This article was downloaded by: [IRSTEA] On: 15 December 2014, At: 06:14 Publisher: Taylor & Francis Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



Bioscience, Biotechnology, and Biochemistry

Publication details, including instructions for authors and subscription information: <u>http://www.tandfonline.com/loi/tbbb20</u>

Prediction of Effective Thermal Diffusivity of Fish and Meats

Jai-Yul Kong^a, Toshimasa Yano^{ab}, Jong-Deog Kim^a, Seoung-Kwon Bae^{ac}, Min-Young Kim^{ad} & In-Soo Kong^a

^a Department of Biotechnology & Bioengineering, National Fisheries University of Pusan, 599-1 Daeyeon-Dong, Nam-Gu, Pusan 608-737, Korea

^b Department of Material Science and Chemical Engineering, Yokohama National University, 156 Tokiwadai, Hodogaya-ku, Yokohama, Kanagawa 240, Japan

^c Department of Agricultural Chemistry, The University of Tokyo, Bunkyo-ku, Tokyo 113, Japan

^d Department of Refrigeration Engineering, Yosu National Fisheries University, 195 Kuk-Dong, Yosu 550-749, Korea

Published online: 12 Jun 2014.

To cite this article: Jai-Yul Kong, Toshimasa Yano, Jong-Deog Kim, Seoung-Kwon Bae, Min-Young Kim & In-Soo Kong (1994) Prediction of Effective Thermal Diffusivity of Fish and Meats, Bioscience, Biotechnology, and Biochemistry, 58:11, 1942-1946, DOI: <u>10.1271/bbb.58.1942</u>

To link to this article: <u>http://dx.doi.org/10.1271/bbb.58.1942</u>

PLEASE SCROLL DOWN FOR ARTICLE

Taylor & Francis makes every effort to ensure the accuracy of all the information (the "Content") contained in the publications on our platform. However, Taylor & Francis, our agents, and our licensors make no representations or warranties whatsoever as to the accuracy, completeness, or suitability for any purpose of the Content. Any opinions and views expressed in this publication are the opinions and views of the authors, and are not the views of or endorsed by Taylor & Francis. The accuracy of the Content should not be relied upon and should be independently verified with primary sources of information. Taylor and Francis shall not be liable for any losses, actions, claims, proceedings, demands, costs, expenses, damages, and other liabilities whatsoever or howsoever caused arising directly or indirectly in connection with, in relation to or arising out of the use of the Content.

This article may be used for research, teaching, and private study purposes. Any substantial or systematic reproduction, redistribution, reselling, loan, sub-licensing, systematic supply, or distribution in any form to anyone is expressly forbidden. Terms & Conditions of access and use can be found at http://www.tandfonline.com/page/terms-and-conditions

Prediction of Effective Thermal Diffusivity of Fish and Meats

Jai-Yul Kong, Toshimasa Yano,* Jong-Deog Kim, Seoung-Kwon BAE,** Min-Young Kim,*** and In-Soo Kong

Department of Biotechnology & Bioengineering, National Fisheries University of Pusan, 599–1 Daeyeon-Dong, Nam-Gu, Pusan 608–737, Korea

* Department of Material Science and Chemical Engineering, Yokohama National University, 156 Tokiwadai, Hodogaya-ku, Yokohama, Kanagawa 240, Japan

** Department of Agricultural Chemistry, The University of Tokyo, Bunkyo-ku, Tokyo 113, Japan

*** Department of Refrigeration Engineering, Yosu National Fisheries University, 195 Kuk-Dong, Yosu 550–749, Korea Received February 4, 1994

Theoretical formulas for the prediction of effective thermal diffusivity on one-dimensional unsteady heat conduction of fish and meats in unfrozen and frozen states were investigated. Theoretical results agreed well with measured values in the temperature range of $9 \sim -22^{\circ}$ C. Theoretical results also agreed with published data from Dickerson and Read within the standard deviation of 7.8%. To predict the effective thermal diffusivity of fabricated food materials, a ternary contour diagram was prepared.

Manipulations involving heat transfer are often required during the processing and/or storage of foods. Information on thermal properties should therefore be available reduce energy cost before a heating or cooling apparatus is designed or operated. In heat conduction, the effective thermal diffusivity values are essential for predicting freezing times in cooling and freezing processes. However, only a little work has been done on thermal diffusivities of fish flesh. Dickerson and Read¹⁾ have measured the thermal diffusivity of fish flesh. Dickerson and Read¹⁾ have measured the thermal diffusivity of flatfish in the temperature range of $40 \sim 63^{\circ}$ C. Lentz and van den Berg²⁾ have reported thermal diffusivities of fish flesh using codfish $(3 \sim 9^{\circ}C)$ and salmon $(-10^{\circ}C)$. The effective thermal diffusivity of food depends on three physical properties, thermal conductivity, specific heat capacity, and density. Unlike thermal conductivity, specific heat capacity, and density are relatively well understood because of their additive properties. The thermal conductivity of heterogeneous food depends on the intrinsic thermal conductivities of individual food components as well as the complicated spatial distribution of these components.³⁾ For this reason, thermal conductivity of heterogeneous food is hardly understood theoretically. 4^{-7}

In previous works, 8^{-12} the effective thermal conductivities of fish and meats were measured in frozen and unfrozen states using a Series model and the intrinsic thermal conductivities of food proteins were also estimated.

In this work, a prediction model for the effective thermal diffusivity of fish and meats has been developed. The effective thermal diffusivities predicted by the model were compared to the measured values in frozen and unfrozen states and compared to published values¹³⁾ in Table I. For predicting the effective thermal diffusivity of food materials, a ternary contour diagram was prepared using intrinsic thermal conductivities of water, ice, protein, and fat.

Theory

The basic differential equations for one-dimensional unsteady heat conduction¹⁴⁾ are:

$$q = -\lambda A \frac{d\theta}{dx} \tag{1}$$

$$\partial \theta / \partial t = K(\partial^2 \theta / \partial x^2) \tag{2}$$

From Eq. (2),

$$K \equiv \lambda / (\rho \cdot C_{p}) \tag{3}$$

where, K is thermal diffusivity of food. The initial and boundary conditions are defined as shown in Eqs. (4), (5), and (6).

[Intial Condition]

$$\theta(x,0) = \theta_0 \tag{4}$$

[Boundary Condition]

$$\theta(0, t) = \theta_{\rm p} \tag{5}$$

$$\theta(\infty, t) = \theta_0 \tag{6}$$

Equation (2) can be transformed to obtain Eq. (7), which can be used to calculate thermal diffusivity, by using Eqs. (4) to (6) as the initial condition and boundary conditions, respectively.

$$(\theta - \theta_0) / (\theta_p - \theta_0) = 1 - \operatorname{erf}(x / \sqrt{4K \cdot t})$$
(7)

where the definition of error function is

$$\operatorname{erf}(Z) = 2/\sqrt{\pi} \int_0^Z \mathrm{e}^{-\eta^2} \,\mathrm{d}\eta$$

As shown in our previous papers, 8^{-12} the effective thermal diffusivity can be defined as follows:

$$K_{\rm e} = \lambda_{\rm e} / (\rho_{\rm m} \cdot C_{\rm m}) \tag{8}$$

where the effective thermal conductivity, λ_{e} , of foodstuffs composed of three components can be applied to the Series model.¹⁵⁾

$$\lambda_{\rm e} = 1/(X_{\rm w}^{\rm v}/\lambda_{\rm w} + X_{\rm p}^{\rm v}/\lambda_{\rm p} + X_{\rm F}^{\rm v}/\lambda_{\rm F}) \tag{9}$$

Due to their additive properties, the specific heat capacity and density can be expressed as Eqs. (10) and (11),

Foodstuffs	Moisture content (wt%)	Fat content (wt%)	Tempera- ture (°C)	Thermal diffusivity (m ² /h) × 10 ⁻⁴
Meats:				
Beef	75		0	4.35
Beef	76		5	4.32
Beef	76		65	5.10
Beef, chuck	66	16	43-66	4.44
Beef, round	71	4	43-66	4.80
Beef, tongue	68	13	43-66	4.74
Ham, smoked	64		5	4.26
Ham, smoked	64		65	4.61
Ham, smoked	64	11	43-66	4.92
Ham, smoked	64	14	43-66	4.98
Fish:				
Codfish	81		5	4.38
Codfish	81		65	5.10
Halibut	76	1.0	43-66	5.28
Halibut	78	0.6	43-66	4.92
Fruits:				
Cherry, flesh	-		0-30	4.74
Peaches		_	2-32	5.04
Apple, whole	85		0-30	4.92
(Red delicious)				
Strawberry, flesh	92		5	4.56
Apple, sauce	37	_	5	3.78
Apple, sauce	90		5	4.49
Bananas, flesh	76		5	4.26
Plum, jam	43		5	3.78
Marmalade	44	_	5	3.78
Vegetables:			U U	0110
Tomato	60–70	_	25-65	4.50
Tomato, marrow	67		5	4.26
Potato, mashed	78	affective.	5	4.44
Potato, whole	83.6		24.6	6.05
(Monona)				0100
Potato, squash	88.5		22.9	5.87
(Golden	0010			0101
delicious)				
Cereals:				
Wheat	15		_	3.70
Dough	45			6.90
Peanuts,				
Ground, kernels	8 (d.b)		4.5	5.11
Ground, hulls	5 (d.b)		4.5	4.00
Peanuts pods	7 (d.b)		4.5	2.61
Others:	, (1.2)			
Sugar beets		-	060	4.56
Egg	88	_	5	4.60
Curd, lean	55		5	3.90
Curd, lean	82		5	4.38
Beef, fat			0-30	3.70
Pork, fat	5		-10^{-10}	4.10
Butter	_		-10-35	5.20
Water			0	4.73
			-10	44.89

 Table I. Thermal Diffusivity of Foodstuffs from Published Reports¹³⁾

 Values for water and ice were included for comparison.

respectively.

$$C_{\rm m} = C_{\rm w} \cdot X_{\rm w}^{\rm w} + C_{\rm p} \cdot X_{\rm p}^{\rm w} + C_{\rm F} \cdot X_{\rm F}^{\rm w} \tag{10}$$

and

$$\rho_{\mathbf{m}} = \rho_{\mathbf{w}} \cdot X^{\mathbf{v}}_{\mathbf{w}} + \rho_{\mathbf{p}} \cdot X^{\mathbf{v}}_{\mathbf{p}} + \rho_{\mathbf{F}} \cdot X^{\mathbf{v}}_{\mathbf{F}} \tag{11}$$

Therefore, the effective thermal diffusivity can be

 Table II.
 Thermal Properties Used to Predict the Effective Thermal Diffusivity for Fish and Meats

	Unfrozen state	Frozen state
λ _P	0.342 [W/m·°C]	0.581 [W/m·°C]
λ _F	0.190 [W/m·°C]	0.190 [W/m·°C]
λ _w	Eq. (A) $[W/m \cdot °C]$	Eq. (B) $[W/m \cdot °C]$
$\rho_{\rm P}$	1340 [kg/m ³]	$1340 [kg/m^3]$
$\rho_{\rm F}$	900 [kg/m ³]	930 $[kg/m^3]$
$\rho_{\rm w}$	$1000 \left[\text{kg/m}^3 \right]$	919 $[kg/m^3]$
C _{P.P}	1.26 [kJ/kg·°C]	1.26 [kJ/kg·°C]
C _{P,F}	2.09 [kJ/kg·°C]	2.09 [kJ/kg·°C]
C _{P.w}	4.19 [kJ/kg·°C]	2.00 [kJ/kg·°C]

Eq. (A): $\lambda_{\rm w} = 0.565 + 0.008T - 5.8 \times 10^{-6}T^2$.

Eq. (B): $\lambda_{\rm w} = 2.25 - 0.0062T + 1.15 \times 10^{-4}T^2$.

transformed to obtain Eq. (12) by using Eqs. (8), (9), (10), and (11).

$$\frac{1/K_{e} = (X_{w}^{v}/\lambda_{w} + X_{p}^{v}/\lambda_{p} + X_{F}^{v}/\lambda_{F})}{\times (C_{w} \cdot X_{w}^{v} \cdot \rho_{w} + C_{p} \cdot X_{p}^{v} \cdot \rho_{p} + C_{F} \cdot X_{F}^{v} \cdot \rho_{F})}$$
(12)

Values of thermal properties for Eq. (8) are presented in Table II.

Materials and Methods

Materials and sample preparation. Thermal diffusivities were measured for fish (codfish, jack mackerel, and sardine) and meats (beef and pork). Fresh fish was purchased from a local market. Thigh and neck from beef and thigh from pork were purchased from a local market. Skin, intestine, and bones were removed, and after thorough washing, muscles were minced using a meat chopper and then placed in the thermal diffusivity cell. After degassing with a vacuum pump, the thermal diffusivity cell was stored in a freezer, until the temperature of the sample reached a certain value.

Approximate composition. Moisture and fat contents were measured by the freeze-drying and Soxhlet method, ¹⁶) respectively. The amount of protein was estimated by subtracting moisture and fat from the total weight of sample.

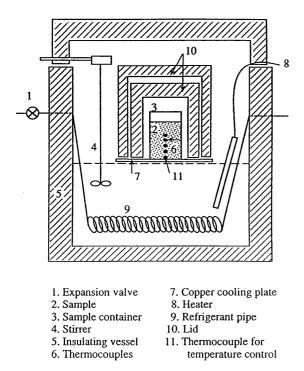
Experimental apparatus. A schematic diagram of the thermal diffusivity cell is shown in Fig. 1. The thermal diffusivity cell was made from a cylindrical acrylic resin pipe (55 mm in inside diameter, 120 mm in length, and 3 mm in thickness) on a copper plate (1 mm thick). Inside the cell, five copper constantan thermocouples (diameter = 0.3 mm) were installed along the center line, 10 mm from the bottom plate, to measure the temperature variation in each section. The radial temperature profile was checked and there was no difference in it. The outside of the cell was insulated twice to ensure a one-dimensional heat flow. The inner part was isolated by 7-cm thick styrofoam and 2-cm thick glass fibers.

The reliability of the measurement made from the apparatus has been analyzed and showed good results compared to the previous report.⁸) Crank–Nicolson's implicit method¹⁷ was used, in case the boundary conditions for Eq. 7 could not be used for the estimation of thermal diffusivity due to initially delayed heat flux of some samples.

Results and Discussion

To evaluate the themal diffusivity, temperature changes were monitored with unfrozen beef $(9 \sim 12^{\circ}C, Fig. 2)$ and with frozen codfish flesh $(-16.5 \sim -21.0^{\circ}C, Fig. 3)$. Moisture contents of beef and codfish were 75% and 81%, respectively. Predicted values agreed with the measured values, indicating that heterogeneous materials could be regarded as equivalent homogeneous materials in terms of heat conduction. Effective thermal diffusivities of beef and codfish were $4.50 \times 10^{-4} (m^2/h)$ and $3.45 \times 10^{-3} (m^2/h)$, respectively.

Downloaded by [IRSTEA] at 06:14 15 December 2014





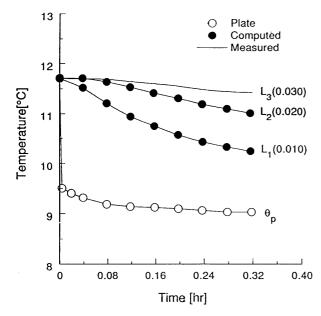


Fig. 2. Thermal Diffusivity of Round Beef with Moisture Content of 75.0% between 9° C to 12° C.

 $L_1, 0.010~{\rm m};~L_2,~0.020~{\rm m};~L_3,~0.030~{\rm m}$ from the bottom plate. $\theta_{\rm p},$ temperature of the cooling plate.

Thermal diffusivities of meats and fish flesh were measured unfrozen $(8 \sim 11^{\circ}\text{C})$ and frozen $(-18.0 \sim -21.0^{\circ}\text{C})$ and compared to those predicted by Eq. (8). Tables III and IV agreed between measured and predicted values with a standard deviation of 3.6% in the unfrozen state. Even though the standard deviation in the frozen state (7.8%) was higher than that in the unfrozen state, it could be taken as good agreement, considering the fact that the thermal diffusivity of ice was about ten times higher than that of water (Table I).

The validity of the prediction method was evaluated by comparing the predicted values from Eq. (8) with published

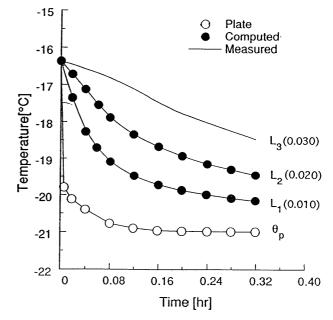


Fig. 3. Thermal Diffusivity of Codfish with Moisture Content of 81.0% between -16.5° C to -21.0° C. Legends, same as Fig. 2.

Table III. Comparison of Measured and Predicted Values of the Effective Thermal Diffusivity for Fish and Meats between 8° C and 11° C

Product	Moisture content (wt%)	Fat content (wt%)	Measured K_e (m^2/h) $\times 10^{-4}$	Predicted K_e (m^2/h) $\times 10^{-4}$	Difference (%)
Codfish	82.0	0.6	5.08	4.93	-3.0
	80.0	0.8	5.02	4.92	-2.0
	80.0	0.6	4.85	4.93	1.6
Jack mackerel	74.0	3.0	4.65	4.78	2.7
	74.0	2.8	4.64	4.80	3.3
	75.0	3.3	4.72	4.76	0.8
Sardine	70.0	9.0	4.11	4.45	7.6
	69.0	10.0	4.19	4.41	5.0
	72.0	4.5	4.79	4.70	-1.9
Beef round	73.0	4.3	4.73	4.71	-0.4
	74.0	3.4	4.50	4.66	-3.4
Beef chuck	68.0	12.0	4.07	4.33	6.0
	67.1	14.0	3.90	4.23	7.8
Pork leg	71.0	6.1	4.62	4.61	-0.2
· ·	71.0	7.0	4.46	4.56	2.2
Pork ham	69.0	7.8	4.32	4.53	4.6

Standard deviation, $\pm 3.6\%$.

values from Dickerson and Read,¹⁾ using unfrozen fish flesh (Table V). A good agreement was obtained between the predicted values and published values with the differences of about $8.8 \sim -10.8\%$ and a standard deviation of 7.8%. As shown in Table V the thermal diffusivity of halibut was higher than that of pork ham due to the high water content and low fat content of halibut. There was a significant trend that the thermal diffusivity decreased with decreases of water content. However, the published thermal diffusivity showed more deviation with the increase of fat content and the decrease of water content. According to this, the fat content should be taken into consideration for the evaluation of thermal diffusivity. The published thermal diffusivities were

Product	Moisture content (wt%)	Fat content (wt%)	Measured K_e (m^2/h) $\times 10^{-3}$	Predicted K_e (m^2/h) $\times 10^{-3}$	Difference (%)
Codfish	82.0	0.6	3.54	3.25	-9.0
	80.0	0.8	3.36	3.11	-8.0
	80.0	0.6	3.40	3.15	-6.0
Jack mackerel	74.0	3.0	2.65	2.51	-5.6
	74.0	2.8	2.47	2.54	-7.9
	75.0	3.3	2.56	2.50	-2.4
Sardine	70.0	9.0	1.70	1.83	7.0
	69.0	10.0	1.66	1.75	4.9
	72.0	4.5	2.45	2.27	-7.9
Beef round	73.0	4.3	2.52	2.32	-8.8
	74.0	3.4	2.72	2.46	-10.7
Beef chuck	68.0	12.0	1.45	1.61	9.8
	67.1	14.0	1.33	1.50	11.3
Pork leg	71.0	6.1	2.20	2.08	- 5.8
-	71.0	7.0	1.91	2.01	5.0
Pork ham	69.0	7.8	1.76	1.90	7.3

Table IV. Comparison of Measured and Predicted Values of the Effective Thermal Diffusivity for Fish and Meats between -18° C and -21° C

Standard deviation, $\pm 7.8\%$.

Table V. Comparison of Published and Predicted Values of the Effective Thermal Diffusivity for Fish and Meats between 43° C and 66° C^{*a*}

Product	Moisture content (wt%)	Fat content (wt%)	Published K_e (m^2/h) $\times 10^{-4}$	Predicted K_e (m^2/h) $\times 10^{-4}$	Difference (%)
Halibut	78	0.6	4.92	5.30	7.7
	78	0.8	4.86	5.29	8.8
	76	0.2	5.22	5.28	1.1
	76	1.0	5.28	5.27	-0.2
Beef round	67	10	4.80	4.65	-3.1
	71	4	4.80	5.03	4.8
Beef tongue	70	10	4.92	4.64	- 5.7
	68	13	4.74	4.48	- 5.5
Beef chuck	66	12	4.50	4.54	0.8
	66	16	4.44	4.56	2.7
Pork ham	64	11	4.92	4.60	-6.5
	64	14	4.98	4.44	-10.8

^{*a*} Dickerson and Read.¹⁾

Standard deviation, $\pm 7.8\%$.

plotted with various water content of foods such as fruits, juice, jam, beef, pork, and halibut as shown in Fig. 4. The broken line is the predicted values from Riedel's correlation.⁴⁾ These values showed differences of $5 \sim 8\%$ compared to those of Dickerson and Read.¹⁾ Fat content of meats and fish flesh varied from a few percent to more than 10 percent. However, Riedel presumed that moisture content was the only parameter affecting thermal diffusivity. Considering the contribution of high fat content to thermal diffusivity, it might be unreasonable to use Riedel's formula for calculating thermal diffusivities in meats and fish flesh.

To assess the effects of composition on the effective thermal diffusivity of meat sample, calculations were made for temperatures from 10° C to -10° C as shown in Figs. 5 and 6. An arbitrary composition of fish flesh is shown by a corresponding point in Fig. 5. The abscissa and the

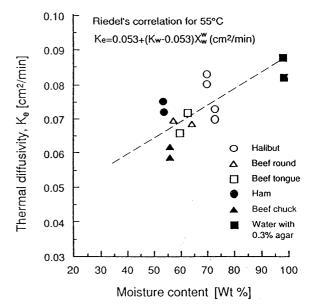


Fig. 4. Thermal Diffusivity of Meat as a Function of Moisture Content.^{1,4)}

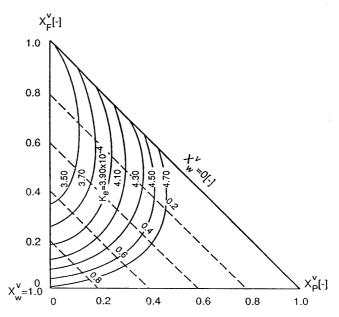


Fig. 5. Prediction of Effective Thermal Diffusivity for Unfrozen Fish as a Function of Protein, Fat, and Moisture Content (10°C). $X_{W}^{v} + X_{F}^{v} + X_{F}^{v} = 1.$

ordinate are contents of protein and fat, respectively. The dotted line in the figures shows the moisture content. The solid contour lines show values of the effective thermal diffusivity. These ternary contour diagrams can be used to predict the effective thermal diffusivities of fish flesh and meats. Moreover, these diagrams can be used for the prediction of thermal properties when new synthetic heterogeneous food stuffs such as surumi or crab meat are fabricated.

A wide-temperature range ternary contour diagram can be prepared and used for practical purposes. The theoretical formula can be easily used for the construction of ternary contour diagrams by obtaining temperature-independent intrinsic thermal conductivities of protein and fat of foodstuffs. J.-Y. KONG et al.

[kJ/kg°C]

[m²] [°C]

[°C]

[—]

۲—٦

[wt%]

 $[m^2/h]$

[W/m °C]

[kg/m³]

 $[W/m^2]$

[m]

[h]

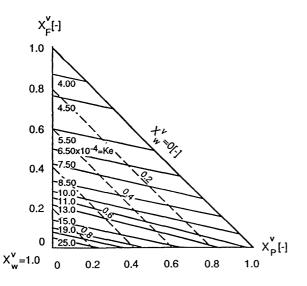


Fig. 6. Prediction of Effective Thermal Diffusivity for Frozen Fish as a Function of Protein, Fat, and Moisture Content $(-10^{\circ}C)$. $X_{W}^{V} + X_{P}^{V} + X_{F}^{V} = 1.$

Nomenclature

$C_{\rm p}$ = specific heat capac	ity
-----------------------------------	-----

- A = area θ
- = temperature θ_{p}
- = temperature of the coolong plate Ŵ
 - = moisture content
- X = volume fraction
- X٣ = weight fraction K
 - = thermal diffusivity
 - = thermal conductivity
 - = density

λ

ρ

 $q \\ X$

t

- = heat flux
- = distance
- = time

- Subscripts
- = effective value e
- = mean or average value m
- F = fat
- Р = protein
- = initial condition 0
- = sample surface s
- = moisture w
- L_1 = distance of 0.010 m from the bottom plate
- L_2 = distance of $0.020 \,\mathrm{m}$ from the bottom plate L_3 = distance of 0.030 m from the bottom plate

References

- R. W. Dickerson, Jr. and R. B. Read, Jr., Trans. ASHRAE, Part I, 1) 356-364 (1975).
- C. P. Lentz and L. van den Berg, Trans. ASHRAE, Part I, 533-540 2) (1977)
- 3) T. Yano, J. Y. Kong, O. Miyawaki, and K. Nakamura, J. Food. Sci., 46, 1357-1361 (1981).
- 4) L. Riedel, Kältetechnik-Klimatiserung, 21, 315-316 (1969).
- R. E. Parker and B. A. Sout, Trans. ASEA, 10, 489-496 (1967). 5)
- A. H. Bennet, C. G. Williams, Jr., and C. H. Randall, Trans. 6) ASHRAE, 75(2), 133-142 (1969).
- 7) D. Suter, K. K. Agrawal, and B. L. Clary, Trans. ASAE, 18, 370-375 (1975).
- 8) J. Y. Kong, O. Miyawaki, and T. Yano, Agric. Biol. Chem., 44, 1905-1910 (1980).
- J. Y. Kong, O. Miyawaki, K. Nakamura, and T. Yano, Agric. Biol. 9) Chem., 46, 783-788 (1982).
- J. Y. Kong, O. Miyawaki, K. Nakamura, and T. Yano, Agric. Biol. 10) Chem., 46, 789-794 (1982).
- 11) J. Y. Kong, O. Miyawaki, K. Nakamura, and T. Yano, Agric. Biol. Chem., 46, 1235-1241 (1982).
- 12) J. Y. Kong, Bull. Korean Fish. Soc., 15, 154-160 (1982).
- R. P. Singh, Food Technol., 36, 87-91 (1982). 13)
- H. S. Carslaw and J. C. Jaeger, in "Conduction of Heat in Solid," 14) 2nd Ed., Clarendon Press, Oxford, 1959.
- 15) J. C. Chato, ASME, Paper 66-WA/HT-37, 1-9 (1966).
- W. Horwitz, in "Official Method of Analysis of the A.O.A.C.," 20th 16) Ed., A.O.A.C., Washington D.C., 1975, p. 234.
- 17) J. Crank and P. Nicolson, Pros. Camb. Phil. Soci., 43, 50-67 (1947).