Transit-time design of earthen barriers

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ABSTRACT

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Transit-time analyses are used to evaluate the design thickness of earth-lined waste containment barriers. Solute transit times are determined from dimensionless charts based on an existing analytical solution for transient solute transport in saturated porous media. Both solute concentration and solute flux are considered in the transit-time analyses. With respect to solute flux, a dimensionless parameter termed the "flux number" is introduced. The results of an example design problem using measured parameters indicate that the effects of diffusion and retardation on solute transit times can be significant in low-permeability (i.e., $\leq 1.0 \cdot 10^{-9}$ m/s), fine-grained barrier materials. The method is relatively simple and can be used for preliminary design of earthen waste containment barriers, evaluation of remedial measures, and/or verification of more sophisticated numerical models.

INTRODUCTION

Current regulations in the United States require that the soil portion of certain surface impoundments and solid-waste landfills consist of a minimum of 0.9 m (3 ft.) of recompacted soil with a hydraulic conductivity (permeability) of $1 \cdot 10^{-9}$ m/s, or less. However, there is a growing belief that these minimum standards will not satisfy the goal of the facility which is to prevent the migration of any waste constituents through the barrier during the operational life of the containment facility, including a post-closure monitoring period (usually a total of 40 to 50 years) (e.g., see U.S. Environmental Protection Agency, 1985). As a result, transit-time analyses may be required to determine if the containment facility will be effective in realizing its desired engineering function.

This paper illustrates the use of transit-time analyses for the design of low-permeability ($\leq 1.0 \cdot 10^{-9}$ m/s) earth-lined waste containment barriers. The analyses employ the use of an existing analytical solution to the one-dimensional advection–dispersion equation for transient solute transport in saturated porous media. The transit-time analyses are based on the transport of miscible chemicals (solutes) and consideration is given to both solute concentration and solute flux. The effects of diffusion and retardation on transit-time determinations are illustrated with the aid of a design example.

TRANSIT-TIME ANALYSES

Several performance criteria can be used to provide a definition for "transit time" for the design of earthen waste-containment barriers (Goldman et al., 1986). Two specific transit-time analyses are considered here. The first analysis considers the time required for the concentration of a specific leachate component to reach a specific value at the barrier bottom. The second analysis considers the time required to reach a specified leachate chemical flux at the barrier bottom. Both analyses are based on the following analytical solution to the one-dimensional advection–dispersion equation for the transport of non-reactive solutes (Ogata and Banks, 1961; Ogata, 1970; Bear, 1972, 1979; Freeze and Cherry, 1979), or:

$$\frac{c}{c_0} = \frac{1}{2}[\text{erfc}(z_1) + \exp(z_2)\text{erfc}(z_3)] \tag{1}$$

where c is the concentration of the solute in the liquid phase of the soil (ML^{-3}); c_o, which is assumed to be constant, is the solute concentration at the liner-leachate interface (ML^{-3}); erfc is the complementary error function and z_1, z_2 and z_3 are arguments given by:

$$z_1 = \frac{x - v_s t}{2\sqrt{D^* t}}; \ z_2 = \frac{v_s x}{D^*}; \ z_3 = \frac{x + v_s t}{2\sqrt{D^* t}} \tag{2}$$

In eqs.2, v_s is the seepage velocity (LT^{-1}), D^* is the effective diffusion coefficient for diffusion in soil ($L^2 T^{-1}$), x is the distance within the soil barrier (L), and t is time (T). The arguments as defined by eqs.2 apply to relatively thin earthen barriers in which the permeability is relatively low ($\leq 1.0 \cdot 10^{-9}$ m/s) and the dispersion of the solute is dominated by molecular diffusion (e.g., see Rowe, 1987). The complementary error function is given by the following relations (e.g., see Carslaw and Jaeger, 1959; Freeze and Cherry, 1979):

$$\text{erfc}(z) = 1 - \text{erf}(z) \tag{3}$$

and:

$$\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-\xi^2} d\xi \tag{4}$$

where erf is the error function, z is the argument for the error function and ξ is the "dummy" variable of integration.

The initial and boundary conditions for the solution given by eq.1 are:

$c(x \geq 0; \ t = 0) = 0$
$c(x \leq 0; \ t > 0) = c_o$
$c(x = \infty; \ t > 0) = 0$

and the assumptions inherent in the use of eq.1 are: (1) the soil barrier is homogeneous and semi-infinite; (2) saturated, steady-state (Darcian) fluid flow has been established; and (3) solute transport occurs in one dimension. Concentration versus time profiles for a barrier of finite thickness, L, can be estimated by substituting L for x in eqs.2. The soil for waste containment barriers generally is

placed and compacted at a water content which is at or above the optimum water content to ensure the minimum possible value for the hydraulic conductivity of the soil (see Mitchell et al., 1965). Under these conditions, the compacted soil barrier should be nearly saturated (i.e., degrees of saturation $> 80\%$). The assumption of steady-state fluid flow is probably unrealistic, while the assumption of one-dimensional transport is relatively accurate due to the large width-to-depth ratio of most earthen containment barriers.

The assumption of a constant concentration of a specific leachate constituent, c_o, on the containment side of the barrier has been criticized (Rowe and Booker, 1985) as being overly conservative in that the attenuation capacity of the soil is neglected after steady-state solute transport has been established. In reality, the leachate concentration on the containment side of the barrier probably fluctuates widely over the operational life of the containment facility. As a result, there is no simple boundary condition for the containment side of the barrier for use with relatively simple analytical solutions. However, in most cases, the operational life of the containment barrier will be much less than the time required to reach steady-state solute transport conditions. In addition, it seems prudent to rely on conservative assumptions for design of containment barriers given the practical difficulty associated with the determination of accurate values for the transport properties, e.g., the hydraulic conductivity of the soil (Daniel, 1984; Daniel and Shackelford, 1988).

Specified leachate concentration

Eq. 1 can be used directly to determine the concentration, c, of a specified leachate constituent at the bottom of a clay barrier of thickness L. For given values of c, c_o, v_s, D^*, and L $(=x)$, all variables in eqs.1 and 2 are defined except for the time, t. An iterative procedure must be used to solve for t. The iterative procedure is facilitated with the use of a series solution for the error function defined by eq.3 (Carslaw and Jaeger, 1959, p.482; and Abramowitz and Stegun, 1972, p.257), or:

$$\text{erf}(z) = \frac{2}{\sqrt{\pi}} \sum_{m=0}^{\infty} \frac{(-1)^m z^{2m+1}}{m!(2m+1)} \tag{5}$$

A small computer program (~ 100 lines) can be written to perform the iterative procedure. Fifty terms ($m = 50$) in eq.5 are sufficient to achieve an accuracy to six decimal places for values of erf(z). The same iterative procedure may be used for reactive solutes subject to reversible sorption reactions if the arguments (eqs.2) are defined as follows:

$$z_1 = \frac{x - v_R t}{2\sqrt{D_R^* t}}; \; z_2 = \frac{v_R x}{D_R^*} = \frac{v_s x}{D^*}; \; z_3 = \frac{x + v_R t}{2\sqrt{D_R^* t}} \tag{6}$$

where $v_R = v_s/R_d$, $D^*_R = D^*/R_d$, and R_d is the retardation factor which represents the ratio of the seepage velocity, v_s, to the average velocity of the reactive solute, v_R.

Specified leachate flux

The advective flux for a saturated soil is given by:

$$J_A = vc = nv_s c = Kic \tag{7}$$

where v is the Darcian velocity $[LT^{-1}]$, K is the hydraulic conductivity or permeability of the soil $[LT^{-1}]$, i is the hydraulic gradient (dimensionless), and n is the total porosity of the soil [dimensionless]. If the concentration, c, in eq.7 is given by eq.1, the advective flux becomes:

$$J_A = \frac{1}{2} nv_s c_0 [\text{erfc}(z_1) + \exp(z_2) \, \text{erfc}(z_3)] \tag{8}$$

For diffusion in saturated soil, the diffusive flux is given by (Shackelford, 1988):

$$J_D = -D^* n \frac{\partial c}{\partial x} \tag{9}$$

The porosity term is required in eq.9 since the flux, J_D, is defined with respect to the total cross-sectional area of the soil. In order to determine the diffusive flux, the gradient in the solute concentration must be determined. The concentration gradient is determined by employing the chain rule for differentiation on eq.1, or:

$$\frac{\partial c}{\partial x} = \frac{1}{2} c_0 \left[\frac{d \, \text{erfc}(z_1)}{dz_1} \frac{dz_1}{dx} + \exp(z_2) \frac{d \, \text{erfc}(z_3)}{dz_3} \frac{dz_3}{dx} + \frac{d \exp(z_2)}{dz_2} \frac{dz_2}{dx} \text{erfc}(z_3) \right] \tag{10}$$

The derivative of the complementary error function is (e.g., see Carslaw and Jaeger, 1959, p.483):

$$\frac{d \, \text{erfc}(z)}{dz} = \frac{d[1 - \text{erf}(z)]}{dz} = -\frac{d \, \text{erf}(z)}{dz} = -\frac{2}{\sqrt{\pi}} \exp(-z^2) \tag{11}$$

and the derivatives of the arguments, eqs.6, are given by:

$$\frac{dz_1}{dx} = \frac{dz_3}{dx} = \frac{1}{2\sqrt{D_R^* t}}; \frac{dz_2}{dx} = \frac{v_s}{D^*} \tag{12}$$

When eqs.11 and 12 are substituted into eq.10, the expression for the concentration gradient becomes:

$$\frac{\partial c}{\partial x} = \frac{1}{2} c_0 \left[\frac{-\exp(-z_1^2)}{\sqrt{\pi D_R^* t}} - \frac{\exp(z_2) \exp(-z_3^2)}{\sqrt{\pi D^* t}} + \frac{v_s}{D_R^*} \exp(z_2) \, \text{erfc}(z_3) \right] \tag{13}$$

Finally, since $\exp(z_2)\exp(-z_3^2) = \exp(-z_1^2)$, the diffusive flux, eq.9, may be written as:

$$J_D = \frac{1}{2} D^* n c_0 \left[\frac{2 \exp(-z_1^2)}{\sqrt{\pi D_R^* t}} - \frac{v_s}{D^*} \exp(z_2) \, \text{erfc}(z_3) \right] \tag{14}$$

Dimensionless parameters

The arguments (eqs.6) for the analytical solutions can be expressed in terms of dimensionless parameters as follows:

$$z_1 = \frac{1 - T_R}{2\sqrt{T_R/P_L}}; z_2 = P_L; z_3 = \frac{1 + T_R}{2\sqrt{T_R/P_L}} \tag{15}$$

where:

$$T_R = \frac{v_s t}{R_d x} = \frac{v_R t}{x} = \frac{v_R t}{L}\bigg|_{x=L} \tag{16}$$

and:

$$P_L = \frac{v_s x}{D^*} = \frac{v_s L}{D^*}\bigg|_{x=L} \tag{17}$$

The dimensionless parameter, T_R, in eq.16 represents the number of pore volumes of flow divided by the retardation factor, R_d, whereas P_L is the Peclet number defined with respect to the barrier thickness, L, assuming mechanical dispersion is negligible (Bear, 1972, p.661). For non-adsorbing solutes, $R_d = 1.0$ and eqs.6 reduce to the arguments for the non-reactive solutes, eqs.2. Therefore, the dimensionless parameters expressed as eqs.16 and 17 are applicable for transport of both retarded and non-retarded solutes.

In terms of dimensionless parameters, the analytical solution for a specified leachate concentration (eq.1) is:

$$\frac{c}{c_0} = \frac{1}{2}\left[\text{erfc}\left(\frac{1 - T_R}{2\sqrt{T_R/P_L}}\right) + \exp(P_L)\,\text{erfc}\left(\frac{1 + T_R}{2\sqrt{T_R/P_L}}\right)\right] \tag{18}$$

This solution is presented graphically as Fig.1. Similar plots have been provided elsewhere (Goldman et al., 1986; Ogata and Banks, 1961; Ogata, 1970). An iterative procedure can be used with Fig.1 to estimate the barrier thickness required to provide the necessary transit time for a specified leachate concentration, c. The iterative procedure involves the following steps: (1) assume a liner thickness, L; (2) calculate P_L (eq.17); (3) determine T_R from Fig.1 for the desired value of c/c_0 and the calculated value of P_L; and (4) use eq.16 to determine the transit time, t. This procedure is repeated until t is greater than or equal to the design life of the containment barrier.

A similar approach to the one presented above for a specified leachate concentration can be developed for a specified leachate flux. When the advective and diffusive flux equation (eqs.8 and 14, respectively) are written in terms of the dimensionless parameters, the following equations result:

$$J_A = \frac{1}{2} n v_s c_0 \left[\text{erfc}\left(\frac{1 - T_R}{2\sqrt{T_R/P_L}}\right) + \exp(P_L)\,\text{erfc}\left(\frac{1 + T_R}{2\sqrt{T_R/P_L}}\right)\right] \tag{19}$$

and

$$J_D = \frac{1}{2}\frac{D^*}{L} n c_0 \left\{ \frac{2 \exp\left[-\left(\frac{1 - T_R}{2\sqrt{T_R/P_L}}\right)^2\right]}{\sqrt{\frac{\pi T_R}{P_L}}} - P_L \exp(P_L)\,\text{erfc}\left(\frac{1 + T_R}{2\sqrt{T_R/P_L}}\right)\right\} \tag{20}$$

In general, the total solute flux, J, will be the desired quantity. The total solute flux is just the sum of the advective and diffusive fluxes, or:
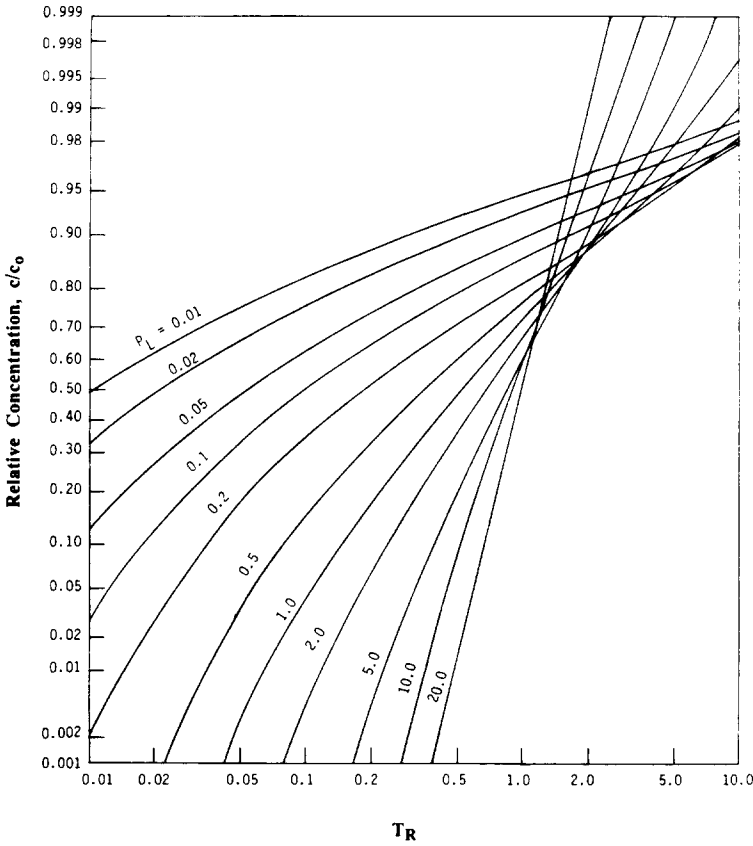
Fig.1. Relative concentration of leachate constituent (c/c_o) as a function of the dimensionless parameters T_R and P_L.

$$J = J_A + J_D = \frac{1}{2} n v_s c_0 (Q_1 + Q_2) + \frac{1}{2} \frac{D^*}{L} n c_0 (Q_3 - P_L Q_2) \tag{21}$$

where Q_1, Q_2, and Q_3 are dimensionless terms defined as follows:

$$Q_1 = \mathrm{erfc}\left(\frac{1 - T_R}{2\sqrt{T_R/P_L}} \right); \quad Q_2 = \exp(P_L)\, \mathrm{erfc}\left(\frac{1 + T_R}{2\sqrt{T_R/P_L}} \right);$$

$$Q_3 = \frac{2 \exp\left[-\left(\dfrac{1 - T_R}{2\sqrt{T_R/P_L}} \right)^2 \right]}{\sqrt{\pi T_R/P_L}} \tag{22}$$

Eq.21 may be further reduced as follows:

$$J = \frac{1}{2} \frac{D^*}{L} n c_0 \left[\frac{v_s L}{D^*} (Q_1 + Q_2) + Q_3 - P_L Q_2 \right] \tag{23}$$
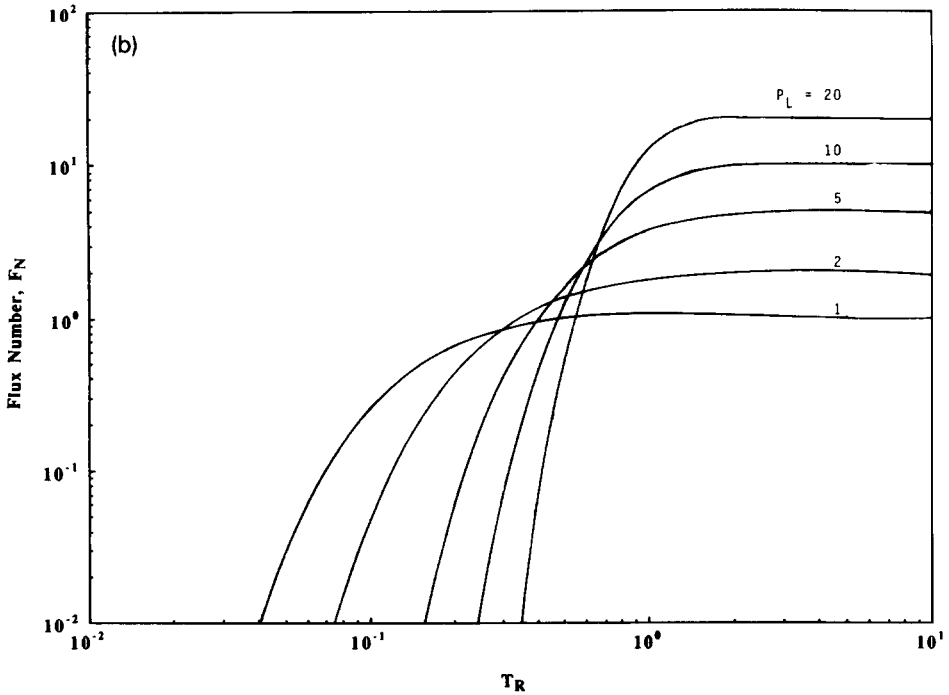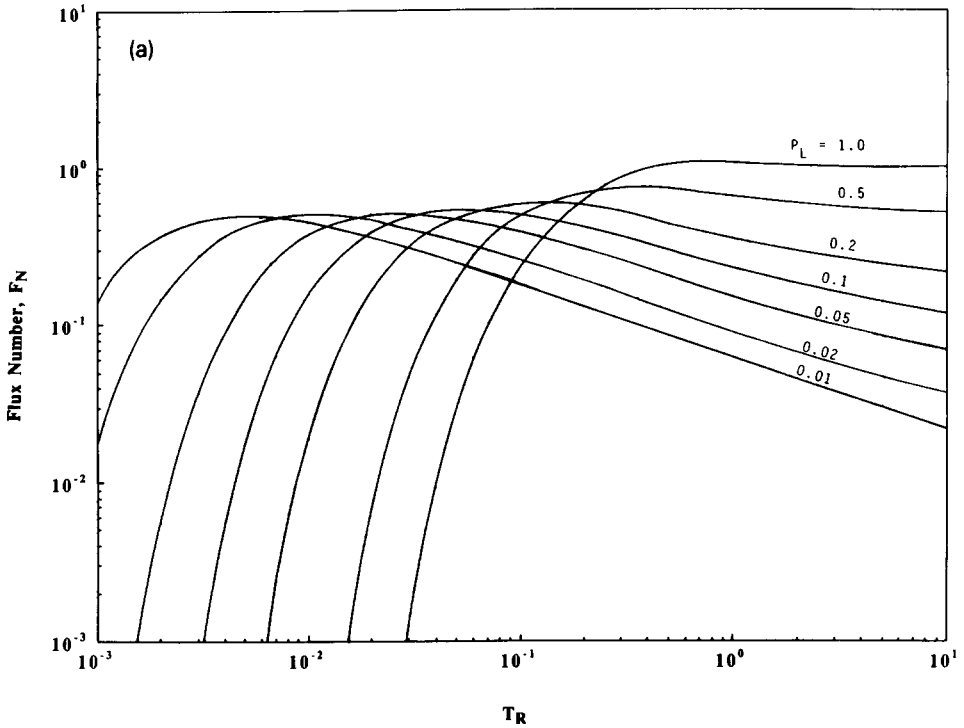
or:

Fig.2. Dimensionless flux number (F_N) as a function of dimensionless parameters T_R and P_L: (a) low P_L; (b) high P_L.

$$F_{\mathrm{N}} = \frac{JL}{nc_0 D^*} = \frac{1}{2}[P_{\mathrm{L}}(Q_1 + Q_2) + Q_3 - P_{\mathrm{L}}Q_2] = \frac{1}{2}(P_{\mathrm{L}}Q_1 + Q_3) \tag{24}$$

where F_{N} is a dimensionless "flux number". The graphical solution to eq.24 is presented as Fig.2. As the value of T_{R} increases, Q_3 approaches zero and z_1 becomes negative. As z_1 becomes increasingly more negative, Q_1 approaches a value of 2.0. Therefore, as T_{R} increases, the value of F_{N} approaches the value for P_{L}, as indicated in Fig.2. The barrier thickness required to provide the necessary transit time for a specified leachate flux may be estimated with Fig.2 by substituting F_{N} for c/c_{o} in the procedure outlined for the specified leachate concentration.

ILLUSTRATIVE DESIGN EXAMPLE

An example analysis is presented using Figs.1 and 2 to illustrate (1) the procedure for estimating the required thickness of a compacted soil barrier, and (2) the effects of diffusion and retardation on calculated results. The design will be based on specified concentrations of leachate constituents (i.e., Fig.1) for a 40-year operational life (transit time) of a sanitary landfill. The soil to be used for the facility is Lufkin clay, a naturally occurring soil from central Texas. The physical properties of Lufkin clay are provided in Table I. For the purposes of the analysis, a total porosity of 0.37 and a dry (bulk) density, ρ_{b}, of 1.60 g/cm^3 are assumed for the compacted soil barrier. A hydraulic gradient of 1.33 and a measured hydraulic conductivity of $5.0 \cdot 10^{-11}$ m/s (Foreman, 1984) are used to calculate the seepage velocity.

Values for the chemical properties used in the analysis are presented in Table II. The concentrations for the leachate constituents were taken from measured values presented by Griffin et al. (1976) for the Du Page County, Illinois, sanitary landfill. One non-reactive (Cl$^-$) and three reactive (K$^+$, Zn^{2+}, and Cd^{2+}) solutes were selected as representative constituents for the leachate.

TABLE I

Properties of Lufkin clay

Property	Value	Method of measurement
Natural water content (g/g)	23%	ASTM D2216
Clay mineralogy	Smectite, kaolinite, illite	X-ray diffraction
Optimum water content (g/g)	20%	ASTM D698
Maximum dry density (g/cm^3)	1.64	ASTM D698
Liquid limit (g/g)	56%	ASTM D4318
Plasticity index (g/g)	42%	ASTM D4318
Specific gravity of solids, G_{s}	2.69	ASTM D854
< No. 200 (0.075 mm) sieve (g/g)	82%	ASTM D1140
Cation exchange capacity (meq./100 g)	25	(*)

*From Daniel and Liljestrand (1984).

TABLE II ·

Chemical properties used for example problem

Chemical constituent	Concentration in leachate, c_o (mg/l)[1]	Drinking water standard (mg/l)	c/c_o	D^* (m²/s)[2]	R_d[3]
Chloride (Cl⁻)	3484	250	0.072	$4.7 \cdot 10^{-10}$	1.0
Potassium (K⁺)	501	250[4]	0.5	$2.0 \cdot 10^{-9}$	26.7
Zinc (Zn²⁺)	18.8	5.0	0.27	$1.7 \cdot 10^{-10}$	92.7
Cadmium (Cd²⁺)	1.95	0.01	0.005	$3.0 \cdot 10^{-10}$	371

[1]From Griffin et al. (1976).
[2]From Shackelford (1988) and Shackelford et al. (1989).
[3]Based on c_o values, $\rho_b = 1.60$ g/cm³ and $n = 0.37$, and data from Shackelford (1988) and Shackelford et al. (1989).
[4]Assumed to be the same as chloride since actual value not established.

Experimental program

The effective diffusion coefficients presented in Table II for each of the chemical constituents were measured using compacted samples of Lufkin clay with the apparatus shown schematically in Fig.3. The test specimens of soil were compacted, using the standard Proctor compaction procedure (ASTM D698), at a water content of 22% (i.e., 2% wet of optimum water content), soaked in $0.01N$ CaSO₄ solution until advective flow was negligible, and then exposed to a synthetic leachate containing $0.01N$ concentrations of the specified ions (Cl⁻, K⁺, Cd²⁺, and Zn²⁺). The diffusion coefficients (D^*) were determined from reservoir concentrations measured during 76 days of exposure to the leachate. The D^* values for cadmium (Cd²⁺) and zinc (Zn²⁺) are based on reservoir concentrations which were modified to account for precipitation of the heavy metals during the experiment. The retardation factors were calculated using Freundlich isotherm parameters determined from batch-equilibrium tests using Lufkin clay, the assumed physical properties of the soil (n and ρ_b), and the concentration, c_o, for each of the constituents given in Table II. A 1:4 soil:solution ratio, by weight, was used in the batch-equilibrium tests. Further details of the diffusion and batch-equilibrium experiments are provided by Shackelford (1988) and Shackelford et al. (1989).

Non-reactive solute

The seepage velocity of the fluid is:

$$v_s = \frac{Ki}{n} = \frac{(5.0 \cdot 10^{-11} \text{ m/s})(1.33)}{0.37} = 1.8 \cdot 10^{-10} \text{ m/s}$$

If the minimum thickness of the barrier is assumed to be 0.9144 m (3 ft.), the Peclet number for chloride (Cl⁻) is:
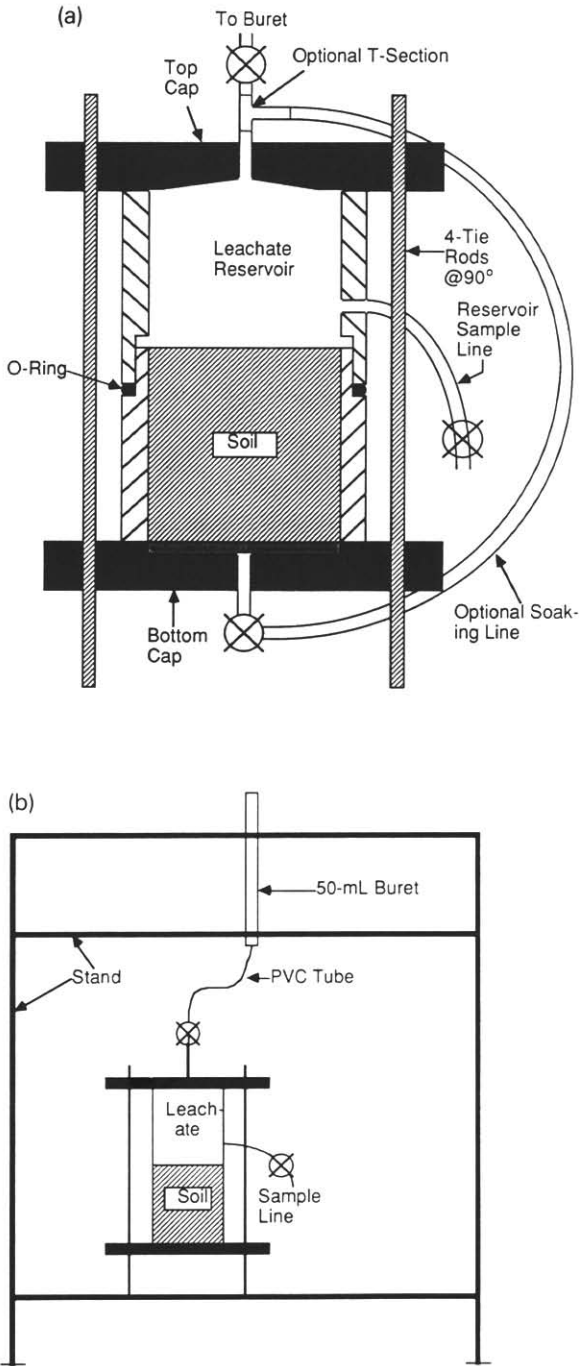
Fig.3. Diffusion test apparatus: (a) diffusion cell; (b) diffusion system.

$$P_L = \frac{(1.8 \cdot 10^{-10} \text{ m/s})(0.9144 \text{ m})}{4.7 \cdot 10^{-10} \text{ m}^2/\text{s}} = 0.35$$

With $c/c_o = 0.072$, T_R is approximately 0.05 (Fig.1). Therefore, the transit time (eq.16) is:

$$t = \frac{T_R R_d L}{v_s} = \frac{(0.05)(1.0)(0.9144 \text{ m})}{(1.8 \cdot 10^{-10} \text{ m/s})(31.536 \cdot 10^6 \text{ s/yr})} = 8.1 \text{ yr}$$

Since a transit time of 8.1 years is much less than the desired design life of 40 years, a 0.9144-m (3-ft.) barrier is too thin to contain the chloride within the specified concentration limit. For the second iteration, a barrier thickness of 1.829 m (6 ft.) will be assumed. The corresponding calculations result in the following values for the required parameters:

$P_L = 2 \times 0.35 = 0.70$
$T_R \sim 0.09$ (Fig.1)
$t \sim 30$ years

Since $t = 30$ years is still less than the desired time, try $L = 2.134$ m (7.0 ft.), or:

$$P_L = \frac{2.134}{0.9144} \times 0.35 \sim 0.82$$

$T_R \sim 0.105$ (Fig.1)
$t \sim 39.5$ years

Therefore, a barrier thickness of about 2.134 m (7.0 ft.) will be required to satisfy the specified leachate concentration criterion for chloride.

The corresponding flux of chloride at 40 years can be determined with Fig.2. For a P_L value of 0.82 and a T_R value of 0.105, F_N is about 0.36. Therefore, the total flux for chloride (eq.24) is:

$$J = \frac{F_N n c_0 D^*}{L}$$

$$= \frac{(0.36)(0.37)(3484 \text{ mg/l})(4.7 \cdot 10^{-10} \text{ m}^2/\text{s})(31.536 \cdot 10^6 \text{ s/yr})}{(2.134 \text{ m})(0.001 \text{ m}^3/\text{l})}$$

$$= 3220 \text{ mg m}^{-2} \text{ yr}^{-1}$$

Reactive solutes

Of the three reactive solutes, potassium (K^+) has the highest diffusion coefficient and the lowest retardation factor and, therefore, will probably control the design based on the reactive solutes. For a 0.9144-m (3-ft.)-thick barrier, P_L for potassium is:

$$P_L = \frac{(1.8 \cdot 10^{-10} \text{ m/s})(0.9144 \text{ m})}{2.0 \cdot 10^{-9} \text{ m}^2/\text{s}} = 0.082$$

With $c/c_o = 0.5$, T_R from Fig.1 is about 0.09. Therefore:

$$t = \frac{(0.09)(26.7)(0.9144 \text{ m})}{(1.8 \cdot 10^{-10} \text{ m/s})(31.536 \cdot 10^6 \text{ s/yr})} = 387 \text{ yr} \gg 40 \text{ yr}$$

So, a 0.9144-m (3-ft.) barrier thickness is more than sufficient to meet the criterion for the specified leachate concentration of potassium. The corresponding flux number, F_N, for potassium is 0.48 (Fig.2a). Therefore, the total flux is:

$$J = \frac{(0.48)(0.37)(501 \text{ mg/l})(2.0 \cdot 10^{-9} \text{ m}^2/\text{s})(31.536 \cdot 10^6 \text{ s/yr})}{(0.9144 \text{ m})(0.001 \text{ m}^3/\text{l})}$$

$$= 6140 \text{ mg m}^{-2} \text{ yr}^{-1}$$

The calculations for cadmium and zinc are similar to those for potassium. The example results for all the chemical constituents are summarized in Table III.

Based on the analyses considering all of the constituents, the 2.134-m (7-ft.)-thick barrier required for chloride controls the design. The controlling influence of chloride can be attributed to the extremely high concentration of chloride in the leachate and the relatively high adsorptive capacity of the Lufkin clay for the reactive solutes. Failure to account for the attenuation (adsorptive) capacity of soil can lead to extremely conservative estimates for the required thickness of a waste containment barrier. However, if diffusion of the reactive solutes is not included in the transit-time analyses, then the resulting transit times for the reactive solutes will be overestimated leading to an unconservative estimate for the required thickness of the earthen barrier.

A 0.9144-m (3-ft.)-thick barrier could be used successfully if the concentration of the chloride in the leachate could be reduced by some type of chemical treatment process. Again, Fig.1 can be used to estimate the required reduction in the chloride concentration. For example, the Peclet number for chloride for a 0.9144-m (3-ft.) barrier was 0.35. The value for T_R for a 40-year operational life is:

$$T_R = \frac{v_s t}{R_d L} = \frac{(1.8 \cdot 10^{-10} \text{ m/s})(40 \text{ yr})(31.536 \cdot 10^6 \text{ s/yr})}{(1.0)(0.9144 \text{ m})} = 0.248$$

For these values of P_L and T_R, the relative concentration, c/c_o, from Fig.1 is about 0.473, so the required chloride concentration in the leachate is:

$$c_0 = \frac{c}{0.473} = \frac{250}{0.473} = 529 \text{ mg/l}$$

TABLE III

Results from example design problem for a 40-year sanitary landfill

Chemical constituent	Required barrier thickness, L (m (ft.))	F_N (Fig.2)	Total flux, J (mg m^{-2} yr^{-1})
Chloride (Cl$^-$)	2.134 (7.0)	0.36	3220
Potassium (K$^+$)	<0.9144 (<3.0)	0.48*1	6140
Cadmium (Cd^{2+})	<0.9144 (<3.0)	0.81*1	60.0
Zinc (Zn^{2+})	<0.9144 (<3.0)	0.048*1	20.0

*1For a 0.9144-m (3.0-ft)-thick barrier.

Therefore, a 0.9144-m (3-ft.)-thick barrier can be used if the chloride concentration is reduced from 3484 mg/l to 529 mg/l.

Finally, the transit time for chloride based solely on the seepage velocity for a 0.9144-m (3-ft.)-thick barrier is:

$$t = \frac{L}{v_s} = \frac{(0.9144 \text{ m})}{(1.8 \cdot 10^{-10} \text{ m/s})(31.536 \cdot 10^6 \text{ s/yr})} = 161 \text{ yr}$$

This value of 161 years is significantly greater than the 8.1 years for chloride using Fig.1. The difference in the two values is due to the diffusive transport of the chloride.

DESIGN CONSIDERATIONS

There are a number of factors related to the design of earthen barriers which should be considered when evaluating the solute transit times determined by the methods presented in this paper. Some of these considerations are discussed below.

Unsaturated conditions

In the design example, the earthen barrier was assumed to be completely saturated. However, in reality, earthen barriers typically are unsaturated. The hydraulic gradient for an unsaturated barrier will be relatively higher than that for a saturated barrier, due to suction in the soil, whereas the hydraulic conductivity, or permeability, typically will be lower for an unsaturated soil. In addition, the cross-sectional area available for fluid phase flow or transport will be less in an unsaturated soil than it is in a saturated soil. All of these factors affect the magnitude of the seepage velocity, v_s.

For compacted earthen barriers in which the degree of saturation, S_r is relatively high, i.e., $> 80\%$, the increase in the hydraulic gradient across the barrier due to suction typically is ignored, and the hydraulic gradient based on a saturated condition is used in the analysis. In addition, the saturated hydraulic conductivity typically is measured and used in the analysis in lieu of the unsaturated hydraulic conductivity. While the use of a hydraulic gradient based on a saturated condition clearly is unconservative in the case of an unsaturated earthen barrier, it is commonly felt that the concurrent use of the saturated hydraulic conductivity will result in a conservative estimate for the value of the seepage velocity. However, this may not be the case for barriers with S_r values much less than 80%. In such cases, an unsaturated analysis employing a different method than those presented herein may be required to provide relatively accurate estimates of solute transit times.

The reduction in the cross-sectional area available for flow may be accounted for in the present methods by using the volumetric moisture content, θ, in place of the soil porosity, n. The volumetric moisture content can be calculated using the following relationship:

$$\theta = nS_r \tag{25}$$

where θ represents the volume of moisture per total volume of soil, and the degree of saturation, S_r, is expressed in decimal form.

Compaction control

Due to variations in compaction which inevitably occur from point to point across the surface of an earthen barrier during construction, there may be significant variations in material properties, e.g., the permeability of the soil. In addition, compacted earthen barriers typically consist of several layers, or lifts, of compacted soil. As a result, there also may be variations in material properties between lifts. If such variations exist, the earthen barrier will resemble more a heterogeneous mass than it does the homogeneous mass assumed by the analysis, and the resulting transit times may be radically different than those predicted by the methods presented in this paper.

Also, it often is difficult to control the moisture content of the soil during compaction, especially in arid climates. If the compacted soil is allowed to dry out, desiccation cracks leading to preferential flow channels between the compacted lifts of soil can form (e.g, see Daniel, 1984). The result of such flow channels typically is to increase the transport rates and decrease the transit times relative to those that would be predicted by the methods employed in this paper.

Time-dependent changes in material properties

The barrier materials may be subject to forces which could lead to significant changes in material properties over the lifetime of the containment facility. For example, long-term creep of the compacted soil on the inclined portion of a barrier may tend to alter the permeability of the soil and, thus, the transit times of the solutes. Also, the leachate from the buried waste may not be compatible with the soil. For example, some strong acids and bases may interact adversely with soil, causing increases in the permeability (e.g., see Daniel and Liljestrand, 1984). Such increases may result in drastic reductions in the solute transit times predicted by the design engineer.

Coupled flow processes

The derivation presented in this paper is based on an analytical solution to the advection–disperson equation, which does not consider the effect of coupled flow processes, i.e., fluid flow due to concentration, thermal, or electrical gradients. While neglecting coupled flow processes is reasonable for flow through highly permeable, granular soils, it may not be reasonable for the relatively low-permeability soils used for waste containment barriers. The importance of coupled flow processes on the determination of solute transit times should be evaluated by the design engineer before proceeding with the transit-time analyses presented in this paper.

CONCLUSIONS

Transit times based solely on the advective transport (i.e., seepage velocity) of solutes in low-permeability, fine-grained barrier materials are unconservative. Such determinations reflect the physical properties (e.g., permeability and porosity) of the

flow system but neglect the chemical properties (e.g., diffusion coefficients of the solutes and the adsorptive capacity of the soil). Models which include both the physical and the chemical properties of the flow system, such as the ones presented in this paper for determining solute transit times, are preferred.

Diffusion is a significant solute transport process in low-permeability barrier materials. Diffusive transport of solutes can result in significantly reduced transit times relative to those based solely on advective transport. As a result, breakthrough of contaminants may occur much sooner than expected if diffusion is neglected in the transit-time analysis for the design of earthen barriers.

Failure to account for the attenuation capacity of the soil will lead to overly conservative (low) estimates of solute transit times resulting in excessively thick earth-lined waste containment barriers.

The methods employed in this paper are based on a simple analytical solution to the advective–dispersive equation for solute transport in saturated soil and, thus, are limited by the boundary and initial conditions and the assumptions inherent in the derivation of the analytical solution. However, the same procedures as developed in this paper may be used with other analytical solutions to the advective–dispersive equation (e.g., see Bear, 1972, 1979). Due to the uncertainties associated with determining accurate transport properties and boundary conditions, relatively simple analytical solutions may be preferred over more complicated and expensive numerical procedures (e.g., finite element and finite difference methods), especially for preliminary design of earth-lined waste containment barriers.

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REFERENCES

Abramowitz, M. and Stegun, I.A., 1972. Handbook of Mathematical Functions. Dover Publishers, New York, N.Y., 1046 pp.

Bear, J., 1972. Dynamics of Fluids in Porous Media. American Elsevier, New York, N.Y., 764 pp.

Bear, J., 1979. Hydraulics of Groundwater. McGraw-Hill, New York, N.Y., 569 pp.

Carslaw, H.S. and Jaeger, J.C., 1959. Conduction of Heat in Solids, 2nd ed. Oxford University Press, Oxford, 510 pp.

Daniel, D.E., 1984. Predicting hydraulic conductivity of clay liners. J. Geotech. Eng., ASCE, 110(2): 285–300.

Daniel, D.E. and Liljestrand, H.M., 1984. Effects of Landfill Leachates on Natural Linear Systems. Chemical Manufacturers Association Report, The University of Texas, Geotechnical Engineering Center, Austin, Texas, 86 pp.

Daniel, D.E. and Shackelford, C.D., 1988. Disposal barriers that release contaminants only by molecular diffusion. Nucl. Chem. Waste Manage. J., 8: 299–305.

Foreman, D.E., 1984. The Effects of Hydraulic Gradient and Concentrated Organic Chemicals on the Hydraulic Conductivity of Compacted Clay. M.S. Thesis, University of Texas, Austin, Texas, 346 pp.

Freeze, R.A. and Cherry, J.A., 1979. Groundwater, Prentice-Hall, Englewood Cliffs, N.J., 604 pp.

Goldman, L.J., Kingsbury, G.L., Northeim, C.M. and Truesdale, R.S., 1986. Technical Resource

Document on Design, Construction, Maintenance, and Evaluation of Clay Liners for Hazardous Waste Facilities. EPA/530-SW-846-007, Cincinnati, Ohio.

Griffin, R.A., Cartwright, K., Shimp, N.F., Steele, J.D., Ruch, R.R., White, W.A., Hughes, G.M. and Gilkeson, R.H., 1976. Attenuation of Pollutants in Municipal Landfill Leachate by Clay Minerals, Part I. Column Leaching and Field Verification. Illinois State Geol. Surv., Environ. Geol. Notes, No. 78.

Mitchell, J.K., Hooper, D.R. and Campanella, R.G., 1965. Permeability of compacted clay. J. Soil Mech. Found. Div., ASCE, 91(SM4):41–65.

Ogata, A., 1970. Theory of dispersion in granular medium. U.S. Geol. Surv. Prof. Pap., 411–I.

Ogata, A. and Banks, R.B., 1961. A solution of the differential equation of longitudinal dispersion in porous media. U.S. Geol. Surv. Prof. Pap., 411–A.

Rowe, R.K., 1987. Pollutant transport through barriers. In: R.D. Woods (Editor), Geotechnical Practice for Waste Disposal. ASCE Spec. Publ., 13: 159–181.

Rowe, R.K. and Booker, J.R., 1985. 1-D pollutant migration in soils of finite depth. J. Geotech. Eng., ASCE, 111(4): 479–499.

Shackelford, C.D., 1988. Diffusion of Inorganic Chemical Wastes in Compacted Clay. Ph.D. Dissertation, University of Texas, Austin, Texas, 449 pp.

Shackelford, C.D., Daniel, D.E., and Liljestrand, H.M., 1989. Diffusion of inorganic chemical species in compacted clay soil. J. Contam. Hydrol., 4: 241–273.

U.S. Environmental Protection Agency, 1985. Draft Minimum Technology Guidance on Double Liner Systems for Landfills and Subsurface Impoundments—Design, Construction, and Operation. EPA/530-SW-85-014, Cincinnati, Ohio.