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An Improved Thermal Conductivity Prediction Model for Fruits and Vegetables as a Function of Temperature, Water Content and Porosity

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ABSTRACT

An improved general thermal conductivity prediction model has been developed for fruits and vegetables as a function of water content, porosity and temperature. Thermal conductivity values of apple, pear, corn starch, raisin and potato were used to develop the model using 164 data points obtained from the literature. Raisin has the maximum mean percent deviation of 15.1% (standard deviation 10.1) and pear gave minimum mean percent deviation of 6.8% (standard deviation 7.3). The errors for predicting the thermal conductivity using this improved model for fruits and vegetables are therefore within the range of 6.8–15.1%, which is acceptable for general engineering practice. © 1997 Elsevier Science Limited. All rights reserved

NOTATION

a	Activity
f	Krischer distribution factor
K	GAB parameter
k	Thermal conductivity (W/m K)
M	Mass fraction (dry basis)
MPE	Mean percent error
T	Temperature (K)
t	Temperature (°C)
X	Mass fraction (wet basis)
Y	GAB parameter

Greek letters

ε	Volume fraction
α	Rahman–Chen structural factor
ψ	Relative humidity
ρ	Density (kg/m^3)
υ	Parameter in eqn (11)
σ	Residual standard error

Subscripts

a	Air
ap	Apparent air
e	Effective value
gm	GAB monolayer water content
i	the ith component
o	the full turgor condition
pa	Parallel model
r	Reference temperature (0°C)
s	Solid
se	Series model
w	Water
w0	Initial water content

INTRODUCTION

Thermal conductivity is one of the most important thermophysical properties of foods used to estimate the rate of conductive heat transfer in food processes such as drying, cooking and frying. It is always desirable to be able to predict accurately thermal conductivity using certain general models, if they are available. Currently, the theoretical models have a number of limitations for application in food materials, e.g. difficulties in considering the structural effect of food (i.e. distribution of phases) and in estimating the parameters required by the models. As such, empirical models are popular and widely used for food process design and control. This type of model is, however, valid only for specific food materials under experimental conditions. A number of attempts have been made to overcome this problem by developing generalized correlations for specific types of foods (Sweat, 1974; Rahman, 1992; Lozano *et al.*, 1983).

It is known that, in foods, water plays the most important role in determining thermal conductivity while the non-aqueous part of the food is less important. This may be due to the relative magnitude of conductivities of water and other food constituents (Cuevas & Cheryan, 1978). Thermal conductivity of foods decreases with decreasing water content. The formation of an air phase in foods during processing further decreases the conductivity.

It is common to adopt a linear relationship between thermal conductivity and water content. Mohsenin (1980), Miles *et al.* (1983), Sweat (1986) and Rahman (1995) have given reviews of the water content and thermal conductivity relationship for different types of foods. In these reviews it has been agreed that a linear correlation of thermal conductivity with water content only, is limited to a narrow range of water content, and the correlation parameters vary with the type of food material. Hence, a non-linear correlation is necessary to cover a wider range of water contents (X_w: 0 to 1.0). Mattea *et al.* (1986, 1989) and Lozano *et al.* (1979) used a non-linear relationship between water content and thermal conductivity for apple, pear and potato during drying. Rahman & Potluri (1991) proposed a more general form of non-linear correlation using dimensionless terms.

There has been consistent effort spent in making generalized correlations to predict all properties of food materials for use in process design and optimization (Rahman, 1992). Baroncini *et al.* (1980) compiled a number of generalized correlations to predict the thermal conductivities of liquids. Lozano *et al.* (1983) developed a generalized correlation to predict the bulk shrinkage during drying processes. Sweat (1974) proposed a linear corelation for predicting the thermal conductivity of fruits and vegetables giving predictive results within ±15% for most experimental values. This model, however, is valid for situations where $X_w > 0.60$ and does not account for the temperature and porosity effect. Rahman (1992) developed a more general form of thermal conductivity correlation by introducing a porosity term:

$$\left(\frac{k_e}{k_0}\right)\left(\frac{1}{1-\varepsilon_a}\right)=\left[1.82-1.66\exp\left(-0.85\frac{X_w}{X_{w0}}\right)\right] \tag{1}$$

This model has been applied to apple, pear, squid, beef and potato for small variations in temperature. The above correlation was developed considering thermal conductivity of five food materials as a function of water content and porosity. The water content was varied from 5 to 88% (wet basis), porosity varied from 0 to 0.5 and temperature varied from 20 to 25°C (Rahman & Chen, 1995). When $\varepsilon_a = 1.0$, however, it has later been identified by Rahman & Chen (1995) that the left side of the above correlation becomes infinity which is physically incorrect. The other disadvantages of the above correlation [eqn (1)] are that it does not include any temperature effects on thermal conductivity and it also needs the conductivity values of the fresh foods (i.e. before processing). In fact the correlation is an extension of the parallel model which is realistic in the case of homogeneous food materials. Many investigators have suggested that the series and parallel models can give the lower and upper bound of the effective thermal conductivities of multi-phase mixtures. This is valid only if the components do not change their physico-chemical properties within a wide range of temperature and composition. The series and parallel models do not take into account the natural distribution or arrangement of component phases. Thus, Krischer, cited Keey (1972), proposed a generalized model by combining the parallel and series models using a phase distribution factor as follows:

$$\frac{1}{k_e}=\frac{1-f}{k_{pa}}+\frac{f}{k_{se}} \tag{2}$$

Where:

$$k_{pa}=\sum_{i=1}^{n}\varepsilon_i k_i \quad \text{and} \quad \frac{1}{k_{se}}=\sum_{i=1}^{n}\frac{\varepsilon_i}{k_i} \tag{3}$$

Although it is theoretically common, the drawback of the above model is that it is not possible to have the values of f for different foods without experimental results.

Moreover, the values of f can vary within a wide range for varying water content, porosity and temperature in the case of apple (Rahman & Chen, 1995). Therefore, Rahman & Chen (1995) proposed another model having a parameter α which accounts for residual effect of temperature and structure (i.e. phase distribution of components) of a food material as:

$$\frac{k_e - \varepsilon_a k_a}{(1 - \varepsilon_a - \varepsilon_w) k_s + \varepsilon_w k_w} = \alpha \tag{4}$$

Rahman & Chen (1995) calculated the values of α for apple when porosity varied from 0.482 to 0.512, water varied from 0.20 to 0.60 (wet basis) and temperature varied from 5 to 45°C. It was observed that the values of α varied within a range from 0.371 to 0.575.

The objective of this work was to determine the values of α for more fruits and vegetables and to relate them in a general correlation so that it can be used in design and control of the food processes.

MODEL DEVELOPMENT

The values of k_w for pure water at different temperatures can be expressed according to Choi & Okos (1986) as follows:

$$k_w = 5.7109 \times 10^{-1} + 1.7625 \times 10^{-3} t - 6.7036 \times 10^{-6} t^2 \tag{5}$$

where t is in °C and the correlation is valid from −40 to 150°C. Rahman (1995) correlated thermal conductivity of moist air at different temperatures from the data of Luikov (1964) as:

$$k_a = 0.0076 + 7.85 \times 10^{-4} t + 0.0156 \psi \tag{6}$$

where ψ is the relative humidity (from 0 to 1) and temperature varies from 20 to 60°C. Since fruits and vegetables contain mainly carbohydrate, the thermal conductivity of the solid phase may be estimated from conductivity of carbohydrate according to Choi & Okos (1986) as:

$$k_c = 2.01 \times 10^{-1} + 1.39 \times 10^{-3} t - 4.33 \times 10^{-6} t^2 \tag{7}$$

The volume fraction of air or porosity can be calculated from the densities as:

$$\varepsilon_a = 1 - \frac{\rho_{ap}}{\rho_s} \tag{8}$$

where ρ_{ap} and ρ_s are the apparent and substance density (kg/m^3). The volume fraction of water can be calculated from the mass fractions according to Rahman & Chen (1995) as:

$$\varepsilon_w = \frac{1}{1 + \varepsilon_a} \left[\frac{X_w / \rho_w}{X_w / \rho_w + X_s / \rho_s} \right] \tag{9}$$

The relative humidity (ψ) at equilibrium is equal to the water activity of food. Thus, the water activity of a food at a given moisture content and temperature can be estimated from isotherm and can be considered as ψ. The water activity of food can be estimated from the widely accepted Guggenheim–Anderson–de Boer (GAB) model, named from Guggenheim (1966), Anderson (1946) and De Boer (1953) who all derived the equation independently. The GAB model can be written as:

$$M_w = \frac{M_{gm} Y K a_w}{(1 - K a_w)(1 - K a_w + Y K a_w)} \tag{10}$$

where M_{gm} is the GAB monolayer moisture (kg water/kg dry solid) and Y and K are functions of temperature. The GAB parameters of different food materials are taken from the compilation made by Rahman (1995). Water activity at a given moisture content can be estimated from another form of the GAB equation as:

$$a_w = \frac{\upsilon - \sqrt{[\upsilon^2 - 4(1-Y)]}}{2K(1-Y)} \tag{11}$$

where:

$$\upsilon = \left[\frac{M_{gm}}{M_w} - 1\right] Y \tag{12}$$

At a given moisture content and temperature, the values of υ can be estimated from eqn (12) using GAB parameters at that temperature. Substituting the values of υ in eqn (11), the water activity, which is equal to ψ, can be estimated. Finally, the thermal conductivity of air can be predicted by eqn (6).

RESULTS AND DISCUSSION

The structural factor α is calculated from eqn (4), which is a function of temperature, water content and porosity. Since the left hand side of eqn (1) tends to infinity at $\varepsilon_a = 1.0$, the left hand side of eqn (1) is changed to a different form by adding k_a to the denominator to make a finite value at $\varepsilon_a = 1.0$. The denomenator will be equal to $[k_a/(k_w)_r]$ when ε_a is equal to 1, thus eqn (1) becomes mathematically sound within a range of porosity.

A general power law correlation is developed as:

$$\frac{\alpha}{1 - \varepsilon_a + k_a/(k_w)_r} = 0.996(T/T_r)^{0.713} X_w^{0.285} \quad (\sigma = 0.12) \tag{13}$$

The correlation is developed considering the thermal conductivity of apple, pear, raisin, corn starch and potato as a function of water content, porosity and temperature. The water content is varied from 14 to 88% (wet basis), porosity varied from 0 to 0.56 and temperature varied from 5 to 100°C, respectively. This is a further extension of the previous model having a temperature term in the general form. The values of $\alpha/[1 - \varepsilon_a + k_a/(k_w)_r]$ as a function of $(T/T_r)^{0.713} X_w^{0.285}$ are plotted in Fig. 1 to show the linearity and scatter of the data points from the predicted line.

There might be a question of correlating α rather than k_e directly with the independent variables (i.e. temperature, water content and porosity). The advantage

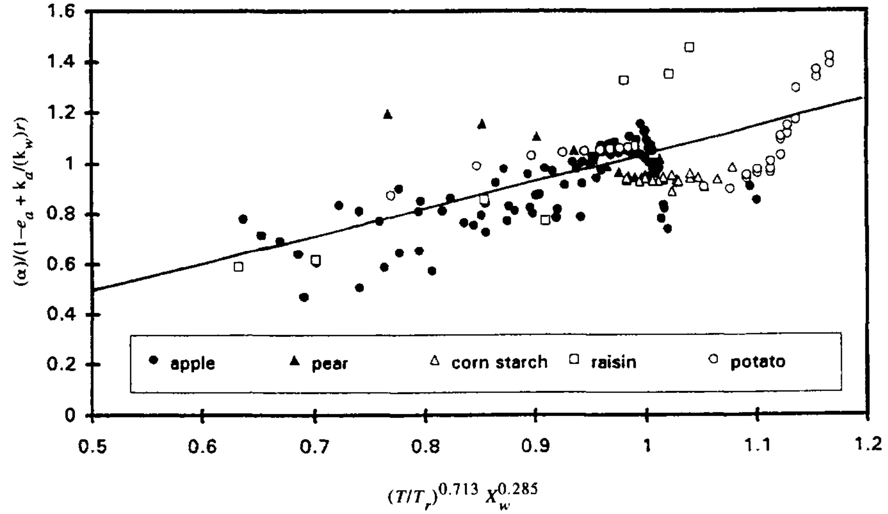

Fig. 1. Plot of $(\alpha)/[1-\varepsilon_a+k_a/(k_w)r]$ *vs* $(T/T_r)^{0.713}X_w^{0.285}$.

of the proposed method is that, if the structural factor is not available for a material, then thermal conductivity can be calculated from eqn (4) considering $\alpha=1$ as a parallel model, since eqn (4) already considers the theoretical effect of composition, porosity and temperature. Thus, further prediction can be improved using eqn (13). If k_e is correlated directly, then the correlation can not be expanded beyond the experimental range and for other fruits and vegetables. Thus, empirical correlation [eqn (13)] is based on eqn (4) which considers the phase distribution by a structural parameter in the parallel model. The advantages of this model compared to a previous model developed by Rahman (1992) are: (1) the present model is based on fruits and vegetables in which similar cell structure is encountered, rather than foods in general; (2) it does not require the thermal conductivity and water content of the fresh product; (3) it is valid for a wide range of temperature, water content and porosity and (4) it is mathematically more sound than eqn (1).

The accuracy of the power law equation is given in Table 1. Raisin gave the maximum mean percent deviation of 15.1 (standard deviation 10.1) and pear gave minimum mean percent deviation of 6.8 (standard deviation 7.3). The developed model can be applied in process design and control purposes where around 15% maximum allowable error in data is permitted.

CONCLUSION

A general model was developed for fruits and vegetables which gave mean percent deviation from 6.8 to 15.1%. The general model can be used in process design and control when thermal conductivity of a specific fruit or vegetable is not available in the literature.

TABLE 1
Percent Error when eqn (13) is used to Predict the Thermal Conductivities of Different Fruit and Vegetables

Material	*T range (°C)*	X_w *range*	ε_a *range*	*N*	*Data source*	*MPE*
Apple	5–60	0.20–0.88	0.18–0.56	90	1,2,3	9.16 (6.69)
Pear	22	0.33–0.88	0.03–0.17	15	1	6.75 (7.26)
Corn starch	20–40	0.80–0.94	0.00–0.01	21	4	10.55 (6.43)
Raisin	44.5	0.14–0.80	0.14–0.40	8	5	15.15 (10.14)
Potato	22–100	0.33–0.81	0.00–0.06	30	1,6	10.95 (5.21)

Note: Values in parantheses are standard deviation.
1, Mattea *et al.* (1986); 2, Singh & Lund (1984); 3, Lozano *et al.* (1979); 4, Drusas *et al.* (1986); 5, Vegenas *et al.* (1990); 6, Califano & Calvelo (1991).

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