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# COMPUTATION OF THE INSTANTANEOUS UNIT HYDROGRAPH AND IDENTIFIABLE COMPONENT FLOWS WITH APPLICATION TO TWO SMALL UPLAND CATCHMENTS

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ABSTRACT

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Our approach is based upon three factors: (i) the representation of total streamflow response as a linear convolution of the instantaneous unit hydrograph with rainfall excess; (ii) approximation of this representation in discrete time by a rational transfer function relationship which involves an efficient and flexible parameterisation; (iii) use of a simple refined version of the instrumental variable method of parameter estimation as the major tool to determine the number of identifiable flow components in the representation and to estimate their dynamic contributions to the instantaneous unit hydrograph. Stream hydrograph separation is undertaken by convoluting rainfall excess with the identified components of the unit hydrograph. The procedure is applied to two small upland catchments in Wales. The results demonstrate that at sampling intervals of the order of one hour, successful separation of quick and slow flow components can be achieved with short time series of rainfall and streamflow.

## INTRODUCTION

Several approaches have been developed to construct models of streamflow response to catchment precipitation. These have been categorised by Chapman (1975) and more recently by Todini (1988) on the basis that the justification for a particular approach depends on the intended use of the model and the type of information available for its construction. The direct utility of any approach will also hinge on several factors including the information available for its use in prediction or simulation mode, the acceptability of its associated performance, the computer resources and time available, and the calibration skills required of the user.

In this paper we are concerned with approaches which rely on approximations of the convolution integral:

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$$y(t) = \int_0^t h(t - s)\, u(s)\, \mathrm{d}s \tag{1}$$

where point or spatially-averaged rainfall excess $u(s)$ is operated on by $h(t - s)$ and integrated over time $t$ to yield flow $y(t)$ at some stream location. The function $h(t)$ is well known as the instantaneous unit hydrograph (IUH) (Chow, 1964). In our paper it is the total streamflow response resulting from unit rainfall excess applied to the catchment over an infinitesimally short period. The concept of the unit hydrograph was introduced by Sherman (1932). Its representation in eqn. (1) assumes a linearity between rainfall excess and streamflow response. Models based on the unit hydrograph concept attempt to describe the lumped dynamic response of the catchment to rainfall and are generally not used to represent the complexities of streamflow generation processes. They require that observational data only on $u(t)$ and $y(t)$ be available for estimation of model parameters. In addition to this advantage of minimal data requirements, it will be argued that the transfer function form of conceptual model for the instantaneous unit hydrograph possesses the following properties: it has a plausible and adaptable physical analogy of linear reservoirs configured in series and/or parallel; in combination with an adequate statistical estimation procedure it can reproduce total streamflow and its dominant quick and slow components with convincing accuracy; and despite not representing a physically detailed approach it still has impressive utility.

Instantaneous unit hydrograph modelling of rainfall–streamflow behaviour has a long history, a review of which, up to 1964, can be found in Chow (1964), including the important reservoir conceptualisations discussed in a later section. Eagleson et al. (1966) used a transformation of the convolution integral leading to the Wiener–Hopf equation with constraint conditions for which an approximate solution was obtained using linear programming. Subsequent methods which rely on a constrained optimisation to realise a non-negative, non-oscillatory hydrograph include those of Deininger (1969), Mays and Taur (1982), Bruen and Dooge (1984) and Raynal Villasenor and Campos Aranda (1988). Papers deriving or applying a system identification approach include those of Kashyap and Rao (1973), Clarke (1974), Whitehead (1979), Whitehead et al. (1979), Mahendrarajah et al. (1982), Jakeman et al. (1984) and Rao and Mao (1987).

The contribution and plan of our paper are as follows. In the next section the assumptions and role for an IUH approach in characterising rainfall–streamflow behaviour are summarised. However, a major aim is to present and apply an objective and accurate procedure for determining the instantaneous unit hydrograph and the number of separate flow components which it is possible to identify from the information in the rainfall–streamflow time series data.

Our system identification procedure of flow component resolution and parameter estimation relies heavily on the choice of a suitable conceptual model of the unit hydrograph and a simple refinement of the instrumental variable method of parameter estimation. Some background and further references to the model and the estimation method are presented in two separate sections. Depending on the properties of the stochastic errors in the IUH model the method may yield asymptotically efficient parameter estimates but the estimates are always consistent. Estimation methods previously recommended in the literature, such as least squares or basic instrumental variables, can be ineffective. They do not minimise an appropriate error criterion such as the optimal generalised equation error (OGEE) of Jakeman and Young (1983). Examples from two small upland catchments in Wales illustrate the applicability of the procedure. Discussion and conclusions are presented together in the final section of the paper.

## THE IUH APPROACH: ASSUMPTIONS AND ROLE

The general assumptions associated with the unit hydrograph approach are well known (e.g. Chow, 1964) but are briefly reviewed here for completeness. In addition to the linearity of streamflow response to rainfall excess, these are homogeneity in the spatial distribution of infiltration capacity, rainfall and rainfall intensity. Clearly these conditions improve the theoretical adequacy of the convolution model and facilitate the construction of conceptually simpler transformations of rainfall to rainfall excess. However, variability in these quantities is a practical reality and it is accepted that this variability generally increases with catchment size.

To cope with the problem of a non-uniform areal rainfall, Chow (1964), among others, has proposed that a basin may have to be divided into smaller sub-basins each of which may be subject to separate hydrograph analysis for rainfall that can be assumed to be representative of the whole sub-basin. The basin hydrograph can then be obtained by routing of available or IUH computed hydrographs for the different tributaries. Liang and Nash (1988) have recently used this general approach and obtained good results for flood routing on the Yangtze River in China.

This procedure would also help ease problems associated with variable infiltration capacity. However, Ward (1975) comments that despite inhomogeneity of infiltration capacity in practice, the (lumped) unit hydrograph technique works well. This is partly explained by the variable source area concept of Hewlett (1961) which recognises that the quickflow-producing areas within a catchment at the beginning of a storm will be dependent on antecedent conditions. The variable source area concept also takes into account that quickflow-producing areas may expand during storms. This increase probably occurs in a systematic way depending on initial conditions and the temporal pattern of rainfall intensity. The lumped IUH approach cannot of course model within-catchment spatial variations of infiltration capacity during storms but, since variations are likely to be systematic, it can account for them on a catchment average basis.

As far as the assumption of spatial uniformity of rainfall intensity is concerned, we would argue that the errors deriving from any variability in this respect for small catchments will often be small in comparison to errors arising from other assumptions. And, as with other errors, if a number of storms are analysed the resultant unit hydrograph can be chosen to reflect a weighted averaging of those errors.

The wide ranging role of the IUH approach will now be considered. A major rationale for its use may be to interpolate streamflow records or to predict streamflow. Real-time forecasting is also possible with the IUH approach. The simple recursive nature of the conceptual model and the potential to model the errors stochastically allows for temporal updating, for example through Kalman filtering of the equivalent state-space formulation (e.g. Todini, 1978).

The results presented in this paper indicate that the unit hydrograph approach based on the convolution integral (eqn. (1)) can also be employed to understand at catchment scale the relative importance of the dominant flow components of the rainfall–runoff process and their dynamics. A minimum by-product can be conceptualisation of travel or storage times and dynamic contributions to streamflow hydrographs of the dominant quick and slow flow systems of particular catchments.

Another modelling objective which extends the utility of IUH models is to relate the derived parameters of unit hydrographs to physiographic and climatic catchment variables. In this way information can be amassed for predicting streamflow behaviour that is not directly observable, for example in ungauged catchments or in catchments which may be subject to land use changes. Two of the earliest attempts at synthesising unit hydrographs from catchment and climatic characteristics were carried out by McCarthy (1938) and Snyder (1938).

A prerequisite for success in modelling with respect to any of these objectives is system identification of a dynamic model which reproduces existing behaviour well. In general it is also important that the model parameters are estimated with significantly small standard errors. In this connection, Jakeman and Young (1980a), among others, have argued the parametrically efficient virtues of a rational transfer function approximation to the convolution integral in eqn. (1). This dynamic model form is the one adopted in our paper and its mathematical representation and physical interpretation are presented in the next section.

## THE MODEL

There are two basic components to our model of the rainfall–streamflow process. One is a non-linear rainfall filter (RF) which is used to produce a rainfall excess or "effective rainfall" which takes into account both short-term conditions, mainly the soil moisture status, and longer-term effects such as evapotranspiration and storage. The concept of effective rainfall is well developed in the hydrological literature (e.g. Chow, 1964). Basically, the RF we use in this paper provides a transformation which allows linearisation of the rainfall–streamflow relationship by considering, as model input $u(t)$ in eqn. (1), only that part of the rainfall available to contribute to streamflow during the period of interest. Second order effects such as depression storage and interception by vegetation are not accommodated explicitly in our rainfall excess model used here for moorland catchments. Being a crude model, it may need refinement in some instances, for example, when afforested catchments are subject to intermittent light rainfall leading to significant interception losses. However, it can be regarded as a useful exploratory starting point for a model construction exercise. The second component of the model presented in this paper identifies the remaining linear relationship using eqn. (1) as the underlying mathematical representation.

*The rainfall filter (RF)*

The rainfall filter model involves three simple operations. The first is a modulation of the measured rainfall $r_k$ at time step $k$ by a temperature dependent factor to compensate for evapotranspiration losses. A simple operation that has worked well (e.g. Whitehead et al., 1979; Mahendrarajah et al., 1982) is given by:

$$r_k^* = t_m^{-1}(t_m - t_k)r_k \qquad (2)$$

where $t_m$ is a reference temperature greater than the recorded maximum for the location in question and, for calculating $r_k^*$ in any given month, $t_k$ is the observed mean temperature for that month.

If model calibration of the IUH and subsequent prediction is undertaken over a period of reasonably constant temperature, eqn. (2) may not be needed. For example, for periods of the order of a couple of months, temperature variation will generally be insignificant compared with other model approximations. On the other hand, when such variation is significant, models more complicated than eqn. (2) may sometimes be required. Evapotranspiration in upland forested catchments, for example, can be quite large and prediction of its effect on streamflow may require more physical detail than provided in eqn. (2).

The second operation in our RF model is an adjustment which allows for antecedent precipitation effects on the soil moisture. A discrete first-order filter of the temperature modulated rainfall $r_k^*$ assesses the soil moisture content as:

$$s_k = s_{k-1} + \tau^{-1}(r_k^* - s_{k-1}) \qquad (3)$$

which, with $z^{-1}$ as the backward shift operator ($z^{-1}s_k = s_{k-1}$), can be expressed as an operation which is more obviously linear, viz.

$$s_k = \{\tau^{-1}/[1 - (1 - \tau^{-1})z^{-1}]\} r_k^*$$

As with $t_m$ in eqn. (2), the term $\tau$ is a constant to be optimised. The larger the value of $\tau$ the longer the effect of antecedent conditions on the soil moisture status.

Like the well-known Antecedent Precipitation Index (API) the operation in eqn. (3) attempts to quantify the current soil moisture status to determine how much of any present rainfall will be available for runoff. As argued by Whitehead et al. (1979), eqn. (3) is a simple parameterisation of an API which achieves exponential weighting with the past very efficiently. Indeed it involves just one parameter, $\tau$.

The effective rainfall $u_k$ is computed using the results of eqns. (2) and (3) as:

$$u_k = \text{const. } r_k^* s_k \quad (4)$$

In this paper the constant term in eqn. (4) is chosen so that the volume of effective rainfall, $u_k$, over the calibration period is equal to the total streamflow volume minus a volume of flow corresponding to the antecedent discharge rate. The ratio of the temporal sum of the output of a system (streamflow) to the temporal sum of the input (effective rainfall) is approximately the steady state gain which we define more rigorously later in the paper. We denote it by ssg (const.) to signify the dependence on the value of the constant term in eqn. (4). Since:

$$\text{ssg(const.)} = \text{ssg}(1)/\text{const.}$$

and

$$\text{ssg(const.)} = crf$$

where $crf$ is the factor which converts rainfall units to flow units for the catchment of interest, then:

$$\text{const.} = \text{ssg}(1)/crf$$

It should be noted that the constant in eqn. (4) is not required for model calibration as the transfer function model estimation procedure to be described can compensate for any missing multiplicative constants.

*The transfer function model*

Consider the discretized version of eqn. (1) with an error compensation term $\zeta_k$ assumed to represent the additive nature of all uncertainties arising from sampling, measurement and model errors, the latter including unrepresented model inputs. It is written as

$$\begin{aligned} y_k &= h_0 u_k + h_1 u_{k-1} + h_2 u_{k-2} + \ldots + h_{k-1} u_1 + \zeta_k \\ &= (\text{h}_0 + h_1 z^{-1} + h_2 z^{-2} + \ldots + h_{k-1} z^{-k+1}) u_k + \zeta_k \\ &= H(z^{-1}) u_k + \zeta_k \; (k = 1, 2, \ldots, N) \end{aligned}$$

It should be noted that the coefficients of the polynomial $H(z^{-1})$ are the desired unit hydrograph ordinates. However, even if the convolution representation is an excellent underlying model of the rainfall excess-streamflow process, the mildly ill-posed nature of eqn. (1) guarantees that even small errors in $u_k$ and $y_k$ lead to instabilities in the $h$ coefficients (Jakeman and Young, 1980a, 1984).

We therefore need to proceed with a stabilisation technique. Rational functions are known to provide efficient approximations of polynomials (e.g. Froberg, 1970), hence the representation:

$$H(z^{-1}) = B_m(z^{-1})/A_n(z^{-1})$$

where we let the as yet unidentified orders of the polynomials in the numerator and denominator be $m$ and $n$, respectively. Thus we define:

$$B_m(z^{-1}) = b_0 + b_1 z^{-1} + \ldots + b_m z^{-m}$$

$$A_n(z^{-1}) = 1 + a_1 z^{-1} + \ldots a_n z^{-n}$$

Then the new discrete version of eqn. (1) becomes:

$$x_k = \{B_m(z^{-1})/A_n(z^{-1})\} u_{k-\delta} \tag{5}$$

$$y_k = x_k + \zeta_k \tag{6}$$

The variable $x_k$ is the deterministic noise-free output of the transfer function or system model $B_m(z^{-1})/A_n(z^{-1})$. This variable is corrupted by error $\zeta_k$ to yield the observed variable $y_k$. Notice that a pure translation term of $\delta$ integral time steps is explicitly inserted in eqn. (5) although this could have been accommodated by the possibility of the first $\delta$ coefficients of $B_m(z^{-1})$ being set to zero. The explicitness allows us to consider $m$ as related to (one less than) the number of parameters in the $B_m(z^{-1})$ polynomial rather than the degree of the polynomial which is less important from an identification viewpoint.

In order to construct a physical analogy of this transfer function model, it is helpful to decompose the right-hand side of eqn. (5) into its constituent components. When all the roots of $A_n(z^{-1})$ are unequal, the decomposition is:

$$x_k = \sum_{i=1}^{n} [\beta_i/(1 + \alpha_i z^{-1})] u_{k-\delta} \tag{7}$$

which represents a model of a linear reservoir with $n$ branches all connected in parallel. For each $i$-th branch the parameter $\alpha_i$ is related to the time (or storage) constant $K_i$ by $K_i = -\Delta/\ln(-\alpha_i)$ with $\Delta$ the discrete time interval width. The time constant is the time it takes the $i$-th order component of the IUH to decay to $\exp(-1)$ or about 37% of its peak value (e.g. Young, 1984). The parameter $\beta_i$ is related to the fractional throughput $f_i$ for each branch where $f_i = \beta_i/(1 + \alpha_i)$. The quantity $f_i$ is also the steady state gain of the $i$-th branch relative to the total rainfall excess. The steady state gain of the dynamic system model (5) is defined as

$$\text{ssg} = B_m(1)/A_n(1) \approx \sum_{k=1}^{N} y_k / \sum_{k=1}^{N} u_k$$

where the approximation improves with $N$. Note that an approximation to the actual physical gain or loss of water through the system requires knowledge of *crf* to convert the rainfall units to the same units as the streamflow.

When all the roots of $A_n(z^{-1})$ are equal, the decomposition of eqn. (5) is

$$x_k = \sum_{i=1}^{n} [\beta_i/(1 + \alpha z^{-1})^i] u_{k-\delta}$$

which is a linear reservoir model where each of the $n$ branches connected in parallel can be further decomposed into first-order polynomial cells. These cells are connected in series and for the $i$-th branch there are $i$ cells. All cells have the same denominator polynomial coefficient $\alpha$ and hence time constant but the fractional throughput can vary.

Of course, various combinations of these two cases are theoretically possible depending on the form of the roots of $A_n(z^{-1})$. It is easily shown (e.g. Spolia and Chander, 1974) that many of the conceptual models such as those of Nash (1957), Diskin (1964) and Kulandaiswamy (1964) proposed in Chow's survey of unit hydrograph methods are special cases of the transfer function model. The model of Dooge (1959) is a little more general, allowing linear channels and reservoirs connecting basin sub-areas.

Like those linear reservoir models, the transfer function model has the analogue of rainfall excess in the catchment following certain broad pathways to the stream, each with a characteristic time constant and steady state gain. It is unlike them in that it need not involve prior judgements on the number and configuration (series vs. parallel) of pathways.

## PARAMETER ESTIMATION METHODS AND THEIR PROPERTIES

A general feature of the application of recent approaches (e.g. Ljung, 1977; Jakeman and Young, 1983) to estimate time series parameters is use of the instrumental variable (IV) technique. The interested reader is referred to Soderstrom and Stoica (1983) and Young (1984) for a comprehensive discussion of the IV technique. The treatments are complementary with the former more theoretical in presentation and the latter more tutorially oriented for the benefit of the practitioner.

The main aim of this section and the appendix is to summarise the basis of a particular IV technique, the simple refined instrumental variables (SRIV), in relation to its major competitors: least squares (LS), basic instrumental variables (BIV) and refined instrumental variables (RIV). SRIV was constructed and used for multivariable discrete-time systems by Jakeman (1979). The recursive form of the basic equations are reported there, while Young (1985) and Jakeman et al. (1989a, b) also indicate how simplification of RIV to SRIV equations can be obtained for the single variable input–output case applicable in this paper.

As with all IV techniques, SRIV yields by definition consistent parameter estimates. Unlike the well-known LS and BIV techniques, SRIV can identify slow IUH components as shown later in this paper. Young (1985) also demonstrates the importance of using SRIV when the input is an impulse obtained, for example, by injection of a tracer. While SRIV is conceptually and computationally simpler and allows more general assumptions on the stochastic term $\zeta_k$ than asymptotically efficient (more refined) versions of IV, it does have their major useful properties. The appendix to this paper provides more details of the properties associated with the often used LS and BIV estimators as well as with the more sophisticated SRIV and RIV estimators.

## MODEL STRUCTURE IDENTIFICATION

The method of identification or specification of the model structure for the rainfall–streamflow process involves two basic components. One is the determination of the most appropriate model parameterisation $(n, m)$ of the system model. The other component is identification of an appropriate rainfall filter (RF) to account for soil moisture conditions and evapotranspiration effects. Consider these two components in reverse order.

### *Determination of RF structure*

The RF structure represented by eqns. (2)–(4) has generally been found to work well by the authors. In addition to the evidence presented later in this paper, satisfactory applications of this rainfall filter structure have been documented in Whitehead (1979), Whitehead et al. (1979), Mahendrarajah et al. (1982) and Rao and Mao (1987). However, other structures may perform satisfactorily. Indeed, different structures may be required in situations where antecedent conditions and water storage processes are exceedingly complex.

Different RF structures may therefore need to be compared in relation to their ability to provide an explanation of the streamflow behaviour of interest with acceptable predictive uncertainty. Predictive uncertainty depends not only on the uncertainty of the parameters in the rainfall filter structure but also that ensuing in the parameters of the transfer function between effective rainfall and streamflow. Therefore, for an hypothesised RF structure, an objective procedure for characterising the overall parametric (and hence predictive) uncertainty is required. This needs to include an assessment of appropriate model structures $(n, m)$ which is the other component of model identification.

All of the above is objectively facilitated by use of a maximum likelihood framework. Conveniently, however, simpler approaches than maximum likelihood yield satisfactory determination of an adequate model structure and estimation of the parameters of a rainfall filter. In this paper, the rainfall filter model structure and its parameter(s) are chosen as a balance between parametric uncertainty and model fit to streamflow observations obtained by SRIV. The result chosen is one which yields a tolerable variance in the model residuals $\hat{\zeta}_k$ and estimates of the transfer function model parameters which are well defined and time-invariant. To estimate the parameter $\tau$, Rao and Mao (1987) minimise an objective function which represents a combination of the total relative standard deviation of the estimated $a_i (i = 1, \ldots, n)$ and $b_j (j = 0, 1, \ldots, m)$ parameters and the model residuals. We would recommend against placing total faith in one combined statistic and prefer to consider a comprehensive range of independent statistics, especially when estimation takes place within a framework that is more arbitrary than maximum likelihood.

*Determination of transfer function model order*

A trade-off between two major statistics is employed to help determine the transfer function model order. The first of these statistics is the average relative parameter error

$$\mathrm{ARPE}(n, m) = (m + n + 1)^{-1}\left[\sum_{i=1}^{n} \hat{\sigma}_i^2/\hat{a}_i^2 + \sum_{i=0}^{m} \hat{\sigma}_{i+n+1}^2/\hat{b}_i^2\right] \tag{8}$$

where the estimated parameter variance $\hat{\sigma}_i^2$ is the estimated variance of the $i$-th element in $\hat{\mathbf{a}} = (a_1, a_2, \ldots, a_n, b_0, b_1, \ldots, b_m)^{\mathrm{T}}$, with the superscript T denoting the vector transpose. These variances are available as byproducts of the SRIV algorithm.

The second statistic for assisting with model order identification is the model fit criterion

$$R^2 = 1 - \hat{\sigma}_\xi^2/\sigma_y^2 \tag{9}$$

with $\hat{\sigma}_\xi^2$ and $\sigma_y^2$ the variance of the residuals and output (streamflow), respectively.

As model parameterisation increases, the value of $R^2$ (model fit) reaches a plateau while the ARPE increases substantially after this plateau has been attained. The model order selected is the lowest one above which no significant improvement in $R^2$ is found but a significant increase in ARPE occurs. This behaviour of these two major first-pass identification criteria is supported by substantial empirical observation (e.g. Young et al., 1980; Jakeman et al., 1989b) in connection with both real data and simulated data from known systems. The ease of objective selection of model order is demonstrated in the results section of this paper.

*A summary of the identification procedure*

Based on the previous discussion and the work of Young et al. (1980), we propose a multi-step procedure for identifying the appropriate rainfall filter parameter(s) and model order for a linear-in-the-parameter transfer function model.

(1) Use prior information to specify the range of both the order of the system model and the RF parameter values. Initially omit the rain filter (RF) transformation(s).

(2) For each system model order candidate $(i, j)$, compute the following:

(a) ARPE $(i, j)$ by SRIV

(b) $R^2$ according to eqn. (9).

(3) Select an initial system model parameterisation $(n, m)$ by trading off the two statistics in 2.

(4) Search the RF parameter space. For each parameter sample, calculate those model fit statistics deemed important and ARPE $(n, m)$.

(5) Select the RF parameter values by trading off the fit and ARPE $(n, m)$ statistics obtained in step 4, and apply the RF to the input rainfall data.

(6) Repeat step 2 to verify that the identified model order $(n, m)$ remains the same.

(7) Perform diagnostic and validation checks on preliminary models.

It should be noted that if the relationship between rainfall and streamflow is highly non-linear, then the parameter values in the RF transformation may affect the model order identified.

## SITE DESCRIPTION AND DATA

The approach was applied to hourly rainfall and streamflow from two small upland catchments in Wales. Catchment CI6 drains 0.72 km$^2$ of moorland eastwards into the Camddwr tributary of the Afon Tywi above Llyn Brianne. It has a main stream slope of 67.1 m km$^{-1}$ and a considerable amount of valley-bottom peat. Catchment CI5 drains 0.34 km$^2$ of moorland westwards into the Afon Camddwr. It has a main stream slope of 113 m km$^{-1}$ and is reported to feature piping, particularly in an area to the south of the stream. The streamflow gauging sites for CI5 and CI6 are a few hundreds of metres apart. The catchments are underlain by Ordovician shales, grits and mudstones; have podzolic and some peat soils; are open pasture used for sheep grazing; and receive about 1800 mm rainfall annually.

Streamflow is measured at thin-plate rectangular-notch weirs and rainfall is measured by single tipping bucket rain gauges at the streamflow gauging sites. Short gaps due to instrument maintenance in the 15-min interval stage data were filled by subjective interpolation. Instantaneous rates of streamflow at the end of each hour were calculated from corresponding stage values and theoretical stage–discharge relations for the weirs. Independent measurement of streamflow by dilution gauging corroborated the stage–discharge relation for CI6 but there was some discrepancy for CI5 (I.G. Littlewood, unpublished data, 1989). Nevertheless, the theoretical stage–discharge relation for CI5 was used for our analysis. Where it was clear from simple inspection of the rainfall records that a tipping bucket gauge had malfunctioned for a short period, rainfall data from another nearby gauge were used.

The periods used for model calibration are given in Table 1. In addition an independent period of data for CI6 was used to examine the validity of the model and this is also given in Table 1. Note that in these applications we did not apply a temperature related modification of rainfall according to eqn. (2). It proved unnecessary to account for temperature dependent variations in evapotranspiration for the periods of calibration and validation considered.

## MODEL ORDER SELECTION RESULTS

Tables 2 and 3 show the results of model order identification for CI6 and CI5 after applying the rainfall filter (eqn. (3)) to obtain effective rainfall $u_k$

TABLE 1

Periods of model calibration and validation

| Catchment (Number of hourly samples, $N$) | Calibration period | Validation period |
|---|---|---|
| CI6 | 1/9/87–18/9/87 (400) | 1/7/87–1/9/87 (1500) |
| CI5 | 5/9/87–30/9/87 (600) | |

TABLE 2

Model order identification results for the CI6 catchment with the rainfall filter parameter in eqn. (3) set at $\tau = 86\,h$

| Parameterisation $(n,m)$ | %ARPE | $R^2$ |
|---|---|---|
| (1,0) | 0.0319 | 0.8191 |
| (2,0) | 21.2976 | 0.8206 |
| (2,1)* | 0.0214 | 0.9455 |
| (3,0) | 0.3637 | 0.8449 |
| (3,1) | 0.8216 | 0.9014 |
| (3,2) | 38.1116 | 0.9078 |

Asterisk denotes identified model structure.

TABLE 3

Model order identification results for the CI5 catchment with the rainfall filter parameter in eqn. (3) set at $\tau = 221\,h$

| Parameterisation | %ARPE | $R^2$ |
|---|---|---|
| (1,0) | 0.0356 | 0.7314 |
| (2,0) | 23.3210 | 0.7344 |
| (2,1)* | 0.0411 | 0.8516 |
| (3,0) | 1.1131 | 0.7539 |
| (3,1) | 2.1571 | 0.7884 |
| (3,2) | Unstable model | |

Asterisk denotes identified model structure.

according to eqn. (4). The rainfall filter parameters were set near their optimal values, $\tau = 86\,h$ and $\tau = 221\,h$, respectively, in order to demonstrate the behaviour of our major criteria. Fortunately, we have found that model order selection according to ARPE and $R^2$, derived from the SRIV estimator, is largely insensitive to the value of the parameter $\tau$, thereby eliminating the need to search the entire space of possible model orders and $\tau$ values jointly. In Table 2, note that the maximum value of $R^2$ corresponds with the minimum ARPE.

This may not always occur, as seen in Table 3. Generally, the level of model parameterisation selected is the one above which either decreases or only small increases in $R^2$ coincide with large increases in ARPE.

## OPTIMISATION RESULTS FOR THE RF PARAMETER

Once the model order is selected, our procedure applies the major criteria to any selected range of rainfall filter parameters. Tables 4 and 5 show that for the two catchments the minimum ARPE and maximum $R^2$ coincide closely and it matters little which value of $\tau$ is chosen within the possible range of coincidence. We have chosen $\tau = 86$ h and $\tau = 221$ h for CI6 and CI5, respectively.

Figures 1 and 2 show plots of the rainfall and effective rainfall for CI6 to demonstrate the effect of the rainfall filter. The effective rainfall can be scaled by equating the volumes of effective rainfall and flow in steady state as shown earlier in the paper. Notice that the damping of rainfall peaks is greater the longer the periods of low antecedent rainfall.

## PARAMETER ESTIMATION RESULTS

The identified system model for CI6 obtained by application of SRIV over the calibration period in Table 1 is:

TABLE 4

Results for the optimisation of the rainfall filter parameter $\tau$ for CI6

| $\tau$ | %ARPE | $R^2$ |
|---|---|---|
| 5 | 0.0305 | 0.9047 |
| 10 | 0.0280 | 0.9164 |
| 15 | 0.0285 | 0.9110 |
| 40 | 0.0255 | 0.9099 |
| 50 | 0.0244 | 0.9178 |
| 60 | 0.0231 | 0.9277 |
| 70 | 0.0220 | 0.9365 |
| 80 | 0.0215 | 0.9429 |
| 82 | 0.0214 | 0.9438 |
| 84 | 0.0214 | 0.9447 |
| 86* | 0.0214* | 0.9455 |
| 88 | 0.0215 | 0.9461 |
| 90 | 0.0215 | 0.9467 |
| 102 | 0.0223 | 0.9484 |
| 104* | 0.0225 | 0.9485* |
| 106 | 0.0227 | 0.9485 |
| 108 | 0.0229 | 0.9484 |
| 110 | 0.0232 | 0.9483 |
| 120 | 0.0247 | 0.9470 |
| 130 | 0.0266 | 0.9448 |

Asterisks denote range of optimum values.

TABLE 5

Results for the optimisation of the rainfall filter parameter $\tau$ for CI5

| $\tau$ | %ARPE | $R^2$ |
|---|---|---|
| 10 | 0.0553 | 0.4632 |
| 20 | 0.0510 | 0.5967 |
| 30 | 0.0519 | 0.4805 |
| 100 | 0.0436 | 0.7232 |
| 150 | 0.0444 | 0.7780 |
| 200 | 0.0456 | 0.7889 |
| 216 | 0.0406 | 0.8430 |
| 218* | 0.0403* | 0.8486 |
| 220 | 0.0406 | 0.8513 |
| 221* | 0.0411 | 0.8516* |
| 222 | 0.0418 | 0.8511 |
| 230 | 0.0560 | 0.8317 |
| 240 | 0.1210 | 0.7996 |

Asterisks denote range of optimum values.

$$\left.\begin{aligned} r_k^* &= r_k \\ s_k &= s_{k-1} + (1/86)(r_k^* - s_{k-1}),\ s_0 = 0 \\ u_k &= \text{const. } r_k^* s_k \end{aligned}\right] \tag{10}$$

$$x_k = \frac{20.4317\ (\pm 0.4056) - 19.9046\ (\pm 0.3840)z^{-1}}{1 - 1.7836\ (\pm 0.0070)z^{-1} + 0.7859\ (\pm 0.0068)z^{-2}}\, u_k \tag{11}$$

$$= \frac{18.8796}{1 - 0.7947\, z^{-1}}\, u_k + \frac{1.5521}{1 - 0.9890\, z^{-1}}\, u_k \tag{12}$$

The quantities in parentheses represent SRIV estimates of the standard errors on parameters. Figure 3 shows the results of SRIV estimation on model fit to

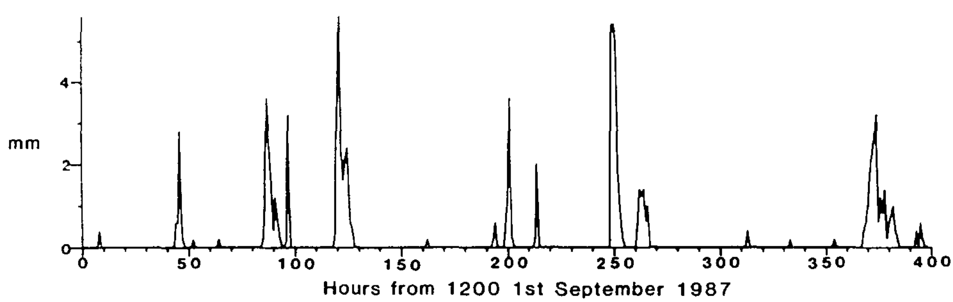


Fig. 1. Hourly rainfall time series for catchment CI6 for calibration period in Table 1.

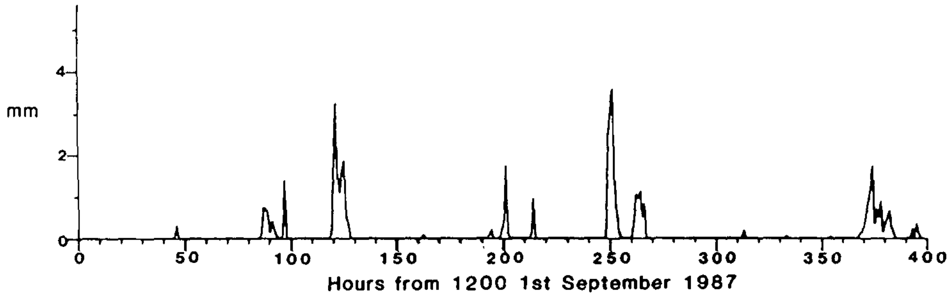


Fig. 2. Effective rainfall corresponding to rainfall in Fig. 1.

the streamflow data. It is interesting to compare the strong definition of the system parameters obtained by SRIV with the results of the better known LS and BIV estimators in Table 6 which show high standard errors on some parameters. In particular, the estimates of $a_2$ in Table 6 are not significantly different from zero. These results from both the LS and BIV algorithms wrongly suggest that there is only one identifiable IUH component. The model fits obtained by both LS and BIV estimation further indicate the inferiority of this identification. Figure 4 displays the corresponding fit obtained by BIV

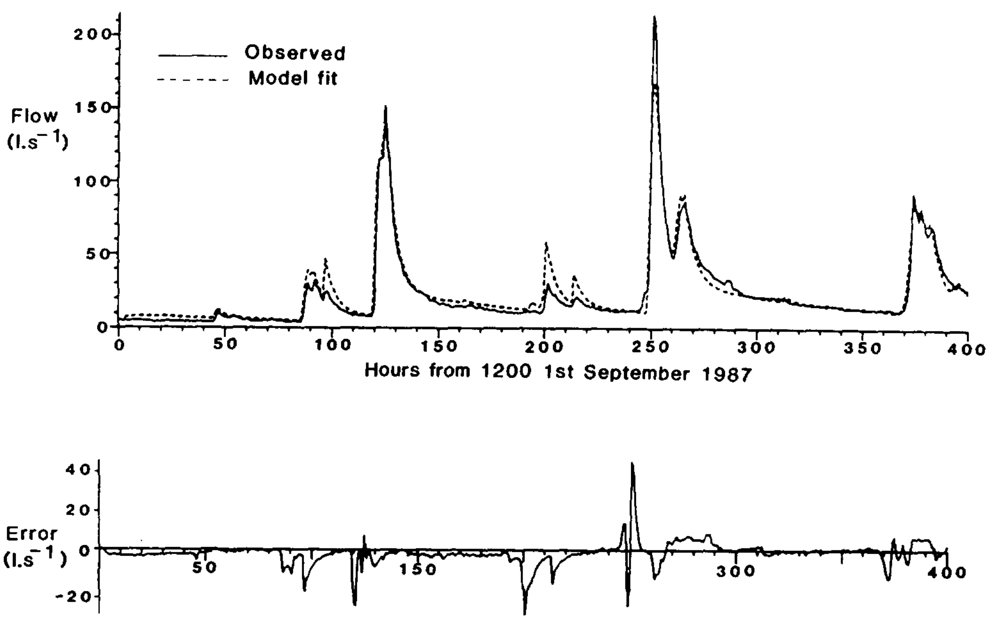


Fig. 3. Model fit to CI6 streamflow data using SRIV estimation over calibration period.

TABLE 6

Identification results for CI6 using LS and BIV

| | $a_1$ | $a_2$ | $b_0$ | $b_1$ | %ARPE | $R^2$ |
|---|---|---|---|---|---|---|
| LS | −0.8804 (±0.1982) | 0.0424 (±0.1709) | 15.3466 (±1.780) | 6.9673 (±4.321) | 14.6269 | 0.7916 |
| BIV | −0.7733 (±0.2364) | −0.0381) (±0.2032) | 15.2651 (±1.606) | 8.7720 (±5.028) | 722.9256 | 0.7947 |

estimation. The fit obtained by least squares estimation was very similar, both BIV and LS estimation techniques performing reasonably well in matching peaks of the streamflow hydrograph but seriously underestimating the recessions.

Equation (12) gives the decomposition of the SRIV-identified model (11) as two parallel first-order systems. The difference in the response coefficients (or time constants) and steady state gains of these two systems should be noted. Let quantities associated with the quicker response (smaller time constant) be denoted by the subscript q and those for the slower response by the subscript s. Then using the definitions for time constant and steady state gain, the relevant quantities for CI6 are:

$$K_q = -1/\ln(0.7947) = 4.4\,\mathrm{h}$$

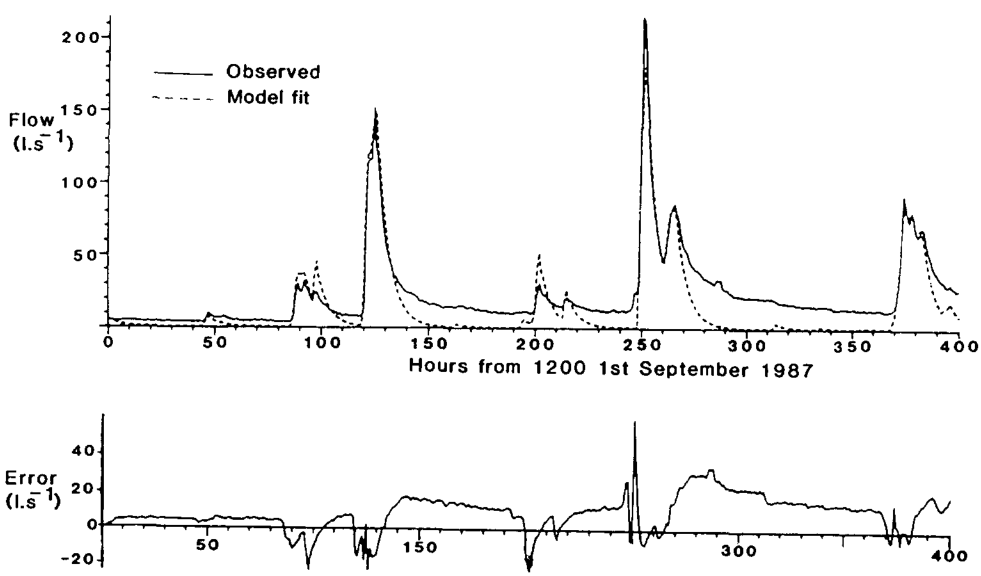


Fig. 4. Model fit to CI6 streamflow data using BIV estimation over calibration period.

$$ssg_q = 18.8796/(1 - 0.7947) = 92$$

$$K_s = -1/\ln(0.9890) = 90.1\,\text{h}$$

$$ssg_s = 1.5521/(1 - 0.9890) = 141$$

Thus the relative amount of effective rainfall discharged from the catchment via the quick response system is, on average, $ssg_q/(ssg_q + ssg_s) = 0.395$ while that through the slower pathway is $ssg_s/(ssg_q + ssg_s) = 0.605$. The relative contributions to the peak of the IUH can be obtained from the numerators in eqn. (12). These yield 0.924 for the quicker pathway and 0.076 for the slower.

The division of the polynomial $B_m(z^{-1})$ by $A_n(z^{-1})$ yields $H(z^{-1})$ whose coefficients are the instantaneous unit hydrograph ordinates. However these ordinates can be obtained more simply by setting $u_k = \delta_{k1}$ (the Kronecker delta function) in eqn. (11) and solving it recursively as:

$$x_1 = b_0 u_1 = 20.4317\,\delta_{k1} = 20.4317$$

$$x_2 = -a_1 x_1 + b_0 u_2 + b_1 u_1 = 1.7836 \times 20.4317 - 19.9046$$

$$x_k = -a_1 x_{k-1} - a_2 x_{k-2} = 1.7836\,x_{k-1} - 0.7859\,x_{k-2} (k = 3, 4, \ldots, N)$$

Similarly the IUH ordinates for the individual quick and slow response systems can be found separately by applying the Kronecker delta function definition to $u_k$ in the separate components of eqn. (12). Note that the peak of the IUH is $x_1$. The components of the IUH and their sum are shown in Fig. 5. The resultant influence of these components on streamflow for the calibration period is displayed in Fig. 6. Notice the more dominant effect of the slow component during low rainfall periods despite its small contribution to peak streamflow as seen in Fig. 5.

The above approach was also applied to catchment CI5 using SRIV estimation. A summary of the characteristics of the system model for CI5 is as follows:

$$\tau = 221\,\text{h}$$

$$x_k = \frac{3.6124\,(\pm 0.1023) - 3.5257\,(\pm 0.0973)z^{-1}}{1 - 1.8813\,(\pm 0.0072)z^{-1} + 0.8822\,(\pm 0.0071)z^{-2}}\,u_k$$

$$= \frac{3.0372}{1 - 0.8888\,z^{-1}}\,u_k + \frac{0.5752}{1 - 0.9925\,z^{-1}}\,u_k$$

$$K_q = 8.5\,\text{h},\ K_s = 133.4\,\text{h},\ ssg_q = 27,\quad ssg_s = 77$$

For the quicker component, the relative volume of effective rainfall and the contribution to the peak of the IUH are 0.2262 and 0.841, respectively. For the slower component, these figures are 0.738 and 0.159. Figure 7 shows the SRIV model fit to streamflow for CI5.

Figure 8 displays the corresponding IUH and component flows. It should be noted that the relative contribution of the slow component to the peak of the

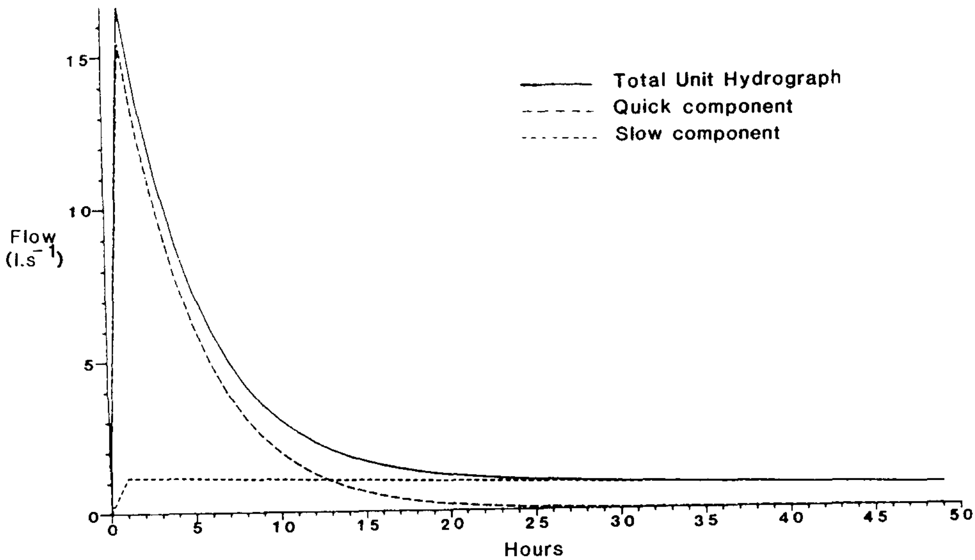


Fig. 5. The IUH and identifiable components for CI6.

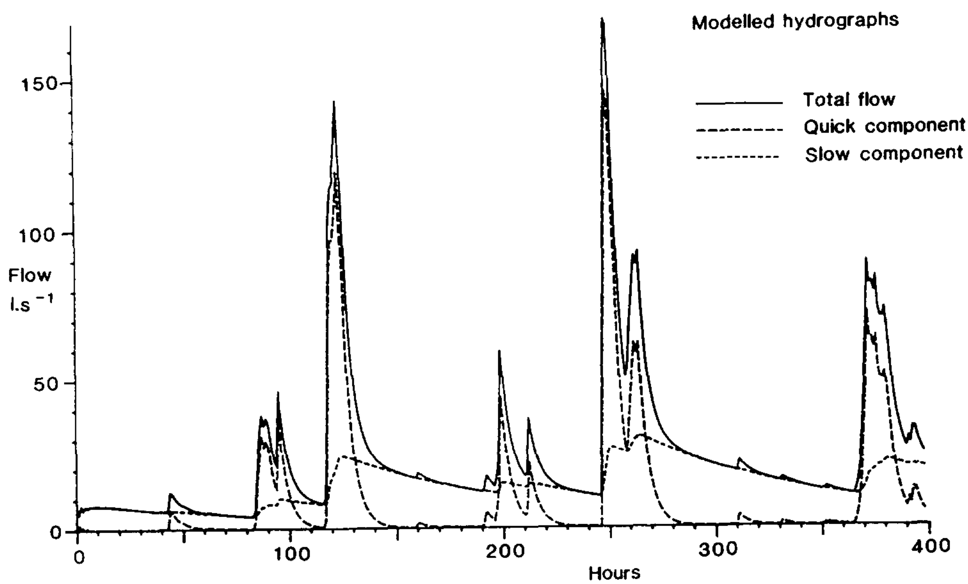


Fig. 6. Contribution of quick and slow IUH components to CI6 streamflow over calibration period.

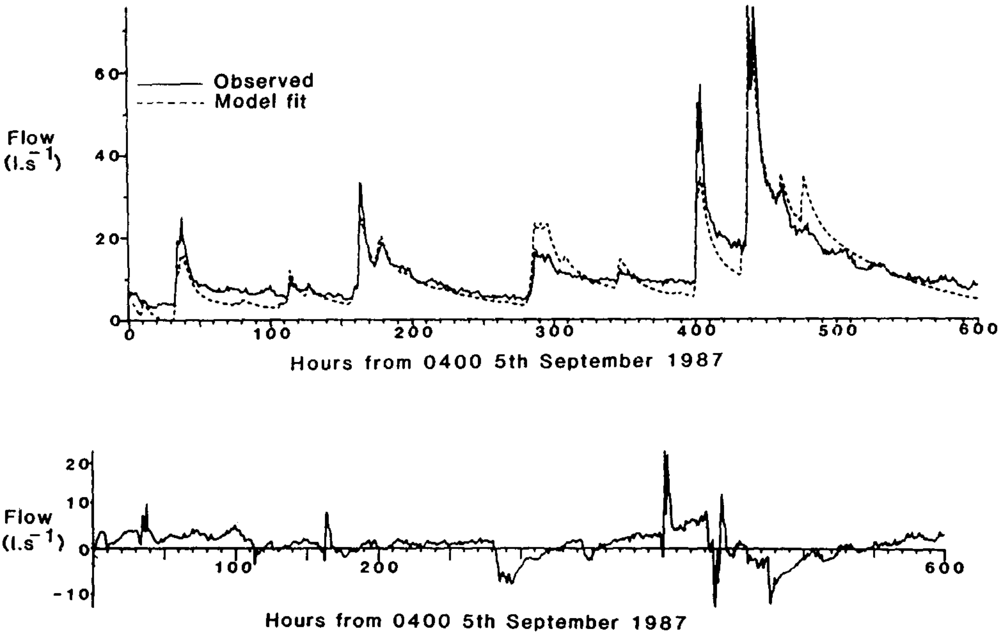


Fig. 7. Model fit to CI5 streamflow over calibration period in Table 1.

IUH is larger for CI5 than for CI6. Figure 9 shows the individual quick and slow flow contributions to CI5 streamflow over the calibration period.

To demonstrate the validity of the model for CI6 the identified model (10) and (11) was applied to an independent series of 1500 rainfall and streamflow observations for the period specified in Table 1. Figure 10 displays the results for the model fit to streamflow over the combined validation and calibration periods.

## DISCUSSION AND CONCLUSIONS

The paper has presented an approach to relate data on rainfall and other climatic variables to streamflow. The resultant model identified for any particular case can have several uses. These concern inference of any one of the following given the other two: system inputs (rainfall), system outputs (streamflow) and system model characteristics at catchment scale (time constant of rainfall filter, and time constant and gain parameters of individual linear reservoirs).

The approach is based upon a mathematical analogy for streamflow generation comprising a configuration of linear reservoirs and an adequate estimation technique. Taking the latter first, the estimator applied in this paper yields a combination of necessary statistical properties: consistency, relative efficiency, and stability. Importantly, the order of the model structure

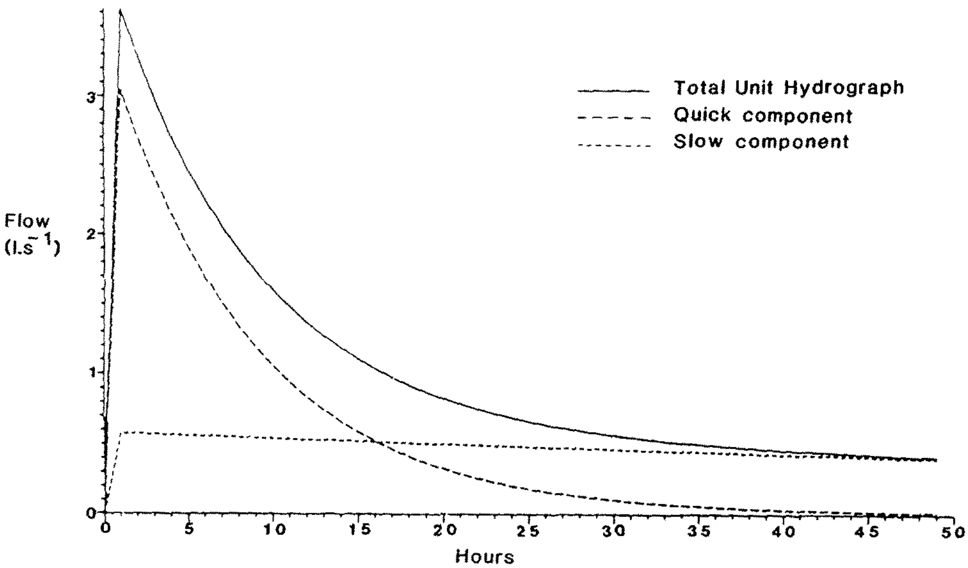


Fig. 8. The IUH and identifiable components for CI5.

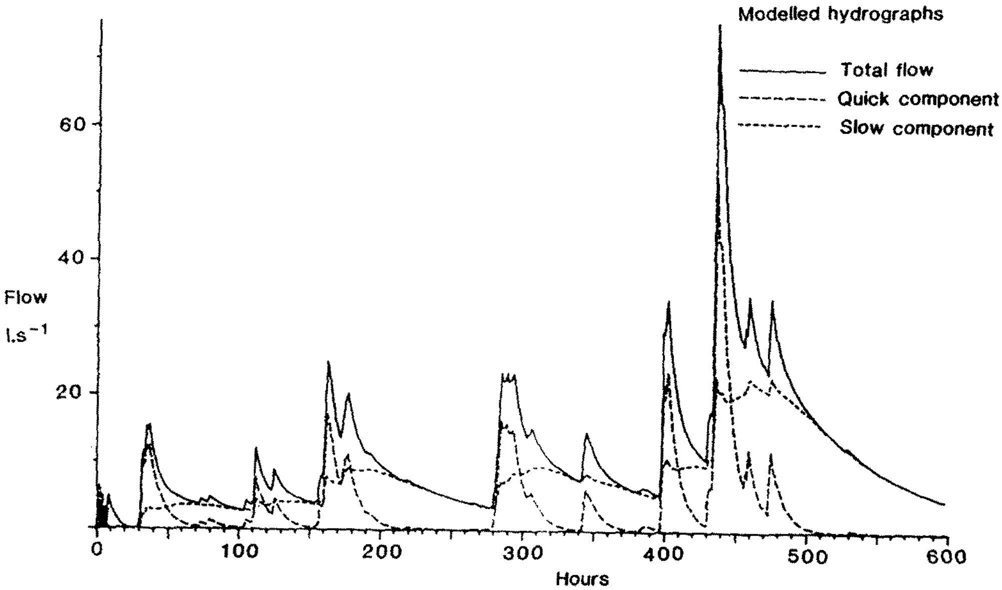


Fig. 9. Contributions of quick and slow IUH components to CI5 streamflow over calibration period.

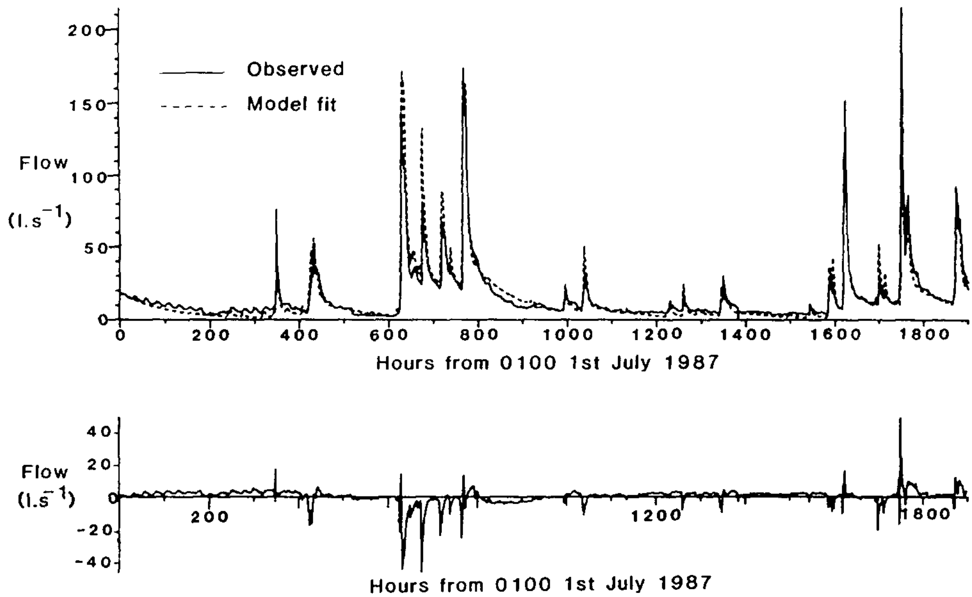


Fig. 10. Model fit to CI6 streamflow over the combined validation and calibration periods.

need not be specified in advance as the SRIV estimator yields statistics which readily allow selection of the structure deemed to be both consistent with streamflow observations and strongly identifiable from the rainfall and streamflow data.

It was demonstrated in the previous section that the use of the well-known least squares and basic instrumental variables estimation techniques can lead to models with poor performance. The LS estimator is biased while the BIV estimator can be unstable. For the BIV this can sometimes result in computational overflow but for LS and BIV it often leads to identification of a lower order model of inferior performance than SRIV or RIV. These results suggest an opposing view to that in Rao and Mao (1987) who regard least squares as adequate at least for their applications. We would recommend use of RIV or SRIV estimators as a matter of course since the additional effort is conceptually minor while the total computational effort is modest enough for implementation on microcomputers.

The mathematical attraction of the IUH approach in this paper is parametric efficiency of the transfer function model and the associated sensitivity of the model output to changes in model structure and parameter values. Its physical plausibility and attraction relates to the basic simplicity of the flexible system of linear reservoirs and their known applicability, from the work of many previous researchers, in simulating the catchment-scale rainfall–runoff process. A forthcoming paper by Young and Jakeman (1990) provides evidence that systems of linear reservoirs in parallel can be used to characterise other basic processes in hydrology.

These mathematical and conceptually attractive physical features add weight to the proposition that if a strongly identified linear structure of quick and slow response systems holds well for a reasonable calibration period then it may also hold when the catchment is subject to different series of rainfall excess excitations. These features cannot be associated with another popular approach based upon direct estimation of IUH ordinates via constrained optimisation. Deininger (1969), Mays and Taur (1982) and Raynal Villasenor and Campos Aranda (1988), for example, minimise various errors between streamflow and the IUH prediction of it subject to constraints imposed on the IUH ordinates. The constraints are necessary to counter problems associated with the high level of parameterisation of the solution, i.e. the number of unit hydrograph ordinates. The over-parameterisation leads to a unit hydrograph solution with oscillatory behaviour and negative ordinates unless the constraints are imposed.

Bruen and Dooge (1984) have effectively specified an improvement to this constrained approach by imposing a smoothness penalty on the IUH ordinates, found by least squares for example, to damp their oscillatory behaviour. The technique is known by mathematicians as regularisation (see Jakeman and Young, 1984 and references therein) and includes the ability to impose smoothness conditions on any linear combination of derivatives of the IUH ordinates. The regularisation approach allows restriction of the class of possible solutions in a systematic way by choice of smoothness condition and level of penalty to the least squares cost function. It still suffers, however, from lack of an attractive conceptual interpretation of the catchment-scale rainfall–runoff process.

There are four other noteworthy advantages of the approach in this paper. Firstly, there is no penalty in terms of additional data requirements compared with competing methods. Indeed, the parametric efficiency of the transfer function representation has a tendency to reduce the quantity of essential data. Instrumental variable estimators of transfer function model parameters can be successful with small time-series data sets of only 40–50 samples (e.g. Mahendrarajah et al., 1982) even with second-order systems (e.g. Jakeman and Young, 1980b). However, while only short data sets are required for model identification of the IUH, the calibrated model will perform better the more information in the data. The information content is basically a function of the range of rainfall and streamflow behaviour in the data set, the number of realisations of individual streamflows in that range and the lack of autocorrelation in the rainfall series.

Secondly, a guide to the sensitivity of the model parameters is directly provided by algorithms like the SRIV. An estimate of the covariance matrix of the parameters is a natural byproduct of the recursive estimation procedure. Therefore, an appreciation of the certainty of any identified model and of the need to collect further data can be obtained.

The third advantage of the approach in this paper is that baseflow (slow response) separation is an integral part of model identification and not, as in some methods which derive a unit hydrograph for quickflow only, a necessary prerequisite. The product of the method described here is an IUH corresponding to total streamflow and this IUH can be resolved into its slow and quick components. Convolution of each IUH component with effective rainfall yields component stream hydrographs. It appears that a lumped, two-component IUH model is conceptually valid for describing streamflow behaviour in the small, humid region, upland catchments examined here. Additional applications of the approach are being undertaken to assess its general ability to identify component flows in a wide range of catchments.

Fourthly, another advantage of the approach presented in this paper is that considerable time need not be spent selecting 'clean', single-peaked events for analysis, as required by some methods. Indeed, it is preferable to employ all the available verified rainfall–streamflow data to improve the precision and representativeness of model parameters.

In conclusion the authors would like to point out that a user-friendly computer package for this approach is at an advanced stage of preparation. It is called IHACRES which is an acronym for 'Identification of unit Hydrographs And Component flows from Rainfall, Evapotranspiration and Streamflow data'. It also signifies joint development of the package by the Institute of Hydrology and the Australian (National University's) Centre for Resource and Environmental Studies.

## ACKNOWLEDGEMENTS

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## APPENDIX

Table A1 summarises the properties of four different estimators of time series model parameters: least squares (LS), basic instrumental variable (BIV), simple refined instrumental variable (SRIV) and refined instrumental variable (RIV) methods. While the least squares estimate possesses the lowest computational complexity of the four, it is generally biased. All the IV estimators circumvent this problem by construction of an appropriate IV vector. However, we demonstrate in the results section that BIV shares a major drawback with the least squares estimator in that it generally fails to identify second-order systems ($n = 2$) and presumably higher order systems where one of the two roots of the $A_n(z^{-1})$ polynomial is near the unit circle. This means that the unit hydrograph response has a component, which may be quite important, that decays slowly. The results of Jakeman et al. (1989b) illustrate that, compared to the other competitors, BIV is also not as stable to outliers in observations.

On the plus side, BIV and SRIV are relatively efficient. Using simulations with known system parameterisations, Young and Jakeman (1979) demonstrate

TABLE A1

Properties of least squares and IV methods

| Property | Method | | | |
|---|---|---|---|---|
| | LS | BIV | SRIV | RIV |
| Unbiased | × | √ | √ | √ |
| Consistent | × | √ | √ | √ |
| Stable to outliers | √ | × | √ | √ |
| Asymptotically efficient | × | (1) | (1) | √ |
| Ability to identify slow IUH component | Poor | Poor | Strong | Strong |
| Relative computational complexity[2] | $N$ | $N.I$ | $N.I.(1+\varepsilon)$ | $N.2.I\,(1+3\varepsilon)$ |
| Recursive | √ | √ | √ | √ |

[1] BIV and SRIV are asymptotically efficient only if system model residuals are not autocorrelated, have mean zero and constant variance.

[2] $I$ denotes the number of iterations for convergence, $N$ the number of recursions per iteration and the relative complexity of filtering operations. Note that usually, $1 < I < 10$ and $0 < \varepsilon \ll 1$.

this for BIV while Jakeman (1979) demonstrates it for SRIV. A multivariable example in the latter paper provides a rough indication of the performance of the three IV techniques: the standard errors of BIV parameter estimates were about twice those of RIV while the standard errors of SRIV were about twice those of BIV. Importantly, the maximum standard error for any SRIV estimated parameter was about 10%. In other words SRIV can be efficient enough to yield useful parameter accuracy. These results were obtained for a multivariable system with five parameters to be estimated from 1000 samples, a problem of a higher order of difficulty than the ones presented in the results section. Note also that BIV and SRIV will be asymptotically efficient when the system model residuals $\zeta_k$ are not autocorrelated, have mean zero and constant variance.

While SRIV may at times suffer some penalty in terms of efficiency of estimates, it has important advantages over the other methods. It can identify slowly decaying IUH components as can RIV but it does this without knowledge of or major assumptions about the model residuals $\zeta_k$. It only requires that these residuals be uncorrelated with the system output $x_k$. And because it does not require estimation of parameters in a stochastic model as does RIV, there is an associated reduction in computational complexity. This is especially important when carrying out system model order identification as it circumvents the problem of having to consider every stochastic model structure option with every system model order option.

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