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Experimental analysis of a transfer function for an air cooled evaporator

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Abstract

A transfer function to model a direct expansion air cooled evaporator, inserted in a vapor compression refrigeration plant, is deduced by means of experimental analysis. For inlet air temperatures onto the evaporator and refrigerant mass flow rate variable in appropriate ranges, the evaporator dynamic behavior is simulated by a linear model with delay. The results of transfer function are compared with experimental data, obtained by applying both step inputs and periodic changes to the refrigerant mass flow rate. The influence of the hunting, typical of a thermostatic expansion valve, is also estimated experimentally and then validated by the transfer function, obtaining a good agreement. These results could be applied to obtain a control algorithm for the refrigerant mass flow rate feeding the evaporator, by varying the speed of the compressor motor. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Vapor compressor plant; Air cooled evaporator; Experimental analysis; Transfer functions

1. Introduction

This paper analyses the transfer function for a heat exchanger working as an air cooled evaporator in a vapor compression refrigeration plant. The study aims at obtaining a control

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Nomenclature

$\dot{m}$ refrigerant mass flow rate (kg/s)
$\Delta\dot{m}$ refrigerating mass flow rate variation (kg/s)
G transfer function
H unit step-function
K gain of the evaporator (Ks/kg)
s Laplace variable

Greek symbols
ΔT superheating (°C)
t time (s)
δ damping coefficient
θ time constant (s)
τ time delay (s)
ω frequency

Subscripts
n natural
o initial
r refrigerant
1 first-order model
2 second-order model

algorithm for the refrigeration capacity to optimise the energy consumption and particularly, to control the refrigerant mass flow rate by varying the compressor speed, in order to control the refrigerating load.

A number of studies have been done on modelling of the evaporator [1–8]. In particular, in [7], a mathematical model which governs the mechanism of the evaporator (*transparent box*) has been considered and the steady state has been analyzed by means of numerical and analytical linearized methods. In [8], the results of this analysis have been compared with the experimental working data and the compatibility of the model has been estimated.

The study of the dynamic behavior and transient response is complex as the mathematical model is described by nonlinear partial differential equations. For this reason, in this paper, an input–output approach is applied and the evaporator system is schematized as a *black box*. Black box models are commonly used to investigate control problems in refrigeration. With this type of modelling, an evaporator system can be represented by a set of transfer functions with several constants have been identified by experiments.

The experimental component is realized by varying the inlet air temperatures in the range 15–35°C and the refrigerant mass flow rate in the range 5–30 g/s. The experimental outputs caused by step-inputs of the mass flow rate cause the evaporator to act as a linear model with delay.

Finally, in order to estimate the suitability of the transfer function, the *hunting* phenomenon, typical of a thermostatic expansion valve, is experimentally investigated. When the transfer function is applied to this input, the related output is in good agreement with the experimental results.

2. Experimental plant

The test evaporator was installed in an experimental vapor compression plant working with the refrigerant fluid R22. The plant consisted of a semihermetic reciprocating compressor, a water coaxial counterflow condenser, a manual valve as expansion device in parallel with a thermostatic expansion valve, and an air cooled evaporator. An oil separator and a liquid receiver complete the refrigerant circuit (Fig. 1).

The evaporator scheme is shown in Fig. 2. It has two parallelly separated circuits with a total of 32 horizontally arranged tubes. The heat exchanger is plate-finned tube type, with the thermal contact between aluminium fin and the copper tube created by mechanical expansion. The total air side heat transfer surface area is 4.64 m^2. The outer diameter of the tube is 9.52 mm and its thickness is 0.41 mm, the fin pitch and thickness are 4.2 and 0.14 mm, respectively.

The evaporator is located in an insulated vertical channel (Fig. 3), where the air flow is

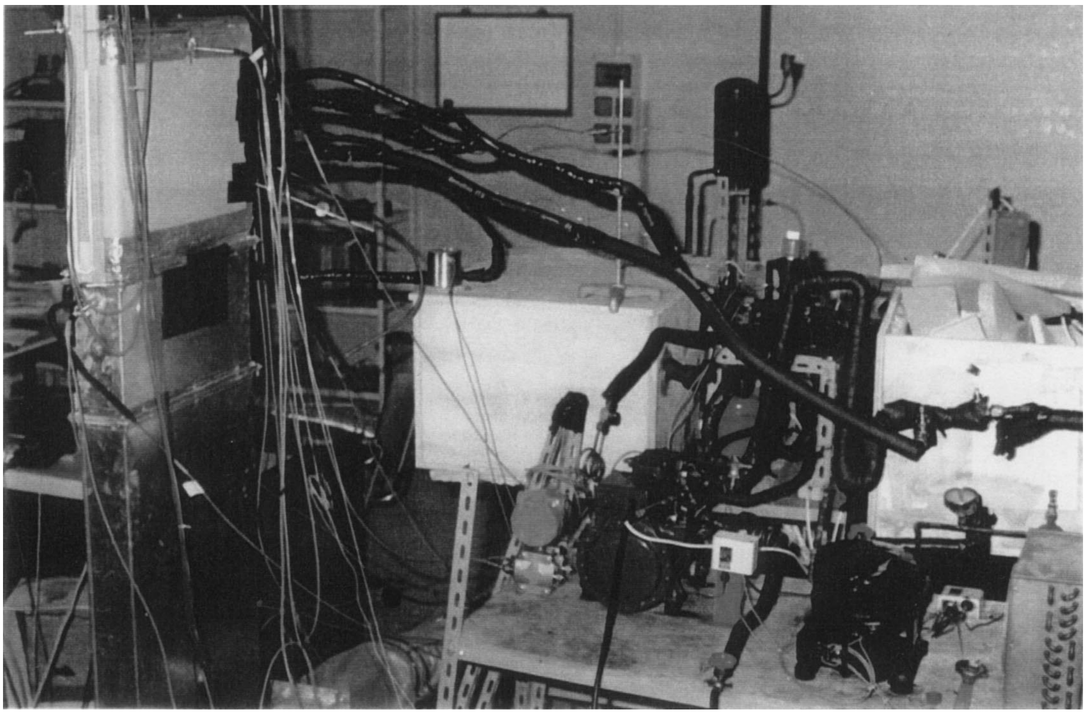

Fig. 1. Plant photo.

effected by a blower. At the channel inlet, the air is heated by means of electrical resistances that allow the air temperature to be varied. An Annubar Pitot tube measures the air velocity. The air then encounters the test evaporator and then is discharged into the atmosphere. A valve is used to control the air flow.

The length of the rectangular channel before the test evaporator is made equal to the entrance length so that the air acts uniformly along the inlet evaporator section. The average dry bulb air temperature at the inlet of the test section has been obtained by one thermoresistance mounted at the middle section, while five thermoresistances have been employed to determine the average air temperature at the outlet of the test evaporator section. To identify the air properties, the relative air humidity is measured at the inlet and at the outlet of the test evaporator by a glass probe cooled by means of the Peltier effect. The uncertainty declared by the manufacturer for the relative humidity is $\pm 1.0\%$.

The refrigerant mass flow rate is measured by a Coriolis-effect flowmeter, while piezoelectric pressure transducers measured the condenser outlet pressure and the test evaporator inlet pressure. To evaluate the enthalpy increase of the refrigerant, the temperatures at the outlet of the condenser and at the outlet of the test evaporator are also measured with two calibrated thermoresistances inserted in the refrigerant flow-stream. The extent of insulation of the experimental plant and the accuracy of the measured quantities allowed a good agreement

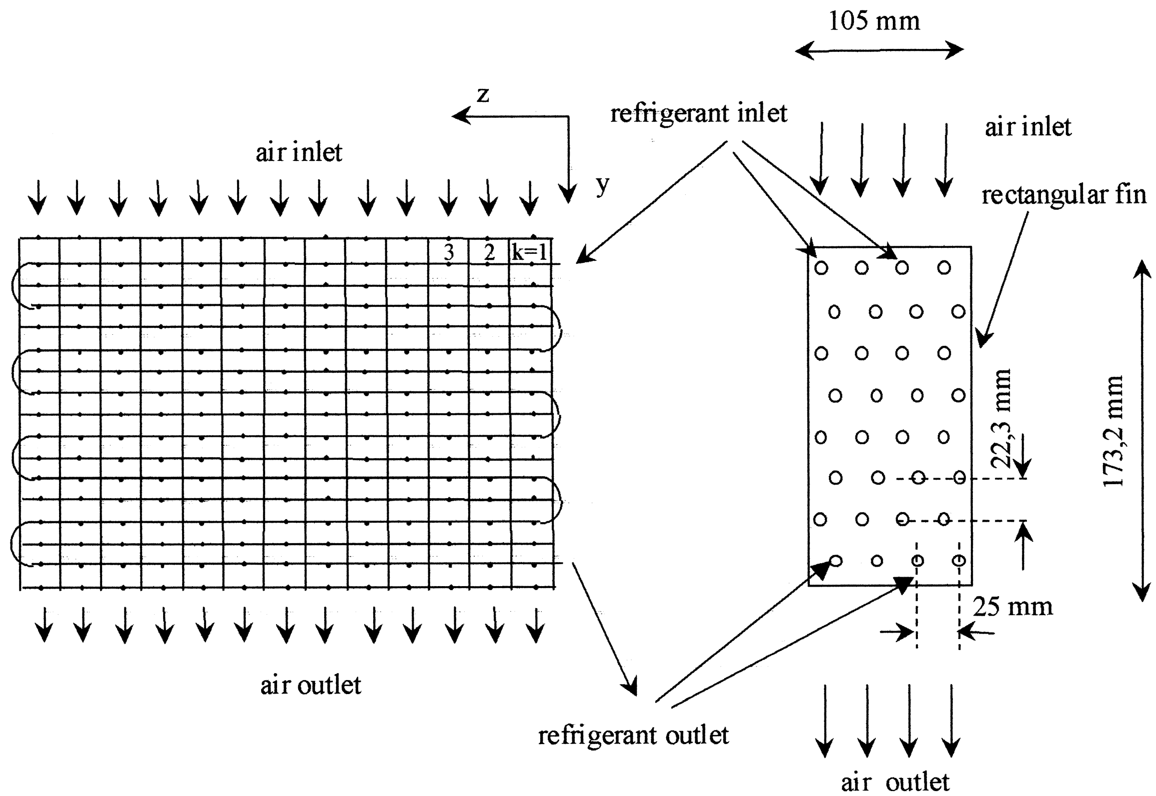

Fig. 2. Plan view of the test evaporator.

between the refrigeration power evaluated at the refrigerant side and the power exchanged by the air flowing onto the evaporator.

The experimental apparatus allows the possibility of varying the compressor motor frequency and voltage by means of an inverter linked to the three-phase electric motor of the compressor. This allowed a fair degree of capacity control.

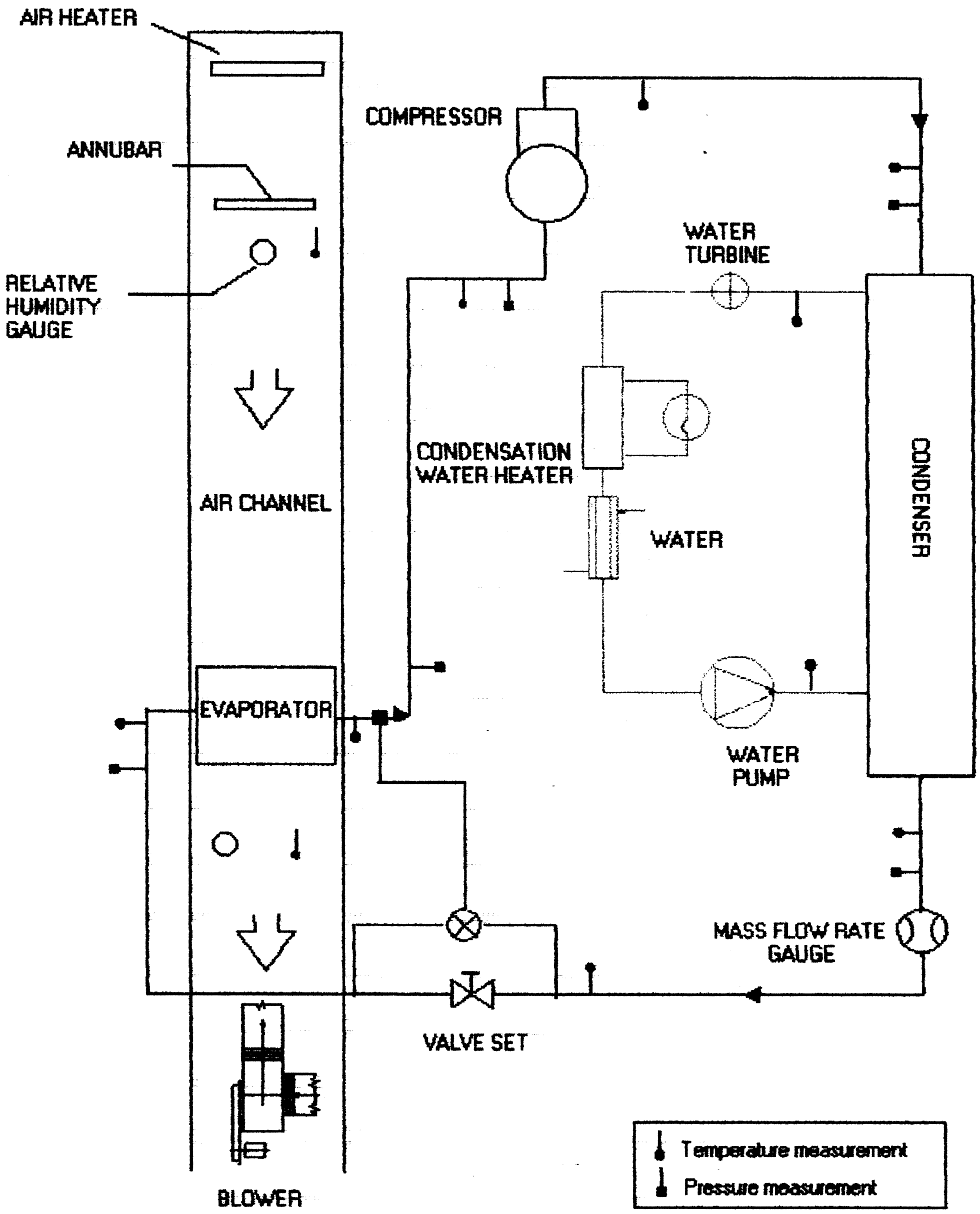

Fig. 3. Sketch of the experimental apparatus.

Table 1 lists the transducers specifications with the uncertainties reported by the manufacturers.

3. Experimental results and parametric identification

The first aspect of the control problem is to identify a possible transfer function for the evaporator, using the input–output approach (*blackbox*). As it is well known [9–14], typical examples of first- and second-order elementary systems with delay are characterized by the following Laplace transforms:

$$G_1(s) = \frac{\Delta T(s)}{\dot{m}_r(s)} = \frac{K e^{-\tau s}}{1 + \theta s} \quad \text{first-order model} \tag{1}$$

$$G_2(s) = \frac{\Delta T(s)}{\dot{m}_r(s)} = K \frac{\omega_n^2 e^{-\tau s}}{s^2 + 2\delta\omega_n s + \omega_n^2} \quad \text{second-order model} \tag{2}$$

These functions of the Laplace variable (s) are equal to the ratio between the transform of the refrigerant superheating at the evaporator outlet and the transform of the refrigerant mass flow rate. In particular, the constant gain K indicates the amount of superheating variation from one steady-state to another, with respect to the refrigerant mass flow rate variation. The delay time (τ) represents the time between the input change (mass flow rate) and the beginning of the outlet change (superheating). Moreover, the time constant (θ), related to the first-order systems (1), denotes the time necessary to reach a superheating of 63.2% of the final value, when a mass flow rate step input occurs. The meaning of the constants ω_n and δ, related to the model of the second-order (2), depends on the poles of $G_2(s)$ on the complex plane.

The experimental tests useful to analyze the evaporator dynamic behavior have been realized by considering the air volumetric flow rate on the evaporator to be a constant (400 m^3/h). On the contrary, the temperature of the air at the inlet of the evaporator has been changed in 5°C steps between 15 and 35°C. The variations in the refrigerant mass flowrate have been determined by means of a step input in the range 5–30 g/s, obtaining as result an evaporation temperature variable in the range −35–0°C. During this time the refrigerating capacity varies in the range 2–4 kW. The experimental apparatus also allows sudden variations of the

Table 1
Transducers specifications

Variables	Transducers	Range	Accuracy
Refrigerant mass flow rate	Coriolis effect	0–2 kg/min	±0.2%
Air volumetric flow rate	Annubar	0–1880 m^3/h	±1.0%
Temperature	RTD-Pt100 4 wires	−200 to 500°C	±0.1 K
Pressure	Piezoelectric effect	0–7 bar; 0–30 bar	±0.5% F.S.
Air relative humidity	Effetto Peltier	4–100%	±1.0%

refrigerant mass flow rate at the inlet of the evaporator, both by the manual valve opening and closing, and by an inverter that acts on the frequency and voltage of the current feeding the compressor motor. In each test, starting from steady-state conditions, a step-input has been set and the transient response has been measured, together with the new steady-state conditions. Generally, the response to the step input is rich in information about the system's dynamic behavior.

In most of the tests, in connection with the aforesaid values of the mass flow rate and the air temperature at the inlet, the gain fluctuates between 2500 and 3000 Ks/kg. Thus, in these cases, the evaporator dynamic behavior can be simulated by a linear model. For instance, the first-order model of Fig. 4 is related to a drop of the mass flow rate of 6 g/s (the evaporation temperature varies from −6.7 to −14.6°C) and the air temperature is of 35°C.

In a few cases, a non linear behavior typical of second-order models has also been observed. For instance, Fig. 5 is related to air temperature of 15°C and a drop of the mass flowrate of 3 g/s (the evaporation temperature varies from −16.2 to −21.1°C).

The constant gain is obtained by varying the mass flow rate at the inlet as mentioned above, waiting for the new steady-state conditions and evaluating the ratio between the obtained superheating variation and the refrigerant mass flow rate variation at the evaporator inlet. The evaporator gain is strictly related to the amount of the refrigerant contained in it. In fact, the higher the amount of the fluid refrigerant is in the evaporator at an instant, the smaller is the influence of the mass flow rate variation at the inlet. Therefore, increasing the refrigerant mass flow rate, the evaporator gain decreases (Fig. 6).

Further, the experimental plant allows, by means of an inverter, some sudden variations of the compressor motor feed frequency and voltage, that cause fast changes, comparable to the step inputs, of the refrigerant mass flow rate at the inlet of the evaporator. Consequently, the

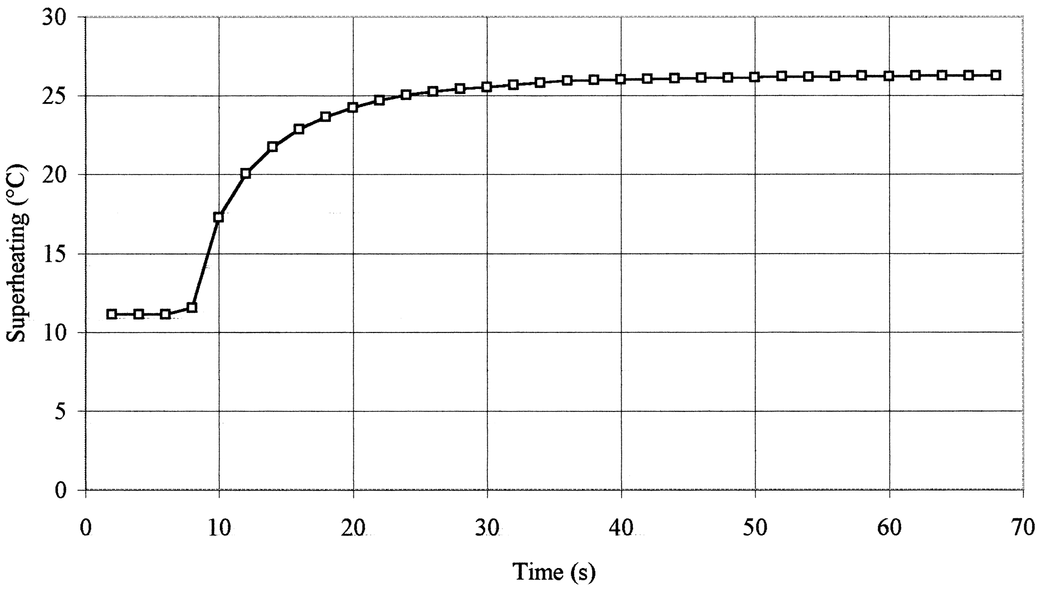

Fig. 4. Evaporator first-order response to step input.

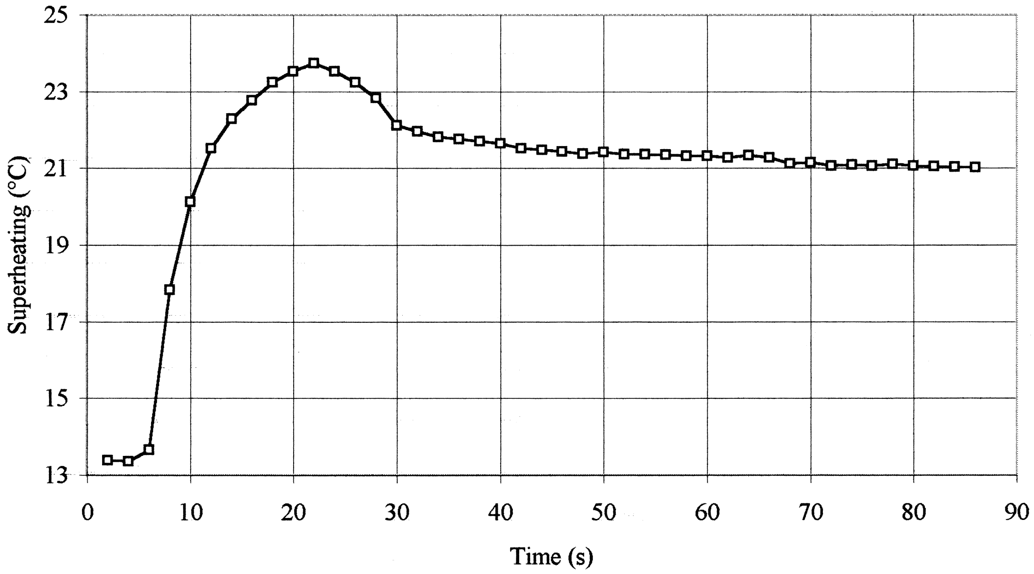

Fig. 5. Evaporator second-order response to the step input.

mass flow rate and the evaporation temperature increase with the compressor speed; therefore the gain decreases with these variables (Fig. 7).

Similar observations can be made to evaluate the time constant (θ) and the delay (τ). In particular, the time constant decreases with an increase in the mass flow rate, because the heat

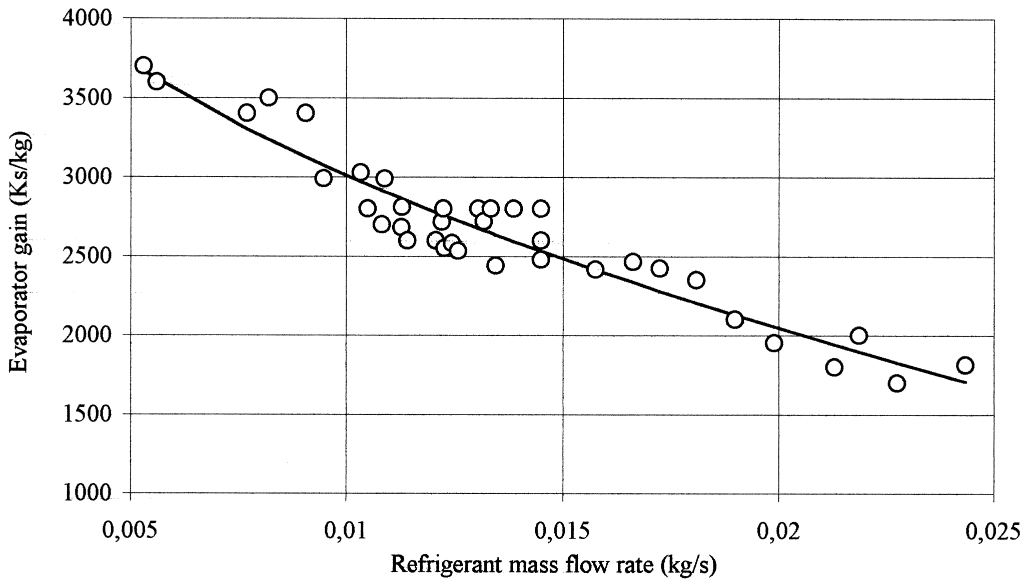

Fig. 6. Evaporator gain as a function of the refrigerant mass flow rate.

exchanged in the evaporator increases. For the time delay, effects are seen. Further, it has been observed that the decrease of θ and τ with the increased mass flow rate and the evaporation temperature is bounded. In fact, when the mass flow rate varies in the range of 5–30 g/s, then the constant time (θ) and the delay time (τ) fluctuate between 5 and 10 and 1 and 4 s, respectively. As for the second-order model, the parameters δ, e, ω_n can be identified by means of interpolating techniques and the use of Matlab.

Allowing $\Delta T - \Delta T_0$ to denote the variation of the superheating caused by a step-input $\Delta\dot{m}$, then the inverse Laplace transform of the transfer function $G_1(s)$ defined in Eq. (1) determines the first-order response, such that for all $t > \tau$ one has:

$$\Delta T = \Delta T_0 - K \cdot \Delta\dot{m}_r \cdot \left(1 - e^{-(1-\tau)/\theta}\right) \tag{3}$$

As for the second-order model characterized by the function $G_2(s)$ defined in Eq. (2), the response depends on the values of the constant δ. When $0 \leq \delta < 1$, the poles of $G_2(s)$ are complex conjugate and one has the damped harmonic response:

$$\Delta T = \Delta T_0 - K \cdot \Delta\dot{m}_r \cdot \left(\frac{e^{-\delta\omega_n(t-\tau)}}{\sqrt{1-\delta^2}}\sin\left[\omega_n(t-\tau) + \arccos\delta\right]\right) \tag{4}$$

where the constants δ and ω_n represent the damping coefficient and the natural frequency. On the contrary, when $\delta > 1$, the evaporator dynamic behavior is of a nonperiodic type and is simulated by the function:

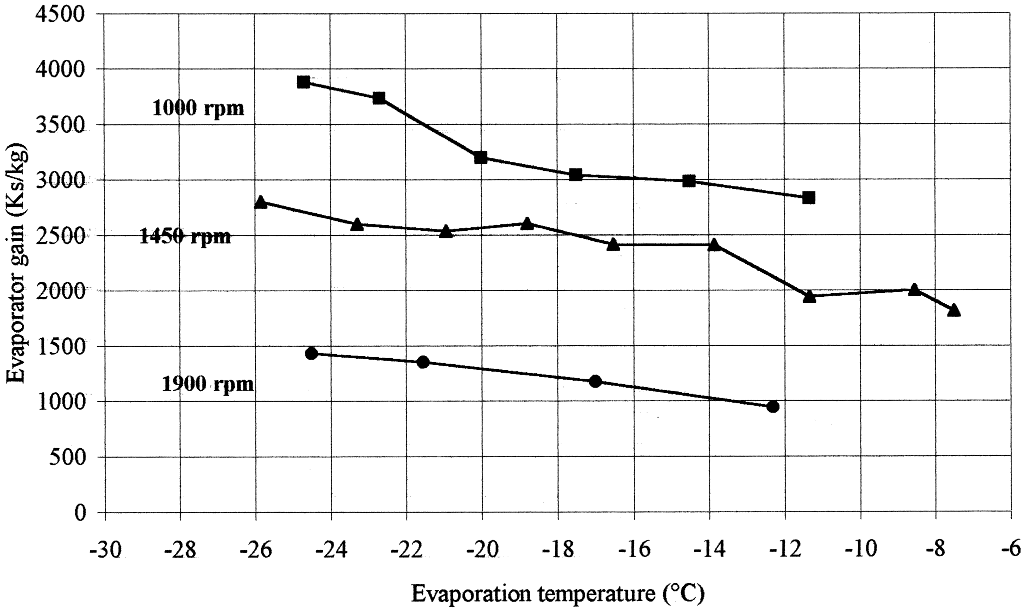

Fig. 7. Evaporator gain as a function of the refrigerant evaporation and of the compressor speed.

$$\Delta T = \Delta T_0 - K \cdot \Delta \dot{m}_r \cdot \left(A_1 \cdot e^{a(t-\tau)} + A_2 \cdot e^{b(t-\tau)}\right) \tag{5}$$

where the constants A_1, A_2, a and b are related to ω_n and δ by means of the known formulae [8]:

$$a = \omega_n\left(-\delta + \sqrt{\delta^2 - 1}\right); \quad b = \omega_n\left(-\delta - \sqrt{\delta^2 - 1}\right); \quad A_1 = \frac{b}{(a-b)}; \quad A_2 = \frac{a}{(b-a)}.$$

4. Analysis of the transfer function — hunting

The functions ΔT defined by Eqs. (3)–(5) represent outputs induced by step-inputs ($\Delta \dot{m}_r$). To estimate the suitability of the transfer functions $G_1(s)$, $G_2(s)$ for simulating the evaporator dynamic behavior, the response of step-inputs and for other types of input must be tested.

As an example, consider the case of the data of Fig. 4. When the linear model (3) is applied, with $\theta = 6$ s, $\tau = 2$ s, $K = 2500$ Ks/kg, then full agreement with the experimental results is achieved (Fig. 8).

To verify the suitability of the transfer function to *other types of inputs*, the hunting phenomenon, typical of a thermostatic expansion valve, has been analyzed. As it is well known, this phenomenon represents a case of real practical interest. In particular, hunting can reduce the capacity of the refrigeration system because the mean evaporator pressure and temperature are lowered and the compressor capacity is reduced. Further, hunting is certainly amplified by the wrong choice of valve for the evaporator capacity; but, even if the valve is correctly dimensioned, it is still present, though to a lesser degree.

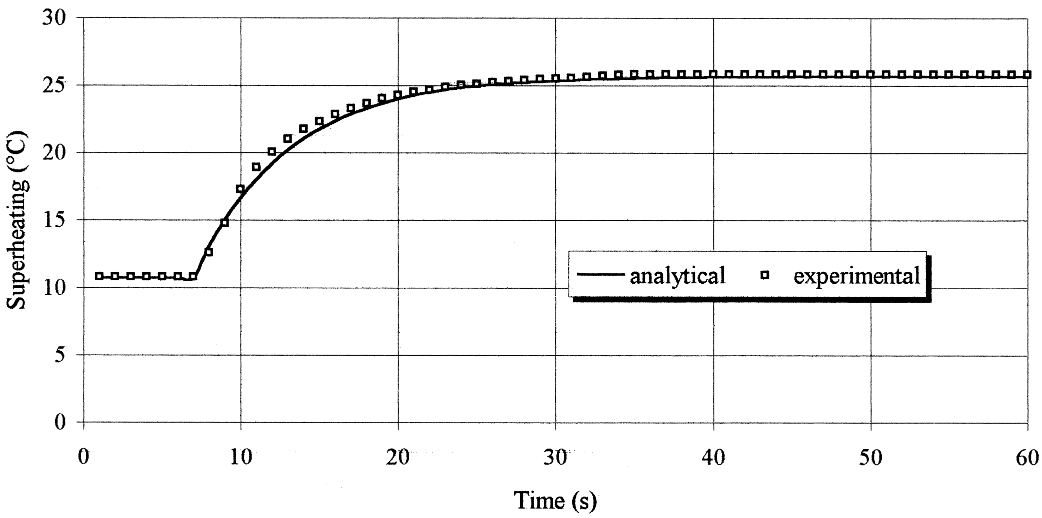

Fig. 8. Analytical and experimental response to the step input.

In order to analyze this phenomenon, the periodic variation of the refrigerant mass flow rate caused by hunting has been experimentally determined and the results are shown in Fig. 9. The order of this oscillation is of 2 g/s and the evaporator dynamic behavior can be fitted by the linear model defined in Eq. (1) by the function $G_1(s)$. The inverse Laplace transform of $G_1(s)$ is:

$$G_1(t) = \frac{K}{\theta} e^{-\frac{t-\tau}{\theta}} H(t-\tau) \tag{6}$$

where H denotes the unit step-function. On the other hand, by interpolating the experimental data of Fig. 9, one has:

$$\Delta \dot{m}_r = a + b \sin(\omega t + c) \tag{7}$$

where a, b, c, ω are appropriate constants.

Then, the superheating due to the hunting is determined by the convolution of the input (7) with the transfer function (6). If one puts:

$$\varphi(\omega) = \mathrm{arctg}(\theta\omega);\ A(\omega) = \left[1 + (\theta\omega)^2\right]^{-1/2};\ \beta = bA;\ B = a + \beta \sin(c - \varphi)$$

for all $t > \tau$, one obtains:

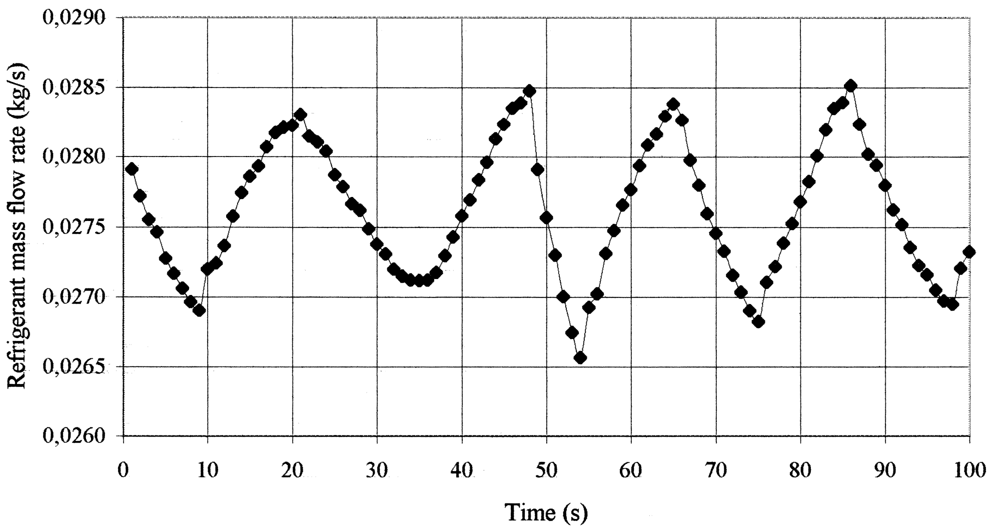

Fig. 9. The refrigerant mass flow rate variation owing to the phenomenon hunting.

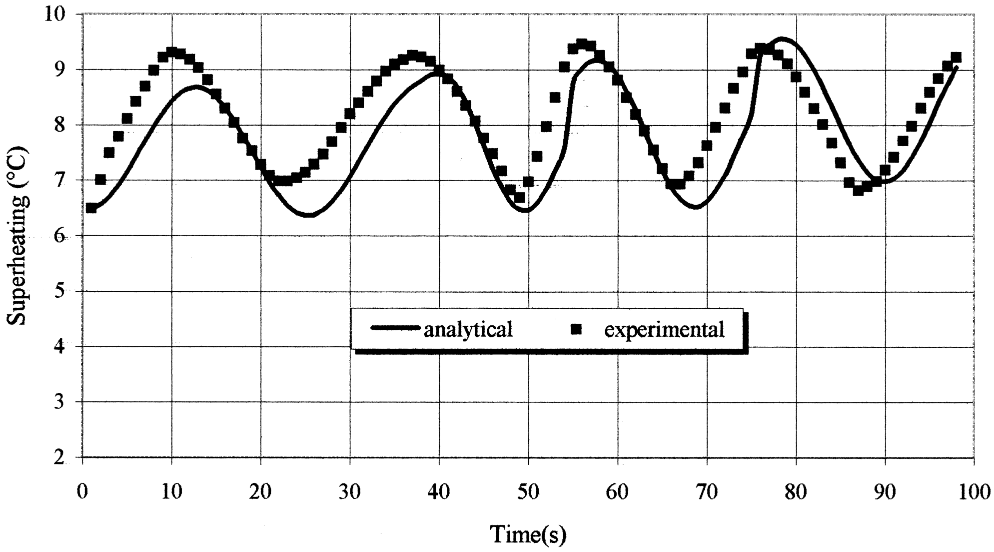

Fig. 10. Superheating analytical and experimental comparison at the evaporator outlet owing to hunting.

$$\Delta T = \Delta T_0 - KB\,\mathrm{e}^{-\frac{t-\tau}{\theta}} + K\big[a + \beta \sin\big[\omega(t-\tau) + c - \varphi\big]\big] \tag{8}$$

The fitness of this function to simulate the experimental output is illustrated in Fig. 10. For $t > 40$ s, the analytical and the experimental behavior almost coincide; for $t < 40$ s, the difference is less than 1°C.

5. Conclusions

For control purposes, the dynamic behavior of a direct expansion air cooled evaporator has been analyzed by means of the input–output approach (*black box*).

For inlet air temperatures varying between 15 and 35°C, in connection with step inputs of the refrigerant mass flow rate, the evaporator outlet superheating has been experimentally determined. Most of the tests have shown that the gain is essentially constant so that the evaporator behavior can be simulated by a linear model. A possible transfer function has been identified, together with the related parameters.

This function has been applied to both step inputs, and to periodic inputs and the results have confirmed the fitness of the model to simulate the evaporator dynamic behavior. Further, as an example of real applications, the influence of the hunting typical of a thermostatic expansion valve, has been estimated experimentally and by the analytical model, with a good agreement between the results.

The transfer function of the evaporator system allows the determination of the appropriate values of the refrigerant mass flow rate when the plant working conditions vary. The next step is to identify a control algorithm and implement it on a vapor compression refrigeration plant, while varying the speed of the compressor motor by means of an inverter that acts on the frequency and voltage of the current.

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