Heat and Mass Transfer 35 (1999) 33–38 © Springer-Verlag 1999

Thermal interaction between free convection and forced convection along a vertical conducting wall

J.-J. Shu, I. Pop

Abstract A theoretical study is presented in this paper to investigate the conjugate heat transfer across a vertical finite wall separating two forced and free convection flows at different temperatures. It is assumed that the heat conduction in the wall is only in the transversal direction. We also assume that countercurrent boundary layers are formed on the both sides of the wall. The governing equations of this problem and their corresponding boundary conditions are all cast into a dimensionless form by using a non-similarity transformation. These resulting equations, which are singular at the points $\xi_c = 0$ and 1, are solved numerically using a very efficient singular perturbation method. The effects of the resistance parameters and of the Prandtl numbers on heat transfer characteristics are investigated and presented in a table and ten figures.

List of symbols

A constant
b thickness of the plate
C constant
f reduced stream function
g acceleration due to gravity
h_x local heat transfer coefficient
k thermal conductivity
L length of the plate
Nu_x local Nusselt number
$\overline{\mathrm{Nu}}$ average Nusselt number
Pr Prandtl number
q_x local heat flux
Q overall heat flux
Re Reynolds number
R_t forced convection thermal resistance parameter
R_t^* free to forced convection parameter
t ambient temperature
Δt characteristic ambient temperature
T fluid temperature
u, v velocity components
U_∞ free stream velocity
x, y Cartesian coordinates

Greek letters

α thermal diffusivity
β coefficient of thermal expansion
θ dimensionless temperature
λ dummy variable
ν kinematic viscosity
ξ, η reduced coordinates
ψ stream function

Subscripts

c cold fluid system
h hot fluid system
s solid wall
w condition at the wall

Received on 8 April 1998

J.-J. Shu
Department of Mechanical Engineering
University of Hong Kong
The Pokfulam Road
Hong Kong

I. Pop
Faculty of Mathematics
University of Cluj
R-3400 Cluj, CP 253
Romania

Correspondence to: J.-J. Shu

The authors greatly appreciate the valuable comments made by the referee which led to the improvement of this paper.

1 Introduction

The study of thermal interaction between two semi-infinite fluid reservoirs at different temperatures through a vertical conductive wall is a very important topic in heat transfer because of its numerous engineering applications. This heat transfer process applies to reactor cooling, heat exchangers, thermal insulation, nuclear reactor safety, etc. Additionally, such interaction mechanisms is, for the most part, inherent in the design of heat transfer apparatus. On the other hand, it is worth mentioning that recent demands in heat transfer engineering have requested researchers to develop new types of equipments with superior performances, especially compact and light-weight ones. The need for small-size units, requires detailed studies on the effects of interaction between the thermal field in both fluids and of the wall conduction, which usually degradates the heat exchanger performance.

The problem of heat exchange between two free convection systems separated by a finite vertical conductive wall has not been studied extensively because of difficulties in solving the developments of flow and thermal boundary layers simultaneously. Lock and Ko [1] applied the local similarity method to investigate theoretically the effect of thermal coupling produced by conduction through a finite vertical wall separating two free convection systems. The study was done under the assumption that the wall conduction is only in the transversal direction. The governing boundary layer equations were transformed by introducing semi-similar variables and then solved numerically using a finite-differences method. This problem was also treated by Viskanta and Lankford [2] following a more simple analysis based on a super-position method. The authors have also conducted interferometric experiments which confirm the validity of their approximated theoretical results. Next, Anderson and Bejan [3] have employed the modified Oseen linearized method to solve such a conjugate problem when the Prandtl number is very large. It was shown that the overall heat transfer rate is relatively independent by the Prandtl number. Sakakibara et al. [4] have recently extended the problem of coupled heat transfer between two free convection systems separated by a finite vertical conducting wall assuming the two-dimensional conduction equation in the wall. In other words, the wall conduction takes place in both axial and transversal directions. Numerical solutions for both free convection systems and the analytical solution for the wall conduction were combined to obtain final solutions for the flow and heat transfer characteristics which fit the conjugate boundary conditions at both sides of the wall. Experiments were also conducted for air-air systems with the conducting wall made of aluminum or glass. It was found that theoretical results describe well the experimental temperature distributions. More recently, in a very interesting paper, Treviño et al. [5] reported numerical and asymptotic solutions of the free convection boundary layers on both sides of a vertical conducting wall for all possible values of two main parameters.

We notice to this end that there were also published several papers by Poulikakos [6], Méndez and Treviño [7], and Chen and Chang [8, 9] on the problem of thermal interaction between laminar film condensation of a saturated vapor and a forced or free convection system separated by a vertical conducting wall.

In this paper we intend to propose a new theoretical (mathematical) method to predict the heat transfer between free convection on one side of a finite vertical conducting wall and forced convection flow on the other side of the wall with the consideration of the wall thermal resistance. It is assumed that the conduction in the wall is in the transversal direction only. Since both the plate temperature and the heat flux through the plate are unknown a priori in this problem, the boundary layer equations on both sides of the wall and the one-dimensional heat conduction equation for the wall are solved simultaneously. The numerical method used is a new comprehensive and non-iterative scheme based on the singular perturbation method described in a recent paper by Shu and Pop [10]. We notice that this method differs by the iterative guessing technique proposed by Chen and Chang [8, 9]. Heat transfer characteristics have been derived for some main parameters entering this problem.

2 Basic equations

The physical model under consideration along with the coordinate systems is shown in Fig. 1, where the vertical plate with length L and thickness b separates two semi-infinite fluid reservoirs at different temperatures. The warmer reservoir contains a stagnant fluid with temperature t_h, while the ambient temperature on the cold side of the plate is t_c. Obviously, t_h is higher than t_c. The upper left corner of the plate coincides with the origin of a Cartesian coordinate system whose y axis points in the direction normal to the plate, while the x axis points downward in the plate's longitudinal direction. Due to gravity, a free convection laminar boundary layer appears on the hot side of the plate and flows downward along the plate. A forced convection flow of the cooling fluid with velocity U_∞ is imposed on the right lateral surface of the plate thus generating a forced convection boundary layer on this surface, which develops with increasing thickness downstream. Accordingly, two fluid streams move in opposite directions. Due to this assumption, the present problem can be formulated in terms of the boundary layer equations for two different heat transfer systems. These governing differential equations need to be considered separately and they are:

Hot fluid

$$\frac{\partial u_h}{\partial x} + \frac{\partial v_h}{\partial y} = 0 \tag{1}$$

$$u_h \frac{\partial u_h}{\partial x} + v_h \frac{\partial u_h}{\partial y} = \nu_h \frac{\partial^2 u_h}{\partial y^2} - g\beta(T_h - t_h) \tag{2}$$

$$u_h \frac{\partial T_h}{\partial x} + v_h \frac{\partial T_h}{\partial y} = \alpha_h \frac{\partial^2 T_h}{\partial y^2} \tag{3}$$

where u_h and v_h denote the velocity components of the hot fluid in the x and y directions, respectively, T_h is the temperature of the hot fluid, g is the gravitational acceleration, β is the thermal expansion coefficient of the hot fluid, and ν_h and α_h are the kinematic viscosity and thermal diffusivity of the heat fluid, respectively.

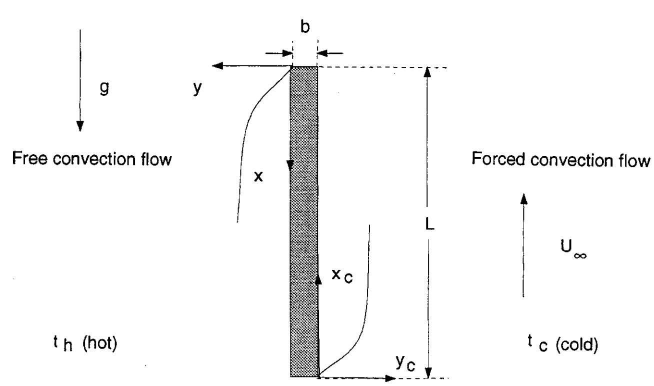

Fig. 1. Schematic diagram of the studied physical model

The boundary conditions for the free convection system are

$$
\begin{aligned}
& u_{\mathrm{h}} = v_{\mathrm{h}} = 0, \quad T_{\mathrm{h}} = T_{\mathrm{wh}}(x) \text{ on } y = 0 \\
& u_{\mathrm{h}} \to 0, \quad T_{\mathrm{h}} \to t_{\mathrm{h}} \text{ as } y \to \infty
\end{aligned}
\tag{4}
$$

where $T_{\mathrm{wh}}(x)$ denotes the wall temperature facing the hot side of the plate.

Cold fluid

$$
\frac{\partial u_{\mathrm{c}}}{\partial x_{\mathrm{c}}} + \frac{\partial v_{\mathrm{c}}}{\partial y_{\mathrm{c}}} = 0 \tag{5}
$$

$$
u_{\mathrm{c}} \frac{\partial u_{\mathrm{c}}}{\partial x_{\mathrm{c}}} + v_{\mathrm{c}} \frac{\partial u_{\mathrm{c}}}{\partial y_{\mathrm{c}}} = \nu_{\mathrm{c}} \frac{\partial^2 u_{\mathrm{c}}}{\partial y_{\mathrm{c}}^2} \tag{6}
$$

$$
u_{\mathrm{c}} \frac{\partial T_{\mathrm{c}}}{\partial x_{\mathrm{c}}} + v_{\mathrm{c}} \frac{\partial T_{\mathrm{c}}}{\partial y_{\mathrm{c}}} = \alpha_{\mathrm{c}} \frac{\partial^2 T_{\mathrm{c}}}{\partial y_{\mathrm{c}}^2} \tag{7}
$$

where x_{c} and y_{c} are Cartesian coordinates on the forced convection side, u_{c} and v_{c} denote the velocity components of the cold fluid in the x_{c} and y_{c} directions, and $T_{\mathrm{c}}, \nu_{\mathrm{c}}$ and α_{c} are the temperature, kinematic viscosity and thermal diffusivity of the cold fluid, respectively.

The boundary conditions for the forced convection system are

$$
\begin{aligned}
& u_{\mathrm{c}} = v_{\mathrm{c}} = 0, \quad T_{\mathrm{c}} = T_{\mathrm{wc}}(x_{\mathrm{c}}) \text{ on } y_{\mathrm{c}} = 0 \\
& u_{\mathrm{c}} \to U_\infty, \quad T_{\mathrm{c}} \to t_{\mathrm{c}} \quad \text{as} \quad y_{\mathrm{c}} \to \infty
\end{aligned}
\tag{8}
$$

where $x_{\mathrm{c}} = L - x$ and $T_{\mathrm{wc}}(x_{\mathrm{c}})$ denotes the wall temperature facing the forced convection side.

It is assumed that heat conduction along the plate is neglected in comparison with transverse heat conduction. Under this condition, the heat flux entering the left face of the plate will be equal to that leaving the right face at any given vertical position x, i.e.

$$
k_{\mathrm{s}} \frac{T_{\mathrm{wh}} - T_{\mathrm{wc}}}{b} = -k_{\mathrm{c}} \left. \frac{\partial T_{\mathrm{c}}}{\partial y_{\mathrm{c}}} \right|_{y_{\mathrm{c}}=0} = k_{\mathrm{h}} \left. \frac{\partial T_{\mathrm{h}}}{\partial y} \right|_{y=0} = q_{xc} \tag{9}
$$

where $k_{\mathrm{s}}, k_{\mathrm{h}}$ and k_{c} denote the thermal conductivities of the solid plate, hot fluid and cold fluid, respectively, and q_{xc} is the local heat flux through the plate. A correlation between T_{wh} and T_{wc} can be obtained from Eq. (9) as

$$
T_{\mathrm{wc}} = T_{\mathrm{wh}} - \frac{b k_{\mathrm{h}}}{k_{\mathrm{s}}} \left. \frac{\partial T_{\mathrm{h}}}{\partial y} \right|_{y=0} . \tag{10}
$$

3 Solution

To solve Eqs. (1)–(3) and (5)–(7), we introduce the following dimensionless variables

$$
\begin{aligned}
& \xi = x/L, \quad \eta = (C_{\mathrm{h}} y)/(L \xi^{1/4}) \\
& \psi_{\mathrm{h}} = 4 \nu_{\mathrm{h}} C_{\mathrm{h}} \xi^{3/4} f_{\mathrm{h}}(\xi, \eta), \; \theta_{\mathrm{h}}(\xi, \eta) = [T_{\mathrm{h}} - (t_{\mathrm{h}} + t_{\mathrm{c}})/2]/\Delta t \\
& \xi_{\mathrm{c}} = x_{\mathrm{c}}/L = 1 - \xi, \; \eta_{\mathrm{c}} = (y_{\mathrm{c}} \mathrm{Re}^{1/2})/(L \xi_{\mathrm{c}}^{1/2}) \\
& \psi_{\mathrm{c}} = (U_\infty \nu_{\mathrm{c}} L \xi_{\mathrm{c}})^{1/2} f_{\mathrm{c}}(\xi_{\mathrm{c}}, \eta_{\mathrm{c}}), \; \theta_{\mathrm{c}}(\xi_{\mathrm{c}}, \eta_{\mathrm{c}}) \\
& \quad = [T_{\mathrm{c}} - (t_{\mathrm{h}} + t_{\mathrm{c}})/2]/\Delta t
\end{aligned}
\tag{11}
$$

where $C_{\mathrm{h}} = [g\beta \Delta t L^3/(4\nu_{\mathrm{h}}^2)]^{1/4}$, $\Delta t = t_{\mathrm{h}} - t_{\mathrm{c}}$, $\mathrm{Re} = U_\infty L/\nu_{\mathrm{c}}$ is the Reynolds number for the forced convection flow, and ψ_{h} and ψ_{c} are the stream functions of the hot and cold fluids, respectively, which are defined as

$$
\begin{aligned}
& u_{\mathrm{h}} = \frac{\partial \psi_{\mathrm{h}}}{\partial y}, \quad v_{\mathrm{h}} = -\frac{\partial \psi_{\mathrm{h}}}{\partial x} \\
& u_{\mathrm{c}} = \frac{\partial \psi_{\mathrm{c}}}{\partial y_{\mathrm{c}}}, \quad v_{\mathrm{c}} = -\frac{\partial \psi_{\mathrm{c}}}{\partial x_{\mathrm{c}}}
\end{aligned}
\tag{12}
$$

Due to the definition of (11) and (12), Eqs. (1)–(3) and (5)–(7) can be transformed into the following form

$$
\begin{aligned}
& \frac{\partial^3 f_{\mathrm{h}}}{\partial \eta^3} + 3 f_{\mathrm{h}} \frac{\partial^2 f_{\mathrm{h}}}{\partial \eta^2} - 2 \left(\frac{\partial f_{\mathrm{h}}}{\partial \eta} \right)^2 - \left(\theta_{\mathrm{h}} - \frac{1}{2} \right) \\
& \quad = 4\xi \left(\frac{\partial f_{\mathrm{h}}}{\partial \eta} \frac{\partial^2 f_{\mathrm{h}}}{\partial \eta \partial \xi} - \frac{\partial^2 f_{\mathrm{h}}}{\partial \eta^2} \frac{\partial f_{\mathrm{h}}}{\partial \xi} \right)
\end{aligned}
\tag{13}
$$

$$
\frac{1}{\mathrm{Pr}_{\mathrm{h}}} \frac{\partial^2 \theta_{\mathrm{h}}}{\partial \eta^2} + 3 f_{\mathrm{h}} \frac{\partial \theta_{\mathrm{h}}}{\partial \eta} = 4\xi \left(\frac{\partial f_{\mathrm{h}}}{\partial \eta} \frac{\partial \theta_{\mathrm{h}}}{\partial \xi} - \frac{\partial \theta_{\mathrm{h}}}{\partial \eta} \frac{\partial f_{\mathrm{h}}}{\partial \xi} \right) \tag{14}
$$

$$
\frac{\partial^3 f_{\mathrm{c}}}{\partial \eta_{\mathrm{c}}^3} + \frac{1}{2} f_{\mathrm{c}} \frac{\partial^2 f_{\mathrm{c}}}{\partial \eta_{\mathrm{c}}^2} = \xi_{\mathrm{c}} \left(\frac{\partial f_{\mathrm{c}}}{\partial \eta_{\mathrm{c}}} \frac{\partial^2 f_{\mathrm{c}}}{\partial \eta_{\mathrm{c}} \partial \xi_{\mathrm{c}}} - \frac{\partial^2 f_{\mathrm{c}}}{\partial \eta_{\mathrm{c}}^2} \frac{\partial f_{\mathrm{c}}}{\partial \xi_{\mathrm{c}}} \right) \tag{15}
$$

$$
\frac{1}{\mathrm{Pr}_{\mathrm{c}}} \frac{\partial^2 \theta_{\mathrm{c}}}{\partial \eta_{\mathrm{c}}^2} + \frac{1}{2} f_{\mathrm{c}} \frac{\partial \theta_{\mathrm{c}}}{\partial \eta_{\mathrm{c}}} = \xi_{\mathrm{c}} \left(\frac{\partial f_{\mathrm{c}}}{\partial \eta_{\mathrm{c}}} \frac{\partial \theta_{\mathrm{c}}}{\partial \xi_{\mathrm{c}}} - \frac{\partial \theta_{\mathrm{c}}}{\partial \eta_{\mathrm{c}}} \frac{\partial f_{\mathrm{c}}}{\partial \xi_{\mathrm{c}}} \right) \tag{16}
$$

where $\mathrm{Pr}_{\mathrm{h}} = \nu_{\mathrm{h}}/\alpha_{\mathrm{h}}$ and $\mathrm{Pr}_{\mathrm{c}} = \nu_{\mathrm{c}}/\alpha_{\mathrm{c}}$ are the Prandtl numbers of hot and cold fluids, respectively.

The boundary conditions (4) and (8) become

$$
\begin{aligned}
& f_{\mathrm{h}} = \frac{\partial f_{\mathrm{h}}}{\partial \eta} = 0, \; \theta_{\mathrm{h}} = \theta_{\mathrm{wh}}(\xi) \text{ at } \eta = 0 \\
& \frac{\partial f_{\mathrm{h}}}{\partial \eta} \to 0, \; \theta_{\mathrm{h}} \to \frac{1}{2} \text{ as } \eta \to \infty
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
& f_{\mathrm{c}} = \frac{\partial f_{\mathrm{c}}}{\partial \eta_{\mathrm{c}}} = 0, \; \theta_{\mathrm{c}} = \theta_{\mathrm{wh}}(\xi) - R_t R_t^* \xi^{-1/4} \left. \frac{\partial \theta_{\mathrm{h}}}{\partial \eta} \right|_{\eta=0} \quad \text{at } \eta_{\mathrm{c}} = 0 \\
& \frac{\partial f_{\mathrm{c}}}{\partial \eta_{\mathrm{c}}} \to 1, \; \theta_{\mathrm{c}} \to -\frac{1}{2} \text{ as } \eta_{\mathrm{c}} \to \infty
\end{aligned}
\tag{18}
$$

where $R_t = b k_{\mathrm{c}} \mathrm{Re}^{1/2}/(k_{\mathrm{s}} L)$ denotes the thermal resistance ratio of the forced convection flow to the wall, $R_t^* = (k_{\mathrm{h}} C_{\mathrm{h}})/(k_{\mathrm{c}} \mathrm{Re}^{1/2})$ can be regarded as the thermal resistance of the hot fluid to the cold fluid, $\theta_{\mathrm{wh}} = [T_{\mathrm{wh}} - (t_{\mathrm{h}} + t_{\mathrm{c}})/2]/\Delta t$ and $\theta_{\mathrm{wc}} = [T_{\mathrm{wc}} - (t_{\mathrm{h}} + t_{\mathrm{c}})/2]/\Delta t$. Substituting variables (11) into Eq. (9), we get

$$
\begin{aligned}
& R_t^* \left(\xi_{\mathrm{c}}^{1/2}/\xi^{1/4} \right) \left. \frac{\partial \theta_{\mathrm{h}}}{\partial \eta} \right|_{\eta=0} + \left. \frac{\partial \theta_{\mathrm{c}}}{\partial \eta_{\mathrm{c}}} \right|_{\eta_{\mathrm{c}}=0} \\
& \quad = 0 \text{ at any given position } x_{\mathrm{c}} .
\end{aligned}
\tag{19}
$$

Based on Eq. (9), the local heat transfer coefficient h_{xc} for the forced convection system can be expressed as

$$h_{xc} = \left[-k_c \frac{\partial T_c}{\partial y_c} \bigg|_{y_c=0} \right] \Big/ [T_c(x_c, 0) - t_c]$$
$$= q_{xc}/[T_c(x_c, 0) - t_c] \ . \tag{20}$$

The local Nusselt number for the forced convection system can be expressed as

$$\mathrm{Nu}_{xc} = \frac{x_c h_{xc}}{k_c} = -\mathrm{Re}^{1/2} \xi_c^{1/2} \frac{\partial \theta_c}{\partial \eta_c} \bigg|_{\eta_c=0} \Big/ \left[\theta_{wc}(\xi_c) + \frac{1}{2} \right] \ . \tag{21}$$

The total heat flux Q through the surface facing the cold fluid is obtained by integrating the local heat flux over the entire height of the plate and can be expressed as

$$Q = k_c \int_0^L \left[-\frac{\partial T_c}{\partial y_c} \bigg|_{y_c=0} \right] \mathrm{d}x_c \ . \tag{22}$$

Substituting (11) into (22) yields the average Nusselt number for the forced convection system as

$$\mathrm{Nu}_c = \frac{Q}{k_c \Delta t} = \mathrm{Re}^{1/2} \int_0^1 \left[-\frac{\partial \theta_c}{\partial \eta_c} \bigg|_{\eta_c=0} \Big/ \xi_c^{1/2} \right] \mathrm{d}\xi_c$$
$$= A\mathrm{Re}^{1/2} \tag{23}$$

where

$$A = \int_0^1 \left[-\frac{\partial \theta_c}{\partial \eta_c} \bigg|_{\eta_c=0} \Big/ \xi_c^{1/2} \right] \mathrm{d}\xi_c \ . \tag{24}$$

To solve Eq. (13) to (16), a comprehensive and non-iterative numerical scheme is proposed, which is in contrast with the iterative process purposed by Chen and Chang [8, 9] using a guessing strategy. Based on Eq. (19), the boundary conditions can be rewritten as

$$f_h = \frac{\partial f_h}{\partial \eta} = 0, \quad \frac{\partial \theta_h}{\partial \eta} = \xi^{1/4} \lambda^*(\xi) \quad \text{at} \quad \eta = 0$$
$$\frac{\partial f_h}{\partial \eta} \to 0, \quad \theta_h \to \frac{1}{2} \quad \text{as} \quad \eta \to \infty \tag{25}$$

$$f_c = \frac{\partial f_c}{\partial \eta_c} = 0, \quad \frac{\partial \theta_c}{\partial \eta_c} = -R_t^* \xi_c^{1/2} \lambda^*(\xi) \quad \text{at} \quad \eta_c = 0$$
$$\frac{\partial f_c}{\partial \eta_c} \to 1, \quad \theta_c \to -\frac{1}{2} \quad \text{as} \quad \eta_c \to \infty \tag{26}$$

and

$$\lambda = 0 \quad \text{at} \quad \eta = 0, \ \eta_c = 0 \tag{27}$$

where the dummy variable λ is defined as

$$\lambda(\xi, \eta) = R_t R_t^* \lambda^*(\xi) - \theta_h + \theta_c \ . \tag{28}$$

The systems (13)–(16) and (28), together with the boundary conditions (25), (26) and (27), are then solved using the singular perturbation method and the difficulties associated with the guessed interfacial conditions have been obviated. Since this procedure was described in a recent paper by Shu and Pop [10], we will not repeat it here. Note that the points $\xi_c = 0$ and 1 are singular which make the problem more difficult.

4
Results and discussion

In this section we discuss the effects of the Prandtl numbers $\mathrm{Pr_h}$ and $\mathrm{Pr_c}$, and resistance parameters R_t and R_t^* on the interface temperatures, heat transfer rates and Nusselt numbers. Figures 2 and 3 show variation of $\theta_{wc}(\xi_c)$ with ξ_c for some values of R_t and R_t^* when the two working fluids have $\mathrm{Pr_h} = \mathrm{Pr_c} = 1$. The results of these figures show that for increasing values of R_t the temperature of the cold side of the wall decreases. It happens because when R_t is increased the wall becomes more effective insulation between the two forced and free convection flows. In contrast, $\theta_{wc}(\xi_c)$ increases as the free

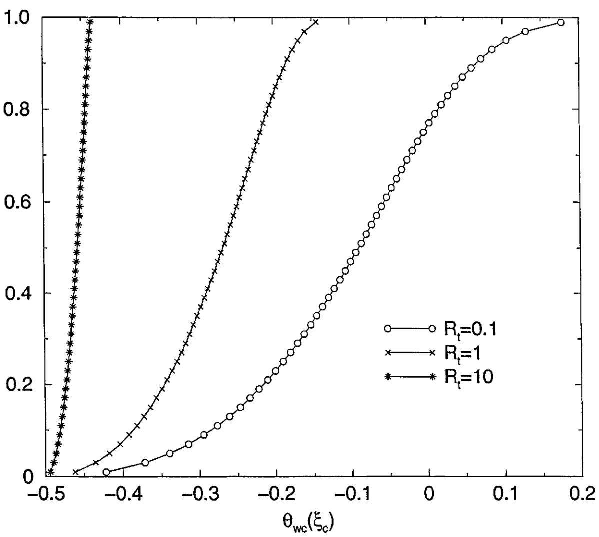

Fig. 2. Effect of R_t on $\theta_{wc}(\xi_c)$ for $R_t^* = \mathrm{Pr_h} = \mathrm{Pr_c} = 1$

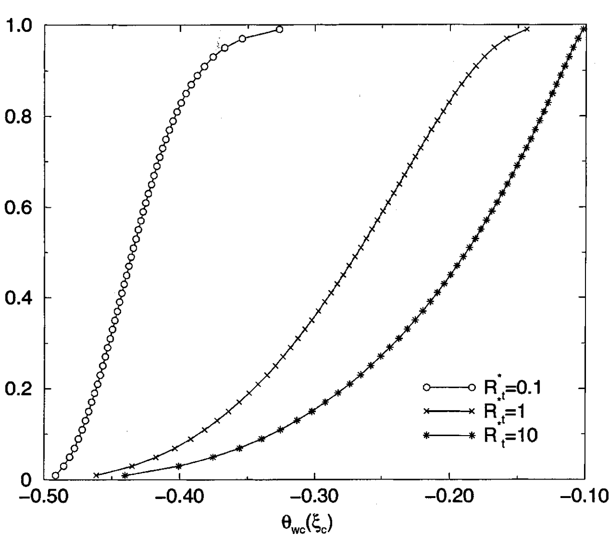

Fig. 3. Effect of R_t^* on $\theta_{wc}(\xi_c)$ for $R_t = \mathrm{Pr_h} = \mathrm{Pr_c} = 1$

convection becomes dominant, i.e. when the parameter R_t^* increases. It turns out that an increase of R_t leads to a reduction of the heat transfer rates at the cold side of the wall and to an increase of these rates with increasing R_t^*, as can be seen from Fig. 4 and 5. It is evident from Fig. 6 and 7 that the effects of R_t and R_t^* on the local Nusselt number are insignificant. Further, Fig. 8 to 11 illustrate the variation of $\theta_{wc}(\xi_c)$ and $\mathrm{Nu}_{xc}(\xi_c)$ with the distance ξ_c along the wall. As can be seen from Fig. 9 and 11 the temperature and local Nusselt number of the cold side of the wall are less affected by $\mathrm{Pr_h}$. To this end it should be mentioned that the same trends persist for the heat transfer characteristics of the free convection system. But, they are not presented here for the sake of space conservation.

Finally, values of the average Nusselt number are given in Table 1 for $R_t = R_t^* = 1$ and some values of $\mathrm{Pr_h}$ and $\mathrm{Pr_c}$. We notice from this table that $\mathrm{Nu_c/Re}^{1/2}$ increases with decreasing $\mathrm{Pr_h}$ or increasing $\mathrm{Pr_c}$.

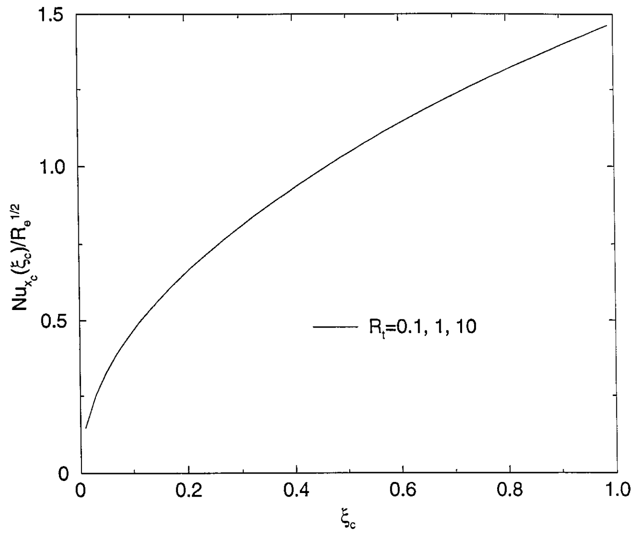

Fig. 6. Effect of R_t on $\mathrm{Nu}_{xc}(\xi_c)/\mathrm{Re}^{1/2}$ for $R_t^* = \mathrm{Pr_h} = \mathrm{Pr_c} = 1$

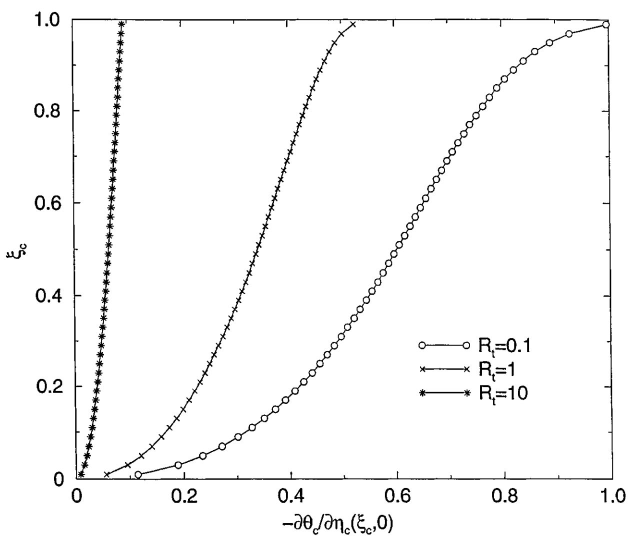

Fig. 4. Effect of R_t on $-\frac{\partial\theta_c}{\partial\eta_c}(\xi_c, 0)$ for $R_t^* = \mathrm{Pr_h} = \mathrm{Pr_c} = 1$

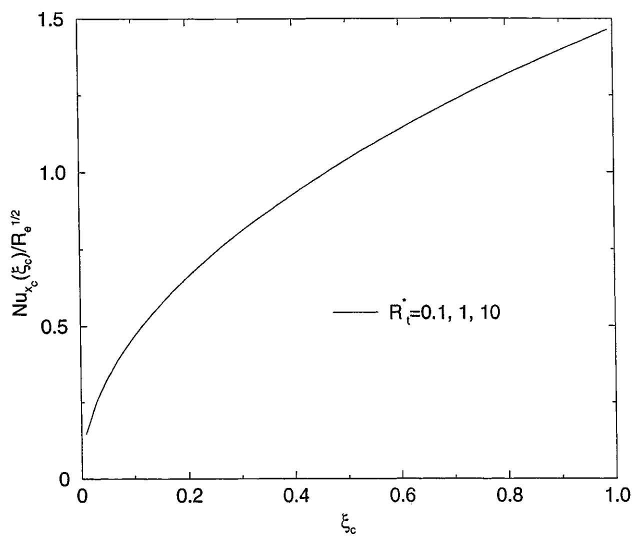

Fig. 7. Effect of R_t^* on $\mathrm{Nu}_{xc}(\xi_c)/\mathrm{Re}^{1/2}$ for $R_t = \mathrm{Pr_h} = \mathrm{Pr_c} = 1$

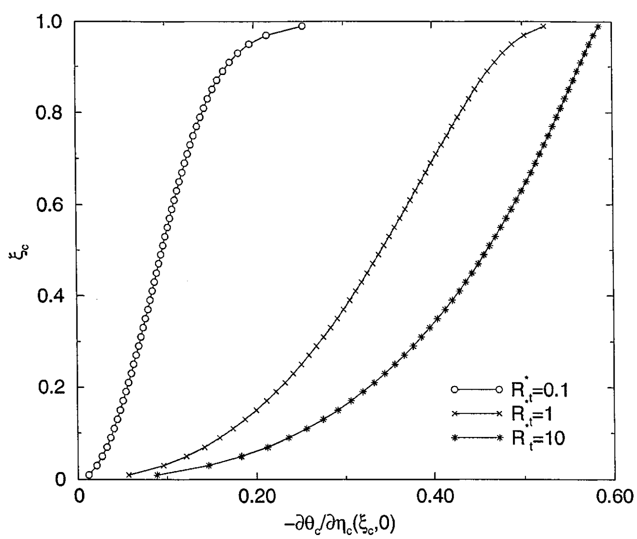

Fig. 5. Effect of R_t^* on $-\frac{\partial\theta_c}{\partial\eta_c}(\xi_c, 0)$ for $R_t = \mathrm{Pr_h} = \mathrm{Pr_c} = 1$

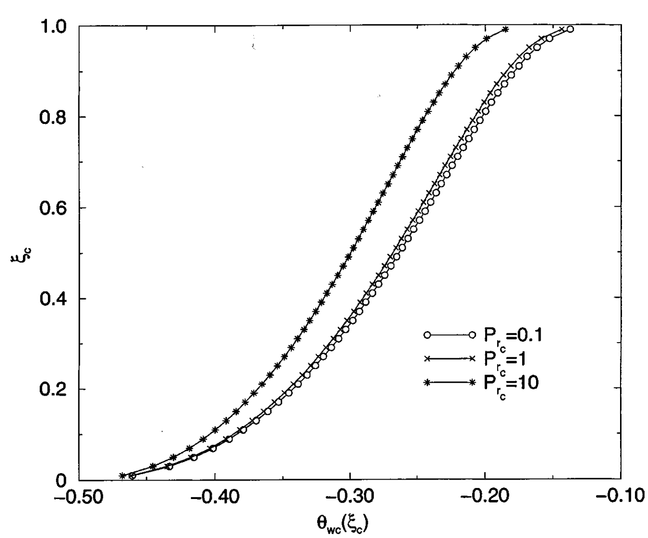

Fig. 8. Effect of $\mathrm{Pr_c}$ on $\theta_{wc}(\xi_c)$ for $R_t = R_t^* = \mathrm{Pr_h} = 1$

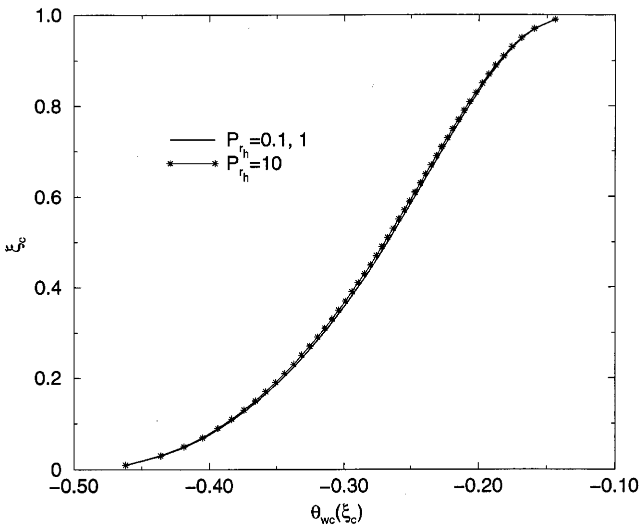

Fig. 9. Effect of $\mathrm{Pr_h}$ on $\theta_{wc}(\xi_c)$ for $R_t = R_t^* = \mathrm{Pr_c} = 1$

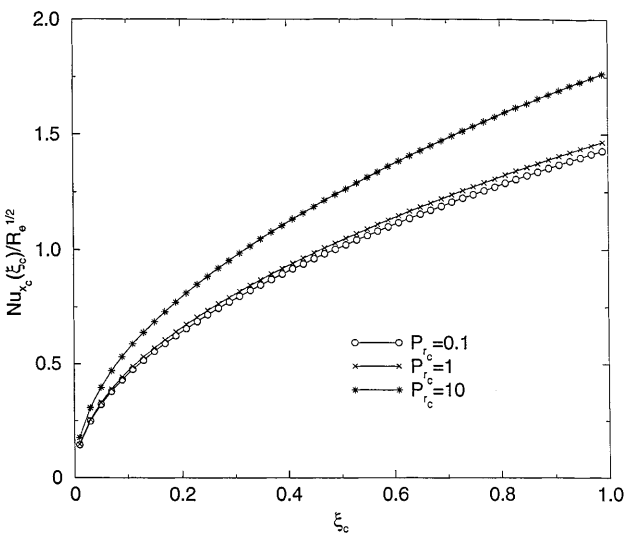

Fig. 10. Effect of $\mathrm{Pr_c}$ on $\mathrm{Nu}_{xc}(\xi_c)/\mathrm{Re}^{1/2}$ for $R_t = R_t^* = \mathrm{Pr_h} = 1$

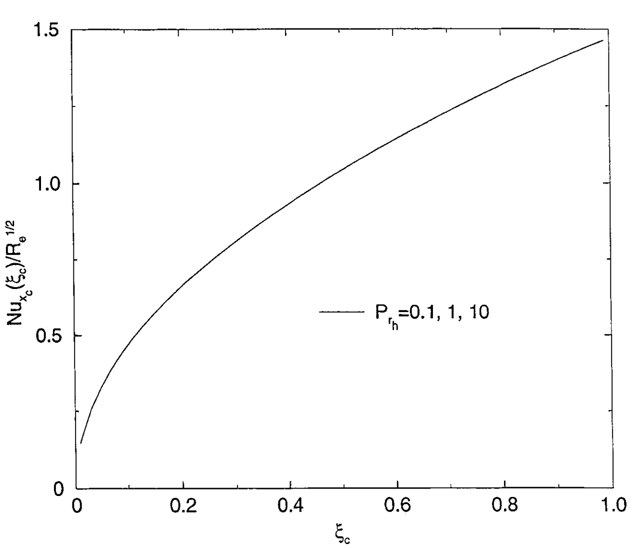

Fig. 11. Effect of $\mathrm{Pr_h}$ on $\mathrm{Nu}_{xc}(\xi_c)/\mathrm{Re}^{1/2}$ for $R_t = R_t^* = \mathrm{Pr_c} = 1$

Table 1. Values of $\mathrm{Nu_c}/\mathrm{Re}^{1/2}$ for $R_t = R_t^* = 1$

	$\mathrm{Pr_h} = 0.1$	$\mathrm{Pr_h} = 1$	$\mathrm{Pr_h} = 10$
$\mathrm{Pr_c} = 0.1$	0.4945	0.4941	0.4899
$\mathrm{Pr_c} = 1$	0.4974	0.4970	0.4927
$\mathrm{Pr_c} = 10$	0.5168	0.5164	0.5116

5 Conclusions

A conjugate problem of heat transfer between a laminar forced convection flow and a laminar free convection separated by a vertical finite wall was studied theoretically. The axial thermal conduction in the wall was neglected. The governing boundary layer equations subject to conjugate boundary conditions were solved numerically using a very efficient method which differs from the one used by other authors. As a result the temperature distributions and heat transfer rates at both sides of the wall have been determined. The results show that the resistance parameters influence substantially the interactive heat transfer characteristics. We have given particular attention to the case when the resistance parameters $R_t = R_t^* = 1$ which include the situation in which the fluids at both sides of the wall are the same.

It is worth mentioning that the results reported in this paper are in general in agreement with those from the open literature. However, the accuracy of these results can be further evidenced through experiments.

References

1. **Lock GSH; Ko RS** (1973) Coupling through a wall between two free convective systems. Int J Heat and Mass Transfer 16: 2087–2096
2. **Viskanta R; Lankford DW** (1981) Coupling of heat transfer between two natural convection systems separated by a vertical wall. Int J Heat and Mass Transfer 24: 1171–1177
3. **Anderson R; Bejan A** (1980) Natural convection on both sides of a vertical wall separating fluids at different temperatures. Transactions of the ASME, J Heat Transfer 102: 630–635
4. **Sakakibara M; Amaya H; Mori S; Tanimoto A** (1992) Conjugate heat transfer between two natural convections separated by a vertical plate. Int J Heat and Mass Transfer 35: 2289–2297
5. **Treviño C; Méndez F; Higuera FJ** (1996) Heat transfer across a vertical wall separating two fluids at different temperatures. Int J Heat and Mass Transfer 39: 2231–2241
6. **Poulikakos D** (1986) Interaction between film condensation on one side of a vertical wall and natural convection on the other side. Transactions of the ASME, J Heat Transfer 108: 560–566
7. **Méndez F; Treviño C** (1996) Film condensation generated by a forced cooling fluid. European J Mechanics, B/Fluids 15: 217–240
8. **Chen H-T; Chang S-M** (1996) Thermal interaction between laminar film condensation and forced convection along a conducting wall. Acta Mechanica 118: 13–26
9. **Chen H-T; Chang S-M** (1997) Coupling between laminar film condensation and natural convection on opposite sides of a vertical plate. Int J Num Meth Fluids 24: 319–336
10. **Shu J-J; Pop I** (1998) Thermal interaction between free convection and forced convection along a conducting plate embedded in a porous medium. Hybrid Methods in Engineering: – Modeling, Programming, Analysis, Animation 1: 55–66