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# A general two-equation macroscopic turbulence model for incompressible flow in porous media

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**Abstract**—In view of the practical and fundamental importance to heat and mass transfer, we present a two-equation turbulence model for incompressible flow within a fluid saturated and rigid porous medium. The derivation consists of time-averaging the general (macroscopic) transport equations and closing the model with the classical eddy diffusivity concept and the Kolmogorov–Prandtl relation. The transport equations for the turbulence kinetic energy ($\kappa$) and its dissipation rate ($\varepsilon$) are attained from the general momentum equations. Analysis of the $\kappa$–$\varepsilon$ equations proves that for a small permeability medium, small enough to minimize the form drag (Forchheimer term), the effect of a porous matrix is to damp turbulence, as physically expected. For the large permeability case the analysis is inconclusive as the Forchheimer term contribution can be to enhance or to damp turbulence. In addition, the model demonstrates that the only possible solution for steady unidirectional flow is zero macroscopic turbulence kinetic energy. The implications of this conclusion are far reaching. Among them, this conclusion supports the hypothesis of having microscopic turbulence, known to exist at high speed flow, damped by the volume averaging process. Therefore, turbulence models derived directly from the general (macroscopic) equations will inevitably fail to characterize accurately turbulence induced by the porous matrix in a microscopic sense.


## INTRODUCTION

Turbulence in porous media is a controversial issue. As most porous materials considered in traditional engineering applications present very small pores and small permeabilities and the fluid speed is relatively small, the predominant regime is the laminar flow regime. However, high speed fluid flow through porous media (high Reynolds number) can lead to turbulent flow within the pores. Dybbs and Edwards [1] and Macdonald *et al.* [2] have detected experimentally turbulent characteristics for flow in a porous medium, that is, highly unsteady chaotic flow within the pores and constant friction coefficient with the increase of Reynolds number, at Reynolds numbers (based on the average pore dimension and the average pore velocity) larger than 300 and 1000, respectively.

Obviously the representative dimension of the largest flow eddy in a porous medium is limited by the pore dimension, commonly much smaller than the macroscopic dimension of the system. This is only one of several characteristics that distinguish turbulence in porous media from turbulence in a clear (of solid porous obstruction) flow.

Another fundamental characteristic of turbulent flow in porous media is the distinction between microscopic and macroscopic turbulence. The only experimental evidence of turbulence in a three-dimensional porous medium is of microscopic turbulence, that is, turbulence detected by point-wise probes placed within the pores of the medium. There has been no attempt to volume-average the local signals within a representative volume to obtain the signature of the macroscopic turbulence. It is possible that when averaging a very large number of local (random) signals within a representative elementary volume, the microscopic turbulence be smoothed out. Therefore, turbulent flow at a macroscopic level might not endure. This is another polemical concept of turbulent flow through porous media [3] in need of consideration.

The research attention given to turbulence in porous media is almost nonexistent. The lack of publications in this area is evidenced by the number of related citations found in two recent monographs of convection in porous media (Kaviany [4] and Nield and Bejan [5]). Bear [6], offered a brief discussion on turbulence, inertia forces and separation. We noticed with interest his assertion that "most experiments indicate that actual turbulence occurs at *Re* values at least one order of magnitude higher than the *Re* at which deviation from Darcy's law is observed" and that "...the deviation from Darcy's law [due to inertial forces]...cannot be attributed to turbulence". This seems to invalidate the model hypothesis of Masuoka

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## NOMENCLATURE

| | |
|---|---|
| $c$ | specific heat at constant pressure [J kg$^{-1}$ K$^{-1}$] |
| $c_F$ | Forchheimer inertia coefficient, equation (2) |
| $c_\mu$ | dimensionless coefficient, equation (14) |
| $c_{\mu *}$ | dimensionless coefficient, equation (27) |
| $d$ | scale of pore dimension [m] |
| $f_1, f_2, f_\mu$ | nondimensional coefficients |
| $J$ | viscosity ratio |
| $K$ | permeability [m$^2$] |
| $k$ | thermal conductivity [W m$^{-1}$ K$^{-1}$] |
| $p$ | pressure fluctuations [Pa] |
| $P$ | time-averaged pressure [Pa] |
| $Re_t$ | turbulence Reynolds number |
| $Q$ | norm of the time-averaged velocity vector, equation (5) [m s$^{-1}$] |
| $t$ | time [s] |
| $T_0$ | reference temperature [K] |
| $u$ | fluctuation fluid velocity [m s$^{-1}$] |
| $U$ | time-averaged fluid velocity [m s$^{-1}$] |
| $x$ | space coordinate [m]. |

Greek symbols

| | |
|---|---|
| $\alpha$ | thermal diffusivity, equation (13) [m$^2$ s$^{-1}$] |
| $\beta$ | isobaric coefficient of thermal compressibility [K$^{-1}$] |
| $\delta_{ij}$ | Kronecker delta operator |
| $\varepsilon$ | depletion of turbulence kinetic energy, equation (14) [m$^2$ s$^{-3}$] |
| $\phi$ | porosity |
| $\kappa$ | turbulence kinetic energy, equation (14) [m$^2$ s$^{-2}$] |
| $\lambda$ | volumetric specific heat, equation (13) |
| $\mu$ | dynamic viscosity [kg m$^{-1}$ s$^{-1}$] |
| $\tilde{\mu}$ | effective viscosity [kg m$^{-1}$ s$^{-1}$] |
| $\nu$ | kinematic viscosity [m$^2$ s$^{-1}$] |
| $\nu_t$ | turbulence (eddy) viscosity, equation (14) [m$^2$ s$^{-1}$] |
| $\nu_{t*}$ | turbulence (eddy) viscosity, equation (27) [m$^2$ s$^{-1}$] |
| $\rho$ | density [kg m$^{-3}$] |
| $\theta$ | temperature fluctuation [K] |
| $\Theta$ | time-averaged temperature [K] |
| $\sigma$ | Schmidt number. |

Subscripts

| | |
|---|---|
| 0 | reference |
| e | effective porous medium (fluid and solid matrix) |
| f | fluid |
| $i, j, m, r$ | direction of a vector component ($i, j, m, r = 1, 2, 3$) |
| t | turbulence |
| $T$ | temperature |
| $\varepsilon$ | turbulence kinetic energy depletion |
| $\kappa$ | turbulence kinetic energy. |

Superscripts

| | |
|---|---|
| $\overline{(\ )}$ | time-averaged. |

and Takatsu [7] who assumed the deviation from Darcy's law (or the Forchheimer flow resistance) to be caused by turbulence (see also Nield [8]).

The consideration leading to our decision to embark on the pursuit of developing a turbulence model for flow in porous media is not only fundamental, but practical as well. In combustion processes, for instance, the use of porous inert media leads to a reduction in gas temperature and, as a consequence, to a reduction of $NO_x$ emissions [9]. Evidently, turbulence affects the transport phenomena and combustion parameters (flame thickness, gas temperature, species concentration distribution, burning speed etc.). Using a simplified form of the standard $\kappa$–$\varepsilon$ model for turbulent flow, Lim and Mathews [10] showed that better agreement between numerical predictions and experimental results is obtained when accounting for turbulence effects on combustion. We note in passing that their turbulence model does not consider the additional drag (viscous and form) effects imposed by the porous solid matrix to the flow.

A similar approach was taken recently by Prescott and Incropera [11] for simulating the solidification process of a binary metal alloy. Turbulence within the mushy (porous) zone can play a significant role in this thermal process. Their proposed $\kappa$–$\varepsilon$ model for simulating the momentum transport within the mushy zone was obtained by adding a sink term in the $\kappa$ equation of turbulent clear flow. This sink term artificially damps turbulence in the mushy zone. Although showing good agreement with their experimental results, their turbulence model is of limited use, as indicated by Lage [12], because it does not take into consideration buoyancy and other drag effects imposed by the porous region.

Another important practical phenomenon, of natural origin, is the contaminant transport by air flow (wind) through forests and crops. This kind of flow is generally modeled as flow through porous media. The geometric dimensions and fluid speed are such that turbulence can be achieved. Wang and Takle [13] presented a turbulence model for boundary layer flow near porous obstacles. They stated that "time averaging followed by spatial averaging implies that obstacle elements interact only with time-averaged flow...turbulence energy-cascade process is precluded under these assumptions". This is a very important statement as it discourages the derivation of a turbulence model for flow in a porous medium by applying the volume averaging technique to a standard (closed) time-averaged clear flow turbulence model.

Rudraiah studied the turbulent natural convection in porous media using a Darcy–Lapwood model, that is, a Darcy momentum equation extended by the inclusion of the convective acceleration term (Rudraiah [14] and Rudraiah *et al.* [15]). Although the inclusion of the full convective acceleration term is still a controversial topic (Nield [16, 17] and Vafai and Kim [18]), the absence of the Forchheimer term when modeling high speed flow in porous media is difficult to justify.

Gratton *et al.* [19] presented a randomly varying morphology-based turbulence model obtained by the hierarchical modeling methodology and an advanced averaging technique developed by the same authors. Subsequently, Travkin and Catton [20] introduced a two-temperature heat and momentum transport model in which the morphology of the medium plays a fundamental role. Their momentum transport model does not include the convective inertia term. Moreover, for irregular media morphology, several complications arise with the inclusion of morphology-specific differential and integral terms in the average transport equations.

A model derived by Lee and Howell [21] is apparently the most elaborated version of a general (porous medium) turbulence transport equation. Their model was derived by including a turbulence eddy diffusivity in the viscous diffusion term of the general momentum equation. They did not time-average the general momentum equation and, therefore, their model neglects an additional contribution of the Reynolds stresses. Furthermore, the absolute value of the velocity vector appearing in the Forchheimer term is treated as turbulence independent, an approximation that simplifies the derivation of the $\kappa$ and $\varepsilon$ equations tremendously.

Our purpose is to present the main steps towards the derivation of a macroscopic turbulence $\kappa$–$\varepsilon$ model for incompressible flow in porous media. The turbulence model is derived by time-averaging the general equations for porous media with all terms: time acceleration, convective inertia, pressure gradient, Darcy, Forchheimer and Brinkman. This endeavor is important because it provides: (1) the correct $\kappa$–$\varepsilon$ formulation, consistent with the general equations; (2) a mathematical model for simulating macroscopic turbulence in complex geometries; and (3) the means to verify the ability of a macroscopic general model to represent the turbulence phenomena known to exist at the microscopic level.

Although compressible flow is of scientific and practical interest, our decision to consider the flow as incompressible is justified: including compressibility effects at this stage would complicate the modeling considerably and obscure some important physical aspects easily observed with the simplified model. Moreover, there are several applications in which compressibility effects are negligible (low Mach number in the case of gas flow and incompressible fluid in the case of liquid flow) and yet turbulence plays a fundamental role in the process (e.g. combustion, Lim and Mathews [10], liquid heat exchangers, Joshi and Webb [22]).

## DERIVING THE TIME-AVERAGED EQUATIONS

It is universally accepted that, for turbulence in a clear flow, the Navier–Stokes equations are valid. The only difficulty in solving them for situations that involve turbulence is the necessity of a very large number of grid points, required by the presence of a large range of scales. Time averaging procedures [23] offer means to model the phenomenon with equations that can be solved numerically in a more economical way (less CPU time).

Harlow and Nakayama [24] seem to have been the first to propose a two equation ($\kappa$–$\varepsilon$) turbulence model. Hanjalić and Launder [25] found a representation of Reynolds stresses in terms of eddy viscosity for the $\kappa$–$\varepsilon$ turbulence model. A few years later a more general representation was proposed by Launder *et al.* [26]. Many other models were developed, but the $\kappa$–$\varepsilon$ model is still commonly used because of its low computational effort requirement.

For deriving a $\kappa$–$\varepsilon$ turbulence model for flow in a fluid saturated porous medium, we start from the general flow equations that include the time acceleration term, the convective inertia term, the pressure gradient term, the Darcy (microscopic viscous drag) term, the Forchheimer (microscopic form drag) term, the Brinkman (viscous diffusion) term and the Boussinesq–Oberbeck (buoyancy) term. Assuming a rigid, isotropic and fixed porous matrix and a constant properties newtonian fluid, the mass, momentum and energy conservation equations [27], written for convenience in tensor notation, are:

$$\frac{\partial}{\partial x_j}(U_j+u_j) = 0 \tag{1}$$

$$\begin{aligned}
&\frac{\partial}{\partial t}(U_i+u_i)+(U_j+u_j)\frac{\partial}{\partial x_j}(U_i+u_i) \\
&= -\frac{1}{\rho_f}\frac{\partial}{\partial x_i}(P+p)-\phi\frac{\nu}{K}(U_i+u_i) \\
&\quad -\phi^2\frac{c_F}{K^{1/2}}[(U_j+u_j)(U_j+u_j)]^{1/2}(U_i+u_i) \\
&\quad +\nu J\frac{\partial^2}{\partial x_j\,\partial x_j}(U_i+u_i)-\delta_{i3}[1-\beta(\Theta+\theta-T_0)]g
\end{aligned} \tag{2}$$

$$(\rho c)_e \frac{\partial}{\partial t}(\Theta+\theta)+\phi(\rho c)_f(U_j+u_j)\frac{\partial}{\partial x_j}(\Theta+\theta) = k_e \frac{\partial^2}{\partial x_j \partial x_j}(\Theta+\theta) \quad (3)$$

where $\rho_f$ is the fluid density, $\phi$ is the porosity of the matrix, $K$ is the permeability of the matrix, $c_F$ is the Forchheimer coefficient, $\nu$ is the fluid kinematic viscosity, $\beta$ is the isobaric coefficient of volumetric thermal expansion, $c$ is the specific heat and $k_e$ is the effective thermal conductivity of the medium. The viscosity ratio $J = \tilde{\mu}/\mu$ for most applications can be assumed to be equal to one, although it was indicated recently by Givler and Altobelli [28] that its value can deviate substantially from one for high porosity media.

In the above equations each of the variables (velocity components, pressure and temperature) is written as a sum of the time-averaged ($U_j$, $P$, $\Theta$) and fluctuating values ($u_j$, $p$, $\theta$). The acceleration of gravity vector is assumed to be aligned with the direction of the third axis, $x_3$. Viscous dissipation and pressure work done on the fluid are ignored and the Boussinesq–Oberbeck approximation is invoked.

In order to facilitate the time averaging process of the momentum equation, the Forchheimer term is expanded as follows:

$$[(U_j+u_j)(U_j+u_j)]^{1/2}(U_i+u_i) = QU_i+Qu_i+\frac{U_jU_i}{Q}u_j \quad (4)$$

where

$$Q = (U_m U_m)^{1/2} \quad (5)$$

In equation (4) only the linear terms of the expansion are kept. The (small) second and higher order terms are dropped.

The time-averaged version of equations (1)–(3) is obtained by using the averaging procedures found, for instance, in Tennekes and Lumley [29]. After substitution of (4) into (2), the time-averaged equations become

$$\frac{\partial U_j}{\partial x_j} = 0 \quad (6)$$

$$\frac{\partial U_i}{\partial t}+U_j\frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho_f}\frac{\partial P}{\partial x_i}+\nu J\frac{\partial^2 U_i}{\partial x_j \partial x_j}-\frac{\partial}{\partial x_j}(\overline{u_j u_i}) - \delta_{i3}[1-\beta(\Theta-T_0)]g-\phi\frac{\nu}{K}U_i-\phi^2\frac{c_F}{K^{1/2}}QU_i \quad (7)$$

$$(\rho c)_e\frac{\partial \Theta}{\partial t}+\phi(\rho c)_f U_j\frac{\partial \Theta}{\partial x_j} = k_e\frac{\partial^2\Theta}{\partial x_j \partial x_j} - \phi(\rho c)_f\frac{\partial}{\partial x_j}(\overline{u_j\theta}). \quad (8)$$

Employing the eddy diffusivity concept, we obtain a formula for the Reynolds stresses

$$\overline{u_i u_j} = -\nu_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)+\tfrac{2}{3}\kappa\delta_{ij}. \quad (9)$$

The turbulence fluctuations are in general nonisotropic, but for simplicity their nonisotropy is neglected. Therefore, here we have considered a scalar eddy viscosity $\nu_t$.

A similar equation is introduced for the second-order correlation between the turbulent fluctuations of velocity and temperature, using the turbulent thermal diffusivity ($\alpha_T = \nu_t/\sigma_T$) [30]

$$\overline{u_j\theta} = -\alpha_T\frac{\partial \Theta}{\partial x_j}. \quad (10)$$

With equations (9) and (10) into equations (7) and (8) and after some manipulation, the average momentum and energy equations become

$$\frac{\partial U_i}{\partial t}+U_j\frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho_f}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\nu_t+\nu J)\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\right] - \delta_{i3}[1-\beta(\Theta-T_0)]g-\frac{2}{3}\frac{\partial \kappa}{\partial x_i}-\phi\frac{\nu}{K}U_i - \phi^2\frac{c_F}{K^{1/2}}QU_i \quad (11)$$

$$\lambda\frac{\partial \Theta}{\partial t}+\phi U_j\frac{\partial \Theta}{\partial x_j} = \frac{\partial}{\partial x_j}\left[(\alpha_e+\phi\alpha_T)\frac{\partial \Theta}{\partial x_j}\right] \quad (12)$$

where

$$\lambda = \frac{(\rho c)_e}{(\rho c)_f} \quad \alpha_e = \frac{k_e}{(\rho c)_f}. \quad (13)$$

The turbulent viscosity, $\nu_t$, which appears in the previous equations is found from a dimensional analysis process [31] and the Kolmogorov–Prandtl relation [32, 33], as a function of the turbulence kinetic energy, $\kappa$ and its rate of dissipation, $\varepsilon$, respectively,

$$\nu_t = c_\mu\frac{\kappa^2}{\varepsilon} \quad \kappa = \frac{\overline{u_m u_m}}{2}$$

$$\varepsilon = \nu\overline{\frac{\partial u_i}{\partial x_m}\frac{\partial u_i}{\partial x_m}}. \quad (14)$$

Further on we derive the average equations for $\kappa$ (turbulence kinetic energy) and for $\varepsilon$ (decay rate of turbulence kinetic energy). Before doing so, we point out that the scale of the decay rate of turbulence kinetic energy, $\varepsilon$, can be obtained by considering the eddy length scale of turbulent flow (Bejan [34], p. 272). As the largest eddy within the porous matrix is constrained by the pore size, $d$, the $\varepsilon$ scale becomes

$$\varepsilon \sim \frac{\left(\frac{2}{3}\kappa\right)^{3/2}}{d} \quad (15)$$

where $[(2/3)\kappa]^{1/2}$ represents the scale of the fluctuating velocity $u_j$. Equation (15) can be used in place of the $\varepsilon$ function of equation (14) when estimates of $\varepsilon$ and $\nu_t$ are desired.

## DERIVING THE $\kappa$ AND $\varepsilon$ EQUATIONS

The first step in deriving these equations is to find the momentum fluctuating equation by subtracting the average equation (11) from (2)

$$\frac{\partial u_i}{\partial t}+U_j\frac{\partial u_i}{\partial x_j}+u_j\frac{\partial U_i}{\partial x_j}+\frac{\partial}{\partial x_j}(u_ju_i-\overline{u_ju_i}) = -\frac{1}{\rho_f}\frac{\partial p}{\partial x_i}+\nu J\frac{\partial^2 u_i}{\partial x_j\partial x_j}+\delta_{j3}\beta\theta g-\phi\frac{\nu}{K}u_i -\phi^2\frac{c_F}{K^{1/2}}\left(Qu_i+\frac{U_jU_i}{Q}u_j\right). \quad (16)$$

Multiplying equation (16) by $u_i$, doing the summation in $i$ and averaging it in time, produces the following equation in $\kappa$

$$\frac{\partial\kappa}{\partial t}+U_j\frac{\partial\kappa}{\partial x_j} = -\frac{\partial}{\partial x_j}\left[\frac{1}{\rho_f}\overline{pu_j}+\tfrac{1}{2}\overline{(u_iu_i)u_j}\right] -\overline{u_ju_i}\frac{\partial U_i}{\partial x_j}+\nu J\frac{\partial^2\kappa}{\partial x_j\partial x_j}+\beta\overline{\theta u_3}g -2\phi\frac{\nu}{K}\kappa-\phi^2\frac{c_F}{K^{1/2}}\left(2Q\kappa+\frac{U_jU_i}{Q}\overline{u_ju_i}\right)-J\varepsilon. \quad (17)$$

Using the assumption that the turbulent transport processes are parallel with the molecular ones and the time and length scales for turbulence are much smaller than the ones for the main flow, the turbulence term in the right side of equation (17) is modeled with the gradient transport hypothesis [25]

$$\frac{1}{\rho_f}\overline{pu_j}+\tfrac{1}{2}\overline{(u_iu_i)u_j} = -\frac{\nu_t}{\sigma_\kappa}\frac{\partial\kappa}{\partial x_j}. \quad (18)$$

Using equations (9) and (18) in (17), the final $\kappa$ equation is obtained as

$$\frac{\partial\kappa}{\partial t}+U_j\frac{\partial\kappa}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\nu J+\frac{\nu_t}{\sigma_\kappa}\right)\frac{\partial\kappa}{\partial x_j}\right] +\nu_t\frac{\partial U_i}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)-\beta g\alpha_T\frac{\partial\Theta}{\partial x_3}-J\varepsilon-2\phi\frac{\nu}{K}\kappa -\phi^2\frac{c_F}{K^{1/2}}\left(\frac{8}{3}Q\kappa-2\nu_t\frac{U_jU_i}{Q}\frac{\partial U_j}{\partial x_i}\right). \quad (19)$$

Notice the last two terms of equation (19). These are the contributions of the Darcy and Forchheimer terms, respectively, to the equation. The Darcy contribution, $-(2\phi\nu\kappa)/K$, is always negative and proportional to $\kappa$. Therefore, this term enhances the depletion of turbulence kinetic energy within the medium for any value of $\kappa\neq 0$ and it increases with an increase in $\kappa$ or a decrease in the permeability $K$. The same can be said of the first part of the Forchheimer contribution. We cannot be definite about the second part of the Forchheimer contribution because the velocity components and their derivatives can be either positive or negative, in general.

The equation for the dissipation rate of turbulence kinetic energy $\varepsilon$ is obtained by differentiating equation (16) with respect to $x_r$, multiplying the result by $\nu$ $(\partial u_i/\partial x_r)$ and averaging it in time. An intermediate result is

$$\frac{\partial\varepsilon}{\partial t}+U_j\frac{\partial\varepsilon}{\partial x_j}=\nu J\frac{\partial^2\varepsilon}{\partial x_j\partial x_j} \quad (20a)$$

$$+\frac{\partial}{\partial x_i}\left(-\frac{2\nu}{\rho_f}\overline{\frac{\partial p}{\partial x_r}\frac{\partial u_i}{\partial x_r}}-\nu\overline{u_i\frac{\partial u_j}{\partial x_r}\frac{\partial u_j}{\partial x_r}}\right) \quad (20b)$$

$$-2\nu^2J\overline{\left[\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_r}\right)\right]^2} \quad (20c)$$

$$-2\nu\frac{\partial U_i}{\partial x_j}\left(\overline{\frac{\partial u_j}{\partial x_r}\frac{\partial u_i}{\partial x_r}}+\overline{\frac{\partial u_r}{\partial x_j}\frac{\partial u_r}{\partial x_i}}\right) \quad (20d)$$

$$-2\nu\overline{u_j\frac{\partial u_i}{\partial x_r}}\frac{\partial^2 U_i}{\partial x_j\partial x_r} \quad (20e)$$

$$-2\nu\overline{\frac{\partial u_i}{\partial x_r}\frac{\partial u_j}{\partial x_r}\frac{\partial u_i}{\partial x_j}} \quad (20f)$$

$$+2g\beta\nu\overline{\frac{\partial\theta}{\partial x_j}\frac{\partial u_3}{\partial x_j}} \quad (20g)$$

$$-2\phi^2\frac{\nu c_F}{K^{1/2}}\overline{\frac{\partial}{\partial x_r}\left(Qu_i+\frac{U_jU_i}{Q}u_j\right)\frac{\partial u_i}{\partial x_r}} \quad (20h)$$

$$-2\phi\frac{\nu}{K}\varepsilon. \quad (20i)$$

The term (20b), which represents the turbulent diffusion of $\varepsilon$, is modeled from the gradient transport hypothesis as

$$(20b)=\frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial\varepsilon}{\partial x_i}\right). \quad (21)$$

The destruction term (20c) is determined by the length and time scales only (or any linear independent combination of these two scales; here the independent combinations are $\kappa$ and $\varepsilon$). Using a simple dimensional analysis we can conclude that

$$(20c)=c_{\varepsilon 2}\frac{\varepsilon^2}{\kappa}. \quad (22)$$

The production terms (20d), (20e) and (20f) were modeled by Hanjalić and Launder [25] based on the physics of turbulence: the production of dissipation can be approximated in terms of $\overline{u_iu_j}$, $\varepsilon$ and the mean rate of strain. The same result was derived using the coordinate invariance and dimensional analysis methods [35]. Using the same modeling one obtains

$$(20\text{d})+(20\text{e})+(20\text{f}) = c_{\varepsilon 1}\frac{\varepsilon}{\kappa}\nu_t\frac{\partial U_i}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right). \tag{23}$$

A similar approach is used for the buoyancy term (20g), taken to be a function of $\overline{u_3\theta}$, $\varepsilon$ and $\kappa$, which leads to

$$(20\text{g}) = -c_{\varepsilon 3}\beta g\frac{\nu_t}{\sigma_T}\frac{\partial\Theta}{\partial x_3}. \tag{24}$$

In the $\varepsilon$ equation the set that corresponds to the Forchheimer contribution (20h) needs also to be modeled in terms of average quantities. After differentiation and some manipulation we get

$$(20\text{h}) = -2\nu\phi^2\frac{c_F}{K^{1/2}}\left[Q\frac{\varepsilon}{\nu}\right. \tag{25a}$$

$$+\frac{U_jU_i}{Q}\overline{\frac{\partial u_j}{\partial x_r}\frac{\partial u_i}{\partial x_r}} \tag{25b}$$

$$+\frac{\partial Q}{\partial x_r}\overline{u_i\frac{\partial u_i}{\partial x_r}} \tag{25c}$$

$$\left.+\frac{\partial}{\partial x_r}\left(\frac{U_jU_i}{Q}\right)\overline{u_j\frac{\partial u_i}{\partial x_r}}\right]. \tag{25d}$$

For (25b), an expression analogous with (9) is proposed:

$$\overline{\frac{\partial u_j}{\partial x_r}\frac{\partial u_i}{\partial x_r}} = -\nu_{t*}\left(\frac{\partial^3 U_i}{\partial x_j\partial x_r\partial x_r}+\frac{\partial^3 U_j}{\partial x_i\partial x_r\partial x_r}\right)+\frac{1}{3}\frac{\varepsilon}{\nu}\delta_{ij} \tag{26}$$

where

$$\nu_{t*} = c_{\mu*}\frac{\kappa^2}{\varepsilon}. \tag{27}$$

The term (25c) can be written as

$$(25\text{c}) = \frac{1}{2}\frac{\partial Q}{\partial x_r}\frac{\partial}{\partial x_r}(\overline{u_iu_i}) = \frac{1}{2}\frac{\partial Q}{\partial x_r}\frac{\partial\kappa}{\partial x_r} \tag{28}$$

and (25d) becomes:

$$(25\text{d}) = \frac{1}{2}\frac{\partial}{\partial x_r}\left(\frac{U_jU_i}{Q}\right)\left(\overline{u_j\frac{\partial u_i}{\partial x_r}}+\overline{u_i\frac{\partial u_j}{\partial x_r}}\right) = \frac{1}{2}\frac{\partial}{\partial x_r}\left(\frac{U_jU_i}{Q}\right)\frac{\partial}{\partial x_r}(\overline{u_ju_i}). \tag{29}$$

By using the Reynolds stresses equation (9), the previous expression becomes:

$$(25\text{d}) = -\tfrac{1}{2}\nu_t\frac{\partial}{\partial x_r}\left(\frac{U_jU_i}{Q}\right)\left(\frac{\partial^2 U_i}{\partial x_j\partial x_r}+\frac{\partial^2 U_j}{\partial x_i\partial x_r}\right)+\frac{1}{3}\frac{\partial\kappa}{\partial x_r}\frac{\partial Q}{\partial x_r}. \tag{30}$$

Using equations (20)–(30) the final $\varepsilon$ equation is obtained as

$$\frac{\partial\varepsilon}{\partial t}+U_j\frac{\partial\varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu J+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x_j}\right]+c_{\varepsilon 1}\frac{\varepsilon}{\kappa}\nu_t\frac{\partial U_i}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)-Jc_{\varepsilon 2}\frac{\varepsilon^2}{\kappa}-c_{\varepsilon 3}\beta g\frac{\nu_t}{\sigma_T}\frac{\partial\Theta}{\partial x_3}-2\phi\frac{\nu}{K}\varepsilon-2\phi^2\frac{c_F}{K^{1/2}}\left[\tfrac{4}{3}Q\varepsilon+\frac{5\nu}{6}\frac{\partial\kappa}{\partial x_r}\frac{\partial Q}{\partial x_r}-\nu\nu_t\frac{\partial}{\partial x_r}\left(\frac{U_jU_i}{Q}\right)\left(\frac{\partial^2 U_i}{\partial x_j\partial x_r}+\frac{\partial^2 U_j}{\partial x_i\partial x_r}\right)-\tfrac{1}{2}\nu\nu_{t*}\left(\frac{\partial^3 U_i}{\partial x_j\partial x_r\partial x_r}+\frac{\partial^3 U_j}{\partial x_i\partial x_r\partial x_r}\right)\right]. \tag{31}$$

Notice in equation (31) that the Darcy contribution, $-(2\phi\nu\varepsilon)/K$ always negative and linearly proportional to $\varepsilon$, leads to a decrease in the depletion of turbulence kinetic energy. Again, the Forchheimer contribution might be to increase or decrease $\varepsilon$, depending upon the sign of the velocity components and their derivatives.

Because of the damping effect of the porous matrix, flow with low turbulence Reynolds number $Re_t = \nu\kappa^2/\varepsilon$ might be the norm and not the exception. According to Jones and Launder [36] and Launder and Sharma [37] new terms have to be added in the $\kappa$ and $\varepsilon$ equations to extend the model to low Reynolds number values. These terms resolve also the problem of boundary conditions for $\varepsilon$, which further on will denote only the isotropic part of the dissipation rate ($\varepsilon$ is set to zero at the solid walls).

The $\kappa$ and $\varepsilon$ equations with the new terms included are, respectively,

$$\frac{\partial\kappa}{\partial t}+U_j\frac{\partial\kappa}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu J+\frac{\nu_t}{\sigma_\kappa}\right)\frac{\partial\kappa}{\partial x_j}\right]+\nu_t\frac{\partial U_i}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)-\beta g\alpha_T\frac{\partial\Theta}{\partial x_3}-J\varepsilon-2\phi\frac{\nu}{K}\kappa-\phi^2\frac{c_F}{K^{1/2}}\left(\frac{8}{3}Q\kappa-2\nu_t\frac{U_jU_i}{Q}\frac{\partial U_j}{\partial x_i}\right)-2\nu\frac{\partial\kappa^{1/2}}{\partial x_r}\frac{\partial\kappa^{1/2}}{\partial x_r} \tag{32}$$

$$\frac{\partial\varepsilon}{\partial t}+U_j\frac{\partial\varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu J+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x_j}\right]+c_{\varepsilon 1}f_1\frac{\varepsilon}{\kappa}\nu_t\frac{\partial U_i}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)-Jc_{\varepsilon 2}f_2\frac{\varepsilon^2}{\kappa}-c_{\varepsilon 3}\beta g\frac{\nu_t}{\sigma_T}\frac{\partial\Theta}{\partial x_3}$$

$$-2\phi\frac{2\nu}{K}\varepsilon-2\phi^2\frac{c_F}{K^{1/2}}\left[\frac{4}{3}Q\varepsilon+\frac{5\nu}{6}\frac{\partial\kappa}{\partial x_r}\frac{\partial Q}{\partial x_r}\right.$$
$$-\nu\nu_t\frac{\partial}{\partial x_r}\left(\frac{U_jU_i}{Q}\right)\left(\frac{\partial^2U_i}{\partial x_j\partial x_r}+\frac{\partial^2U_j}{\partial x_i\partial x_r}\right)$$
$$\left.-\tfrac{1}{2}\nu\nu_{t*}\left(\frac{\partial^3U_i}{\partial x_j\partial x_r\partial x_r}+\frac{\partial^3U_j}{\partial x_i\partial x_r\partial x_r}\right)\right]$$
$$+2\nu\nu_t\frac{\partial^2U_j}{\partial x_r\partial x_m}\frac{\partial^2U_j}{\partial x_r\partial x_m}. \quad (33)$$

It is worth mentioning that the added term in the $\varepsilon$ equation (last term of equation (33)) was presented in a wrong form in Jones and Launder [36] and corrected later on by Launder and Sharma [37].

The precise evaluation of coefficients $f_1$, $f_2$ and $f_\mu$ (defined next in equation (35)) of equation (33) requires comparison with experimental results. In a first approximation they can be assumed to have values close to the values for clear flow at low turbulence Reynolds number ($Re_t$ approaching zero) as the variations of these coefficients at low Reynolds numbers are small. Therefore, we suggest for preliminary tests

$$f_1=1.0 \quad f_2=0.7 \quad \text{and} \quad f_\mu=0.08. \quad (34)$$

Here $f_\mu$ is the correction coefficient for the turbulent viscosity equation (14):

$$\nu_t=f_\mu c_\mu\frac{\kappa^2}{\varepsilon}. \quad (35)$$

The present model assumes no influence of $Re_t$ on $\nu_{t*}$.

The final proposed model is composed of several time- and space-averaged equations: (6) continuity; (11) momentum; (12) energy; (32) kinetic turbulence; (33) dissipation turbulence; (35) eddy viscosity; and (27) $\nu_{t*}$ equation. Notice that the resulting turbulence porous-model reduces to the well known $\kappa$–$\varepsilon$ model of clear (of solid matrix) turbulent flow when the permeability of the porous medium tends to infinity.

## FULLY DEVELOPED FLOW

Experimental data of flow in porous media are not very numerous and usually they apply to a particular fully developed flow. In order to facilitate the comparison between the proposed model and experimental results, we consider a particularization of the general equations (6), (11), (32), (33) and (35). In it, we assume fully developed flow aligned with $x_1$. This assumption leads to $U_2=U_3=0$ (velocity along $x_1$ direction only) and $\partial P/\partial x_2=\partial P/\partial x_3=0$ (pressure function of $x_1$ coordinate only).

Under these assumptions the simplified balancing equations, neglecting buoyancy, become:

$$\frac{\partial U_1}{\partial x_1}=0 \quad (36)$$

$$\frac{\partial U_1}{\partial t}=-\frac{1}{\rho_f}\frac{\partial P}{\partial x_1}+\frac{\partial}{\partial x_2}\left[(\nu_t+\nu J)\frac{\partial U_1}{\partial x_2}\right]$$
$$+\frac{\partial}{\partial x_3}\left[(\nu_t+\nu J)\frac{\partial U_1}{\partial x_3}\right]-\frac{2}{3}\frac{\partial\kappa}{\partial x_1}$$
$$-\phi\frac{\nu}{K}U_1-\phi^2\frac{c_F}{K^{1/2}}|U_1|U_1 \quad (37)$$

$$0=\frac{\partial\nu_t}{\partial x_1}\frac{\partial U_1}{\partial x_2}-\frac{2}{3}\frac{\partial\kappa}{\partial x_2} \quad (38)$$

$$0=\frac{\partial\nu_t}{\partial x_1}\frac{\partial U_1}{\partial x_3}-\frac{2}{3}\frac{\partial\kappa}{\partial x_3} \quad (39)$$

$$\frac{\partial\kappa}{\partial t}+U_1\frac{\partial\kappa}{\partial x_1}=\frac{\partial}{\partial x_1}\left[\left(\nu J+\frac{\nu_t}{\sigma_\kappa}\right)\frac{\partial\kappa}{\partial x_1}\right]$$
$$+\frac{\partial}{\partial x_2}\left[\left(\nu J+\frac{\nu_t}{\sigma_\kappa}\right)\frac{\partial\kappa}{\partial x_2}\right]+\frac{\partial}{\partial x_3}\left[\left(\nu J+\frac{\nu_t}{\sigma_\kappa}\right)\frac{\partial\kappa}{\partial x_3}\right]$$
$$+\nu_t\left[\left(\frac{\partial U_1}{\partial x_2}\right)^2+\left(\frac{\partial U_1}{\partial x_3}\right)^2\right]$$
$$-J\varepsilon-2\phi\frac{\nu}{K}\kappa-\frac{8}{3}\phi^2\frac{c_F}{K^{1/2}}U_1\kappa$$
$$-\frac{\nu}{2\kappa}\left[\left(\frac{\partial\kappa}{\partial x_1}\right)^2+\left(\frac{\partial\kappa}{\partial x_2}\right)^2+\left(\frac{\partial\kappa}{\partial x_3}\right)^2\right] \quad (40)$$

$$\frac{\partial\varepsilon}{\partial t}+U_1\frac{\partial\varepsilon}{\partial x_1}=\frac{\partial}{\partial x_1}\left[\left(\nu J+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x_1}\right]$$
$$+\frac{\partial}{\partial x_2}\left[\left(\nu J+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x_2}\right]$$
$$+\frac{\partial}{\partial x_3}\left[\left(\nu J+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x_3}\right]$$
$$+c_{\varepsilon1}f_1\frac{\varepsilon}{\kappa}\nu_t\left[\left(\frac{\partial U_1}{\partial x_2}\right)^2+\left(\frac{\partial U_1}{\partial x_3}\right)^2\right]$$
$$-Jc_{\varepsilon2}f_2\frac{\varepsilon^2}{\kappa}-2\phi\frac{\nu}{K}\varepsilon-2\phi^2\frac{c_F}{K^{1/2}}$$
$$\times\left[\frac{4}{3}U_1\varepsilon+\frac{5\nu}{6}\left(\frac{\partial\kappa}{\partial x_2}\frac{\partial U_1}{\partial x_2}+\frac{\partial\kappa}{\partial x_3}\frac{\partial U_1}{\partial x_3}\right)\right.$$
$$\left.-\tfrac{1}{2}\nu\nu_{t*}\left(\frac{\partial^3U_1}{\partial x_2\partial x_3^2}+\frac{\partial^3U_1}{\partial x_3\partial x_2^2}+\frac{\partial^3U_1}{\partial x_2^3}+\frac{\partial^3U_1}{\partial x_3^3}\right)\right]$$
$$+2\nu\nu_t\left[\left(\frac{\partial^2U_1}{\partial x_2^2}\right)^2+\left(\frac{\partial^2U_1}{\partial x_3^2}\right)^2+2\left(\frac{\partial^2U_1}{\partial x_2\partial x_3}\right)^2\right]. \quad (41)$$

Further simplification can be made by assuming that the temporal derivatives of the time-averaged quantities are zero, that $x_2$ and $x_3$ derivatives of the time-averaged velocity component $U_1$ are negligible, that $U_1$ is positive and that $\varepsilon$ is function of $x_1$ only. These assumptions are reasonable when considering steady turbulent flow through a bounded porous medium. Using these assumptions, and dropping the subscript '1', equations (36)–(41) are transformed into:

$$0 = -\frac{1}{\rho_f}\frac{dP}{dx} - \frac{2}{3}\frac{d\kappa}{dx} - \phi\frac{\nu}{K}U - \phi^2\frac{c_F}{K^{1/2}}U^2 \quad (42)$$

$$U\frac{d\kappa}{dx} = \frac{d}{dx}\left[\left(\nu J + \frac{\nu_t}{\sigma_\kappa}\right)\frac{d\kappa}{dx}\right] - J\varepsilon - 2\phi\frac{\nu}{K}\kappa - \frac{8}{3}\phi^2\frac{c_F}{K^{1/2}}U\kappa - \frac{\nu}{2\kappa}\left(\frac{d\kappa}{dx}\right)^2 \quad (43)$$

$$U\frac{d\varepsilon}{dx} = \frac{d}{dx}\left[\left(\nu J + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{d\varepsilon}{dx}\right] - Jc_{\varepsilon 2}f_2\frac{\varepsilon^2}{\kappa} - 2\phi\frac{\nu}{K}\varepsilon - \frac{8}{3}\phi^2\frac{c_F}{K^{1/2}}U\varepsilon. \quad (44)$$

Equation (42) is the simplified momentum equation (37) in the $x_1$ direction, equation (43) is the simplified turbulence kinetic energy equation (40) and finally, equation (44) is the simplified $\varepsilon$ equation (41). Observe that equations (38) and (39) are used to simplify equation (40).

It is worth noting that the new momentum equation (42) is the macroscopic Forchheimer-extended Darcy equation modified for unidirectional turbulent flow

$$-\frac{1}{\rho_f}\frac{dP}{dx} = \phi\frac{\nu}{K}U + \phi^2\frac{c_F}{K^{1/2}}U^2 + \frac{2}{3}\frac{d\kappa}{dx}. \quad (45)$$

The turbulence effect is revealed by the last term of equation (45), that is, the longitudinal variation of turbulence kinetic energy. Consider a situation in which the initial flow is slow enough to be laminar within the porous medium. The fluid speed is then increased in discrete increments and let stabilize. From a macroscopic point-of-view, one would expect the last term of equation (45) to be positive when considering the transition from laminar to turbulence regime in a low porosity medium (a medium consisting mainly of interconnected conduits) because, at the same fluid speed, the turbulent regime should yield higher pressure drop than the laminar regime (we conceive that one can have both regimes at the same speed). In this case, the turbulence kinetic energy (and the pressure gradient) would increase continuously along the flow direction until a certain constant limiting value is achieved at a certain $x$ location.

In the same experiment, a lower pressure drop (than the laminar flow case) can be achieved when considering a very high porosity medium, one in which the solid matrix would act similarly to isolated solid obstacles (e.g. spheres positioned in a lattice far apart from each other). In this case the last term of equation (45) would be negative and the pressure drop would decrease continuously in $x$ until a limiting value is achieved beyond which $\kappa$ would no longer change.

These two conceptual experiments are important to realize that $\kappa$ must be bounded by a certain limiting value so that the pressure gradient is also bounded. With this in mind, we went further on and analyzed equations (43) and (44), in which $\nu_t$ was substituted by $c_\mu\kappa^2/\varepsilon$, to determine what value $\kappa$ should tend to. When $\kappa$ is constant, equation (43), rewritten as

$$J\varepsilon + 2\phi\frac{\nu}{K}\kappa + \frac{8}{3}\phi^2\frac{c_F}{K^{1/2}}U\kappa = 0 \quad (46)$$

implies that $\varepsilon$ must be constant also. When $\kappa$ and $\varepsilon$ values do not change in $x$ anymore, equation (44) reduces to

$$2\phi^2\frac{\nu}{K}\varepsilon + \frac{8}{3}\phi^2\frac{c_F}{K^{1/2}}U\varepsilon = 0 \quad (47)$$

indicating that $\varepsilon$ must be zero. From equation (46), $\kappa$ must be zero also. Then, the only equilibrium solution is the trivial solution $(\kappa, \varepsilon) = (0, 0)$. Hence, a constant level of macroscopic turbulence kinetic energy different than zero is unlikely to persist in fully developed unidirectional flow through a porous medium. This conclusion, born out of our turbulence model, confirms Nield's [16] prediction "...that true turbulence, in which there is a cascade of energy from large eddies to smaller eddies, does not occur on a macroscopic scale in a dense porous medium" and also the statement by Nield and Bejan [5] that "...it does not make sense to talk about turbulence on a macroscopic scale in a natural porous medium because one cannot have unimpeded eddies of arbitrary size".

Notice that when dropping the porous medium contributions to equations (46) and (47), an equilibrium solution with $\varepsilon = 0$ and any $\kappa$ is possible. Therefore, it is the porous medium drag effect that leads to the zero $\kappa$ solution. For a medium of low porosity, the turbulence pressure gradient is expected to be higher than the laminar equivalent and so $\kappa$ will not decrease with $x$. Since $\kappa$ is nonnegative and $\kappa = 0$ is the only equilibrium value of $\kappa$, this implies that $\kappa$ increases without limit as $x$ increases if $\kappa$ is initially nonzero, but this is not physically realistic. It follows that, in this case, the only valid solution to equations (46) and (47) is one where $\kappa$ is zero for all values of $x$. If a high porosity medium is considered, the $x$ derivative restriction is lifted ($\kappa$ can increase or decrease with $x$). Nevertheless the pressure gradient must be bounded also in this case and the system will inevitably tend to $\kappa = 0$, that is, the macroscopic turbulence kinetic energy is damped out no matter the situation.

Therefore, the only possibility of turbulence affecting the macroscopic pressure drop within a porous medium is when the flow does not satisfy the fully developed assumptions invoked previously. This effect can be to increase or to decrease the macroscopic pressure drop compared to the laminar case. This is most likely to occur near the interface region (entrance and exit) between the porous medium and the clear flow. Within the entrance interface region, for example, where the flow from the clear fluid region must accommodate (develop) to the porous medium presence, the fully developed assumptions do not hold.

It is interesting to contrast the previous conclusions with existing ideas of turbulence in porous media. Published work by Fand *et al.* [38] and Kececioglu and Jiang [39] indicates that by increasing the fluid speed the flow evolves from the Forchheimer regime to a regime referred to by them as the *turbulent* regime. The measurements made by Fand *et al.* [38] and by Kececioglu and Jiang [39] are macroscopic pressure measurements, that is, measurements not made at the pore level. Therefore, it would have been impossible for them to detect microscopic effects (in a single pore).

The evidence of turbulent flow presented by Fand *et al.* [38] and Kececioglu and Jiang [39] is not direct, but inferred from experimental results presented in terms of pressure drop and fluid speed. We notice that when plotting their data in terms of pressure gradient divided by fluid speed versus fluid speed (e.g. Fig. 5, p. 168 of Kececioglu and Jiang [39]), the slope of the curve decreases as the regime evolves from Forchheimer to the regime called turbulent. This leads to the following conjecture: if one is capable of maintaining the laminar regime beyond the transition zone (like extending the laminar curve into the turbulence regime) the pressure drop of the laminar regime would be greater than the pressure drop of the turbulence regime, at the same fluid speed. This result, according to our model, would be typical of a high porosity medium.

We used our simplified (fully developed) turbulence model to simulate the pressure drop versus velocity relationship presented by Fand *et al.* [38] and Kececioglu and Jiang [39]. Solving numerically equations (42)–(44), together with equation (14), we observed that any combination of positive turbulence kinetic energy and depletion levels set at the initial location of the porous medium ($x = 0$) leads to the unrestricted growth of $\kappa$ and $\varepsilon$ with $x$ and the growth of the pressure drop. Since this conclusion is physically unrealistic, the hypothesis of nonzero $\kappa$ and $\varepsilon$ must be incorrect. The implication is, therefore, that $\kappa$ and $\varepsilon$ must be zero.

The anomaly of pressure variation is most likely to be related to their experimental configuration as other experimental works (e.g. Dudgeon [40], Lage *et al.* [41]) with high speed incompressible flow through porous media indicate a very different behavior, that of the pressure gradient relationship with fluid speed evolving from a quadratic to a cubic polynomial function. This tendency, according to our model, is more in line with the expected effect of turbulence in low porosity media because it denotes an increase in the pressure gradient as the flow evolves from laminar to turbulence regime. Notice that Dudgeon [40] referred to strong local pressure fluctuation measured at the pore level, that is, his macroscopic measurements correspond to a microscopic turbulent flow regime.

The transition from quadratic to cubic function, detected by Dudgeon [40] and Lage *et al.* [41] using macroscopic measurements, is most likely to be related to turbulence at the macroscopic level produced near the fluid-porous interface, as the fluid flows from the clear region to the porous medium region (notice that in both cases the pressure measurements are made outside the porous region). As this effect is very small (because the transition from quadratic to cubic, as shown experimentally, is continuous and smooth) there is very little hope that macroscopic turbulence persists downstream of the interface, where the flow eventually becomes unidirectional and fully developed. This aspect supports our previous conclusion.

## SUMMARY AND CONCLUSIONS

It is interesting to compare the $\kappa$–$\varepsilon$ turbulence model for clear flow (Hanjalic and Launder [25] and Speziale [35]) and the general model obtained for incompressible flow through a porous medium. We can easily notice in equations (11), (32) and (33), the appearance of new terms as a result of including the Darcy and Forchheimer terms in the Navier–Stokes equation.

We can now look at the consequences of adding the Darcy and Forchheimer terms. There is one obvious feature. The effect of the Darcy term is quite clear. The presence on the RHS of equation (32) of a term proportional to $\kappa$ and with a negative coefficient is to cause an additional decay in $\kappa$, the turbulence kinetic energy. Likewise, the presence of a similar term for $\varepsilon$ on the RHS of equation (33) shows that the rate of dissipation of $\kappa$ also will decay with time. It is easy to verify that the effect on $\kappa$ is stronger than the effect on $\varepsilon$. Thus turbulent fluctuations are damped out. (Incidentally, this provides further justification for our linearization of the Forchheimer term: even if the turbulence level is high initially it will tend to decay.) The effect of the Forchheimer term need not be similar because the drag effect represented by the Forchheimer term is of different origin (form drag) and the shape of the medium can enhance or damp turbulence.

The energy equation (12) is very similar, the only difference being the appearance of the porous medium properties instead of the ones for fluid. In addition to this the transport of energy by convection is reduced by a factor of $\phi$. The constant $\sigma_T$ (included in $\alpha_T$) should be close to the one determined for the clear flow case. Its precise value needs to be determined by fine tuning the results of the general model to reliable experimental results.

We point out that, depending on the characteristics of the fluid and solid porous medium, and the flow speed, two separated energy equations might be necessary, one for the fluid and one for the solid. Unfortunately, additional work needs to be done for obtaining a general and accurate correlation of heat transfer coefficient between solid matrix and fluid.

Existing equations correlating Nusselt number with porosity, Reynolds number and fluid Prandtl number are restricted to a very specific medium (spherical particles in a packed bed) and limited to low or high Reynolds number [42]. Achenbach [43] recently proposed to combine the asymptotic laminar and turbulent heat transfer correlations for pebble beds via an empirical arrangement factor, a function of porosity.

Referring to the constants $f_1, f_2, f_\mu, c_{\varepsilon 1}, c_{\varepsilon 2}, c_{\varepsilon 3}, c_\mu, \sigma_\varepsilon, \sigma_\varepsilon'$ and $\sigma_\kappa$ we can say that they should have values close to the ones for clear fluid turbulence as long as the medium presents high porosity, but they might be function of porosity as well. In the last case for large porosities ($\phi \to 1$ corresponds to clear flow) their limits need to be the corresponding clear flow values. The new constant $c_\mu^*$ needs to be determined by matching with experimental data.

The analysis of a simplified turbulence model for steady unidirectional fully developed flow indicates that macroscopic turbulence can not persist in a porous medium once this flow configuration is achieved. This is not at all unexpected. Even when having turbulence at the pore level, if one measures the fluctuations of a flow parameter (e.g. fluid speed) at several points within a representative volume (space occupied by several pores), the volumetric averaging of all these quantities might lead to a smooth macroscopic result. This hypothesis needs to be verified experimentally.

We mention in passing the first attempt to obtain macroscopic turbulence quantities from microscopic quantities by Kuwahara *et al.* [44]. Using a low-Reynolds two-equation turbulence model they simulated the microscopic flow field within a two-dimensional periodic porous medium. Their results show, for unidirectional flow, the microscopic turbulence level within the pores. Notice also that the macroscopic pressure gradient obtained by volume averaging the microscopic pressure is very close to the one given by the Forchheimer-extended Darcy equation. Our present model (equation (45)) would predict the value zero for the macroscopic turbulence kinetic energy gradient (aligned with the flow direction). This is exactly what Kuwahara *et al.* [44] imposed in their simulations: notice that they used periodic boundary conditions for microscopic $\kappa$, what leads to zero gradient for macroscopic $\kappa$.

It is important to point out that time averaging does not commute with volume averaging. Therefore, one can not use the results of microscopic $\kappa$ and $\varepsilon$ presented by Kuwahara *et al.* [44] to compute the macroscopic turbulence kinetic energy because the microscopic $\kappa$ represents the local time-averaged turbulence intensity. To get the real macroscopic $\kappa$ one has to obtain the local values of the fluctuation fluid velocity within a representative elementary volume (rev), take the volume average of it and then time average the result. Notice that the fluctuation fluid velocity can take positive or negative values within a rev, in contrast with the positive microscopic $\kappa$ that is always positive.

Finally, it is convenient to comment on an observation made by one of the reviewers that "...the order of time averaging and spatial averaging is independent of the final form of turbulence model so long as the contributions of microscopic eddies to macroscopic turbulence field is justifiably modeled in averaging (filtering) process". The observation is pertinent. However, the situation is more complex than the simple problem of commuting space and time averaging. If one can prove that the time-integral commutes with the space-integral of the momentum equations, then we would agree with the reviewer's comment. The closure problem after each averaging is what creates the difficulty in proving that the integrals commute. Even if one does not close the problem after each averaging, the final model obtained by the sequence time- and space-averaging will present different terms when compared with the terms obtained by space- and time-averaging the equations, so the closure problem will be different in each case. Therefore, in principle, the final form of the turbulence model depends on the averaging (space and time) order.

The comment by Wang and Tackle [13], quoted in the seventh paragraph of the introduction section, then holds if the closure model for the Reynolds stress does not account for the interaction with the solid matrix in a space-averaged sense (this can only happen if the closure of the time-averaged equations is postponed for after the space averaging process). If, say, the time-averaged equations are closed without regard to the solid matrix, then there would be no term in the average equations to represent the interactions between instantaneous quantities and the solid matrix. This lost information can no longer be recovered by the space averaging process and turbulence, as caused by the solid matrix, is totally precluded from the model.

Our model suffers from similar difficulties. When time averaging the balance equations after closing the space averaging problem, the model inevitably loses the information of how local (in space) quantities contribute individually to the time-averaged quantities. This is in line with the concept of space averaging and the result does not preclude turbulence induced by the porous matrix in a space average sense. It does, however, preclude the accurate characterization of turbulence induced by the porous matrix in a microscopic sense. As pointed out in our manuscript, this is the main deficiency of a general turbulence model obtained from the space-averaged equations.

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tion of equation (26), a closure for the Forchheimer contribution to the ε equation.

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