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SIMILARITY SOLUTIONS OF FREE CONVECTION BOUNDARY LAYERS OVER VERTICAL AND HORIZONTAL SURFACES IN POROUS MEDIA WITH INTERNAL HEAT GENERATION

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ABSTRACT

New similarity solution for the problem of free convection boundary layer over vertical and horizontal surfaces which are embedded in a porous medium are reported in this paper. These solutions are valid for a fluid-porous medium with an exponential decaying heat generation term and wall temperature distribution proportional to x^{λ}. It was found that similarity solutions exist for a large range of values of λ as those of classical problems without internal heat generation rates.

Introduction

Convective heat transfer in porous media has received great attention during the last decades due to the importance of this process which occurs in many engineering and natural systems of practical interest such geothermal energy utilization, thermal energy storage and recoverable systems, petroleum reservoirs, industrial and agricultural water distribution to name just a few applications. Very recent reviews by Nield and Bejan [1], and Ingham and Pop [2] give the extent of research information in the area of convective flow in porous media. A literature survey reveals that a large member of similarity solutions have been devoted to free and mixed convection flow over vertical and horizontal surfaces embedded in porous medium which are useful in understanding the interaction of the flow and temperature fields.

In this paper, we present further similarity solutions for the free convection boundary layer flow over vertical and horizontal surfaces embedded in a porous medium with interval heat generation. The effect of interval heat generation is important in several applications that include reactor safety analyses, metal waste form development for spent nuclear full, fire and combustion studies, and storage of radioactive materials. As in the classical papers by Cheng and Minkowycz [3], and Cheng and Chang [4], we assume that the temperature distribution of the plate varies as x^{λ}, where x is the distance along the plate measured from its leading edge and λ is a constant. It is worth mentioning that for the vertical plate in a porous medium without heat generation term, Ingham and Brown [5] have shown that a solution exist only for $\lambda > -1/2$, while Merkin and Zhang [6] found that for a horizontal surface in a porous medium the parameter m should satisfy the range $\lambda > -2/5$. We have, however, shown that for the present problem similarity solutions exist for larger range values of λ.

Mathematical analysis

Consider the problem of steady free convection boundary layer from a heated vertical or horizontal surface embedded in a porous medium of uniform ambient temperature T_∞ and internal heat generation q'''. We assume that the Darcy-Boussinesq approximation holds and that the temperature distribution of the heat varies as x^{λ}. Two flow configurations will be considered, normally, the vertical and horizontal situations, respectively.

a) Vertical surface

The bounder-layer equations governing the free convection over a vertical surface in a porous medium with an internal heat generation are, see Nield and Bejan[1],

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u=\frac{gK\beta}{\nu}(T-T_\infty) \tag{2}$$

$$(\rho C_p)f\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_m\frac{\partial^2 T}{\partial y^2}+q''' \tag{3}$$

which are subjected to the boundary conditions

$$v = 0,\ T_w(x) = T_\infty + Ax^\lambda \text{ on } y = 0 \tag{4}$$
$$u \to \infty,\ T \to T_\infty \qquad \text{as } y \to \infty$$

where the Cartesian coordinates x and y are measured along the plate and normal to it, respectively.

If the heat generation rate q''', is of the form

$$q''' = \frac{k_m(T_w - T_\infty)}{x^2} Ra_x e^\eta \tag{5}$$

Equations (1)-(4) admit then the following similarity solution

$$\psi = \alpha_m Ra_x^{1/2} f(\eta),\ \theta(\eta) = (T - T_\infty)/(T_w - T_\infty),\ \eta = Ra_x^{1/2}(y/x) \tag{6}$$

where ψ is the stream function which is defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, and $Ra_x = gk\beta(T_w - T_\infty)x(\alpha_m V)$ is the local Rayleigh number.

Substituting (6) into Eq. (30) becomes

$$f''' + \frac{\lambda+1}{2} ff'' - \lambda f'^2 + e^{-\eta} = 0 \tag{7a}$$

$$f(0) = 0,\ f'(0) = 1,\ f'(\infty) = 0 \tag{7b}$$

We notice that without the heat generation term, $e^{-\eta}$, Equations (7) coincide with that of Cheng and Minkowycz [3].

Equations (7) were solved numerically for a range values of λ between -1 and 3/4 using the Keller-box method along with an under-relaxation procedure, as proposed by Rees [8].

The non-dimensional velocity or temperature profiles f' with internal heat generation rates is shown for some values of λ in Fig. 1. It is seen from this figure that the heat generation effect is to overshoot the velocity or temperature profiles close to the plate for $\lambda \leq 0$. This can be physically explained because the interval energy generation results in an increase in the buoyancy forces, which in turn induce more flow along the plate.

Values of the local Nusselt number

$$Nu/Ra_x^{1/2} = -f'(0) \tag{8}$$

are given in the Table 1 for some λ. We see that, as expected, the values of the local Nusselt number are higher for $\lambda < 0$ when there is an internal heat generation than those when the internal heat generation is absent. However, for $\lambda > 0$, the heat transfer takes place from the fluid to the plate, i. e. the plate is cooled by internal heat generation rates.

b) Horizontal surfaces

For this configuration Eqs. (1) and (3) still apply, but Eq. (2) is given now by

$$\frac{\partial u}{\partial y} = -\frac{gK\beta}{\nu}\frac{\partial T}{\partial x} \tag{9}$$

We now seek a solution of Eqs. (1), (3) and (9) of the form

$$\psi = \alpha_m Ra_x^{1/3} F(\xi),\ \phi(\xi) = (T - T_\infty)/(T_w - T_\infty),\ \xi = Ra_x^{1/3}(y/x) \tag{10}$$

Where q''' should have the form

$$q''' = \frac{k_m(T_w - T_\infty)}{x^2} Ra_x^{2/3} e^{\xi} \tag{11}$$

After some algebra, we obtain

$$F'' + \lambda\phi + \frac{\lambda - 2}{3}\xi\phi' = 0 \tag{12}$$

$$\phi'' + \frac{\lambda + 1}{3}F\phi' - \lambda\phi + e^{-\xi} = 0 \tag{13}$$

With the boundary conditions

$$F(0) = 0,\ \phi(0) = 1,\ F'(\infty) = 0,\ \phi(\infty) = 0 \tag{14}$$

Without heat generation term, $e^{-\xi}$, Eqs. (12) and (13) reduce to those of Cheng and Chang [3]. These equations were solved numerically for $-0.3 \leq \lambda \leq 2$ using again the Keller-box method. It was found that Eqs. (12) and (13) have solutions only for $\lambda \leq -0.4$ as in the case when the internal heat generation term is absent see Merkin and Zhang [4].

The velocity and temperature profiles with internal heat generation for the horizontal plate is displayed for some values of λ of interest in Fig. 2 and 3. Also, values of local Nusselt number, which for the present problem are given by

$$Nu/Ra_x^{1/3} = -\phi'(0) \tag{15}$$

are given in Table 2. It is clearly seen that the effect of internal heat generation is especially pronounced for negative values of λ.

TABLE 1
Values of $-f''(0)$ for different values of λ

λ	Cheng and Minkowycz [3] Without q'''	Present With q'''
-1/3	0	0.99961
-1/4	0.1621	0.67917
-0.19	-	0.52822
-0.15	-	0.44666
-0.1	-	0.35895
0	0.4440	0.21524
1/4	0.6303	-0.04515
1/3	0.6788	-0.11415
1/2	0.7615	-0.23610
3/4	0.8926	-0.39121
1	1.001	-0.52409

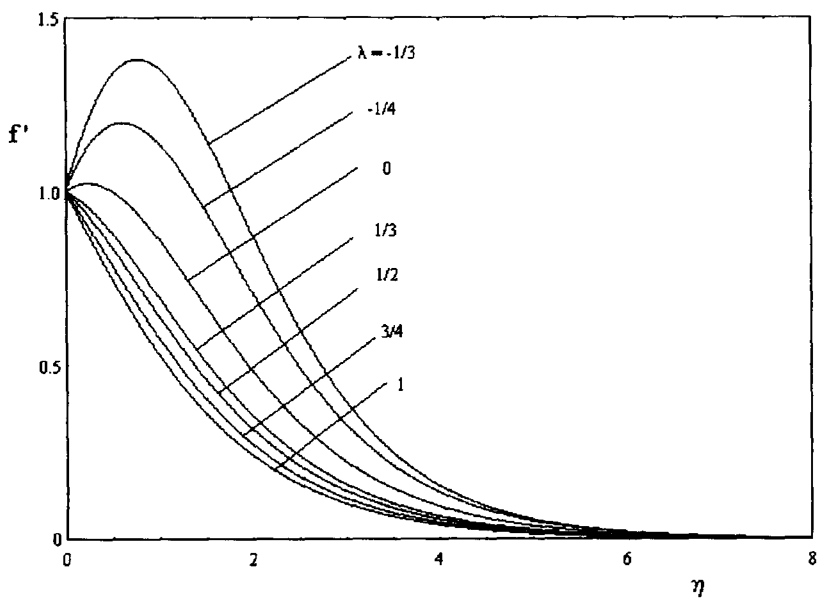

FIG. 1
Velocity or temperature profiles with internal heat generation for the vertical plate

TABLE 2
Values of $-\phi'(0)$ for different values of λ

λ	Cheng and Chang [4] Without q'''	Present with q'''
-0.35	-	3.06570
-0.3	-	1.51211
-0.2	-	0.72532
-0.1	-	0.40463
0	-	0.20020
0.5	-0.8164	-0.39361
1	-1.099	-0.79298
1.5	-1.351	-1.16250
2	-1.571	-1.42250

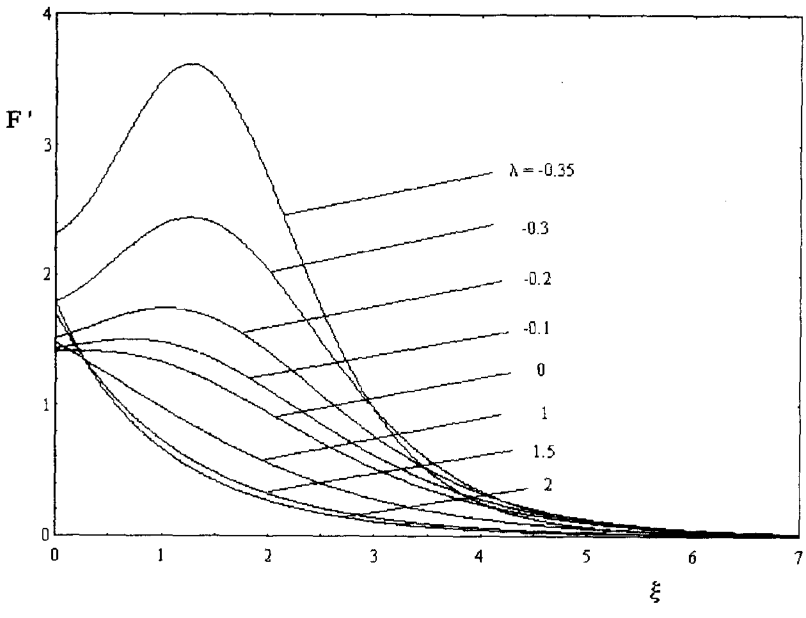

FIG. 2
Velocity profiles with internal heat generation for the horizontal plate

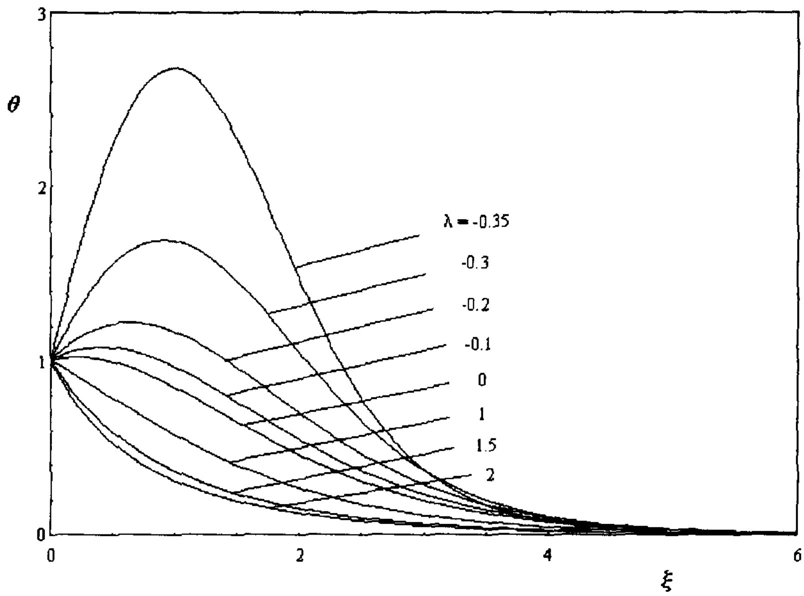

FIG. 3
Temperature profiles with internal heat generation for the horizontal plate

Nomenclature

A	constant
C_p	specific heat at constant pressure of the fluid
f, F	dimensionless reduce stream functions
g	acceleration due to the gravity
k_m	thermal conductivity of porous medium
K	permeability of porous medium
Nu	local Nusselt number
q'''	internal heat generation per unit volume
Ra_x	local Rayleigh number
T	temperature
u, v	velocity components in x and y directions

x, y Cartesian coordinates along the plate and normal to it respectively

Greek symbols

α_m effective thermal diffusivity of the fluid-saturated porous medium

β coefficient of thermal expansion

λ constant

ξ, η similarity variables

θ, ϕ dimensionless temperature functions

ρ density

ν kinematic viscosity

ψ stream function

Subscripts

f fluid condition

w wall condition

∞ ambient condition

Superscripts

/ differentiation with respect to η or ξ

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