Review

Recent Advances in Research on Water-Freezing and Ice-Melting Problems

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■ A comprehensive review of recent developments in research on a variety of water-freezing and ice-melting problems is presented that covers such subjects as analytical and numerical methods on freezing and melting problems, freezing of water with and without main flow, atmospheric and marine icings, and various types of melting of ice. An attempt is also made to review the thermophysical properties of pure ice and sea ice.

Keywords: *freezing heat transfer, melting heat transfer, phase change, cold climates, ice layer*

INTRODUCTION

The problems of water-freezing and ice-melting heat transfer have received much attention because they are closely related to human life and the development of industrial plants in cold regions. The growth or decay of an ice layer on a river or lake and the freezing or melting of a water pipe are practical phenomena that are commonly encountered in cold regions. Freezing of water flow in a water pipe, for example, may cause a variety of detrimental effects such as an increase in hydraulic pressure loss, diminution of flow rate, and damage to the pipe resulting from flow blockage caused by ice formation. Recently, melting of icebergs in Arctic or Antarctic waters due to the so-called greenhouse effect has received much attention because it may strongly influence the supply of nutrients to surface water as well as the weather on Earth.

The objective of the present article is to review briefly the results of studies that have been carried out on water-freezing and ice-melting characteristics under a variety of conditions and also to demonstrate the need for additional investigations. Attention is mainly focused on the interesting physical phenomena that have been observed experimentally.

THERMOPHYSICAL PROPERTIES OF ICE AND MATHEMATICAL ASPECTS OF PHASE-CHANGE PROBLEMS

Thermophysical Properties of Ice*

Thermal Conductivity of Ice The measured values of the thermal conductivity of ice at 0°C and atmospheric pressure range from 2.09 W/(m · K) [2] to 2.26 [3]. For practical purposes, it was suggested by Sakazume and Seki [4] that one can use

$$\lambda_i = 1.16(1.91 - 8.66 \times 10^{-3}\theta + 2.97 \times 10^{-5}\theta^2) \quad [W/(m \cdot K); \theta, °C] \tag{1}$$

to calculate λ_i at temperatures ranging from 100 to 273 K at atmospheric pressure. The thermal conductivity of ice at temperatures below 100 K was determined by Klinger [5], whose data are shown in Fig. 1 along with previous data [2, 3, 6]. It should be noted that the λ_i value increases with decreasing temperature, passes through a maximum at about 7 K (about 100 times as large as that at 273 K), and then decreases as the temperature decreases further.

Specific Heat of Ice Reviews and summaries of many measurements were reported by Dorsey [7]. Heat capacity measurements at very low temperatures were carried out by Flubacher et al [8]. Sugisaki et al [9] determined the specific heats of amorphous, cubic, and hexagonal ice. Previous data at atmospheric pressure can be represented by

$$Cp_i = 0.185 + 0.689 \times 10^{-2}T \quad [kJ/(kg \cdot K)], \quad 273 \geq T \geq 90\ K \tag{2}$$

and

$$Cp_i = 0.895 \times 10^{-2}T \quad [kJ/(kg \cdot K)], \quad 90 \geq T \geq 40\ K \tag{3}$$

Density of Ice The density of pure ice was determined by a number of investigators. Ginnings and Corruccini [10] reported a value of 916.71 ± 0.05 kg/m^3 at 0°C, which is in good agreement with the value of 916.4 kg/m^3 derived from previous data [11, 12]. The data reported at atmo-

* This section is mainly from [1].

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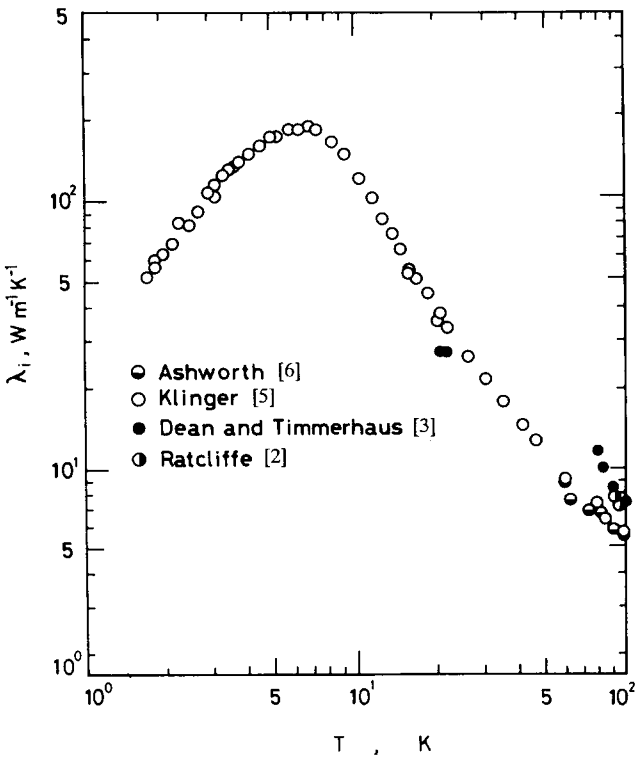

Figure 1. Thermal conductivity of pure ice [5].

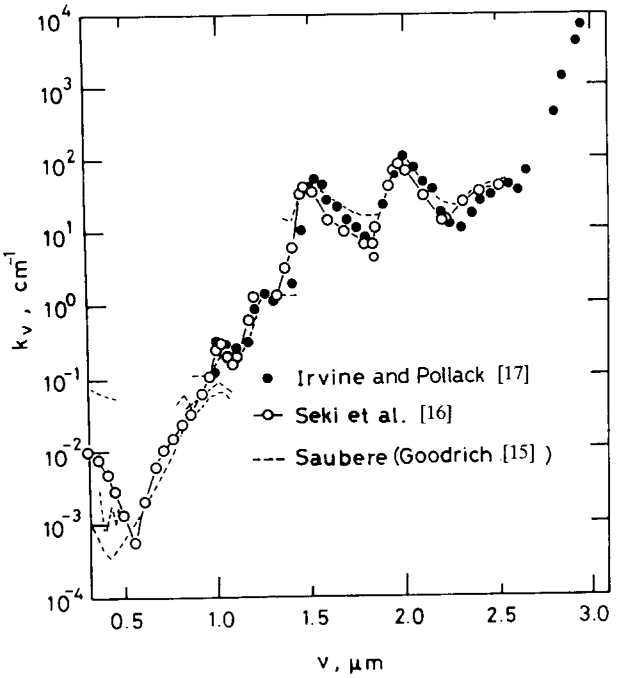

Figure 2. Radiant absorption coefficient of ice [16].

spheric pressure can be represented by

$$\rho_i = 917(1 - 1.17 \times 10^{-4}\theta) \quad [kg/m^3], \quad 0 \geq \theta \geq -140°C \qquad (4)$$

$$\rho_i = 930(1 - 1.54 \times 10^{-5}\theta) \quad [kg/m^3], \quad -140 \geq \theta \geq -260°C \qquad (5)$$

Thermal Expansion There are two coefficients of thermal expansion, namely, the coefficient of linear expansion α_l and the coefficient of cubic expansion α_v.

All previous data at atmospheric pressure indicate that the coefficient of linear expansion decreases with a decrease in temperature. For the value at 0°C, Butkovich [13] gave 52.33×10^{-6} $(1/T)$ for polycrystalline normal to the *c* axis. Based on a number of reported data, Yen [14] gave an expression for α_{vi} as

$$\alpha_{vi} = (0.67T - 24.86) \times 10^{-6} \quad [K^{-1}] \qquad (6)$$

Absorption Coefficient of Ice Absorption coefficients for clear ice in the spectral region 0.3–3 μm were reviewed by Goodrich [15]. Seki et al [16] determined experimentally the k_ν values for wavelengths between 0.3 and 2.5 μm. The results at atmospheric pressure are shown in Fig. 2.

Mathematical Aspects of Freezing and Melting Problems

A great number of studies dealing with analytical or numerical aspects of particular problems have appeared in the literature. Useful summaries of these have been published by Bankoff [18], Muelbauer and Sunderland [19], Mori and Araki [20], Saitoh [21], Cheung and Epstein [22], Fukusako and Seki [23], Lunardini [24], and Yao and Prusa [25]. In Ref. 23, the original features of the analytical and numerical methods that have been applied to a variety of freezing or melting heat transfer problems were extensively demonstrated.

FREEZING OF WATER WITHOUT MAIN FLOW

Freezing of Water in Cylinder Tube

It is well known that a pipe with no main flow can be blocked by ice formation, to the extent that flow cannot be resumed, much sooner than previously predicted by London and Seban [26], whose analysis was based on simple annular growth of ice. Kanayama [27] and Gilpin [28] clarified that this was because dendritic ice forms when there is no main flow through the pipe during the freezing process. Figure 3 shows the flow (left half) and temperature (right half) characteristics in a tube when quiescent water is cooled. The cooling rate, θ_v, is defined as the decrease in the temperature of the tube wall per unit time. It is observed that complicated flow patterns may appear owing to the effect of density inversion in the water. Just after the water starts to be cooled, the cooled water flows down along the tube wall (see plate 1 of Fig. 3). After 12 min, a clockwise eddy appears due to the density inversion effect (see plate 2 of Fig. 3). Ice growth does not nucleate until the water temperature near the pipe wall has been supercooled 4–6°C below fusion temperature.

Figure 4 demonstrates the growth of dendritic ice from the nucleation center near the top of the cylinder that will eventually block the cross section. The total time required

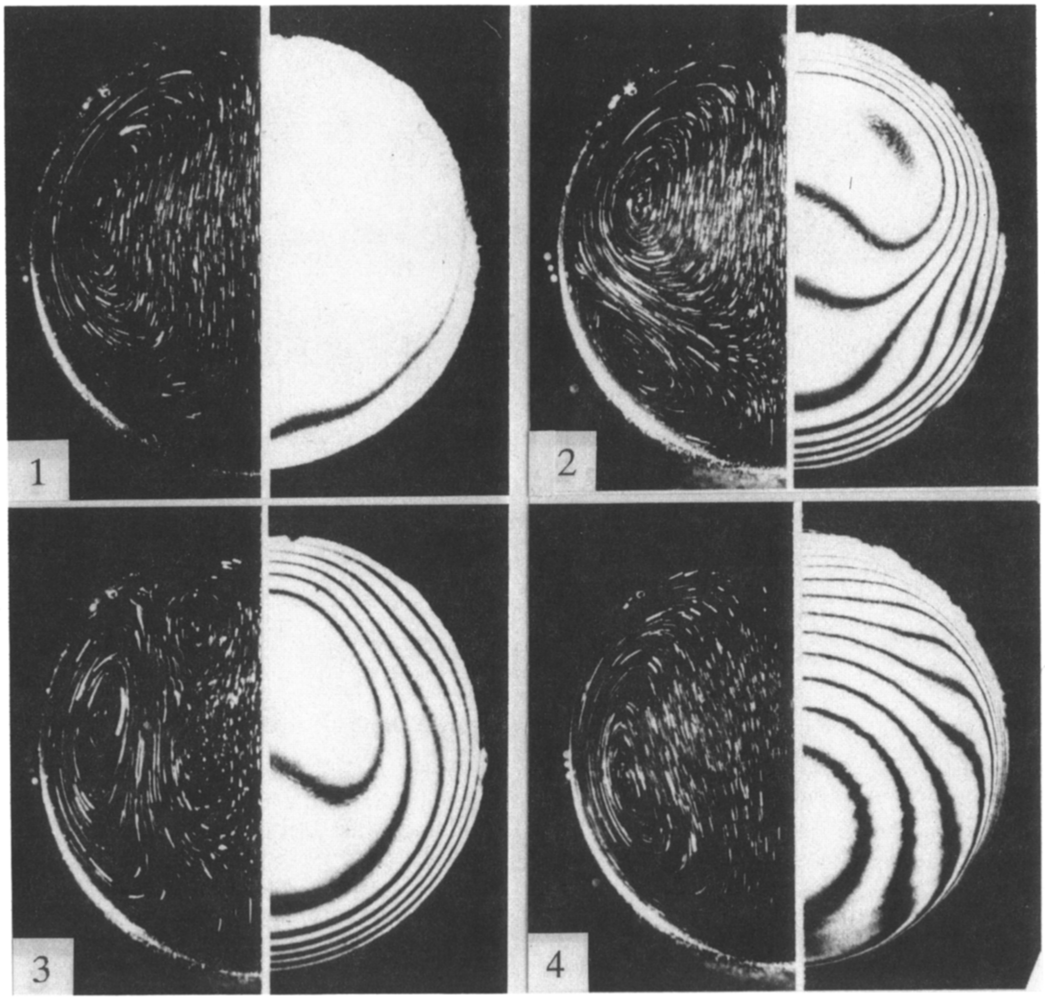

Figure 3. Flow and temperature characteristics in a tube [29]. $T_{\text{ini}} = 5°C$, $\theta_v = 0.27°C/min$. Elapsed time: (1) 0 min, (2) 12 min, (3) 16 min, (4) 48 min.

to complete this dendritic ice growth was about 3 s. The temperature of the water remaining in the pipe returned to 0°C because of the release of the latent heat of fusion. Finally, the growth of an ice annulus began from the pipe wall. Then it is revealed that there are two stages of ice blockage: blockage of the pipe cross section by the formation of dendritic ice as a result of supercooling in the water and subsequent blockage of the pipe by the inward growth of a solid annulus of ice from the wall.

The effects of free convection [30] and cooling rate [31] on the formation of dendritic ice in quiescent water were reported. Fukusako and Takahashi [29] measured the local heat transfer coefficient along the pipe wall.

Effect of Free Convection

Because of the nonlinear relationship between density and temperature, the cooling of water through its maximum density point at an initially uniform temperature greater than 4°C causes complex free convection, which introduces an additional complication to freezing problems.

Vertical or Horizontal Freezing Ozaki et al [32] studied the effect of density inversion on free convection along a vertical frozen front. They found that dual flow (up and down) existed in the boundary layer along the vertical ice surface. The role of free convection in the formation of ice when water is cooled from below was determined by Tankin and Farhadieh [33]. They found that the critical Rayleigh number, $\text{Ra}'_c (= g\beta\Delta T^3/\kappa\nu$, where g is gravitational acceleration, β is the coefficient of thermal expansion, ΔT the temperature difference between the top and bottom of the unstable layer, κ the thermal diffusivity, and ν the viscosity), marking the onset of free convection for the case of freezing from below was about 480. Brewster and Gebhart [34] observed the flow characteristics on the ice surface facing downward. Saitoh et al [35] investigated the effect of a variety of parameters on the supercooling process. The effect of free convection on the freezing of supercooled water was experimentally studied by Kashiwagi et al [36].

Freezing in a Horizontal Tube Gilpin [37] carried out an experimental and analytical study on the cooling of water in a horizontal tube, with the wall temperature decreasing at a constant rate through the maximum density point at 4°C. He measured temperature distributions, boundary layer velocity profiles, and flow patterns, which were compared to predictions of a quasi-steady boundary layer

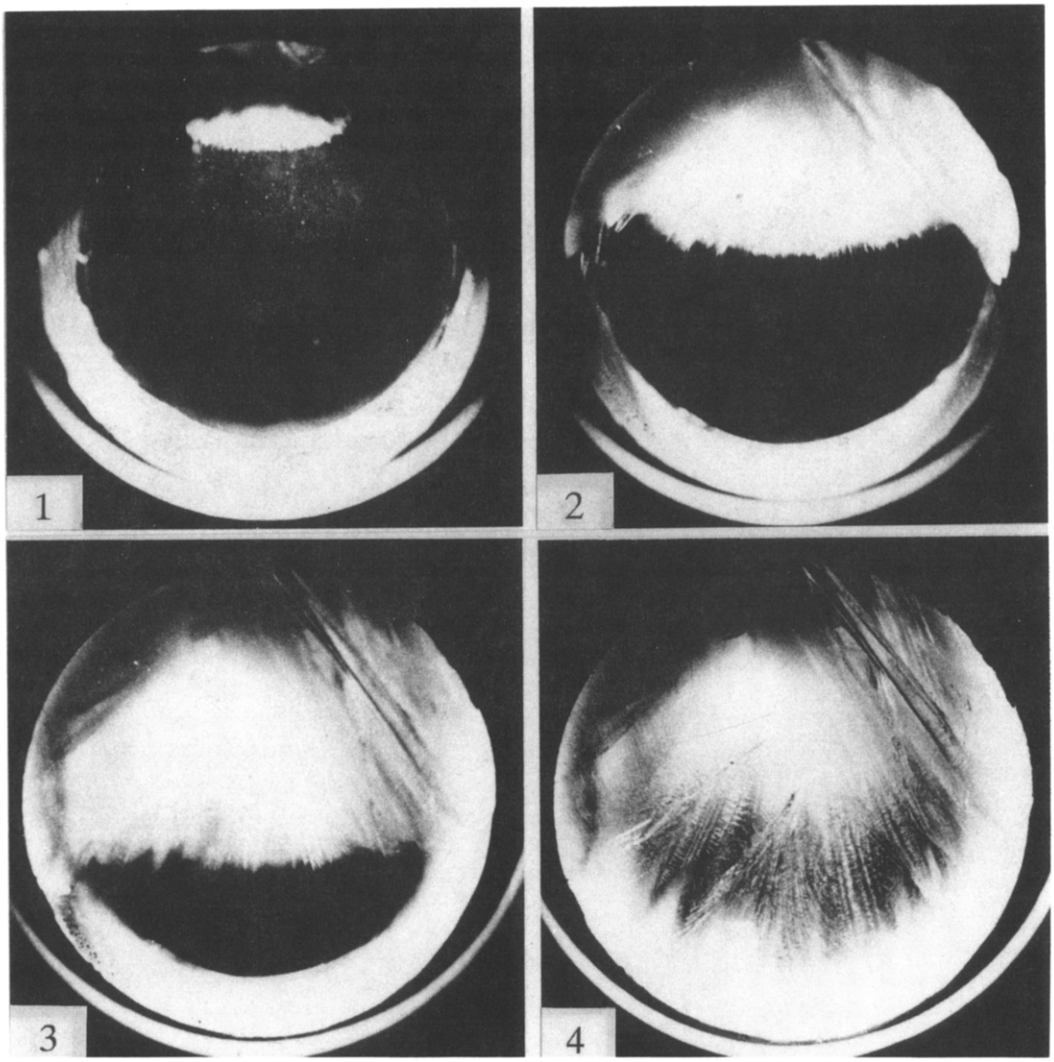

Figure 4. Occurrence of dendritic ice in a tube [29]. Elapsed time: (1) 112 min 0.12 s, (2) 112 min 1.55 s, (3) 112 min 2.25 s, (4) 112 min 3.00 s.

model. The analytical model predicted the existence of three quasi-steady modes of convection for some temperatures below 4°C. Giplin [37] concluded that the reversal of the flow pattern observed involved a transition from one quasi-steady mode to another. Numerical investigations on transient free convection of water in a horizontal pipe with a constant cooling rate through 4°C were performed by Cheng and Takeuchi [38] and Cheng et al [39].

Freezing in a Rectangular Cavity Transient free convection flow and heat transfer in a rectangular cavity filled with cold water, where the water was initially at a uniform temperature greater than 4°C and cavity walls were maintained at a temperature of 0°C, was studied by Vasseur and Robillard [40]. More recently, Dutton and Sharan [41] measured the flow and temperature distributions in a cavity filled with cold water.

Freezing of Air–Water Layer in a Horizontal Circular Tube

The free-convection heat transfer of air–water layers and dendritic ice formation in a horizontal tube with uniformly decreased wall temperature were recently studied by Fukusako and Takahashi [42, 43], who used holographic interferometry to determine the time-dependent temperature distribution in the tube. Figure 5 shows the temperature distributions, time-dependent sequential flow patterns, and a schematic diagram of the flow patterns, where T_{ini} is initial water temperature, H the water depth, and θ_v the cooling rate of the tube wall. One minute from the start of the cooling, there are dense interference rings near the tube wall (a_1). The water temperature near the tube wall is above 4°C, and the streamlines form a counterclockwise eddy (c_1) as the water density increases with decreasing water temperature. After 11 min, the interference rings move from the edge of the air–water interface to the center of the tube (a_2), and there are both clockwise and counterclockwise eddies (b_2, c_2). The water temperature near the tube wall is below 4°C, and water ascends along the tube wall to create a clockwise eddy. At 15 min, the clockwise secondary eddy has enlarged, and the counterclockwise eddy is disappearing (b_3, c_3). The interference rings reach further from the edge of the air–water interface toward the center of the tube (a_3). After 25 min, the interference rings have spread through-

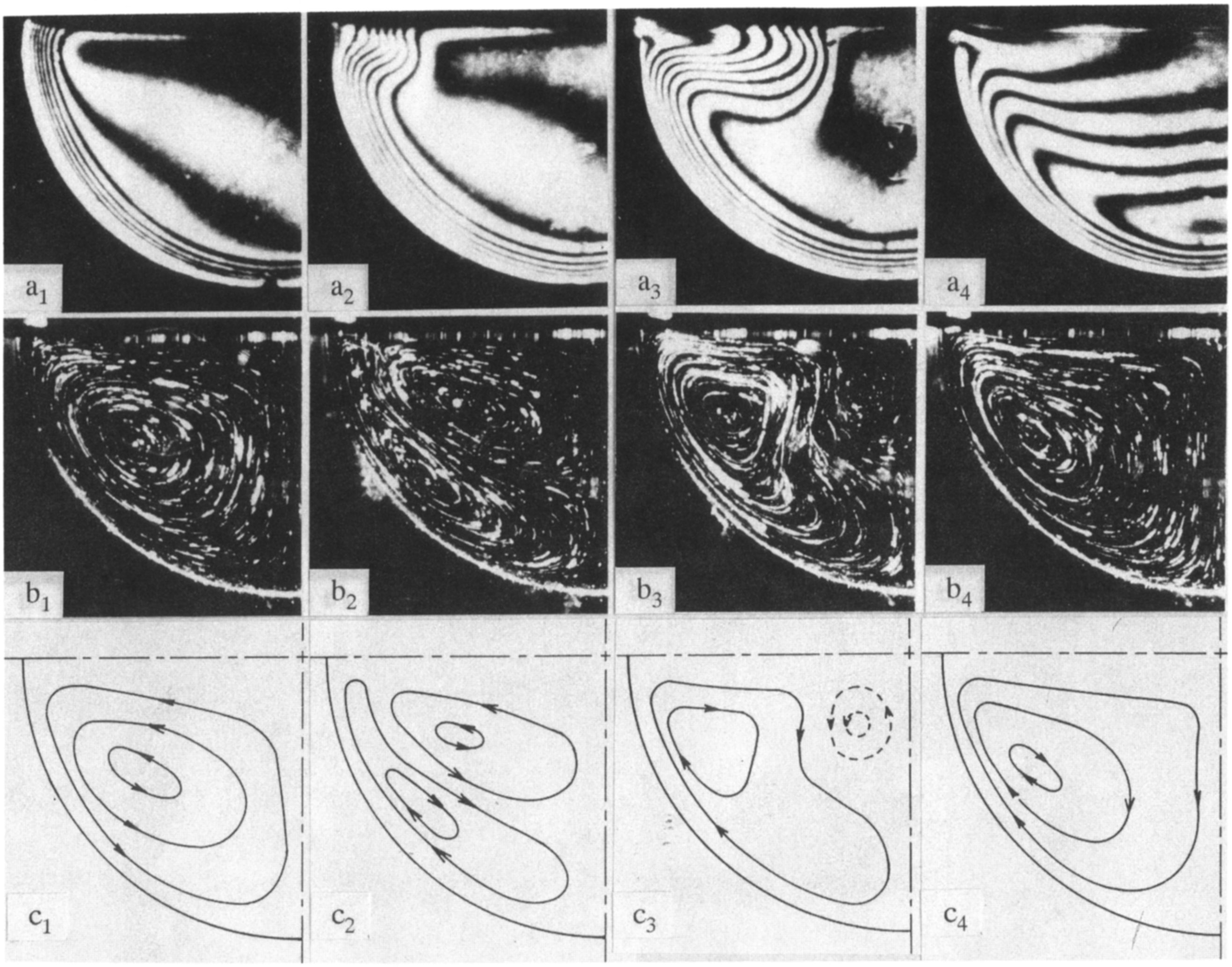

Figure 5. Flow and temperature characteristics [43]. H = 32 mm, T_{ini} = 5°C, θ_v = 0.27°C/min. (a) Temperature distribution; (b) streamline; (c) sketch of stream line. Elapsed time: (1) 1 min, (2) 11 min, (3) 15 min, (4) 25 min.

out the tube. The water temperature has dropped below 4°C, and only the clockwise eddy remains.

Figure 6 shows the characteristics of dendritic ice growth. At 37 min and 56.04 s (1) into the experiment, dendritic ice starts to form from the edge of the air–water interface. A second later [37 min 57.09 s (2)] it starts spreading toward the tube bottom, and at 37 min 58.12 s (3) there is thick dendritic ice in the water. The dendritic ice growth is sudden after the water has been supercooled considerably (−4 to −8°C). The supercooling disappears when the dendritic ice appears, and the temperature of the remaining water turns to 0°C instantaneously as latent heat is released by the growth of the ice.

Freezing of Porous Media

Freezing heat transfer of liquid-saturated porous media is of practical importance in relation to a variety of engineering problems such as natural freezing of soil in cold regions, artificial freezing of the ground, freeze-treatment of sewage, and refrigeration of food. Katayama and Hattori [44] and Weaver and Viskanta [45] treated the freezing of a water-saturated porous medium as a heat condition problem. Okada [46] examined the freezing around a row of pipes embedded in a water-saturated porous layer. Sugawara et al [47] studied the freezing of a horizontal water-saturated porous layer. Viskanta and coworkers [48–50] determined both experimentally and analytically the freezing of a liquid-saturated porous bed in a rectangular cavity. Recently, transient freezing heat transfer of water-saturated porous media in a rectangular cavity was extensively investigated both experimentally and numerically by Sasaki et al [51–53], who pointed out that there were a variety of parameters that might control the freezing characteristics within the liquid-saturated porous bed and examined the effects of Darcy number based on particle size and Prandtl number, which is assessed by thermophysical properties of particles. It was found that the numerical predictions based on the boundary-fixing method were in good agreement with the experimental data.

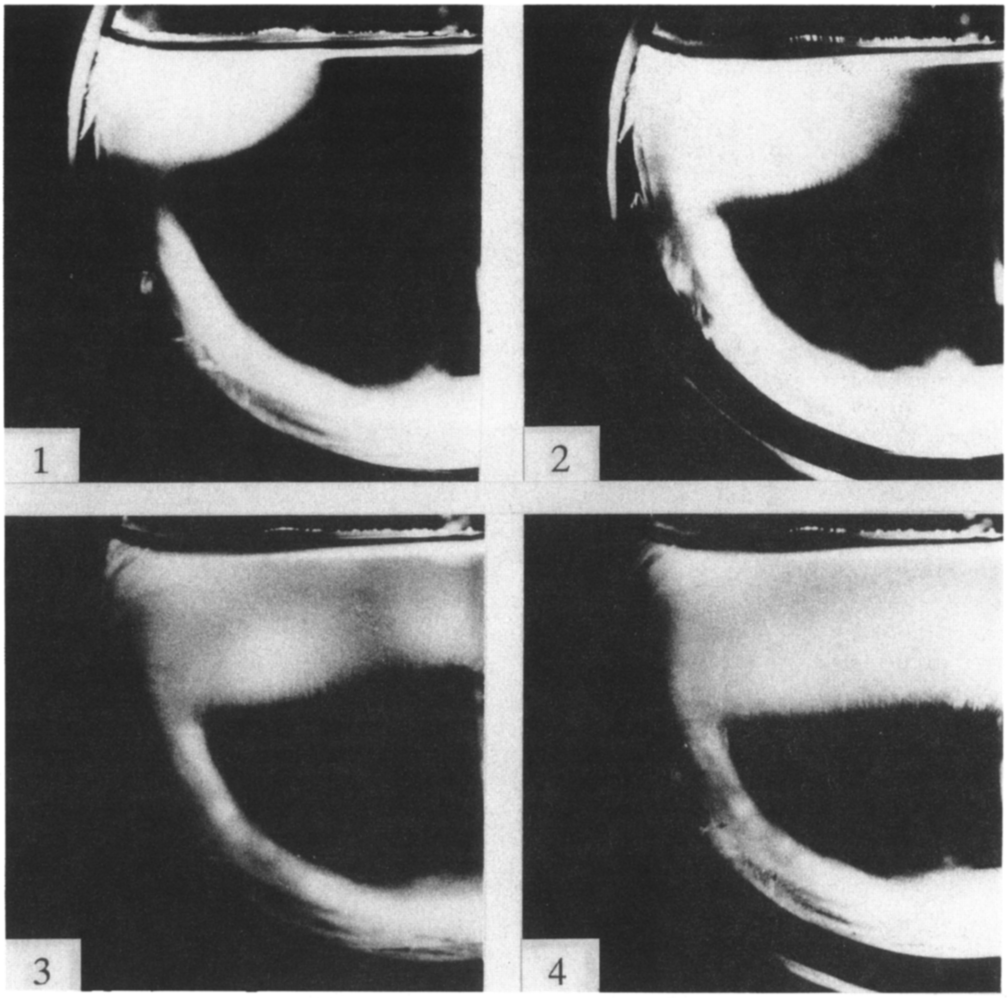

Figure 6. Growth of dendritic ice [43], Elapsed time: (1) 37 min 56.04 s, (2) 37 min 56.54 s, (3) 37 min 57.09 s, (4) 37 min 58.12 s.

FREEZING OF WATER WITH MAIN FLOW

Freezing Problems with Convection

In freezing problems where a convective flow exists at the phase-change interface, two additional complications with the basic nonlinearity of the transient phenomenon may take place. The first complication will arise from the fact that the phase change at the interface is equivalent to an effective blowing based on the difference in the specific volumes of the liquid and solid, which may alter the heat transfer coefficient at the interface. It has been found [54, 55] that when the Stefan number (St = $C\,\Delta T/L$, where C is the specific heat of liquid, ΔT is the reference temperature difference, and L is the latent heat of fusion) is less than about 0.1, this effect on the heat transfer coefficient can be neglected. The second complication is that there is mutual interaction among the shape of the solid–liquid interface, the flowfield next to it, and the temperature field.

Prusa and Yao [141] determined numerically the effect of density change on the unsteady two-dimensional heat transfer process of melting around a horizontal cylinder and concluded that the density change taking place upon melting has a negligible effect on the temperature field and heat transfer rates.

Freezing on a Horizontal Flat Plate

Transient freezing of a forced laminar flow over a cooled flat plate was theoretically studied by a number of investigators [56–58]. It was in general assumed in their studies that the ice layers formed were thin enough that streamwise heat conduction within the ice and the effects of the ice layer on the flow over the plate could be neglected.

Hirata et al [59, 60] and Gilpin et al [61] observed phenomena quite different from those assumed in the analysis. In the laminar regime, it was found that under steady-state conditions the ice layer thickness increased monotonically with increasing distance from the leading edge of the plate. In the transition regime, they found that there were two distinctly different transition modes (smooth and step transitions) depending on the mutual interactions among the shape of the ice surface, the fluid flow over the surface, and the heat transfer. In the turbulent regime, it was observed that for values of $\theta_c [= (T_f - T_w)/(T_\infty - T_f)$, where T_f is the fusion temperature, T_w the cooled wall temperature, and T_∞ the fluid temperature] greater than 12, the wavy ice–water interface became unstable due to an interaction between the ice surface and the fluid flow. (The ice layer does not develop monotonically, but its thickness presents a cyclic variation between increasing and decreasing like a water wave surface.)

Freezing Between Horizontal Parallel Plates

The freezing of liquids in a forced flow between parallel plates was studied analytically by a number of researchers [62–64]. In the turbulent flow prediction, as in laminar cases, it was tacitly assumed that a smooth and stable ice–water interface might exist.

Seki et al [65–67] carried out a series of experimental studies on ice-formation phenomena and heat transfer characteristics for water flow between horizontal parallel plates. They [65] observed two different types of ice formations: a transition type and a smooth type, as shown in Fig. 7, where Re_H is the Reynolds number defined as HU_m/ν (H is the depth between parallel plates, U_m the mean velocity, and ν the kinematic viscosity) and θ_c is the cooling temperature ratio. It was found that the transition ice formation type occurred for $Re_H/\theta_c^{0.741} < 10^4$, while the smooth ice formation type occurred for $Re_H/\theta_c^{0.741} > 10^4$. Seki et al [67] also studied the effect of an orifice installed at the leading edge of the horizontal parallel plates on ice formation and pressure loss characteristics and found that the relative increase in the pressure loss resulting from ice formation was markedly less than without the orifice.

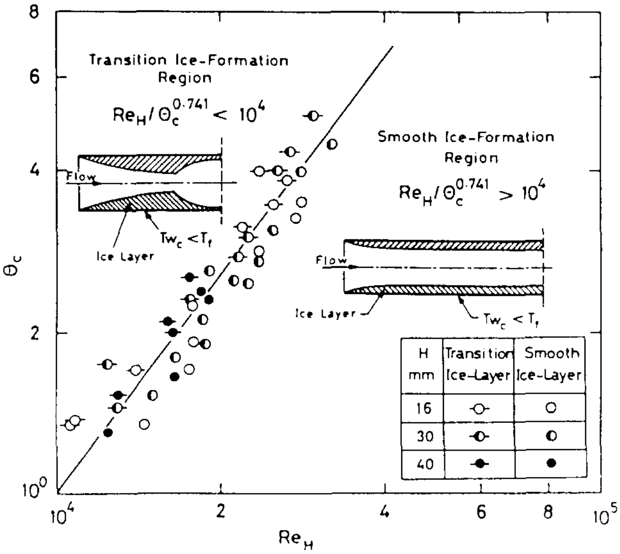

Figure 7. Ice-layer profile [65].

Freezing in a Horizontal Circular Tube

Analysis of Zerkle and Sunderland The problem of steady-state freezing and pressure drop in a horizontal circular tube with laminar flow was first analyzed by Zerkle and Sunderland [68], who assumed that the entrance effects could be neglected and that a parabolic velocity profile was maintained throughout the entire region and then reduced the problem to the classical Graetz problem. In the framework of the analysis, a number of studies were carried out for laminar flow [69–71] and for turbulent flow [72–75]. Ozisik and Mulligan [76] and Cho and Ozisik [77] extended their analysis to transient freezing by assuming a slug-flow velocity profile and quasi-steady-state heat conduction in the ice layer.

Observed Phenomena Gilpin [78–80] observed extensively the ice formation phenomena in a cold pipe containing flows and found that three growth modes might exist depending on the flow velocity. The first growth mode, which was observed for relatively high main flow, was similar to that analytically predicted by previous investigators. However, near the transition Reynolds number, Gilpin [80] found that the final steady-state ice did not exhibit a uniformly tapered flow passage but that there was a flow passage with a cyclic variation in the cross section along the length of the tube, as shown in Fig. 8. There are five cycles of expansion and contraction of the flow passage (the ice band; S is the distance between separation points on consecutive ice bands, and D is the inner tube diameter) for $\theta_c = 11.8$. Ice-band spacing decreases sharply with θ_c for small values of θ_c and then approaches a constant value of about 6 for $\theta_c > 10$.

Freeze-off Condition Epstein and coworkers [81, 82] analytically and experimentally studied the solidification of a liquid at its fusion temperature as it penetrates an initially empty tube cooled on the outside. A mathematical model for the criteria that predicts the conditions of freeze-off was proposed by Sampson and Gibson [83, 84]. Hirata and Ishihara [85] analytically and experimentally studied the condition for the onset of freeze-off in a pipe containing a flow of water using copper and steel tubes. They found that the pipe freeze-off first took place at the contraction region of the ice bands, and its condition was given as a relationship between the cooling temperature ratio θ_c and the Reynolds number. The freeze-off condition for laminar water flow in a tube was determined by Chida [86]. Thomason [87] investigated the effect of a variety of parameters on the onset of freeze-off in water flow in a tube.

Freezing of a Curved Tube The freezing of a curved tube is of great importance because of many kinds of uses in industrial equipment. Inaba et al [88, 89] studied the freezing characteristics of a variety of curved pipes. Chida and Tajima [90] also determined the freeze-off conditions for a 90° curved tube.

Freezing in a Return Bend

Freezing heat transfer in curved ducts is of considerable importance because of its wide application in piping design. Freezing behavior and freezing heat transfer characteristics in curved ducts were studied experimentally by Fukusako et al [91] and Tago and Fukusako [92], who determined the effects of a variety of the parameters (fluid velocity, duct depth, cooling temperature ratio, and Dean number). They found that the "smooth" ice transition occurred on the concave wall, while the "step" ice transition occurred on the convex wall, and that the mean heat transfer coefficients increased with both increasing

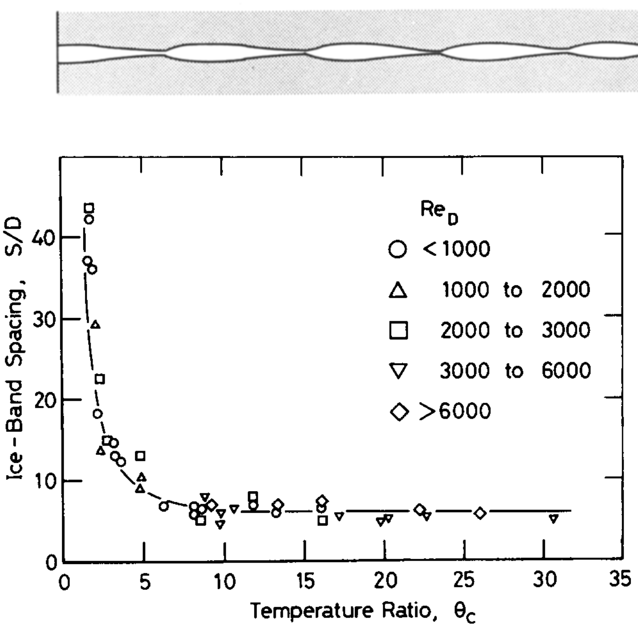

Figure 8. Ice-band structure [80].

duct height and decreasing radius of curvature in the return bend. Figure 9 shows that the grooves with the regular wave number form along the ice surface formed on the concave wall, which appears to be based on Görtler vortices.

Freezing of Layered Air–Water Flow

For a large-scale pipeline and a drain pipe, in which an air phase and liquid phase coexist, little was known about the freezing characteristics of layered air–water flow in a pipe. Fukusako and Takahashi [93–95] investigated extensively the effects of water depth, inlet water and air temperatures, wall temperature, and water-flow and airflow rates on both freezing behavior and freeze-off characteristics in horizontal tubes.

ICE ACCRETION

Atmospheric Icing

Ice accretion is well known as atmospheric icing on trees and towers, which is essentially caused by the freezing of small water droplets carried by an airstream. Aircraft and helicopter icing can occur on the ground as well as in the air under particular meteorological conditions with temperatures ranging from 0 to −40°C. The physics of the ice accretion process, its causes, and preventive measures appear to be well investigated. The literature on atmospheric icing is extensively reviewed by Cheng and Yen [96].

The factors affecting the rate of ice accretion are air temperature, atmospheric water content, number and size distribution of water droplets, and wind velocity. Kuroiwa [97] found that accretions of ice on ground structures occur in four forms as shown in Fig. 10: (1) air hore forms when wet air impinges on an object cooled to below 0°C, (2) soft rime is observed when the air temperature and wind velocity are low and the diameter of the supercooled water droplets is small, (3) the condition for hard rime (clear ice) is between that of soft rime and glaze (4), which forms when the air temperature and wind velocity are high and the diameter of supercooled water droplets is small. Recent literature on atmospheric icing on structures is well reviewed by Minsk [98].

Marine Icing

In northern winter seas, ice accretion on fishing boats takes place frequently due to freezing seawater spray. These accumulations can greatly affect the stability of the ship and the safety of the crew. There have been more than 25 serious accidents due to seawater spray icing since 1960 in the sea near Hokkaido, taking the lives of over 300 crew members [99].

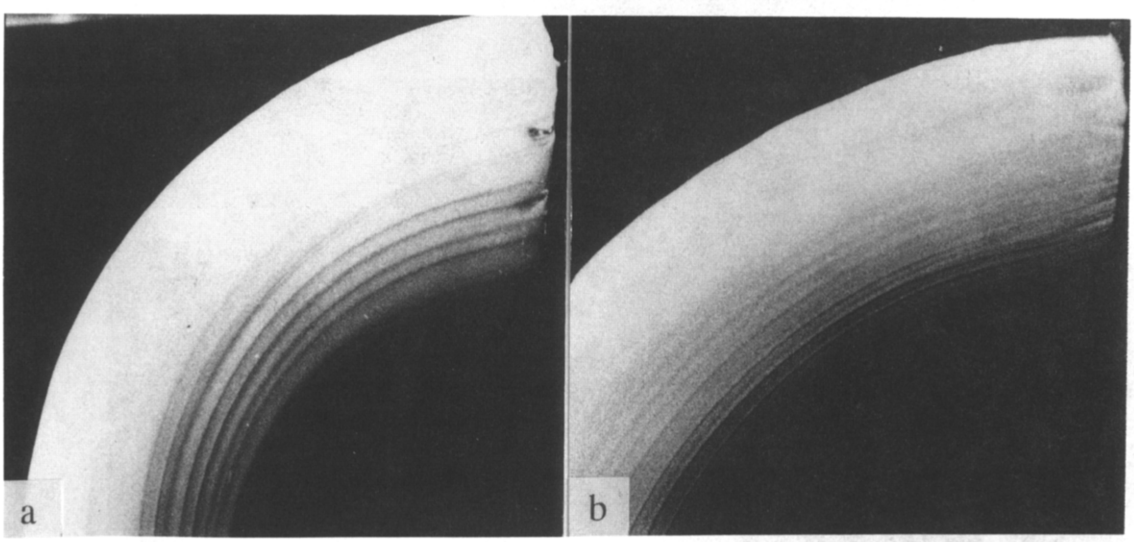

Figure 9. Ice-layer along a concave wall [91]. (a) $U = 0.01$ m/s, $\theta_w = -10°C$, $\theta_{in} = 3°C$; (b) $U = 0.05$ m/s, $\theta_w = -9°C$, $\theta_{in} = 1°C$.

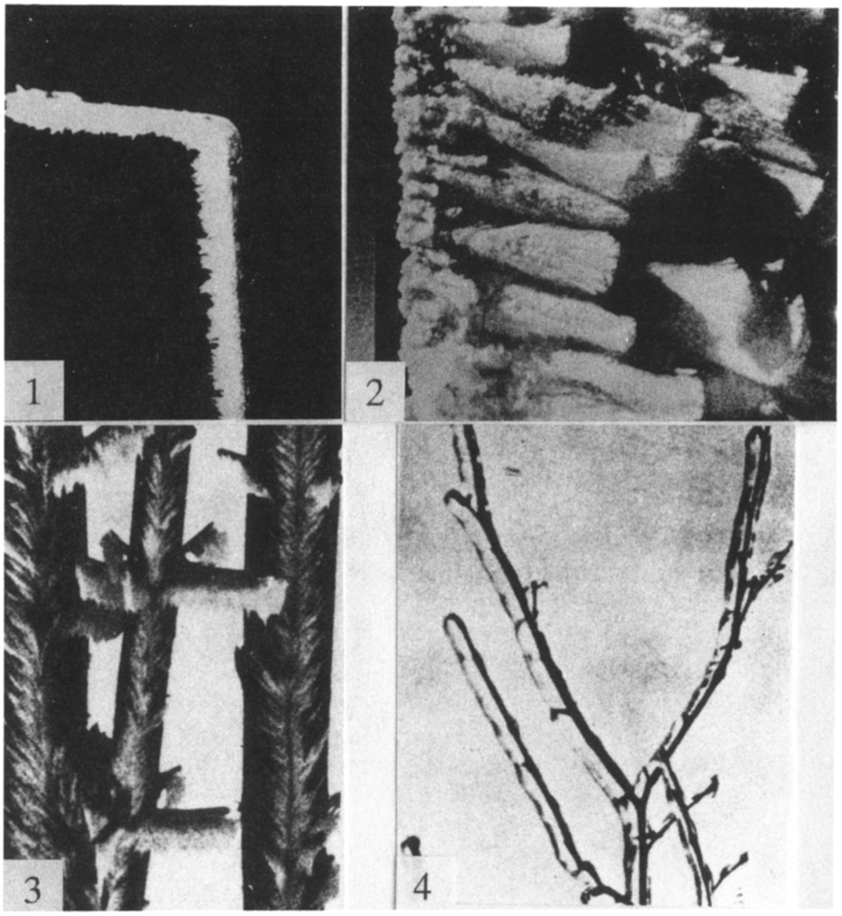

Figure 10. Icing phenomena [97]. (1) Air hore; (2) soft rime; (3) hazard rime; (4) glaze.

The literature on marine icing is reviewed by Lock [100] and Lozowski and Gates [101]. A comprehensive account of ice accumulation on ships is given by Itagaki [102], Horjen [103], and Makkonen [104]. Fukusako et al [105] studied experimentally the characteristics of marine icing on a horizontal circular cylinder immersed in a cold airstream accompanied by seawater spray. They determined the effects of airstream velocity, air temperature, droplet diameter, initial droplet temperature, and mass flow rate of droplets. Figure 11 shows the effect of airstream velocity on the ice accretion characteristics along a cylinder. As the airstream velocity increases, the ice morphology tends to become a reverse triangle, while the icicle hanging from the bottom of the cylinder tends to disappear gradually. This may be due to the fact that (1) the brine droplets tend to avoid the cylinder owing to the

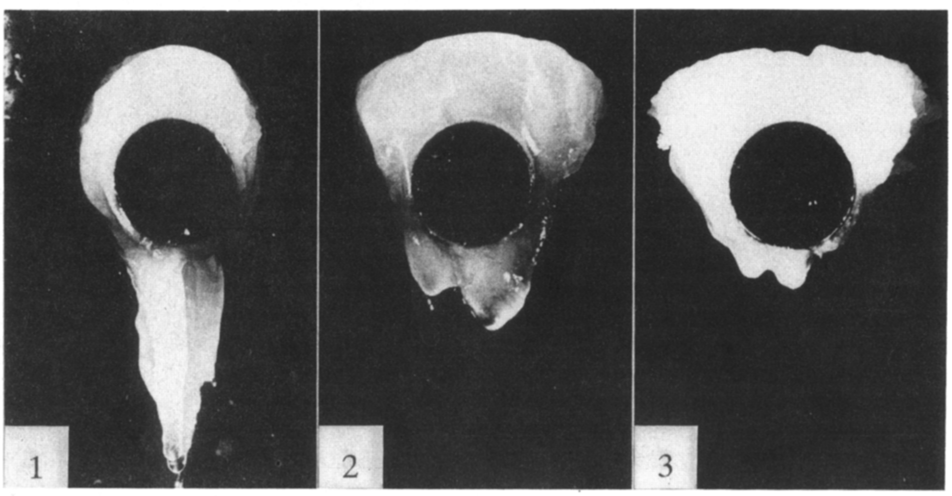

Figure 11. Effect of air velocity on icing [105]. (1) 6 m/s; (2) 10 m/s; (3) 20 m/s.

drag force operating in the direction of the wind streamline and consequently impinge on the ice surface downstream from the point where they should impinge on it, (2) the dispersing rate in the droplets impinging on the cylinder increases, and (3) the heat transfer coefficient at the air–ice interface increases.

MELTING OF ICE

Melting of Horizontal Ice Layer

Melting Forms In a horizontal melt layer of ice heated from above or below, four typical situations may occur due to the peculiar characteristics of water, which exhibits a density inversion at 4°C, as shown in Fig. 12. When heated from above, the fluid layer consists of both a potentially stable and a potentially unstable layer if the surface temperature is higher than 4°C, whereas the entire fluid layer is potentially unstable if the surface temperature is in the range of 0–4°C, as shown in Figs. 12a and 12b. On the other hand, when heated from below, there will be an unstable liquid layer only when the lower boundary temperature is greater than 4°C, as shown in Fig. 12d.

Melting with Free Surface When a horizontal melt layer of lake or river ice is heated from above, for instance by solar energy, it corresponds to Figs. 12a and 12b. Wu and Cheng [106] and Seki et al [107] determined the critical Rayleigh number, $Ra_c [= g|\beta|(T_m - T_1)/\kappa\nu$ for $T_2 > 4°C$, or $= g|\beta|(T_2 - T_1)/\kappa\nu$ for $T_2 \leq 4°C$, where β is the coefficient of thermal expansion, T_m the temperature at maximum density, T_1 the ice–liquid interface temperature, and T_2 the water surface temperature], marking the onset of free convection in a horizontal melt water layer with a free surface. A graph comparing the experimental and theoretical critical Rayleigh number Ra_c is presented in Fig. 13. For $T_2 > 8°C$, the values of Ra_c are nearly uniform because of the considerable thickness of the upper stable layer, while the critical Rayleigh number for $T_2 < 8°C$ increases and takes the value $Ra_c = 3000$ for $T_2 = 4°C$. For the normal liquid without density inversion,

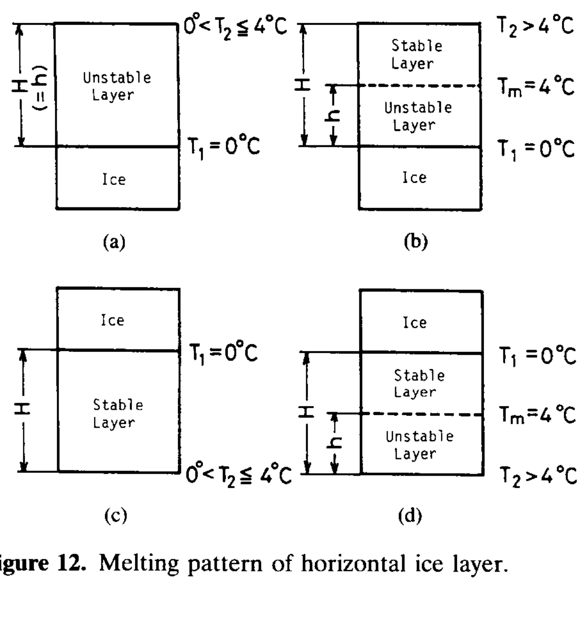

Figure 12. Melting pattern of horizontal ice layer.

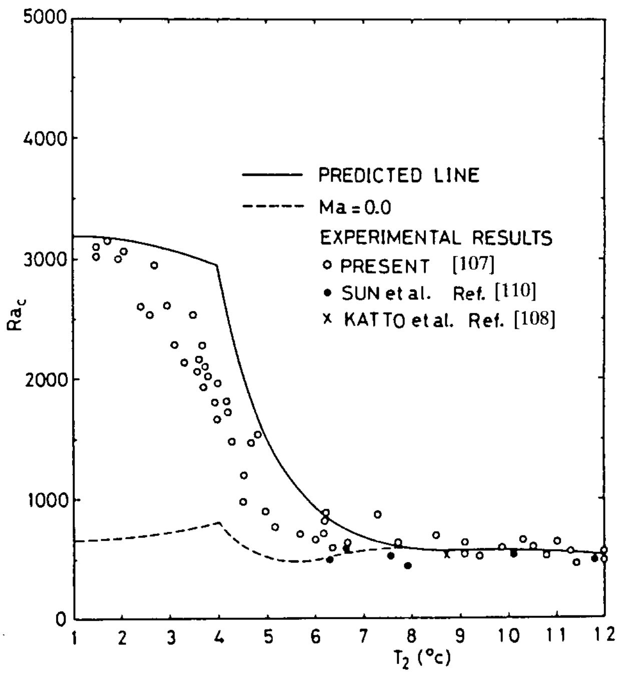

Figure 13. Onset of free convection with free surface [107].

the direction of the surface tension force is identical to that of the liquid flow based on the onset of free convection; thus it may promote the onset of free convection. On the other hand, for the melt-water layer with density inversion, the direction of the surface tension force is opposite to that of liquid flow; thus the surface tension may retard the onset of free convection.

Melting by Heating the Wall from Above or Below Katto and Iwanaga [108] showed that the critical Rayleigh number that both they and Seki et al [109] obtained agreed well with that given by a modification of the analytical results obtained by Sun et al [110]. Seki et al [109] found both experimentally and numerically that the onset of free convection and free convection heat transfer were markedly affected by the temperature of the upper wall for $T_2 < 8°C$, unlike the results obtained for common fluids that do not undergo a density inversion. Merker et al [111, 112] determined the critical Rayleigh number by defining a modified Rayleigh number that used the height of the complete fluid layer rather than the height of the unstable layer.

Melting by a Radiant Heat Source The melting problem of a horizontal ice layer sticking to a substrate was studied experimentally and analytically by Gilpin et al [113], Seki et al [114, 115], and Cho and Ozisik [116] using short-wave radiant energy. Figure 14 shows a typical pattern of radiative melting in a horizontal clear ice layer sticking to insulation with a radiative black surface. It is clearly seen in the figure that there exist both an upper melt layer and a lower melt layer based on "back melting." Chan et al [117] proposed a melting model to account for two-phase zones induced by internal thermal radiation between the liquid layer and the pure solid layers. They did not, however, take the "back melting" phenomena into consideration in the analytical model.

Melting of a Vertical Ice Layer

Melting in Cold Water A number of studies on melting heat transfer of a vertical ice layer immersed in cold water were carried out. Wilson and Lee [118] reported that their analysis did not yield stable results for $4.5°C < T_\infty < 5.7°C$ because this flow regime might be transitory. Carey and Gebhart [119] observed that as the ambient water temperature increased from 3.9 to 8.4°C, the regimes of upward, locally bidirectional, and downward flows took place. They claimed that the experimentally obtained velocity profile and heat transfer for $T_\infty < 4.05°C$ and $T_\infty > 5.9°C$ were in good agreement with the analytical results of Carey et al [120].

Melting with Vapor Condensation When an ice surface is exposed to hot air containing some amount of vapor, melting of the ice will occur because of both the heat released by vapor condensation on the ice surface and the heat transferred from the ambient hot air. Thus, the heat transfer situation becomes somewhat complicated because of simultaneous vapor condensation and melting of the condensing surface. This phenomenon was initially studied by Tien and Yen [121]. Epstein and Cho [122] determined the two parameters that mainly control the phenomenon. The effects of noncondensable gas [123] and immiscibility [124] on the vapor condensation melting were reported.

Melting by Radiant Heat Source Seki et al [125] studied radiative melting of a vertical ice layer adhering to a substrate both experimentally and analytically. They used a halogen lamp having a large fraction of short-wave beams and found that heating by short-wave radiation produced a peculiar melting pattern on the rough melting surface, caused by the internal melting at the grain boundary of the ice surface.

Melting of Spherical or Cylindrical Ice

Merk [126] calculated the local heat transfer around a melting sphere of ice and predicted that for an environmental water temperature of $T_\infty = 5°C$ the Nusselt number was at a minimum and the direction of the flow changed from upward at lower temperatures to downward at higher temperatures. Dumore et al [127] carried out an experiment on melting ice spheres and found that the flow was upward for $T_\infty < 4.8°C$ and downward for $T_\infty > 4.8°C$ and that a convective inversion took place at $T_\infty = 4.8°C$. Measurements of free convective heat transfer for an ice sphere for T_∞ ranging from 0 to 10°C were performed by Schenk and Schenkels [128], who claimed that a dual flow existed for T_∞ ranging from 4 to 6°C. Vanier and Tien [129] determined experimentally the melting characteristics of a sphere of ice in water between 0 and 20°C and found that a convective inversion occurred at 5.35°C.

The problem of free convection heat transfer of a horizontal ice cylinder immersed in water at its maximum density was studied both numerically and experimentally by Saitoh [130], who reported that a minimum Nusselt number existed around $T_\infty = 6°C$.

MELTING OF ICE IN SALINE WATER

When an ice slab melts in saline water, complicated transport phenomena arise in the boundary-induced flow

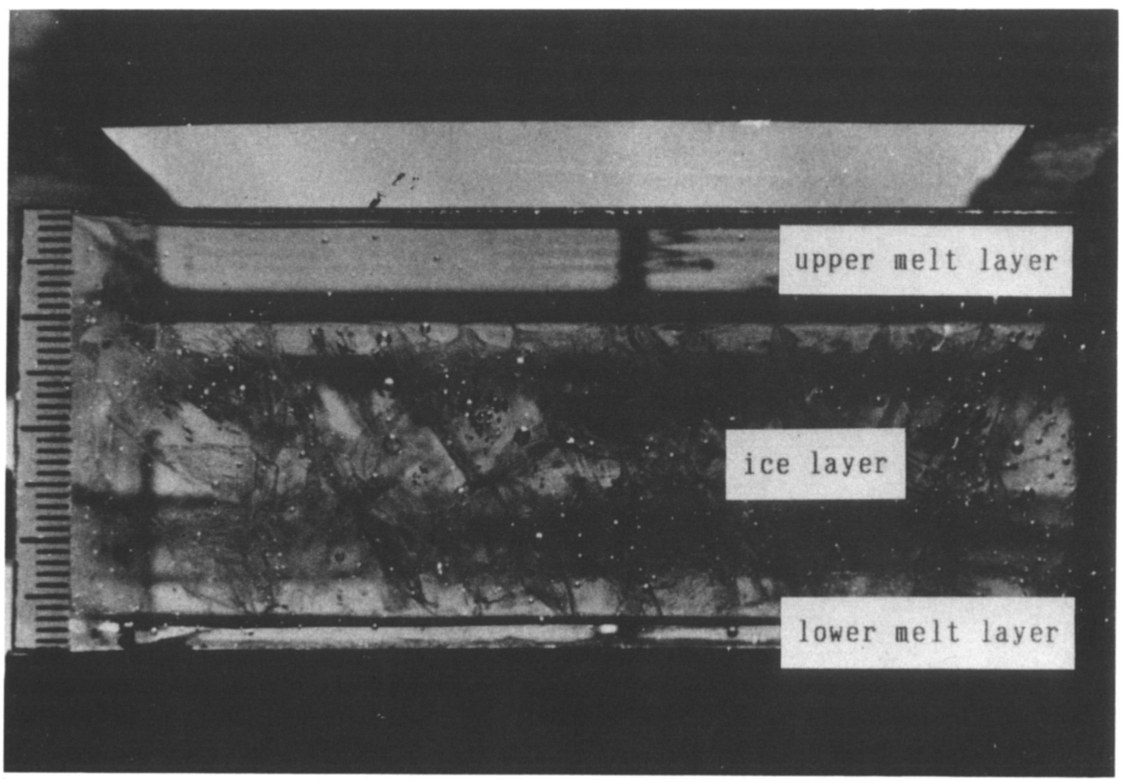

Figure 14. Back-melting phenomenon of horizontal ice layer [114].

adjacent to the ice, owing to the coupled effects of thermal and saline diffusion on the motion-causing buoyancy force, resulting in considerable added complexity. It is well known that upwelling produced by melting icebergs can strongly influence the supply of nutrients to Antarctic surface water.

Melting with Main Flow

The forced convection heat transfer process along melting horizontal flat ice in saline water under laminar flow conditions was studied by Griffin [131] using the heat-balance integral method. He estimated the relative thickness of the momentum and mass diffusion layers for a variety of flow and thermal parameters.

Melting with No Main Flow

Melting of a Vertical Ice Plate Huppert and Turner [132] investigated experimentally the effect of ambient stratification on the flow and transport adjacent to a vertical ice surface melting in seawater. The same problem was analytically studied by Marschall [133]. Johnson [134] determined experimentally the transport characteristics of a vertical flat ice slab melting in seawater. He used a schlieren system and found that the flow was upward and laminar at low ambient temperatures and that at higher ambient temperatures there was a transition to turbulence. Josberger and Martin [135] carried out an excellent and extensive observation of the flow along the melting vertical ice surface in saline water. They found that for ambient temperatures T_∞ less than about 18°C the flow was laminar near the ice surface and bidirectional near the bottom, while near the top of vertical ice the flow was fully upward and turbulent. The same problem was studied both experimentally and numerically by Carey and Gebhart [136], Sammakia and Gebhart [137], and Johnson and Mollendorf [138].

Melting of a Horizontal Ice Cylinder Melting heat transfer characteristics of a horizontal ice cylinder immersed in seawater were determined experimentally by Fukusako et al [139]. Figure 15 shows the variations of the ice cylinder profiles and flow patterns with time elapsed from the start of the experiment. The deviation of the ice–liquid interface from the circular shape results from strong free convection effects. Along the lower portion of the ice cylinder, the laminar velocity field is bidirectional, consisting of a quite narrow upward inner flow adjacent to the ice cylinder inside a wider downward outer flow. Along

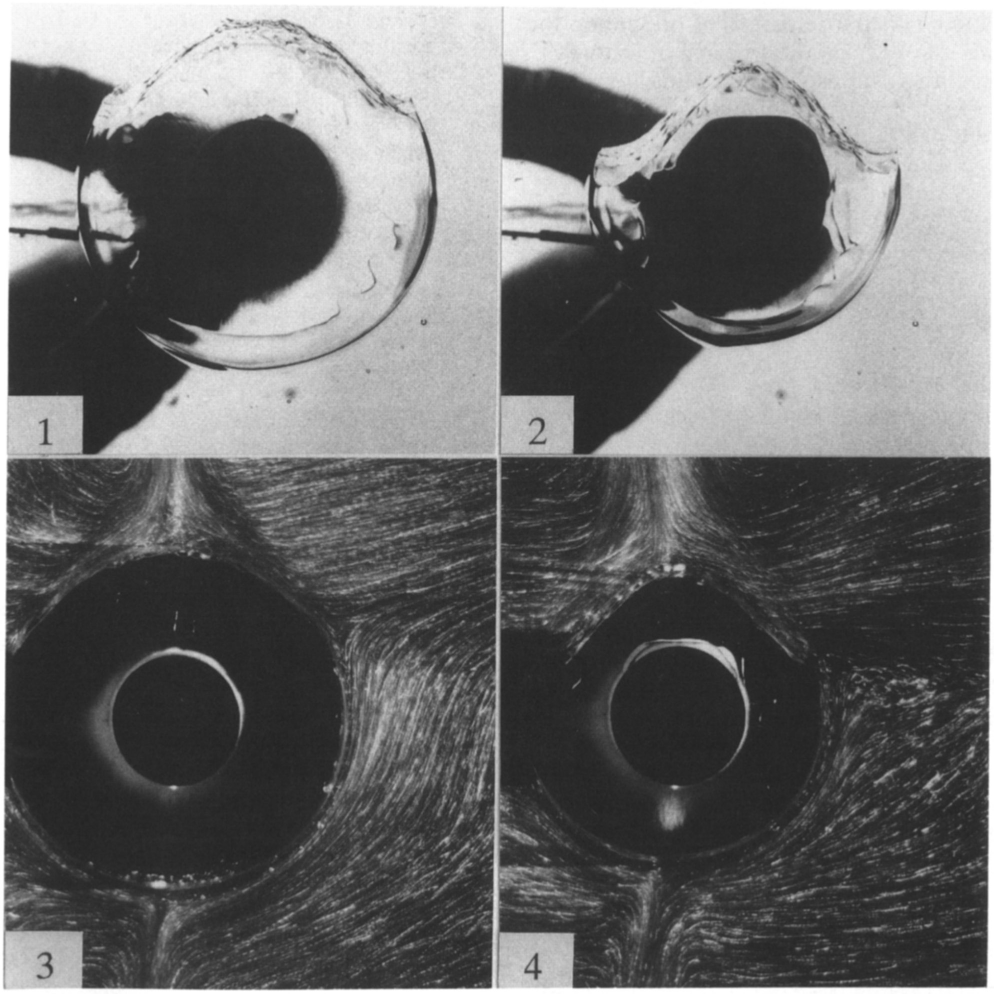

Figure 15. Melting ice profile and flow pattern [139]. T_∞ = 2.8°C. Elapsed time: (1) 40 min, (2) 80 min, (3) 15 min, (4) 60 min.

the upper portion of the ice cylinder, the velocity field is unidirectional and there is upward turbulent flow above the step, which probably results from the abrupt increase in the heat transfer coefficient owing to the transition from laminar to turbulent flow. At the level of the transition to turbulence, it appears that turbulent diffusion may transport both dilute saline water and upward momentum away from the ice over a horizontal distance that is comparable to the thermal boundary layer thickness. This upward buoyancy and momentum may overcome the downward buoyancy so that the net result is an upward-flowing turbulent boundary layer. Then it seems that the resultant divergence between the upward turbulent flow and the downward laminar thermal flow may produce a horizontal jet of ambient saline water flowing toward the ice cylinder.

Melting of an Inclined Ice Plate The melting of an inclined ice plate was studied experimentally by Fukusako et al [140], who found that the melting characteristics of an inclined ice plate facing upward were quite different from those of an inclined ice plate facing downward.

CONCLUDING REMARKS

A brief review was carried out of water-freezing and ice-melting problems under a variety of conditions. This article was prepared with the intention of presenting the recent advances in research on the freezing of water with and without main flow, atmospheric and marine icings, and the melting of ice in a variety of situations. Attention was also directed to the thermophysical properties of pure ice and sea ice. Further developments in this field are expected.

NOMENCLATURE

- C_p specific heat, kJ/(kg · K)
- D diameter of tube, m
- g gravitational acceleration, m/s^2
- H depth between parallel plates or thickness of melt layer, m
- h thickness of unstable layer, m
- k_ν absorption coefficient, cm^{-1}
- L latent heat of fusion, kJ/kg
- Ra_c critical Rayleigh number [$= g|\beta|(T_m - T_1)h^3/\kappa\nu$ for $T_2 > 4$°C; $= g|\beta|(T_2 - T_1)H^3/\kappa\nu$ for $T_2 \le 4$°C], dimensionless
- Ra'_c critical Rayleigh number ($= g\beta\Delta T h^3/\kappa\nu$), dimensionless
- Re_D Reynolds number ($= U_m D/\nu$), dimensionless
- Re_H Reynolds number ($= U_m H/\nu$), dimensionless
- S ice-band spacing, m
- Ste Stefan number ($= C_p\ \Delta T/L$), dimensionless
- T temperature, K
- ΔT temperature difference between top and bottom of unstable layer, K
- U velocity, m/s

Greek Symbols

- α_ν coefficient of cubic thermal expansion, K^{-1}
- β thermal expansion coefficient, K^{-1}
- θ temperature, °C
- θ_c cooling-temperature ratio [$= (T_f - T_w)/(T_\infty - T_f)$], dimensionless
- κ thermal diffusivity, m^2/s
- λ thermal conductivity, W/(m · K)
- ν wavelength, μm or kinematic viscosity, m^2/s
- ρ density, kg/m^3

Subscripts

- 1 ice-band interface or lower wall
- 2 water surface or upper wall
- f fusion
- i ice layer
- in inlet
- ini initial
- m mean or maximum
- w wall
- ∞ environment

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