Lebensm.-Wiss. u.-Technol., 30, 624–626 (1997)

Research Note

Methods for the Thawing Time Prediction of Ellipses

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(Received September 23, 1996; accepted March 17, 1997)

Methods for predicting the thawing times of ellipses are proposed. A finite difference numerical model was used for numerical data generation. The thawing time predictions of the methods were compared with numerical results. With the analytical methods considered, it was possible to predict the numerical results with errors lower than 5.4%.

Keywords: thawing time; ellipse; equivalent diameter; thermal properties

Introduction

Accurate thawing time prediction methods are available for unidirectional heat transfer (1, 2). Thawing time prediction becomes more complicated when heat transfer becomes multidirectional. Cleland and Earle (3) defined an equivalent heat transfer dimension (shape factor) E as:

$$E = \frac{\text{Freezing (thawing) time of infinite slab}}{\text{Freezing (thawing) time of complex-shaped object}} \qquad \text{Eqn [1]}$$

where the slab has the same thickness as the object. For complex-shaped objects, thawing time prediction reduces to the determination of E for the object.
Cleland *et al*. (4) proposed a general method for the prediction of freezing and thawing times of foods having irregular shape. However, the method is complicated and not very accurate.
Ilicali and Engez (5) replaced bricks and finite cylinders by an equivalent sphere with a diameter:

$$D_{eq} = \frac{D_v + \beta \times D_{vs}}{1 + \beta} \qquad \text{Eqn [2]}$$

Ilicali *et al*. (6) used the equivalent diameter in a modified form to predict the freezing times of ellipses.
Pham (7) derived a new set of shape factors for the freezing time prediction of ellipses and ellipsoids. For ellipsoids, the proposed model was:

$$E = \frac{1 + \mathrm{Bi}/4}{\dfrac{1}{F} + \dfrac{\mathrm{Bi}/4}{1 + \alpha_1^2 + \alpha_2^2}} \qquad \text{Eqn [3]}$$

Pham (7) observed that Eqns [2] and [3] did not give close agreement with numerical results for the freezing of ellipses or ellipsoids. Therefore, a curve-fitting expression was proposed (7):

$$E = 1 + \left(\frac{F - 1}{\alpha_1 + \alpha_2}\right)^P (\alpha_1^q + \alpha_2^q) \qquad \text{Eqn [4]}$$

where:

$$P = \frac{1}{1 + \mathrm{Bi}} \qquad \text{Eqn [5]}$$

$$q = \frac{1 + \mathrm{Bi}/2}{1 + \mathrm{Bi}/4} \qquad \text{Eqn [6]}$$

If one of the dimensions of an ellipsoid is much longer than the other two dimensions, this geometry will be an ellipse and α_2 will be equal to zero.
The validity of Eqns [3] and [4] has only been tested for freezing time predictions. Equation [2] has been tested against thawing data (5). However the data were limited. In this paper Eqns [2], [3] and [4] will be tested against numerical data for the thawing time of ellipses. The accuracies of these methods will be assessed. The methods discussed in this paper will be also useful practically since objects like animal carcasses, offals, fish and some frozen processed foods can be represented, at least approximately, by elliptic shapes.

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Materials and Methods

Initially, the effects of the initial temperature of the food and the ambient temperature on the shape factor E were investigated by using an analytical model (1). It was observed that the shape factor E depended only weakly on the initial temperature and the ambient temperature.

Finite difference models were used for the assessment of Eqns [2], [3] and [4]. The thawing times of ellipses were calculated by a numerical model which is described in detail elsewhere (6). Thermophysical properties of lean beef were calculated from the empirical formulae proposed by Succar and Hayakawa (8). Thawing times for an infinite slab having the same thickness as the ellipse and the equivalent infinite cylinder were calculated from a previously developed accurate numerical model. The thermal data used in these computations were taken from Cleland and Earle (9). The equivalent finite cylinder diameter was calculated from Eqn [2].

The initial temperature and the ambient temperature were taken as –20 °C and 20 °C, respectively. Since E was found to be independent of the initial temperature and ambient temperature, these two temperatures were taken as constant in all runs. The shortest side of the ellipse was taken as 0.06 m. The calculations were carried out for β values of 1.41, 2, 4 and 6, and Biot numbers 0.59 (h = 5 W/ (m^2.K)), and 1.77, 10.04 and 100.4 (h = 850 W/ (m^2.K)). Thawing was assumed to be completed when the centre temperature reached 0 °C.

Results and Discussion

Table 1 shows the comparison of the predictions of the numerical model using Eqns [2], [3] and [4]. Thermal conductivity in the thawed state was used in Eqns [3] and [4]. **Table 2** shows the effect of the aspect ratio on the errors between Eqns [2], [3] and [4] and the numerical model.

As can be observed from **Tables 1** and **2**, Eqn [4] gave the smallest absolute mean error for all Biot numbers and aspect ratios. However, Eqn [2] gave the narrowest error range. Absolute mean errors were also small for Eqn [3]; however the error range was large. For small aspect ratios, all of the models had satisfactory performance. The Biot number had a very slight effect on the error range. As the Biot number increases, a decrease in the absolute mean error was observed in all cases.

Since a three-dimensional numerical model was not available, Eqns [2], [3] and [4] could not be tested for the thawing of ellipsoids. However, the observed accuracy in the prediction of thawing times hints that these equations can also be used for the prediction of the thawing times of ellipsoids.

Nomenclature

A	Area (m^2)
Bi	hD/k_1 for thawing, hD/k_f for freezing
D	Smallest dimension (m)
D_{eq}	Diameter of equivalent sphere (or cylinder) (m)

Table 1 Shape factors for two-dimensional ellipses[a]

β	Bi	Shape factor (E) Numerical	Eqn [2]	Eqn [3]	Eqn [4]
1.41	0.59	1.75	1.65	1.69	1.68
1.41	1.77	1.66	1.59	1.65	1.64
1.41	10.04	1.53	1.50	1.56	1.55
1.41	100.4	1.48	1.46	1.51	1.51
2.00	0.59	1.51	1.42	1.50	1.46
2.00	1.77	1.41	1.34	1.44	1.40
2.00	10.4	1.27	1.20	1.32	1.30
2.00	100.4	1.22	1.13	1.26	1.26
4.00	0.59	1.22	1.18	1.32	1.27
4.00	1.77	1.13	1.07	1.26	1.19
4.00	10.04	1.06	1.00	1.13	1.09
4.00	100.4	1.04	1.00	1.07	1.07
6.00	0.59	1.13	1.11	1.27	1.16
6.00	1.77	1.07	1.01	1.21	1.12
6.00	10.4	1.03	1.00	1.09	1.05
6.00	100.4	1.02	1.00	1.04	1.03

[a]For symbols and equations used see text and nomenclature.

Table 2 Effect of aspect ratio on the error range and absolute mean errors between the analytical numerical models (%E = (thawing time for the analytical model/thawing time for the numerical model –1) × 100)

	Eqn (2)		Eqn (3)		Eqn (4)	
	Range	Abs.M.Er.	Range	Abs.M.Er.	Range	Abs.M.Er.
β ≤ 6	–7.4, –1.4	+4.2	–3.4, +13.0	+4.9	–4.0, +5.3	+2.7
β ≤ 4	–7.4, –1.4	+4.6	–3.4, +11.5	+3.8	–4.0, +5.3	+2.8
β ≤ 2	–7.4, –1.4	+4.7	–3.4, –3.9	+2.3	–4.0, +3.3	+2.3

Abs.M.Er. = absolute mean error.

D_v	Diameter of sphere (or cylinder) with same volume (m)
D_{vs}	Diameter of sphere (or cylinder) with same volume/area ratio (m)
E	Equivalent heat transfer dimension, shape factor, defined by Eqn [1]
F	Geometric ratio ($AD/(2V)$)
h	Heat transfer coefficient (W/ (m^2.K))
k	Thermal conductivity (W/ (m.K))
P	Function defined by Eqn [5]
q	Function defined by Eqn [6]
t	Time (s)
V	Volume (m^3)
α_1	Ratio of smallest dimension to second smallest dimension of an ellipsoid
α_2	Ratio of smallest dimension to largest dimension of an ellipsoid
β	Ratio of largest dimension to smallest dimension of an ellipsoid

Subscripts

f	Frozen state
1	Thawed state

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