fluid flow

k_{es} = effective thermal conductivity of a packed bed in the absence of fluid flow

k_g = thermal conductivity of gas

log = natural logarithm

$\log_{10}$ = logarithm to base 10

N = number of samples

$p(\theta)$ = probability density function

s_l = log population standard deviation

T = temperature

T_b = bulk temperature of the bed

T_w = wall temperature of the heater

V = superficial air velocity

V_{mf} = minimum fluidization velocity

x = distance from heat transfer surface

Greek Letters

δ = interval length

ϵ = void fraction (effective)

ϵ_1 = void fraction of a loosely packed bed

ϵ_2 = void fraction of a densely packed bed

θ = time

θ_e = emulsion (packet) residence time

$\bar{\theta}_e'$ = root-square average emulsion (packet) residence time by Equation (3)

$\bar{\theta}_l$ = log sample mean of residence times

μ_g = viscosity of gas

π = 3.1415926

ρ_e = emulsion (packet) density

ρ_g = density of gas

ρ_s = solid density

σ_l = log sample standard deviation

LITERATURE CITED

Aitchison, J., and J. A. C. Brown, *The Lognormal Distribution*, Cambridge at the University Press, England (1963).

Davidson, J. F., and D. Harrison, *Fluidization*, Academic Press, New York (1971).

Bendat, J. S., and A. G. Piersol, *Measurement and Analysis of Random Data*, J. Wiley, New York (1966).

Kimura, M., et al., *Chem. Eng. (Japan)*, **19**, 397 (1955).

Koya, T., and D. Kunii, "Measurement of Thermal Conductivities of Solid Particles in Packed Beds," *Heat Transfer, Japanese Research*, **1**, 2 (1972).

Kubie, J., and J. Broughton, "A Model of Heat Transfer in Gas Fluidized Beds," *Int. J. Heat Mass Transfer*, **18**, 289 (1975).

Kunii, J. M., and D. Smith, "Heat Transfer Characteristics of Porous Rocks," *AIChE J.*, **6**, 71 (1960).

Lipson, C., and N. J. Sheth, *Statistical Design and Analysis of Engineering Experiments*, McGraw Hill, New York (1973).

Mickley, H. S., and D. F. Fairbanks, "Mechanism of Heat Transfer to Fluidized Beds," *AIChE J.*, **1**, 376 (1955).

———, and R. D. Hawthorn, *Chem. Eng. Progr. Symposium Ser. No. 32*, **51** (1961).

Ozkaynak, T. F., and J. C. Chen, "Packet Residence Time and its Relation with the Heat Transfer Coefficient in Fluidized Beds," *Thermosciences*, TS-742, Lehigh Univ., Pa. (1974).

———, "Average Residence Times of Emulsion and Void Phases at the Surface of Heat Transfer Tubes in Fluidized Beds," *AIChE Symposium Ser. No. 174*, **74**, 334 (1978).

Ranz, W. E., "Friction and Transfer Coefficients for Simple Particles and Packed Beds," *Chem. Eng. Progr.*, **48**, 247 (1952).

Schwartz, C. E., and J. M. Smith, "Flow Distribution in Packed Beds," *Ind. Chem. Eng.*, **45**, 1209 (1953).

Wender, L., and G. T. Cooper, "Heat Transfer Between Fluidized-Solid Beds and Boundary Surfaces—Correlation of Data," *AIChE J.*, **4**, 15 (1958).

Yagi, S. and D. Kunii, "Studies on Effective Thermal Conductivities in Packed Beds," *ibid.*, **3**, 373 (1957).

———, and N. Wakao, "Radially Effective Thermal Conductivities in Packed Beds," *Int. Dev. Heat Transfer, ASME*, **4**, 742 (1961).

Yagi, S., and D. Kunii, "Studies on Heat Transfer in Packed Beds," *ibid.*, 750 (1961).

Yasutomi, T., and T. Yokota, "Heat Transfer from the Surface of a Body Fixed in a Fluidized Bed," *Kagaku Koguku Ronbunshu*, **2**, 205 (1976).

Yoshida, K., D. Kunii, and O. Levenspiel, "Heat Transfer Mechanism Between Wall Surface and Fluidized Bed," *Int. J. Heat Mass Transfer*, **12**, 529 (1969).

Ziegler, E. N., L. B. Koppel and W. T. Brazelton, "Effects of Solid Thermal Properties on Heat Transfer to Gas Fluidized Beds," *Ind. Eng. Chem. Fundamentals*, **3**, 324 (1964).

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The Critical Velocity in Pipeline Flow of Slurries

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In slurry transport, the critical velocity is defined as the minimum velocity demarcating flows in which the solids form a bed at the bottom of the pipe (bed load flows) from fully suspended flows. An analysis based on balancing the energy required to suspend the particles with that derived from dissipation of an appropriate fraction of the turbulent eddies is used to develop a correlation for prediction of the critical velocity. Comparison of the results with available experimental critical velocity data, relating to a rather wide variety of slurry systems, confirms that the present correlation does a superior job of prediction than all previously proposed critical velocity correlations.

SCOPE

The principal objective of this work was to develop an analytical procedure for predicting the critical velocity for slurry transport in pipelines. The prediction of the critical velocity, defined as the minimum velocity demarcating flows in which the solids form a bed at the bottom of the pipe from fully suspended flows, is an eminently practical problem in itself. However, the analytical approach described here derives its broader relevance from the fact that it constitutes an attempt to establish a better understanding of the complex problem of slurry transport.

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CONCLUSIONS AND SIGNIFICANCE

A correlation for the prediction of the critical velocity for pipeline flow of slurries was developed using an analysis based on balancing the energy required to suspend the solid particles with that derived from dissipation of an appropriate fraction of the turbulent eddies of the flow. The significance of the present work resides in the fact that the resulting correlation does a demonstrably better job of predicting the critical velocity than that with previously available correlations. Aside from the practical importance of the results, the analysis presented here represents an attempt to establish an understanding of slurry transport.

There are a number of important problems relating to transport of slurries in pipelines. Among these the essential ones, from a technical viewpoint, are the determination or prediction of the relationship between head loss and throughput, the prediction of the prevailing flow regime given a slurry under prescribed flow conditions and the estimation of the critical velocity. This article is concerned with the last of these, namely, it is concerned with the description of an analysis which leads to a correlation for predicting the critical velocity in slurry transport. The literature on slurry transport has become quite extensive in recent years, and the associated research activity has clearly been motivated by the recognition that slurry transport constitutes a promising alternative surface transportation method for solid commodities, mainly coal.

The quest for developing correlations for predicting pressure drop in slurries dates back more than seven decades to the work of Blatch (1906) who proposed a rudimentary relationship, which has long since been superceded. Subsequent pressure drop correlations were proposed by Howard (1939), Wilson (1942), Newitt et al. (1955), Durand and co-workers (1951, 1952, 1953*a*, *b*), Zandi and Govatos (1967) and Turian and Yuan (1977). The work of Durand and co-workers in the early fifties represents the first major milestone in the area of slurry flows, since it involved exploration of the widest range of pertinent variables up to that time. In particular, they used sand and gravel slurries, with particle sizes ranging from 0.2 to 25 mm, in pipes ranging in diameter from 3.8 to 58 cm, and solids concentrations up to 60% by volume. The resulting correlation from Durand's work, given by

$$\frac{i - i_w}{i_w C} = K\left[\frac{v^2}{gD(s-1)}\sqrt{C_D}\right]^m \qquad (1)$$

does identify the main variables governing the flow but has been found to be inadequate, in the main, in predicting head loss in general even when the adjustable constants K and m are specifically selected. In Equation (1), C_D is the drag coefficient for settling of the particle at its terminal velocity in the stagnant unbounded liquid, and for the equivalent spherical particle of diameter d it is given by

$$C_D = \frac{4}{3}\frac{gd(s-1)}{v_\infty^2} \qquad (2)$$

The next important contribution is due to Zandi and Govatos (1967) who proposed the following improved modification for prediction of slurry pressure drop:

$$\frac{i - i_w}{i_w C} = 280\psi^{-1.93} \qquad \text{for } \psi < 10 \qquad (3)$$

$$\frac{i - i_w}{i_w C} = 6.30\psi^{-0.354} \qquad \text{for } \psi > 10 \qquad (4)$$

with

$$\psi = \frac{v_2}{gD(s-1)}\sqrt{C_D} \qquad (5)$$

The correlation given by Equations (3) and (4) is, according to Zandi and Govatos, valid when the transition criterion given by

$$N_I = \frac{v^2\sqrt{C_D}}{CDg(s-1)} > 40 \qquad (6)$$

is obeyed. According to these authors, data characterized by $N_I < 40$ do not belong to the fully suspended regimes of flow.

In a more recent study, Turian and Yuan (1977) developed an extended pressure drop correlation scheme which accounts for each of the four main flow regimes observed in slurry transport. These were classified according to increasing slurry velocity as flow with a stationary bed, saltation, heterogeneous and homogeneous flow. The classification is only broadly descriptive and is somewhat arbitrary, as explained by Turian and Yuan (1977). Using a total number of 2 848 data points, relating to ranges of the pertinent variables extensive enough to span all four flow regimes, they developed the following correlations:

Flow with a stationary bed (regime 0):

$$f - f_w = 0.4036\, C^{0.7389} f_w^{\,0.7717}\, C_D^{-0.4054}\left[\frac{v^2}{Dg(s-1)}\right]^{-1.096} \qquad (7)$$

Saltation flow (regime 1):

$$f - f_w = 0.9857\, C^{1.018} f_w^{\,1.046}\, C_D^{-0.4213}\left[\frac{v^2}{Dg(s-1)}\right]^{-1.354} \qquad (8)$$

Heterogeneous flow (regime 2):

$$f - f_w = 0.5513\, C^{0.8687} f_w^{\,1.200}\, C_D^{-0.1677}\left[\frac{v^2}{Dg(s-1)}\right]^{-0.6938} \qquad (9)$$

Homogeneous flow (regime 3):

$$f - f_w = 0.8444\, C^{0.5024} f_w^{\,1.428}\, C_D^{0.1516}\left[\frac{v^2}{Dg(s-1)}\right]^{-0.3531} \qquad (10)$$

In addition, these correlations were used to develop a generalized quantitative regime delineation scheme capable of ascertaining the flow regime which might prevail under prescribed conditions (Turian and Yuan, 1977). A detailed comparison among the various pressure drop correlations which have been proposed, and an evaluation of their effectiveness in predicting head loss, is given by Turian and Yuan (1977). Furthermore, the utility of these results and of Zandi and Govatos' (1967) result, Equation

(6), in predicting the critical velocity will be examined later in light of the result of the present work.

THE CRITICAL VELOCITY

The critical velocity v_c is defined variously as the minimum velocity which demarcates flows in which the solids form a bed at the bottom of the pipe (designated here as bed load flows) from fully suspended flows, or as the minimum velocity at which steady state operation, as confirmed by pressure drop data, can be sustained without settling of particles in the pipe. The former, more general, interpretation is adopted here in recognition of the fact that steady state flows of slurries in pipelines are not incompatible with the deposited state. The evident practical significance of the critical velocity has motivated many studies at determining it experimentally or at developing correlations capable of generalizing the data to permit its prediction under more extended conditions. However, presently available correlations usually yield predictions for the critical velocity that are widely divergent. Moreover, there are important qualitative differences among the various critical velocity correlations, many failing to account for one pertinent effect or another.

Under restricted conditions, which includes uniformly

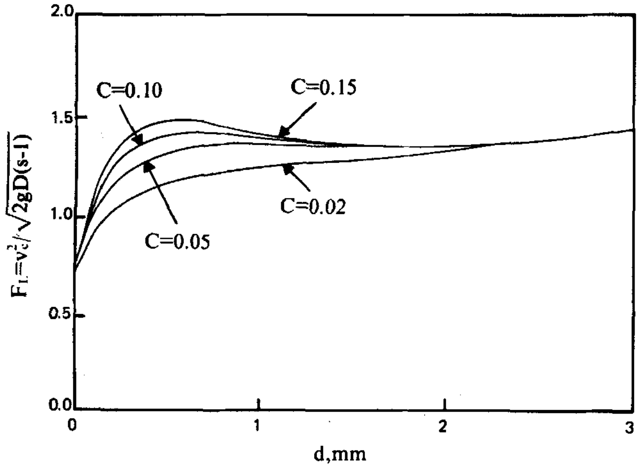

Figure 1. F_L **as function of particle diameter and slurry concentration (Durand and Condolios, 1952).**

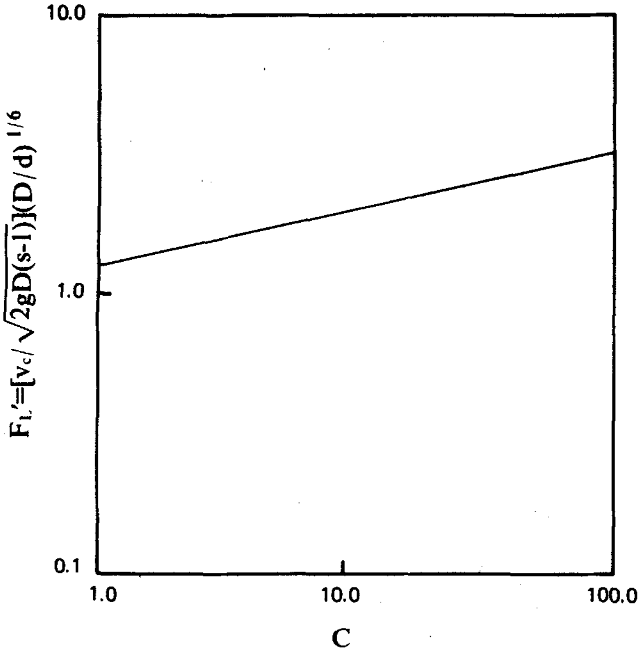

Figure 2. F_L' **as function of slurry concentration (Wasp et al., 1977).**

sized spherical particles and smooth pipe walls, the critical velocity dependence can be expressed by

$$v_c = \bar{f}[d, D, C, (\rho_s - \rho_l)g, \rho_l, \mu] \tag{11}$$

Inclusion of the combined terms $(\rho_s - \rho_l)g$ implies that we consider that the effect of gravity is only manifested through the buoyancy forces on the particles. Rewriting Equation (11) in dimensionless form, we get

$$\frac{v_c}{\sqrt{gd(s-1)}} = f\left[\left(\frac{d}{D}\right), \frac{D\rho_l\sqrt{gd(s-1)}}{\mu}, C\right] \tag{12}$$

The characteristic velocity $\sqrt{gd(s-1)}$ is used here to define the Reynolds number on the right-hand side of Equation (12) and is the counterpart for settling of spheres of the friction velocity $v_0 = \sqrt{\tau_0/\rho_l}$ in pipe flow in which τ_0 is the wall shear stress. For, at steady state, the drag force F_D on a sphere settling in the liquid is equal to the net value of the gravitational and buoyant forces:

$$F_D = \frac{\pi d^2}{4}\left(\frac{1}{2}\rho_l v_\infty^2\right) C_D = \frac{\pi d^3}{6} g(\rho_s - \rho_l) \tag{13}$$

which gives

$$\sqrt{gd(s-1)} = \frac{\sqrt{3}}{2} v_\infty \sqrt{C_D} \tag{14}$$

The analogous expression in pipe flow, of course, states that the friction velocity v_0 is directly proportional to the average velocity and the square root of the friction factor.

Some of the various expressions for the relationship given by Equation (12) that have been proposed are given in Table 1. The entries in Table 1 suggest that the variety of proposed critical velocity relationships is rich, but there is clearly a need to seek some sort of reconciliation among the widely disparate predictions that they yield. Furthermore, those relationships which do not show any dependence on slurry concentration C, and they constitute a majority, are inherently limited because they ignore the critical importance of particle-particle interactions in multiparticle systems on the settling behavior.

The expressions for the regime transition numbers R_{01}, R_{02}, R_{03} for use with the critical velocity expressions due to Turian and Yuan, in Table 1, are given by

$$R_{01} = \frac{v^2}{31.93 C^{1.083} f_w^{1.064} C_D^{-0.0616} Dg(s-1)} \tag{15}$$

$$R_{02} = \frac{v^2}{0.4608 C^{-0.3225} f_w^{-1.065} C_D^{-0.5906} Dg(s-1)} \tag{16}$$

$$R_{03} = \frac{v^2}{0.3703 C^{0.3183} f_w^{-0.8837} C_D^{-0.7496} Dg(s-1)} \tag{17}$$

DEVELOPMENT OF A CRITICAL VELOCITY CORRELATION

We start our analysis by considering the case of a particle settling at its terminal velocity v_∞ in an unbounded fluid medium. The drag force exerted on the particle by the fluid is given by Equation (13), and C_D is the drag coefficient calculated in the usual way. In order to keep a particle stationary and suspended, the fluid must flow upwards at velocity v_∞, and the drag force on the particle is again given by Equation (13). Hence, the rate of dissipa-

TABLE 1. PROPOSED CRITICAL VELOCITY CORRELATIONS

Author	$v_c/\sqrt{gd(s-1)}$	Comments
(1) Durand and Condolios (1952)*	$\sqrt{2}F_L(d/D)^{-1/2}$	$F_L = fn(d,C)$
(2) Kao and Wood (1974)+	$\sqrt{8/3\alpha\beta^2}(d/D)^{1/n}(d/D)^{-1/2}$	$\alpha = 2^{2/n+2}n/(1+n)(2+n)$
		$\beta = (2n+1)(n+1)/2n^2$
(3) Newitt, et al. (1955)	$34/\sqrt{3C_D}$	
(4) Spells (1955)	$\{[0.025](Dv\rho_m/\mu)^{0.775}\}^{1/2}$	
(5) Turian and Yuan (1977)**	$[31.93C^{1.083}f_w^{1.064}C_D^{-0.0616}D/d]^{1/2}$	$R_{02}, R_{03} \le 1$
	$[0.3703C^{0.3183}f_w^{-0.8837}C_D^{-0.7496}D/d]^{1/2}$	$R_{01}, R_{02} \le 1$
	$[0.4608C^{-0.3225}f_w^{-1.065}C_D^{-0.5906}D/d]^{1/2}$	$R_{01}, R_{03} \le 1$
(6) Wasp, et al. (1977)++	$\sqrt{2}F_L'(d/D)^{-1/3}$	$F_L' = fn(C)$
(7) Zandi and Govatos (1967)	$(40C/\sqrt{C_D})^{1/2}(d/D)^{-1/2}$	

* F_L given in Figure 1.
+ With $7 < n < 10$.
** R_{01}, R_{02} and R_{03} given by Equations (15) to (17).
++ F_L' given in Figure 2.

tion of energy to keep the particle suspended, due to drag, is given by

$$P_D = F_D \cdot v_\infty = C_D\left(\frac{1}{2}\rho_l v_\infty^2\right)\frac{\pi d^2}{4}v_\infty \tag{18}$$

We now consider the situation of suspended flow at the incipient critical condition, and we imagine it to be similar to the case of a fluid flow past a stationary particle. We also imagine that the energy required to maintain the particles in suspension is derived from the dissipation of the turbulent eddies. Here, again, the drag force on the particle is given by (13) in the form

$$F_D = C_D\left(\frac{1}{2}\rho_l v_s^2\right)\frac{\pi d^2}{4} \tag{19}$$

where now v_s in Equation (19) is the settling velocity of the particle in the presence of other particles, that is, it is the hindered settling velocity of the particle, and C_D is again the drag coefficient calculated in the usual way. The velocity v_s depends on the concentration of particles in the slurry and can be given by

$$v_s = v_\infty(1-C)^n \tag{20}$$

We shall elaborate on the hindered settling velocity in the next section.

The drag force, given by (19), exerted by an eddy to keep a particle stationary and suspended lasts for the lifetime of that eddy. In other words, this force is exerted by the eddy over a distance equal to the eddy mean length. Assuming that the carrier fluid is flowing at conditions corresponding to the fully turbulent region, and that the turbulence is nearly isotropic, we can estimate the time averaged mean eddy length l_e. The energy per particle dissipated by the eddy to keep the particle stationary and suspended is equal to

$$(E_D)_1 = F_D l_e \tag{21}$$

Accordingly, using Equation (19), we get

$$(E_D)_1 = C_D\left(\frac{1}{2}\rho_l v_s^2\right)\frac{\pi d^2}{4}l_e \tag{22}$$

Assuming a uniform concentration C of particles of size d across the pipe cross section, we can determine the total number of particles in a unit length of pipeline from

$$N_m = \frac{C\dfrac{\pi D^2}{4}}{(\pi d^3/6)} = \frac{3}{2}C\frac{D^2}{d^3} \tag{23}$$

From (22) and (23) we can obtain the total energy required per unit length of pipeline to maintain all the particles in suspension, and this is equal to

$$E_D = N_m(E_D)_1 = \frac{3}{2}C\ C_D\left(\frac{1}{2}\rho_l v_s^2\right)\frac{l_e}{d}\frac{\pi D^2}{4} \tag{24}$$

In the homogeneous flow regime, the turbulence of the fluid is such that the energy usually exceeds that corresponding to E_D. Hence, the particles will follow random movements similar to those of the eddies. As the mean flow velocity is reduced, the energy transferred to the particles decreases, and there is a corresponding decrease in the random motion of the particles. As the velocity is reduced still further, a point is reached when the turbulent energy transferred to the particles just equals E_D. We take this point to correspond to the critical condition and the corresponding velocity to be the critical velocity. If the mean slurry velocity is reduced still further, the energy supplied by fluid turbulence can no longer sustain full suspension, and the particles will begin to settle to the bottom of the pipe to form a bed of solids. The formation of this bed of solids reduces the cross sectional area for flow, which in turn increases the mean slurry velocity locally in the reduced flow region. We presume that the deposition of solids and the associated formation of a stationary bed and reduction of cross-sectional area continue until the energy supplied by the fluid turbulence equals $(E_D)_1$ per particle.

In order to determine the critical mean slurry velocity, it is necessary to determine what fraction of the turbulent energy is effective in suspending particles against gravity. In isotropic turbulence, the total turbulent energy is given by Taylor (1935) as

$$(E_T)_v = \frac{3}{2}\rho_l \bar{v}'^2 \tag{25}$$

where $\tilde{v}' = \frac{1}{3}\overline{\{v_x'^2 + v_y'^2 + v_z'^2\}}^{1/2}$ is the root-mean-squared time averaged velocity fluctuation, defined as the square root of the time average of the squares of the instantaneous velocity fluctuations v_x', v_y', v_z', in the x, y and z directions, respectively. Pertinent details on isotropic turbulence, especially as they relate to the problem under consideration here, are considered later. For the present, we proceed by asserting that of the total energy $(E_T)_v$, which is indifferent to direction and spreads equally in all six directions ($\pm x, \pm y, \pm z$) for isotropic turbulence, only the energy corresponding to the $+y$ direction (that is, the y axis is taken in the direction of the vertical or $-g$ direction) is effective in keeping the particles in suspension. The fraction of turbulent energy corresponding to only the $+y$ direction is, therefore, equal to $(E_T)_v/6$. From Equation (25), this fraction of the turbulent energy is given by

$$(E_T)_v^{+y} = \frac{1}{4}\rho_l \tilde{v}'^2 \tag{26}$$

Further, as shown subsequently, this equation can be expressed as

$$(E_T)_v^{+y} = \frac{1}{4}\rho_l v_0^2 \tag{27}$$

The friction velocity can be calculated from the mean flow velocity v using the following relation due to Blasius (1908), derived on the basis of the $(1/7)^{\text{th}}$ power law velocity profile:

$$v_0 = \left(\frac{\tau_0}{\rho_l}\right)^{1/2} = 0.2 v N_{Re_w}^{-1/8} \tag{28}$$

where

$$N_{Re_w} = \frac{D v \rho_l}{\mu} \tag{29}$$

Substituting (28) into (27), we obtain

$$(E_T)_v^{+y} = \frac{1}{4}\rho_l (0.04 v^2 N_{Re_w}^{-1/4}) \tag{30}$$

Now $(E_T)_v^{+y}$ is the turbulent energy, per unit volume of fluid, used in maintaining the particles in suspension. The corresponding energy based on unit length of pipe is given by

$$(E_T)_l^{+y} = (E_T)_v^{+y} \frac{\pi D^2}{4}(1-C) \tag{31}$$

Combining Equations (30) and (31), we get

$$(E_T)_l^{+y} = 0.01 \rho_l v^2 N_{Re_w}^{-1/4} \frac{\pi D^2}{4}(1-C) \tag{32}$$

As explained earlier, when the condition of critical velocity is attained, the energy required for the particles to remain in suspension E_D must be equal to the fraction of the turbulent energy effective in suspending them, $(E_T)_l^{+y}$; that is, when $v = v_c$

$$E_D = (E_T)_l^{+y} \tag{33}$$

From Equations (24) and (32) we obtain, therefore

$$\frac{3}{2} C\, C_D \left(\frac{1}{2}\rho_l v_s^2\right) \frac{l_e}{d} \frac{\pi D^2}{4}$$

$$= 0.01 \rho_l v_c^2 N_{Re_w}^{-1/4} \frac{\pi D^2}{4}(1-C) \tag{34}$$

The eddy length l_e in Equation (34) is also estimated from the expression derivable using the $(1/7)^{\text{th}}$ power law (Davies, 1972), and this is given by

$$l_e = 0.05 D N_{Re_w}^{-1/8} \tag{35}$$

When the expression (35) is inserted into Equation (34), one gets

$$v_c = \left\{\frac{15}{4} \frac{C}{1-C} v_s^2 C_D \frac{D}{d}\left(\frac{D\rho_l}{\mu}\right)^{1/8}\right\}^{8/15} \tag{36}$$

Further, using Equation (21) for v_s, we get

$$v_c = \left\{\frac{15}{4} \frac{C}{(1-C)^{1-2n}} v_\infty^2 C_D \frac{D}{d}\left(\frac{D\rho_l}{\mu}\right)^{1/8}\right\}^{8/15} \tag{37}$$

In deriving Equation (37), we assumed that all the turbulent energy attributable to the $+y$ direction is used in suspending the particles. It is, however, reasonable to assume that a part of this energy, corresponding to the smaller eddies, is dissipated into heat and will not contribute to suspending the particles. We assume now that only those eddies possessing instantaneous velocities equal to or greater than the terminal settling velocities of the particles are effective in maintaining suspension of the particles. Let x be the fraction of eddies, with $v_y' > v_s$ at anytime. Thus, the turbulent energy used in maintaining suspension of particles is reduced by factor x, and Equation (32) is modified to the form

$$(E_T)_l^{+y} = \left\{0.01 \rho_l v_\infty^2 N_{Re_w}^{-1/4} \frac{\pi D^2}{4}(1-C)\right\} x \tag{38}$$

Hence the corrected correlation for v_c is given by modifying Equation (37) to

$$v_c = \left\{\frac{15}{4} \frac{C}{(1-C)^{1-2n}} C_D v_\infty^2 \frac{D}{d}\left(\frac{D\rho_l}{\mu}\right)^{1/8} / x\right\}^{8/15} \tag{39}$$

The method for estimating x will be discussed later. Finally, using the expression

$$v_\infty^2 C_D = \frac{4}{3} g d (s-1) \tag{40}$$

and substituting into Equation (39), we get

$$\frac{v_c}{\sqrt{gd(s-1)}} = \left\{5C(1-C)^{2n-1}\left(\frac{D}{d}\right)\left(\frac{D\rho_l\sqrt{gd(s-1)}}{\mu}\right)^{1/8} / x\right\}^{8/15} \tag{41}$$

DISCUSSION OF BASIC RESULTS USED IN CORRELATION

Before proceeding to an evaluation of the effectiveness of the relation given in Equation (41), we examine in some detail the various basic results which have been incorporated in its derivation. The results pertaining to the eddy length, hindered settling and to the estimation of the factor x in Equation (41) will be discussed in detail.

In isotropic turbulence, the root-mean-squared time averages of the components of the fluctuating velocity are equal in all three directions. Isotropic turbulence is ap-

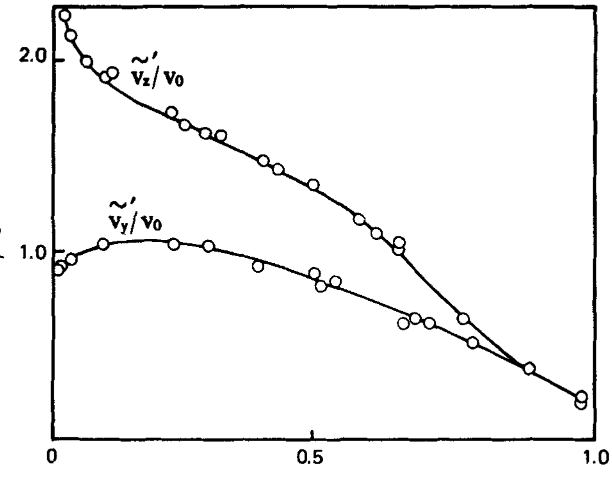

Figure 3. Variation of fluctuating velocities across pipe cross section (Laufer, 1954)

proached in the region about the axis of the pipe in pipe flow. In fact, the ratio $\tilde{v}'_y/v_0$ (where $\tilde{v}'_y$ is the component in the direction pointing radially inwards towards the pipe axis; $y = (D/2) - r = a - r$ is measured from the pipe wall) is never far from unity except close to the pipe wall (Davies, 1972). This is confirmed by the experimental measurements of Laufer (1954) as reproduced in Figure 3. The friction velocity v_0 is estimated in accordance with the relation derived by Blasius (1908) using the empirical $(1/7)^{th}$ power law velocity profile. We recognize that this empirical velocity profile has some fundamental deficiencies and, therefore, so does most probably the friction velocity expression, Equation (28), derived from it. However, in view of the fact that the presence of the solids by itself, particularly at the higher concentrations of practical interest, will undoubtedly affect the turbulent flow in profound though unknown ways, there seems to be little justification for reaching to greater erudition on the selection of the friction velocity expression, if that were possible at all. The objective in the present analysis was to use some relationship for the energy, needed to suspend the particles, that was in some approximate sense intrinsic to the turbulent flow. It is difficult to estimate how the presence of the solids in high concentration alters the flow, but the essential test of our final results is in their ability to predict the critical velocity.

Aside from the foregoing result for the mean fluctuating velocity, we examine the result relating to the mean eddy length. As shown in Figure 4, taken from Davies (1972), the large eddies, which are relatively short lived, contain perhaps as much as 20% of the total kinetic energy of the isotropic turbulence, while medium sized eddies make the main contribution to the kinetic energy. They are commonly called the energy containing eddies, and their characteristic size is denoted by l_e. The small eddies contain around 10% of the energy of the isotropic turbulence, most of which they dissipate by viscous heating. For isotropic turbulence, using Kolmogoroff's (1941*a*, *b*) relation $P^*_M = (\tilde{v}')^3/l_e$, and Blasius Equation (28), Davies (1972) derived the equation for mean eddy length given by Equation (35). Therefore, the same reservations expressed in the preceding paragraph relative to the friction velocity also appy to eddy length expression.

The result for hindered settling in Equation (20) is from Maude and Whitmore (1958) and was developed to account for the effect of the presence of neighboring particles on the settling of the individual particle. The exponent n is found to be a function of the particle Reynolds number N_{Re}, as shown in Figure 5. It is observed from this figure that for Stokes' law region ($N_{Re} < 0.3$), $n = 4.65$,

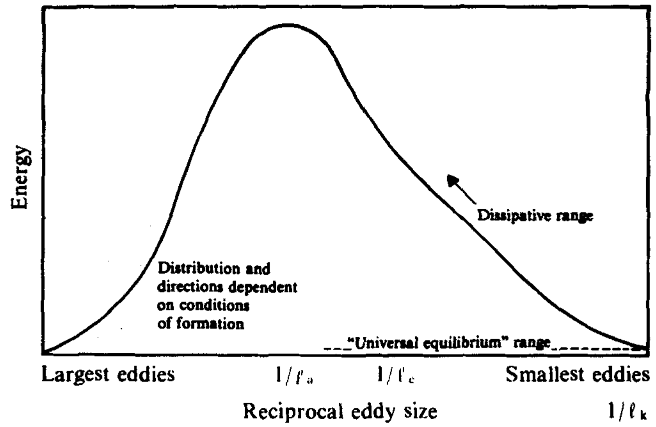

Figure 4. Spectrum of eddies in isotropic turbulence (Davies, 1972).

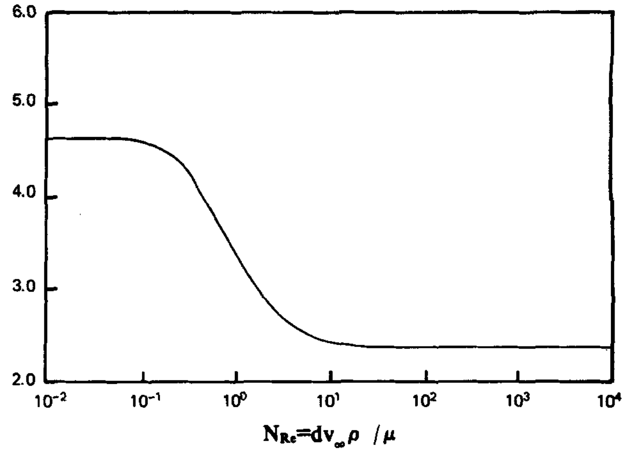

Figure 5. Hindered settling velocity exponent as a function of particle Reynolds number (Maude and Whitmore, 1958).

while for Newton's law region ($N_{Re} > 1\,000$), $n = 2.33$. The concentration effect is more pronounced for nonspherical and for irregular particles than for spherical particles (Steinour, 1944). In the Stokes' law region, because of linearity of the governing equations, the motion in a given direction is independent of the motion in a direction perpendicular to it, and the trajectory of the particle is that resulting from superposition of one motion upon the other. Outside Stokes' law region, motion in one direction will be influenced by velocity components in other directions. Thus, the net vertical velocity of the particle will be lower the higher the horizontal velocity component (Holland-Batt, 1972). In the present analysis we choose $n = 2$ as an approximation which takes into account these various effects on the settling velocity.

We now attempt an examination of the factor x in the critical velocity correlation Equation (41). The factor x was defined as the fraction of the eddies having $\tilde{v}'_y > v_s$ at any time. A definitive theory capable of predicting the relationship for x is difficult. Accordingly, we assume that the distribution of eddy velocities is similar to the distribution of molecular velocities as derived in the kinetic theory of gases (Kennard, 1938). It is found that the fraction of molecules that have speeds above the value v is given by

$$x = 4\pi\left(\frac{\beta^3}{\pi^{3/2}}\right)\int_v^\infty v^2 e^{-\beta^2 v^2}\,dv$$

$$= \frac{2}{\sqrt{\pi}}\beta\left[ve^{-\beta^2 v^2} + \int_v^\infty e^{-\beta^2 v^2}\,dv\right] \quad (42)$$

TABLE 2. EXPERIMENTAL CRITICAL VELOCITY DATA

Source	Slurry	Solid density $kg/m^3 \times 10^{-3}$	Fluid density $kg/m^3 \times 10^{-3}$	Fluid viscosity $kg/m\text{-}s \times 10^3$	Particle size $m \times 10^6$	Pipe diameter $m \times 10^2$	Concentration vol. %	Number of data pts.
SRC Report V (1973)	Coal/water	1.30-1.40	1.0	1.123-1.150	200-370	5.22-31.52	20-50	41
SRC Report VI & VII (1973)	Sand/water	2.65	0.98-1.0	0.470-1300*	175-500	5.22-31.45	5-42	127
SRC Report VI (1973)	Sand/ethylene glycol	2.65	1.10-1.35	5.6-38.0	200-500	5.24	5-42	91
Wasp, et al. (1970)	Sand/water	2.65	1.0	0.98	250-2040	2.67-13.97	1-25	50
Wasp, et al. (1970)	Iron/kerosene	5.245	0.9	1.9-2.0	138	1.905	1-18	12
SRC Report II (1973)	Limestone/water	2.755	1.0	0.98-1.12	100-415	5.245-13.77	12-40	21
SRC Report IV (1973)	Potash/brine	1.984	1.13-1.15	1.14-1.20	300-400	5.22-26.31	30-50	15

*Viscosity changed due to temperature changes.

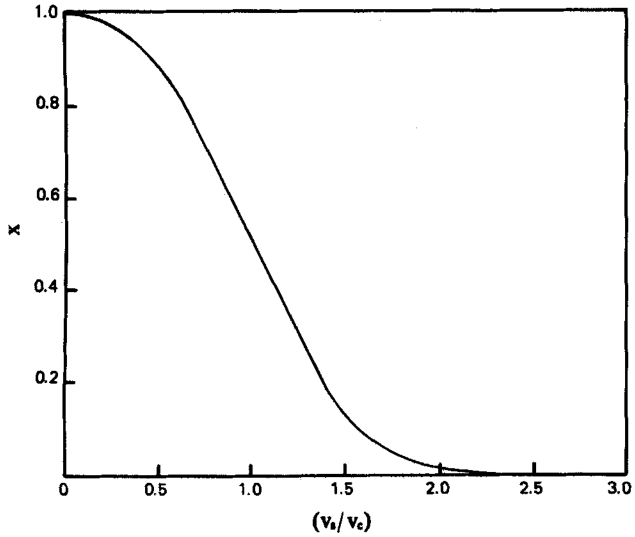

Figure 6. Plot of x vs (v_s/v_c) from Equation (43).

where $\beta v = (2v/\sqrt{\pi}\,\bar{v})$, and $\bar{v}$ is the average velocity of the molecules. In the present situation, $\bar{v}$ corresponds to the mean slurry velocity at the critical velocity condition v_c, and assuming Equation (42) applies here, the fraction of eddies having velocities equal to or greater than the settling velocity is given by

$$x = \frac{2}{\sqrt{\pi}}\left\{\frac{2}{\sqrt{\pi}}\gamma\exp(-4\gamma^2/\pi) + \int_{\gamma}^{\infty}\exp(-4\gamma^2/\pi)\,d\gamma\right\} \quad (43)$$

where we have used $\gamma = v_s/v_c$. The integral on the right-hand side of Equation (43) is recognized as the error function, which is tabulated. It is therefore easy to obtain values of x for different values of the ratio v_s/v_c through use of Equation (43) as shown in Figure 6. It should be mentioned that for the slurries for which critical velocity data were available, and which were used to test our correlation, the value of x was found to be usually close to unity (>0.95).

EXPERIMENTAL DATA AND CORRELATION BY REGRESSION ANALYSIS

We present the results of comparisons of Equation (41) and of previously proposed critical velocity correlations with experimental data in the next section. The experimental critical velocity data used here were collected from the reports documenting the extensive slurry flow research carried out by the Saskatchewan Research Council (SRC)* [see Turian and Yuan (1977)], from Wasp et al. (1977) and from Spells (1955). After the data were screened for redundancies and evident inaccuracies, a collection of 357 data points were gathered. Table 2 contains descriptions of the slurry systems corresponding to this collection of data, and a detailed tabulation is given by Oroskar (1979). For mixed particle sizes, the particle diameter d_{50}, corresponding to that above which 50% of the particles by weight lie, was taken as the equivalent particle diameter.

Comparisons of these data were made with Equation (41), with each of the correlations listed in Table 1, and also with a modification of Equation (41) derived by regression analysis based on the body of 357 data points described above. This latter correlation was motivated by the fact that our main result, Equation (41), in addition to providing prediction does serve to identify the variables determining the critical velocity. This motivated us to explore the possibility of further improved prediction through development of the following extended form of Equation (41):

$$\frac{v_c}{\sqrt{gd(s-1)}} = a_0 C^{a1}(1-C)^{a2}(d/D)^{a3}\tilde{N}_{Re}^{a4}x^{a5} \quad (44)$$

in which the constants a_i ($i = 0, 1, 2, \ldots 5$) are determined by fitting the 357 critical velocity data by regression. In Equation (44) we have used $\tilde{N}_{Re} = D\rho_l\sqrt{gd(s-1)}/\mu$. The result of the regression analysis using Equation (44) and our data collection is given by

$$\frac{v_c}{\sqrt{gd(s-1)}} = 1.85C^{0.1536}(1-C)^{0.3564}(d/D)^{-0.378}\tilde{N}_{Re}^{0.09}x^{0.30} \quad (45)$$

EVALUATION OF CRITICAL VELOCITY CORRELATIONS

In order to compare the various correlations with the experimental data, we calculate for each data point and correlation the percent deviation given by

$$\%\,\text{dev} = D_v = \left[\frac{v_{c\,(\text{calc})} - v_{c\,(\text{exp})}}{v_{c\,(\text{exp})}}\right] \times 100 \quad (46)$$

*SRC reports on hydraulic transport of solids are available through the Saskatchewan Research Council, Saskatoon, Saskatchewan, Canada.

TABLE 3. COMPARISON OF CRITICAL VELOCITY CORRELATIONS WITH EXPERIMENTAL DATA

Correlation	Overall % rms deviation
Present analysis, Equation (41)	25.94
Regression result, Equation (45)	21.82
Turian and Yuan (1977)	67.33
Zandi and Govatos (1967)	58.86
Durand and Condolios (1953*a*, *b*)	51.26
Kao and Wood (1974)	61.90
Wasp et al. (1977)	49.80
Newitt et al. (1955)	73.2
Spells (1955)	62.0

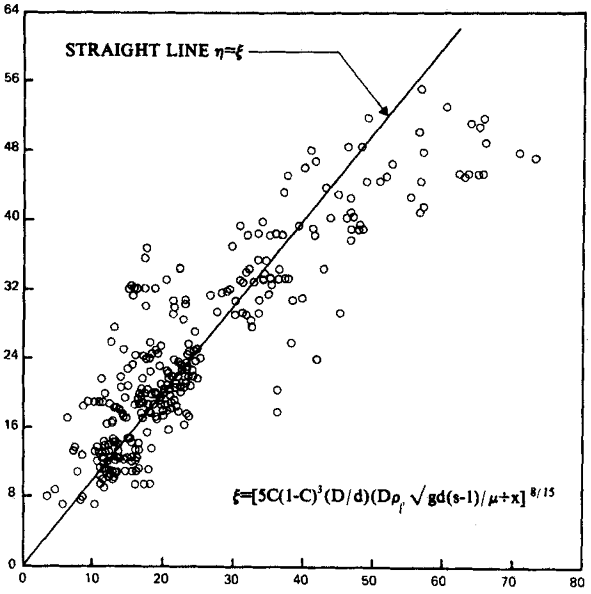

Figure 7. Comparison of critical velocity correlation, Equation (41), with experimental data.

From the calculated values of D_{vi} for each data point and correlation, an overall percent root mean square deviation was calculated for each correlation using

$$\% \text{rms dev} = \left[\sum_{i=1}^{N} D_{vi}^2/N \right]^{1/2} \tag{47}$$

The results of the foregoing calculations are given in Table 3, which clearly indicate that Equation (41) provides satisfactory prediction of the critical velocity. Furthermore, these comparisons also demonstrate that the extended correlation, Equation (45), derived using regression analysis results in only marginal improvement in prediction over that using Equation (41). Nonetheless, both Equations (41) and (45) do a superior job of prediction as compared to correlations proposed by others before. Graphical comparisons of the data with Equations (41) and (45) are given in Figures 7 and 8, respectively.

CONCLUSIONS

The semitheoretical analysis used in the foregoing yields results capable of very satisfactory prediction of the critical velocity, and the results do clearly constitute a marked improvement over available critical velocity correlations. In view of the enormous complexity of the problem of slurry transport, the evident success of the present analytical approach is extremely encouraging. Analytical approaches, like the one in this work, capable of coming to terms with complex problems of the sort under consideration here provide us with the potential for

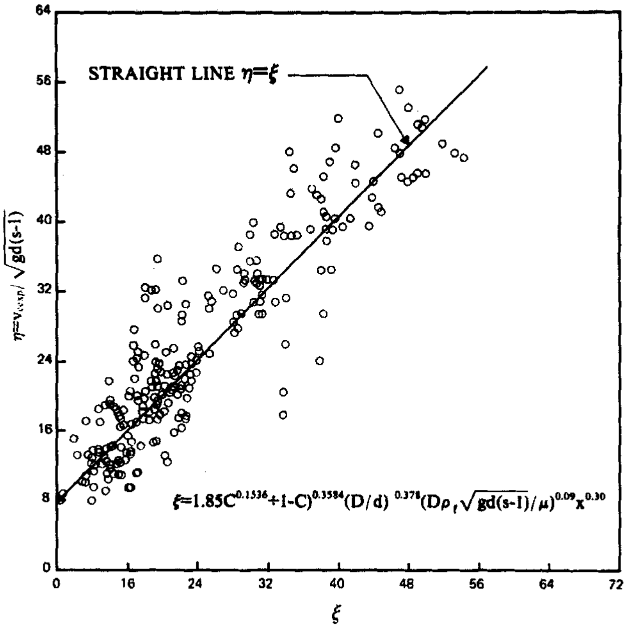

Figure 8. Comparison of critical velocity correlation developed by regression, Equation (45), with experimental data.

greater insight into the problem. The prediction of the critical velocity in slurry transport, is of course, an eminently practical problem, and therefore these results are useful regardless of the broader potentialities of the underlying analysis.

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NOTATION

C	= solids concentration, volume fraction
C_D	= drag coefficient for sphere settling at terminal velocity in quiescent fluid, Equation (2)
D	= diameter of pipe, m
d	= diameter of particle, m
D_v	= percent deviation, Equation (46)
d_{50}	= diameter corresponding to 50% on cumulative curve for particle size distribution, m
E_D	= total energy required to maintain particles suspended in a unit length of pipe, Equation (24), J
$(E_D)_1$	= energy dissipated by eddies to maintain one particle suspended, Equation (22), J
$(E_T)_v$	= turbulent energy per unit volume of fluid, Equation (25), J/m³
F_D	= drag force on a particle, Equation (13), N
F_L, F_L'	= factors used in critical velocity correlations, Table 1
f	= $[(-\Delta p)D]/[2\rho_l L v^2]$, friction factor for slurry flow
f_w	= friction factor for pure carrier liquid
g	= acceleration due to gravity, m/s²
i	= headloss for slurry flow
i_w	= headloss for pure carrier fluid
l_e	= mean eddy size, m
N_I	= $[v^2\sqrt{C_D}]/[CDg(s-1)]$, Equation (6)
N_m	= total number of particles in unit length of pipe
N_{Re}	= $dv_\infty \rho_l/\mu$, particle Reynolds number
N_{Re_w}	= $Dv\rho_l/\mu$, Reynolds number for carrier fluid in pipe
$\tilde{N}_{Re}$	= $[D\rho_l\sqrt{gd(s-1)}]/\mu$, modified Reynolds number
P_D	= rate of dissipation of energy to keep particle suspended, Equation (19), J/m³s

R_{ij}	=	regime numbers, equations (15) to (17)
s	=	(ρ_s/ρ_l), ratio of solid to liquid densities
v	=	mean slurry velocity, m/s
v_c	=	critical velocity of slurry, m/s
v_∞	=	settling velocity of solid in unbounded quiescent fluid, m/s
v_s	=	hindered settling velocity of solid, m/s
v_0	=	$\sqrt{\tau_0/\rho_l}$, friction velocity in pipe flow, m/s
$\tilde{v}_i$	=	root mean of time averaged square of velocity fluctuation in direction i, m/s
x	=	fraction of eddies with velocities exceeding v_s, Equation (43)

Greek Letters

γ	=	v_∞/v_c, Equation (43)
μ	=	carrier liquid viscosity, kg/m − s
ρ_l	=	liquid density, kg/m^3
ρ_m	=	slurry density, kg/m^3
ρ_s	=	solid density, kg/m^3
τ_0	=	shear stress at pipe wall, N/m^2
ψ	=	$[v^2\sqrt{C}]/[gD(s-1)]$, Equation (5)]

LITERATURE CITED

Blasius, H., "Grenzschichten in Flüssigkeiten mit Kleiner Reibung," *Zeit. Math. Phys.*, **56**, 1-37 (1908).

Blatch, N. S., "Discussion: Water Filtration at Washington, DC," ASCE, **57**, 400-408 (1906).

Davies, J. T., *Turbulence Phenomena*, Academic Press, New York (1972).

Durand, R., "Transport Hydraulique de Graviers et Galets en Conduite," *La houiulle Blance*, **6**, 609-169 (1951).

———, "Basic Relationships of the Transportation of Solids in Pipes — Experimental Research," *Proceedings Minnesota International Hydraulics Convention*, 89-103 (1953*a*).

———, and E. Condolios, "Communication de R. Durand et E. Condolios," *Compte Rendu des Deuxiemes Journees de L'Hydraulique* (Paris, Societe Hydrotechnique de France) 29-55 (June, 1952).

———, "Hydraulic Transport of Coal and Solid Materials in Pipes," *Proceedings of a Colloquim on the Hydraulic Transport of Coal* (Nov. 5-6, 1952); pp. 39-52, National Coal Board, London, England (1953*b*).

Holland-Batt, A. B., "Behaviour of Particles Accelerating in Fluids," *Trans. Inst. Chem. Engrs.* (*London*) **50**, (1972).

———, "Two-Dimensional Motion of Particles Accelerating in Fluids," *ibid.*, **50**, 156-167 (1972).

Howard, G. W., "Transportation of Sand and Gravel in a Four-Inch Pipe," *Trans. ASCE*, **104**, 1334-1348 (1939).

Kao, T.-Y., and D. J. Wood, "Incipient Motion of Solids in Solid-Liquid Transport System," *Trans. Soc. Min. Engrs., AIME*, **255**, 39-44 (1974).

Kennard, E. M., *Kinetic Theory of Gases*, McGraw-Hill, New York (1938).

Kolmogoroff, A. N., "The Local Structure of Turbulence in Incompressible Viscous Fluid for Very Large Reynolds Numbers," *Compt. Rend. Acad. Sci. URSS*, **30**, 301-305 (1941*a*).

———, "Dissipation of Energy in Locally Isotropic Turbulence," *ibid.*, **32**, 16-18 (1941*b*).

Laufer, J., "The Structure of Turbulence in Fully Developed Pipe Flow," *NACA Rept. No. 1174* (1954).

Maude, A. D., and R. L., Whitmore, "Hindered Settling," *Brit. J. Appl. Phys.*, **9**, 477-482 (1958).

Newitt, D. M., J. F. Richardson, M. Abbott and R. B. Turtle, "Hydraulic Conveying of Solids in Horizontal Pipes," *Trans. Inst. Chem. Engrs.*, **33**, 93-110 (1955).

Oroskar, Anil R., "Flow of Slurries in Horizontal Pipelines," M. S. thesis, Syracuse Univ., NY (1979).

Spells, K. E., "Correlations for Use in Transport of Aqueous Suspensions on Fine Solids through Pipes," *Trans. Inst. Chem. Engrs.*, **33**, 79-84 (1955).

Steinour, H. H., "Rate of Sedimentation-Suspension of Uniform Sized Angular Particles," *Ind. Eng. Chem.*, **36**, 840-847 (1944).

Taylor, G. I., "Statistical Theory of Turbulence," *Proc. Royal Soc. London, A151*, 421-478 (1935).

Turian, R. M., and T.-F. Yuan, "Flow of Slurries in Pipelines," *AIChE J.*, **23**, 232-243 (1977).

Wasp, E. J., T. C. Aude, J. P. Kenny, R. H. Seiter, P. B. Williams and R. B. Jacques, "Deposition Velocities and Spatial Distribution of Solids in Slurry Pipelines," presented at the 1st International Conference on the Hydraulic Transport of Solids in Pipes, Coventry, England (Sept., 1970).

———, J. P. Kenny and R. L. Gandhi, "Solid-Liquid Flow-Slurry Pipeline Transportation," *Trans. Tech. Publ.*, Rockport, Mass. (1977).

Wilson, W. E., "Mechanics of Flow, with Noncolloidal, Inert Solids," *Trans. ASCE*, **107**, 1576-1594 (1942).

Zandi, I., and G. Govatos, "Heterogeneous Flow of Solids in Pipelines," *J. Hydr. Div., ASCE*, 93, HY3, 145-159 (1967).

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The Continuous Membrane Column

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