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Research Note

Improved Formulations of Shape Factors for the Freezing and Thawing Time Prediction of Foods

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Improved formulations of shape factors for the freezing and thawing time of infinite cylinders, spheres and ellipses have been proposed. Finite difference models were used for data generation. The generated data were correlated by regression analysis to yield equations which expressed the shape factors as a function of the Biot number, the initial and final temperature of the food, the ambient temperature and the dimensions of the food. It was possible to predict the numerical results with absolute mean errors lower than 4.0% with the shape factor expressions proposed.

Keywords: freezing and thawing time; shape factors

Introduction

Food products of various shapes are frozen or thawed industrially. Therefore, there is a need for the development of simple and accurate prediction methods. The utilization of shape factors to take into account the product shape has been proposed (1) and generally accepted (2). The shape factor (equivalent heat transfer dimension) defined by Cleland and Earle (1) is

$$E = \frac{\text{Freezing (thawing) time of infinite slab}}{\text{Freezing (thawing) time of object}} \quad \text{Eqn [1]}$$

where the slab has the same thickness and thermal properties with the object and is frozen (thawed) under the same conditions. The prediction of the freezing or thawing time of the object reduces to the determination of the shape factor.
Accurate freezing and thawing time prediction equations are available for foods undergoing unidirectional heat transfer. Some of these equations are for a specific shape (3, 4, 5). However, when one general equation is used to predict the freezing or thawing time of foods undergoing unidirectional transfer, V/A ratio is used to characterize the food shape (6, 7) which means that the shape factor E is 2 for an infinite cylinder and 3 for a sphere. This would be only true for the case of negligible internal conductive resistance.
Various shape factors have been proposed in literature for foods undergoing two or three dimensional heat conduction. Hossain (8) reported shape factors for infinite rectangular rods, finite cylinders and rectangular bricks. The proposed shape factors are function of the Biot number and the dimensions of the objects, that is, the shape factor for an infinite rectangular rod is (8)

$$E = \frac{\left(1+\frac{2}{Bi}\right)\left(1+\frac{1}{\beta_1}\right)}{\left[\frac{2}{Bi}+\frac{2}{3\left(1+\frac{1}{\beta_1}\right)}+\frac{(1+\beta_1)}{3}\right]} \quad \text{Eqn [2]}$$

Pham (9) proposed a curve fitting expression for ellipsoids:

$$E = 1 + \left(\frac{F-1}{\alpha_1+\alpha_2}\right)^P (\alpha_1^q + \alpha_2^q) \quad \text{Eqn [3]}$$

where

$$P = \frac{1}{1+Bi} \quad \text{Eqn [4]}$$

$$q = \frac{1+Bi/2}{1+Bi/4} \quad \text{Eqn [5]}$$

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The shape factor given by Eqn [2] reduces to $E = 2$ for an infinite cylinder. Similarly Eqn [3] reduces to $E = 2$ for an infinite cylinder and $E = 3$ for a sphere. For an ellipse, $\alpha_2 = 0$.
Freezing or thawing times of foods are dependent on the initial and the final temperatures of the food, the ambient temperature, the shape and the dimensions of the food, its physical properties and the heat transfer coefficient.
The effects of initial and final temperatures in the food and the ambient temperature on the shape factor have so far been ignored (6, 8, 9). Only the effect of the Biot number and dimension ratio that characterize the food have been taken into account. Therefore, it has been considered worthwhile to investigate quantitatively the effect of the so far ignored parameters, the initial and final temperatures of the food, and the ambient temperature, on the shape factors. Improved correlations of shape factors for the freezing and thawing time prediction of some basic shapes will be proposed.

Material and Methods

The geometries considered were infinite cylinder, spheres and two dimensional ellipses. Previously developed accurate finite difference models were used to calculate the freezing and thawing times. Lean beef was chosen as the test material. The thermal data used in the computations were taken from Cleland and Earle (10), and Succar and Kayakawa (11). These numerical models have been successfully used in predicting the freezing and thawing time of ellipses (12, 13).
The test conditions for the freezing and thawing of infinite cylinders and spheres for fixed Biot numbers are given in **Tables 1** and **2**, respectively. For freezing time predictions, the Biot numbers considered were 0.2, 2, 10 and 200. Similarly, for thawing time calculations, the Biot numbers considered were 0.12, 1.2, 6 and 120. The freezing (thawing) time for the centre to reach a definite final temperature were recorded. For freezing or thawing of infinite cylinders and spheres, 108 data points were obtained for each geometry.
For two dimensional ellipses, the aspect ratios were taken as 1.41, 2, 4 and 6. The computer programs were run for the conditions stated in **Tables 1** and **2** for each aspect ratio and the Biot number, and the freezing (thawing) times were recorded. A total of 432 data points were obtained for the freezing or thawing times of ellipses. After the determination of the freezing and thawing times, the shape factors were calculated from Eqn [1].

Table 1 Test conditions for the freezing of infinite cylinders and spheres at a fixed Biot number

T_i(°C)	T_a(°C)	Final	Centre	Temperature (°C)
0.0	− 20.0	− 5.0	− 10.0	− 18.0
0.0	− 30.0	− 5.0	− 10.0	− 18.0
0.0	− 40.0	− 5.0	− 10.0	− 18.0
15.0	− 20.0	− 5.0	− 10.0	− 18.0
15.0	− 30.0	− 5.0	− 10.0	− 18.0
15.0	− 40.0	− 5.0	− 10.0	− 18.0
30.0	− 20.0	− 5.0	− 10.0	− 18.0
30.0	− 30.0	− 5.0	− 10.0	− 18.0
30.0	− 40.0	− 5.0	− 10.0	− 18.0

Table 2 Test conditions for the thawing of infinite cylinders and spheres at a fixed Biot number

T_i(°C)	T_a(°C)	Final	Centre	Temperature (°C)
− 40.0	5.0	− 2.0	− 1.0	0.0
− 40.0	20.0	− 2.0	− 1.0	0.0
− 40.0	35.0	− 2.0	− 1.0	0.0
− 25.0	5.0	− 2.0	− 1.0	0.0
− 25.0	20.0	− 2.0	− 1.0	0.0
− 25.0	35.0	− 2.0	− 1.0	0.0
− 10.0	5.0	− 2.0	− 1.0	0.0
− 10.0	20.0	− 2.0	− 1.0	0.0
− 10.0	35.0	− 2.0	− 1.0	0.0

Results and Discussion

Linear multiple regression method was used to correlate the computed shape factors. The multiple regression program from SAS Institute Inc., Cary, NC, U.S.A. was used. The regression equations obtained for infinite cylinders and spheres, their absolute mean errors from the numerical results and the error ranges are shown in **Tables 3** and **4**. For comparison, the performance of constant values (2 for an infinite cylinder and 3 for a sphere) are also shown in these tables. It can be clearly seen from these tables that the proposed equations are more accurate than using the ratio DA/2V as the shape factor. Using DA/2V as the shape factor resulted in poor predictions, especially, when the phase change was far from being completed ($T_{cf} = -5\,°C$ for freezing and $T_{cf} = -2\,°C$ for thawing).
For predicting the shape factors for the freezing and thawing times of ellipses, first the curve fitting equations proposed by Pham (9) were used (Eqn [3], [4] and [5]). Similar to the results obtained for infinite, cylinders and spheres, Pham's model (9) gave accurate predictions, except for the cases of high final centre temperature in freezing ($-5\,°C$) and low final centre temperature in thawing ($-2\,°C$). Therefore, in obtaining regression equations for ellipses, Pham's model (9) was used as an input to the regression equations whose forms were suggested by the equations obtained for infinite cylinders and spheres. The regression equations obtained for ellipses, their absolute mean errors from the numerical values and the error ranges are shown in **Table 5**. For comparison the performance of Eqn [3] is also shown in the same Table. As can be observed from **Table 5**. The proposed equations for ellipses are more accurate than Eqn [3].
Investigation of **Table 5** reveals that the shape factors are relatively weak functions of the initial temperature Ti and the ambient temperature Ta. With a slight loss of accuracy, the following regression equations which do

Table 3 Shape factors for predictions the freezing and thawing times of infinite cylinders, absolute mean errors and the error ranges (%E = freezing (thawing) shape factor for the regression equation/freezing (thawing) shape factor for the numerical model − 1) × 100)

	Equation	Abs. M. Err.	Range
Freezing	$E_c = 1.9621–0.0104\,T_{cf} + 0.0015\,T_i + 0.0045\,T_a + 0.0112/Bi$	2.2	− 7.2 + 11.8
	$E_c = 2$	4.0	− 7.8 + 17.0
Thawing	$E_c = 2.0422 + 0.0857_{cf} - 0.00076\,T_i - 0.00356\,T_a + 0.0099/Bi$	2.7	− 6.8 + 6.8
	$E_c = 2$	5.1	− 5.6 + 19.2

Table 4 Shape factors for predicting the freezing and thawing times of spheres, absolute mean errors between the proposed equations and numerical models, and the error ranges

	Equation	Abs. M. Err.	Range
Freezing	$E_S = 2.8842–0.0271\,T_{cf} + 0.00457\,T_i + 0.0113\,T_a + 0.0341/Bi$	2.8	− 7.9 + 10.9
	$E_S = 3$	6.4	− 14.1 + 22.5
Thawing	$E_S = 3.127 + 0.1857_{cf} - 0.00168\,T_i - 0.00818\,T_a + 0.02034/Bi$	4.0	− 10.7 + 10.2
	$E_S = 3$	7.5	− 9.9 + 28.1

not contain Ti and Ta as independent variable were obtained. The result are shown in **Table 6**. For freezing and thawing time prediction the simplified equations were also found to be accurate equations.

To test the proposed equations in **Table 5** outside the range in which they have been developed, these equations have been used to predict the freezing times of infinite cylinders by letting $\beta = 1$. The shape factor for the freezing and thawing times of infinite cylinders obtained from **Table 5** were compared with the numerical freezing and thawing shape factors which have been used in the assessment of equations proposed in **Table 3**. For freezing, the absolute mean error and the error range were calculated as 2.9% and −0.7 to +13.6%, respectively. Similarly, for thawing the corresponding values were 4.8, and −5.4 to 13.2%, respectively. Although the predictions of shape factors for infinite cylinders by the equations proposed in **Table 3** are more accurate, the absolute mean errors and the error ranges given above are smaller than those that would be obtained by utilizing a constant shape factor of 2.

The shape factors proposed in this paper coupled with an accurate infinite slab prediction method will enable the calculation of accurate freezing or thawing times for infinite cylinders, spheres and ellipses.

Nomenclature

A Area (m^2)

Bi Biot number, hD/k_l for thawing, hD/k_f for freezing

Table 5 Shape factors for predicting the freezing and thawing times of ellipses, absolute mean errors between the proposed equations and numerical models, and the error ranges

	Equation	Abs. M. Err.	Range
Freezing	$E_{ell} = E_{Pham} - 0.06437–0.0062\,T_{cf} + 0.00036\,T_i + 0.00177\,T_a + 0.01885/Bi\beta$	2.2	− 8.8 + 8.3
	E_{Pham} (Eqn [3])	4.1	− 5.8 + 13.4
Thawing	$E_{ell} = E_{Pham} + 0.01753 + 0.0330\,T_{cf} - 0.00043\,T_i - 0.00212\,T_a + 0.01968/Bi\beta$	2.6	− 9.7 + 10.4
	E_{Pham} (Eqn [3])	4.2	− 6.5 + 18.2

Table 6 Simplified shape factors for predicting the freezing and thawing times of ellipses, absolute mean error between the proposed equations and numerical models, and the error ranges

	Equation	Abs. M. Err.	Range
Freezing	$E_{ell} = E_{Pham} - 0.119–0.0062\,T_{cf} + 0.01885/Bi\beta$	2.3	− 7.1 + 9.8
Thawing	$E_{ell} = E_{Pham} - 0.01407 + 0.03303\,T_{cf} + 0.01968/Bi\beta$	3.0	− 7.4 + 12.0

D Smallest dimension (m)
E Shape factor, defined by Eqn [1]
F Geometric ratio (AD/(2V))
P Function defined by Eqn [4]
q Function defined by Eqn [5]
T Temperature (°C)
V Volume (m^3)
α_1 Ratio of smallest dimension to second smallest dimension of an ellipsoid
α_2 Ratio of smallest dimension to largest dimension of an ellipsoid
β Aspect ratio for an ellipse
β_1 Aspect ratio for an infinite rectangular rod

Subscripts

a ambient
c Infinite cylinder
cf Final center
f Frozen state
ell Ellips
i Initial
l Thawed state
Pham Calculated from Eqn [3]
s Sphere

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