Technical notes

Experiments in Fluids 22 (1997) 348–350 © Springer-Verlag 1997

Advantages of using a power law in a low R_θ turbulent boundary layer

L. Djenidi, Y. Dubief, R. A. Antonia

Abstract At low values of the momentum thickness Reynolds number, R_θ, a relatively accurate estimate of the friction velocity U_τ can be obtained by assuming a power law velocity distribution.

1
Introduction

In turbulent boundary layer studies, the friction velocity U_τ is an important parameter which needs to be measured accurately. The Clauser chart technique is often used to determine U_τ. However, while this technique is appropriate at Reynolds numbers high enough for a log law to exist, it becomes tenuous and perhaps irrelevant at small Reynolds numbers, say $R_\theta \leqslant 1500$. Indeed, at such Reynolds numbers there is no rigorous basis for the log law. Direct numerical simulations (DNS; Spalart 1988) and Laser Doppler Anemometry (LDA; Ching et al. 1995) of a low Reynolds number ($500 \leqslant R_\theta \leqslant 1400$) turbulent boundary layer showed that the log region is either very narrow or non-existent. This is not too surprising since a log region is strictly tenable only at infinite Reynolds numbers (e.g. Sreenivasan 1990), that is to say when the viscosity of the fluid can be neglected. On the other hand, the arguments for a power law, which are as convincing as those for a log law (Barenblatt 1993; Barenblatt and Prostokishin 1993; George and Castillo 1993), are valid at finite Reynolds numbers and should be more relevant for low Reynolds number boundary layers, when the effects of the viscosity cannot be neglected. Note that, like the log-law, the power law cannot be valid in the region $y^+ \leqslant 30$.

The purpose of this note is to show that U_τ is accurately estimated when a power law is used to represent the mean velocity distribution. Values of U_τ are calculated by assuming a power law (see Sect. 2) and compared to the values measured with a Preston tube in a turbulent boundary layer. Comparisons are also made between calculated values of U_τ and those deduced from the pressure gradient measurements in a fully developed turbulent channel flow and in a fully developed turbulent pipe flow.

Received: *13 October 1995*/*Accepted*: *22 October 1996*

L. Djenidi, Y. Dubief, R. A. Antonia
Department of Mechanical Engineering
University of Newcastle, N.S.W., 2308, Australia

Correspondence to: L. Djenidi

The support of the Australian Research Council is gratefully acknowledged. We are grateful to Dr Y. Zhu and Mr. B. R. Pearson for their assistance in processing the data.

2
Power law and determination of U_τ

Applying incomplete similarity (or scaling) assumptions, Barenblatt (1993) obtained the following power law form for the mean velocity distribution of a turbulent boundary layer:

$$U^+ = Cy^{+\alpha} \tag{1}$$

where U is the mean velocity in the streamwise direction, y is the distance to the wall and the superscript + denotes normalization by the wall variables; C is a constant to be determined and α, following Barenblatt, is given by

$$\alpha = \frac{3}{2\ln R_\delta} \tag{2}$$

where R_δ ($\equiv U_1\delta/\nu$) is the boundary layer thickness Reynolds number – note that because of the use of $\ln R_\delta$, the value of α is not significantly affected by the uncertainty of estimating δ. LDA measurements (Ching et al. 1995) in a low Reynolds number boundary layer showed that, beyond the buffer layer, U^+ is well represented by Eq. (1). The arguments for a power law are based on the assumption that viscosity exerts a non-trivial influence on the mean velocity. This influence is reflected in a *finite* value of the Reynolds number. Thus, it would appear that the universal log law, strictly valid as $R_\delta \to \infty$, can be considered as an asymptotic limit for the power law (see Barenblatt 1993; Barenblatt and Prostokishin 1993 for a discussion of this issue). Consequently, from a practical point of view, a power law seems more appropriate than a log law for describing low Reynolds number turbulent boundary layers.

A friction velocity law corresponding to the power law (1) can be obtained (Ching et al. 1995) by extrapolating to $y=\delta$ as

$$\frac{U_\tau}{U_1} = \frac{1}{\exp(3/2\alpha)}\left(\frac{\exp(3/2\alpha)}{C}\right)^{1/(1+\alpha)} \tag{3}$$

Values of U_τ calculated with Eq. (3) have been compared with those determined by measuring the mean velocity gradient at the wall (Djenidi and Antonia 1993). It was found (see Ching et al. 1995) that the experimental values were within ±3% of those calculated with Eq. (3). This suggests that it is possible to use Eq. (3) to estimate U_τ when near-wall measurements are difficult, for example with a hot wire.

3
Experimental details

The experiment was carried out in an open wind tunnel with a rectangular test section (600 mm × 120 mm). The test section was 2.6 m long and the height of the test section was adjusted to compensate for the boundary layer growth so that the static pressure was kept constant along the test section where the data were taken.

The boundary layer was tripped at the exit of the contraction using a 50 mm wide strip of no. 40 grit sandpaper downstream a 1 mm height backward step. Measurements were made at a distance of 1.7 m downstream the roughness strip and for two values of R_θ (540 and 940) corresponding to freestream velocities of about 2 and 3.7 m/s.

A single hot wire was used to measure the streamwise component of the mean velocity. The wire (Pt-10% Rh Wollaston) has a diameter of 2.5 μm and a length of about 0.5 mm. The hot wire signal was digitized using a 12-bit analog-to-digital converter (Boston technology) at a sampling frequency of 5 kHz into a standard 486 PC and the digitized records transfered by Ethernet to a VAX 780 computer for subsequent analysis.

Experimental values of U_τ were inferred from the wall shear stress τ_w, measured with a Preston tube (0.72 mm outer diameter) and a static tube located at approximately the same x position on one of the vertical wall of the tunnel. Pressure differences were measured with a MKS Baratron pressure transducer whose output was averaged after digitising (1 kHz) for approximately 60 s. The Preston tube was calibrated in a fully developed channel flow (see Antonia et al., 1995 for further details) where U_τ can be determined with reasonable confidence and accuracy from the relation $\tau_w = -h(\mathrm{d}p/\mathrm{d}x)$, where h is the channel half-width and p is the static pressure. It was ensured that the calibration range of the Preston tube covered that encountered in the boundary layer study.

4
Results and discussion

In order to calculate U_τ with Eq. (3), C needs to be estimated. Here, we follow Barenblatt and Prostokishin (1993) and use the form

$$C = \frac{1}{\sqrt{3}} \ln R_\delta + \frac{5}{2} = \frac{\sqrt{3} + 5\alpha}{2\alpha} \tag{4}$$

Equation (4) was derived by curve fitting to the experimental data of Nikuradze (Barenblatt and Prostokishin 1993). The LDA measurements of Ching et al. (1995) confirmed it to within the experimental accuracy (±2.5%). In the present study, $C = 7.39$ ($\alpha = 0.177$) and 7.72 ($\alpha = 0.166$) for $R_\theta = 540$ and 940, respectively. The values of U_τ, measured with the Preston tube, are 0.103 and 0.175 m/s while those calculated with Eq. (3) are 0.104 and 0.176 m/s for $R_\theta = 540$ and 940, respectively. There is clearly a very good agreement between calculation and measurement, thus proving strong support for Eq. (3).

Figure 1 compares the measured mean velocity data U^+ versus y^+ (a log-linear plot is used, with normalisation by the calculated U_τ) with the DNS data of Spalart (1988). We also included the power law distributions (Eq. (1)]. The experimental data are in good agreement with the DNS data above $y^+ = 7$. Closer to the wall, the experimental data depart from the simulations due to the uncertainty of spatial locations of the probe and wall conduction. Note that the present data (in agreement with DNS) do not collapse onto a single line in the region $25 \leqslant y^+ \leqslant 250$, suggesting that the relation

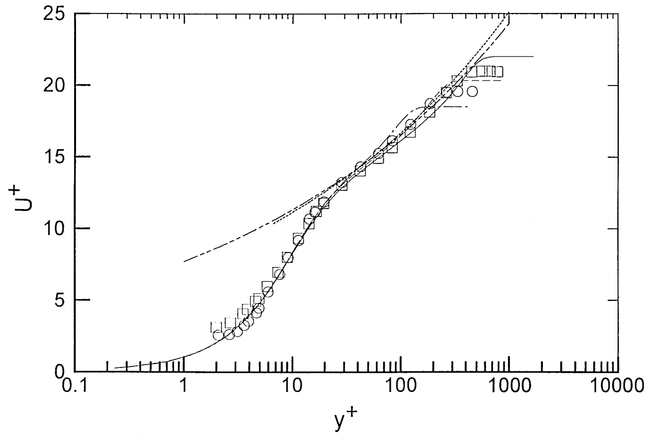

Fig. 1. Mean velocity distributions. Experiment: ○, $R_\theta = 540$; □, $R_\theta = 940$. DNS: — - —, $R_\theta = 300$; - - -, $R_\theta = 670$; —, $R_\theta = 1410$. Power law distribution: — - - —, $R_\theta = 940$; - -, $R_\theta = 540$

$$U^+ = \kappa^{-1} \ln y^+ + B \tag{5}$$

where κ and B are assumed to be Reynolds number independent, is only approximately satisfied (see Ching et al. 1995 for a discussion of the effect of R_θ on the log region).

To further verify the validity of Eq. (3) as well as the universality of the power law, calculations of U_τ were made in a fully developed turbulent channel flow (Zhu 1994) and a fully developed pipe flow (Pearson and Antonia 1995). In both these flows, U_τ was estimated quite accurately (with an uncertainty error of ±1.5%) from pressure gradient measurements. Figure 2a compares the measured values of U_τ with the calculated ones using Eq. (3) where the Reynolds number is based on the radius (or channel half-width) and the centreline velocity, U_1, for the channel (CF) and pipe (PF) flows. The results for the boundary layer (BL) have also been included for completeness. As in the boundary layer calculations, the channel and pipe flow calculations agree well with the measurements.

Relation (3) is based on an empirical observation that the quasi-universal form of the power law (Barenblatt and Prostokishin, 1993)

$$\Psi \equiv \frac{1}{\alpha} \ln \frac{U^+}{C} = \ln y^+ \tag{6}$$

extends from $y^+ = 30$ up to the edge of the boundary layer (Fig. 1; see also Ching et al. 1995). However, in the case of pipe/channel flows, an expression for U_τ can be derived more rigorously (Barenblatt 1993), viz.,

$$\frac{U_\tau d}{\nu} = \left[\frac{R_d 2^\alpha \alpha (1+\alpha)(2+\alpha)}{\sqrt{3} + 5\alpha} \right]^{1/(1+\alpha)} \tag{7}$$

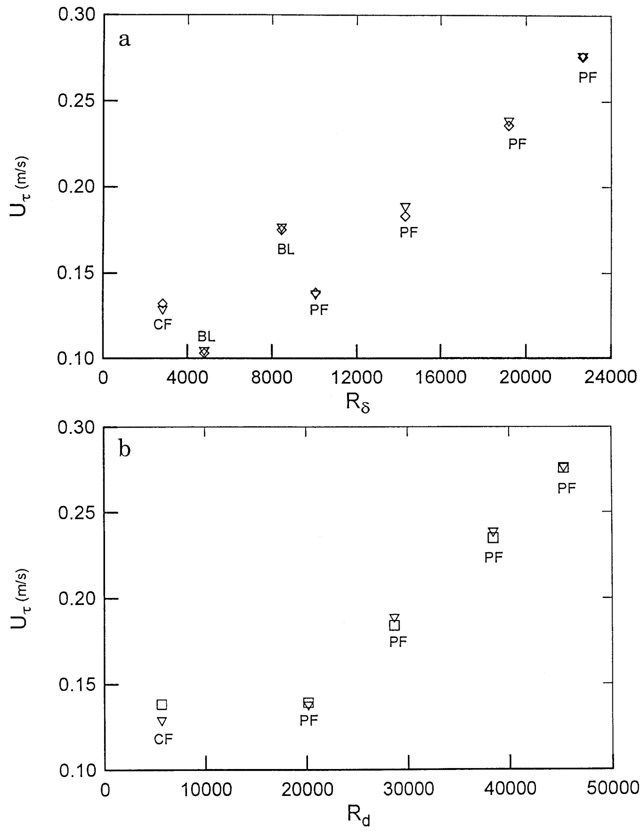

Fig. 2a, b. Calculated and measured friction velocity U_τ *for channel flow* (CF), pipe flow (PF) and boundary layer (BL). **a** ◇, measured; ▽, calculated (Eq. (3)); **b** ▽, calculated (Eq. (3)); □, calculated (Eq. (7))

In Eq. (7) d is either the pipe diameter or the channel width; α is still calculated with Eq. (2) where R_δ is replaced by $R_d = \bar{U}d/\nu$. Equation (7) is obtained by first combining Eqs. (1), (2) and (4) and $A\bar{U}=Q$, where A and Q are the cross-sectional area and volume flow rate respectively. Figure 2b presents the values of U_τ calculated using Eqs. (3) and (7) for the channel and pipe flows. The results quite unambiguously show that both equations yield the same values of U_τ. From a practical point of view, Eq. (3) presents an advantage over Eq. (7): only one measurement (at the centreline) is required for determining U_τ. Of course, when only Q is available and no measurements are possible, Eq. (7) should be used.

5
Conclusions

Reliable estimates of the friction velocity in low Reynolds number turbulent boundary layers can be obtained by assuming a power law distribution for the mean velocity distribution (Eq. (1)). The power law provides a viable alternative to other means, for example the Preston tube, of determining the friction velocity.

References

Antonia RA; Zhu Y; Sokolov M (1995) Effect of concentrated wall suction on a turbulent boundary layer. Phys Fluids 7: 2465–2474

Barenblatt GI (1993) Scaling lows for fully developed turbulent shear flows. Part 1, Basic hypothesis and analysis. J Fluid Mech 248: 513–520

Barenblatt GI; Prostokishin VM (1993) Scaling lows for fully developed turbulent shear. Part 2. Processing of experimental data. J Fluid Mech 248: 521–529

Ching CY; Djenidi L; Antonia RA (1995) Low Reynolds number effects in a turbulent boundary layer. Exp Fluids 19: 61–68

Djenidi L; Antonia RA (1993) LDA measurements in a low Reynolds number turbulent boundary layer. Exp Fluids 14: 280–283. Erratum, Exp Fluids 15: 386

George WK; Castillo L (1993) Boundary layer with pressure gradient: Another look at the equilibrium boundary layer. In: Near-wall turbulent flows, eds. RMC So, CG Speziale and BE Launder, pp. 901–910. Amsterdam: Elsevier

Pearson BR; Antonia RA (1995) Some measurements in a fully developed turbulent pipe flow. Report T. N. FM 95/1, Department of Mechanical Engineering, University of Newcastle

Spalart PR (1988) Direct simulation of a turbulent boundary layer up to $R_\theta = 1410$. J Fluid Mech 187: 61–98

Sreenivasan KV (1990) The turbulent boundary layer. In: Frontiers in experimental fluid mechanics, ed. M Gad-el-Hak, pp. 159–210. Berlin: Springer

Zhu Y (1994) Small scale structure of turbulence in a fully developed channel flow. PhD Thesis, Department of Mechanical Engineering, University of Newcastle