. A pressure sensor fixed at the end of the screw jack and a piston displacement sensor fixed at the rod allow to measure the available volume in the cylinder and the inside pressure. A frequency converter provided with a stroke inverter allows to control the piston-displacement speed during compression or expansion of the foam.

Many solids were tested (Colombian coal, French coal, graphite, resins, etc.), but, only the compression

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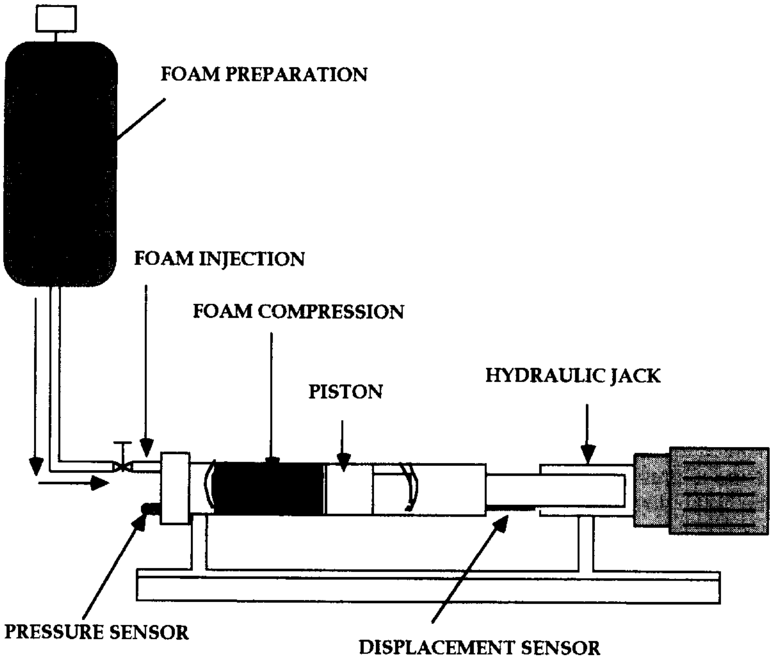

Fig. 1. Sketch of the three-phase foam preparation and compression unit.

of the Colombian coal is discussed in this paper. The compression of the other solids gave results similar to that of this solid (Mbama Gaporaud, 1994). A 0–0.5 mm size fraction was used as the charge of three-phase foam preparation.

When a three-phase foam is prepared as indicated above, the final solid content is first verified by determining the moisture content of the mixture. The stirred steel vessel is then pressurized to about 200 kPa. During this operation, mixing is maintained continuously. The residual air in the cylinder of the compression system is evacuated before feeding three-phase foam in the system by placing the piston at the end of the stroke cylinder. The cylinder is fed by connecting the compression recorder to the batch vessel. When the pressure inside the compression cylinder reaches that of the foam in the vessel, the piston stroke is inverted and the cylinder is filled very slowly as in a syringe. When the piston reaches the end of the cylinder, the compression system is closed by using a valve and the system is then ready to run. For each pressure in the compression system, the feed motion of the piston is recorded and the volume occupied by the foam is obtained by deduction.

There was no temperature rise in the cylinder of compression recorder during the determination of pressure–volume curves. So the test results can be considered as isotherms. The operating temperature, $T$, was 293 K (at the operating temperature of 20°C, the error margin is 3°C).

## RESULTS AND DISCUSSION

The pressure–volume curve of an aqueous foam is presented in Fig. 2. Next, several pressure–volume curves have been established for different coal contents in the foam. The results are also presented in Fig. 2. In order to compare the different curves, the results are plotted by relating the foam pressure vs the foam specific volume. For each solid content, an equation of state is written by using the procedure described below.

*Determination of the three-phase foam equation of state*

Many scientists, such as Derjaguin and Ross, derived an equation of state of foams taking the following shape:

$$PV + \tfrac{2}{3}\sigma A = n_g RT. \tag{1}$$

We assumed that in the case of three-phase foams, Equation (1) becomes

$$P(V_s - V_{si}) + \tfrac{2}{3}\sigma A = n_g RT. \tag{2}$$

If $X_g$, $X_l$, and $X_s$ are the gas, liquid and solid contents in the foam, respectively, it becomes

$$P\left[\frac{1}{\rho_f} - \left(\frac{X_s}{\rho_s} + \frac{X_l}{\rho_l}\right)\right] = X_g\frac{RT}{M} - \tfrac{2}{3}\sigma a. \tag{3}$$

The factors $a$ and $\sigma$ are difficult to determine in the case of three-phase foams because the interfaces are not easily delimited, due to their interaction with the surfaces of the solid particles. So, by taking into account the results presented in Fig. 2, we have postulated a law having the following form:

$$P(V_s - V_{si}) = N_g RT. \tag{4}$$

The number of moles of gas in the foam and the temperature being constant, Eq. (4) can be written as

$$P(V_s - V_{si}) = K. \tag{5}$$

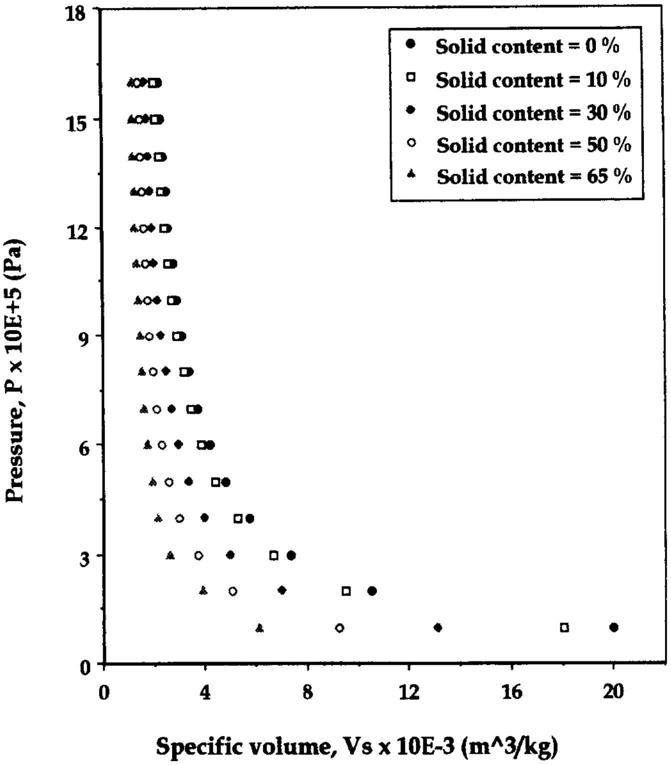

Fig. 2. Pressure–volume diagram of three-phase foams. Influence of the solid content in the foam.

Then

$$V_s = \frac{K}{P} + V_{si}. \tag{6}$$

By plotting $V_s$ vs $1/P$, it is possible to get a straight line whose slope is $K$, and the zero $y$-ordinate $V_{si}$. Thus, $V_{si}$ and $K$ can be directly determined by using experimental results.

*Modelization of the three-phase foam equation of state*

According to the foam composition, it is possible to determine $V_{si}$ and $K$. Given $m$ kg of a foam having a solid content $X_s$ and a liquid content $X_l$, the solid specific weight being $\rho_s$ and that of the liquid $\rho_l$, these incompressible products actually occupy a volume

$$V_i = \frac{mX_s}{\rho_s} + \frac{mX_l}{\rho_l} \quad \text{in m}^3. \tag{7}$$

By translating $V_i$ into a specific volume, it becomes

$$V_{si} = \frac{X_s}{\rho_s} + \frac{X_l}{\rho_l} \quad \text{in m}^3\text{/kg}. \tag{8}$$

The factor $K$ can be determine from normal conditions:

$$P(V_s - V_{si}) = P_0(V_{s0} - V_{si}) = K \tag{9}$$

$$K = P_0(V_{s0} - V_{si}) = P_0\left(V_{s0} - \frac{X_s}{\rho_s} - \frac{X_l}{\rho_l}\right) \quad (\text{Pa m}^3\text{/kg}) \tag{10}$$

$$P\left(\frac{1}{\rho_f} - \frac{X_l}{\rho_l} - \frac{X_s}{\rho_s}\right) = P_0\left(\frac{1}{\rho_{f0}} - \frac{X_l}{\rho_l} - \frac{X_s}{\rho_s}\right). \tag{11}$$

The three-phase foam equation of state has been determined by neglecting the surfactant content and the mass of the gas in the mixture. Then, it will be a function of the solid content, the solid specific weight and the volume of the foam at atmospheric pressure.

Figures 3 and 4 show the influence of solid content, $X_s$, respectively, on the incompressible volume, $V_{si}$, and the constant $K$. Once again, there is a similarity between the experimental results and the theory.

The $V_{si}$ vs the solid content variation is a decreasing straight line whose limits are the following:

- without solid ($X_s = 0$), $V_{si}$ approaches $10^{-3}$ m$^3$/kg, which corresponds to the water-specific volume at ambient temperature;
- without liquid ($X_s = 1$), the specific incompressible volume approaches $1/\rho_s$, which is the solid-specific volume. In the case of the Colombian coal, $V_{si}$ equals $1/1256 = 0.8 \times 10^{-3}$ m$^3$/kg.

The constant $K$ vs the solid content variation corresponds also to a decreasing straight line having the following two limits:

- when $X_s = 0$, $K = P_0[(1/\rho_{f0}) - (1/\rho_l)] = 19 \times 10^2$ Pa m$^3$/kg (the experimental value of $\rho_{f0}$ is 50 kg/m$^3$). This is the difference between the foam-specific volume at atmospheric pressure and the water-specific volume at ambient temperature;
- when $X_s = 1$, the constant $K = P_0[(1/\rho_{f0}) - (1/\rho_s)] = 0.72 \times 10^2$ Pa m$^3$/kg. The constant $K$ is equal to the difference between the foam-specific volume at atmospheric pressure and the solid-specific volume

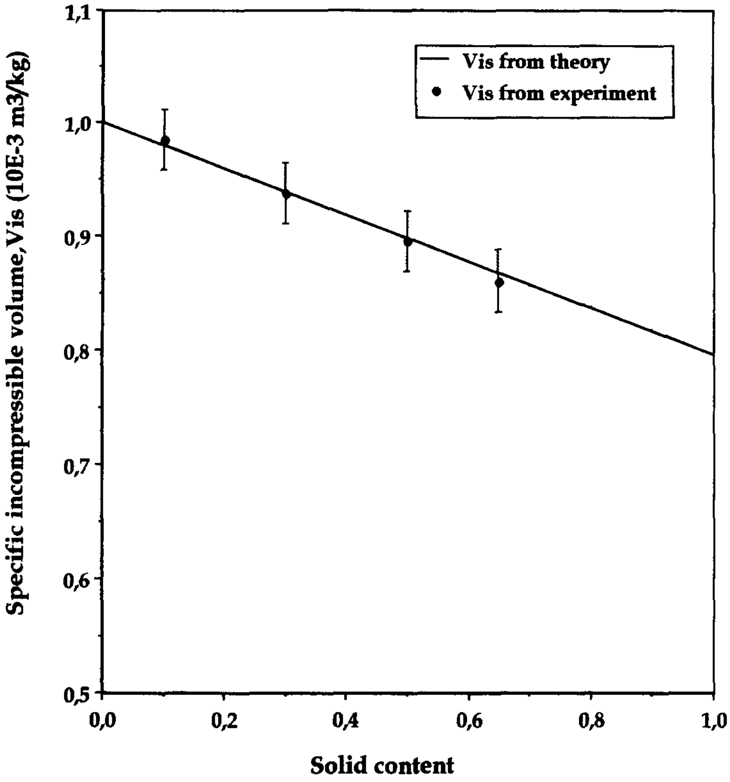

Fig. 3. Three-phase foam equation of state. Specific incompressible volume vs solid content in the foan

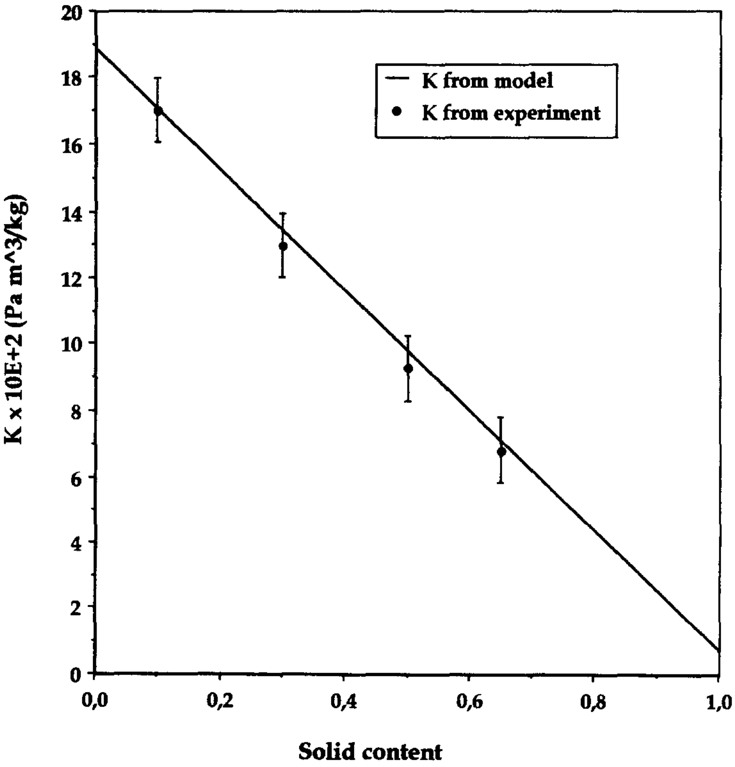

Fig. 4. Three-phase foam equation of state. Constant K vs solid content in the foam.

(when $X_s = 1$, there is no foam but only solid and $\rho_{f0}$ is the apparent specific weight of the solid equal to 660 kg/m$^3$);

It is interesting to know the variations of the specific incompressible volume and the constant $K$ as functions of solid content because these enable the equations of state of three-phase foams having solid contents higher than 65% to be obtained by extrapolation. In fact, due to the high agglomeration of the three-phase foam solid particle at high pressures, it is more difficult to determine the equation of state of foams beyond 65% solid content. The Figs 3 and 4 allow a solution of problem. For example, for a 70% three-phase foam, $V_{si} = 0.85 \times 10^{-3}$ m$^3$ kg$^{-1}$ and $K = 5.41 \times 10^2$ Pa m$^3$ kg$^{-1}$. So, for a 70% three-phase foam of Colombian coal, the equation of state is

$$P(V_s - 0.85 \times 10^{-3}) = 541. \qquad (12)$$

*Validity check of the three-phase foams equation of state.* The difference between the equation of state of a three-phase foam and that of an ideal gas is due to the presence of the specific incompressible volume $V_{si}$. The originality of this work is to demonstrate that the factor 2/3 $\sigma a$ of Eq. 3 can be 'neglected' to assume that the equation of state of three-phase foams is finally

$$P\left[\frac{1}{\rho_f} - \left(\frac{X_s}{\rho_s} + \frac{X_l}{\rho_l}\right)\right] = X_g \frac{RT}{M}. \qquad (13)$$

That is to say

$$V_{si} = \frac{X_s}{\rho_s} + \frac{X_l}{\rho_l} \quad \text{and} \quad K = X_g \frac{RT}{M}.$$

For example, with a 10% three-phase foam, we have 0.200 kg of liquid, and 0.023 kg of solid. The mass of the foam is then 0.223 kg. The experimental total volume of the foam $V = 3.75 \times 10^{-3}$ m$^3$, corresponding to a specific weight of 59 kg/m$^3$, and that of the liquid $V_l = 0.2 \times 10^{-3}$ m$^3$, the solid specific weight being 1256 kg/m$^3$ and the solid volume in the foam $V_{\text{sol}} = 0.0183 \times 10^{-3}$ m$^3$. The gas volume in the foam is then $V_g = 3.5317 \times 10^{-3}$ m$^3$.

For an ideal gas at atmospheric pressure, the number of moles is $0.1495 \times 10^{-3}$ kg mol of gas. For a 10% three-phase foam of Colombian coal, the equation of state obtained by the experiment is $P(V_s - 0.9842 \times 10^{-3}) = 16.83 \times 10^2$ and the corresponding number of moles of gas is then $0.69 \times 10^{-3}$ kg mol of gas/kg of foam.

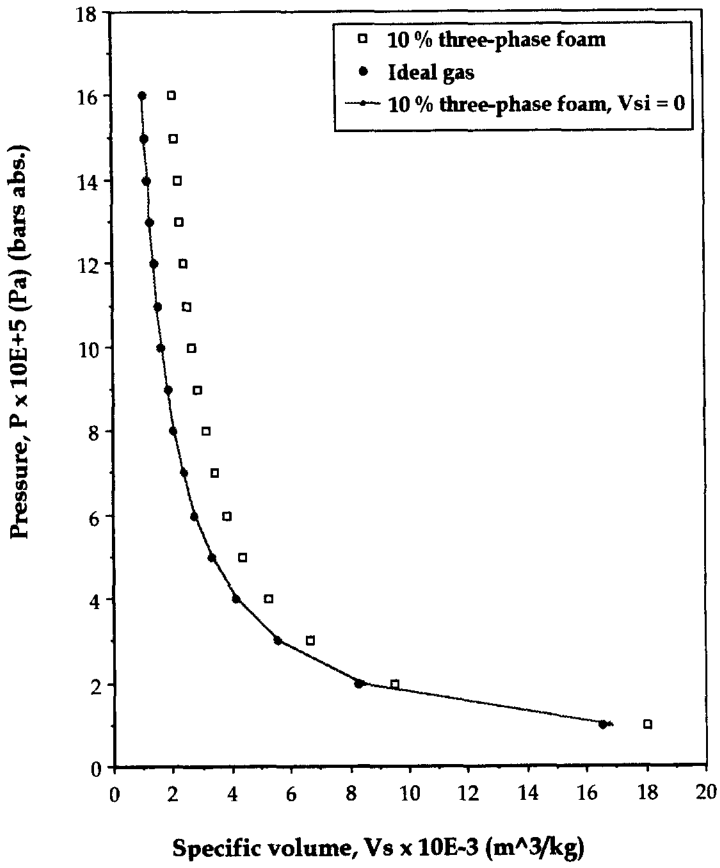

Fig. 5. Three-phase foam equation of state. Comparison between the pressure–volume curve of a three-phase foam and that of an ideal gas.

Finally, for 0.223 kg of foam, the moles of gas in the foam is $0.1542 \times 10^{-3}$ kg mol. The error between this value and that of the ideal gas is 3.1%. So, the calculation of the of quantity of gas in the foam applied to the particular case of ideal gases gives a result close to the theoretical value obtained from the equation of state of ideal gases. Fig. 5 represents the comparison of equation of state of a 10% three-phase foam of Colombian coal and an ideal gas. We note that the difference between the two curves becomes significant when they reach the incompressible volume (which corresponds to the values of highest pressures). This figure indicates little difference between the compressibility of a foam and that of a gas in the ideal conditions. If the factor $2/3\sigma a$ is deducted from the equation of the foam, the two curves become more closer as indicated by the full line in Fig. 5. On the other hand, if the specific incompressible volume is deducted from the equation of state of the foam, there is a superimposition of the results of the ideal gas and that of the gas in the foam.

## CONCLUSIONS

The use of three-phase foams can be a competitive way for the utilization and transport of solids (for example, in feeding pressurized fluidized-bed combustors) and the use of pressurized systems can be apprehended better by knowing the equation of state of three-phase foams. An original compression system has been developed to determine the three-phase foam law. It has been demonstrated that three-phase foams are governed by an equation of state which depends on pressure, temperature, foam-specific volume, foam-specific incompressible volume due to solid and liquid in the foam and moles of gas in the foam. This equation can be written as $P(V_s - V_{si}) = N_g RT$ and will be, in addition way, a function of the formulation conditions: the solid content and, especially, the expansion ratio of the foam. Thus, the results of a same foam can vary depending on whether the foam volume at atmospheric pressure is important. It has been shown that the theoretical model of equation of state coincides well with the experimental results, which shows its reliability.

## NOTATION

| | |
|---|---|
| $a$ | total specific surface of the foam ($= A$/mass of the foam), m$^2$/kg |
| $A$ | surface area of the liquid, m$^2$ |
| $K$ | constant depending on temperature and gas content in the foam, Pa m$^3$/kg |
| $m$ | mass of the foam, kg |
| $n_g$ | moles of gas, kg mol |
| $N_g$ | moles of gas per unit of foam mass, kg moles/kg |
| $P$ | absolute pressure of the foam, Pa |
| $P_0$ | atmospheric pressure, Pa |
| $R$ | universal ideal gas constant ($0.0831 \times 10^5$ Pa m$^3$/kg-mol K) |
| $T$ | absolute temperature, K |
| $V$ | total volume of the foam, m$^3$ |
| $V_g$ | gas volume, m$^3$ |
| $V_l$ | liquid volume, m$^3$ |
| $V_{\text{sol}}$ | solid volume, m$^3$ |
| $V_i$ | incompressible volume of the foam, m$^3$ |
| $V_s$ | specific volume of the foam at pressure $P$, m$^3$/kg |
| $V_{s0}$ | specific volume of the foam at atmospheric pressure $P_0$, m$^3$/kg |
| $V_{si}$ | specific incompressible volume of the foam, m$^3$/kg |
| $X_g$ | gas content in the foam |
| $X_l$ | liquid content in the foam |
| $X_s$ | solid content in the foam |
| $\rho_f$ | specific weight of the foam at pressure $P$, kg/m$^3$ |
| $\rho_{f0}$ | specific weight of the foam at atmospheric pressure $P_0$, kg/m$^3$ |
| $\rho_l$ | specific weight of the liquid, kg/m$^3$ |
| $\rho_s$ | specific weight of the solid, kg/m$^3$ |
| $\rho_{as}$ | apparent specific weight of the solid, kg/m$^3$ |
| $\sigma$ | surface tension of the foaming solution, N/m |
| $\Phi$ | volume fraction of the gas in the foam |

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