ELSEVIER

Theoretical and Applied Fracture Mechanics 31 (1999) 19–29

theoretical and applied fracture mechanics

Morphology of horizontal cracks in swelling soils [1]

V.Y. Chertkov *, I. Ravina

Faculty of Agricultural Engineering, Technion, Haifa 32000, Israel

Abstract

Horizontally formed cracks in the network of cracked swelling soils tend to influence water and solute transport. An approach is suggested for estimating the mean width and volume of horizontal cracks. It is assumed that the nearly horizontal cracks appear as a result of inhomogeneous soil subsidence caused by rapid drying and shrinkage of thin layers at the walls of vertical cracks. Compared with the moist soil matrix, at the same soil depth, horizontal cracks originate as ruptures in stretched layers of the drying walls of vertical cracks. A characteristic of the average inhomogeneity of soil subsidence, i.e., the *mean potential relative subsidence* (MPRS) depending on the soil depth is defined. It is calculated on the basis of linear shrinkage in the clay soil matrix and at the walls of vertical cracks of different depths, and on two geometrical parameters of crack networks. They are namely the maximum crack depth and the thickness of the upper intensive-cracking layer. The absolute value and sign of the derivative of the MPRS function with respect to soil depth determine the specific volume of horizontal cracks (horizontal-cracks porosity), and their mean width as functions of depth. Model predictions are obtained using published data on variation of linear shrinkage with depth in 19 soil profiles. For lack of data specific to horizontal-crack characteristics model, predictions were compared with data on vertical cracks and subsidence at the soil surface. Satisfactory agreement was obtained for all soil depths up to the maximum crack depth.

1. Introduction

Flow and solute transport in a drying clay soil depends on the geometry of a network of cracks in the soil. Such a network consists of cracks in different directions. However, it can be identified mostly with cracks close to vertical or horizontal directions. Generally, the width of the horizontal cracks at any depth is appreciably smaller than that of the largest vertical cracks at the same depth. But the horizontal cracks connect vertical ones and together form continuous and tortuous paths for water flow. The horizontal cracks also contribute to the total crack volume. Hence, these cracks should be considered in determining the hydraulic properties of clay soil. Discussed in Ref. [1–5] is the relation between the soil subsidence and crack system in clay soils. However, the model neglects horizontal cracks. Only the volume of the vertical cracks were considered, and they are assumed to be of the same depth with uniform spacings. The purpose of this work is to determine the relation between vertical cracks, inhomogeneous subsidence and horizontal cracks in a clay soil. An approach is also developed for estimating the mean width and volume of horizontal cracks as functions of soil depth, especially in the upper intensive-cracking layer where inhomogeneous

* Corresponding author.Tel.: +972 4 8293331; fax: 972 4 8221529; e-mail: agvictor@tx.technion.ac.il

[1] This paper has been accepted for the presentation to the MESOMECHANICS'98 Conference, Tel-Aviv, Israel, 1–4, June 1998.

0167-8442/99/$ – see front matter
PII: S 0 1 6 7 - 8 4 4 2 (9 8) 0 0 0 6 3 - 9

subsidence is strong. The concept and result of a general model of crack network geometry [6,7] are used. This model was applied to predict the morphology of vertical cracks in swelling soils and showed satisfactory agreement with data available in literature [6,7].

2. Model formulation

According to the work [1–5], the shrinkage volume is affected by the soil subsidence and vertical cracks. The subsidence was assumed to be homogeneous. However, evaporation from walls of the formed vertical cracks disturbs the homogeneity of subsidence at any horizontal plane. Thin drying soil layers along vertical crack walls tend to contract but the moist soil matrix opposes it. As a consequence the drying layers are subject to tensile stresses which cause development of horizontal (or close to horizontal) cracks starting from the walls of vertical cracks. This inhomogeneity and the evaporation from the surfaces of the horizontal cracks influence their growing and widening behavior. There is a large number of vertical cracks, which have statistically homogeneous distribution at the soil surface. Their depths vary from zero to a maximum which is the depth of the shrinkage layer, z_{m}. It can be assumed that, on the average, the distributions of volume and width of horizontal cracks will be the same at any vertical section.

2.1. Mean potential relative subsidence (MPRS)

The linear shrinkage, $\varepsilon_{\mathrm{o}}(z)$ and surface shrinkage, $\delta_{\mathrm{o}}(z)$ of a soil matrix are functions of soil depth, z and are related to each other as

$$\delta_{\mathrm{o}} = \varepsilon_{\mathrm{o}} \cdot (2 - \varepsilon_{\mathrm{o}}). \tag{1}$$

Linear vertical shrinkage at a point on the wall of a vertical crack, $\varepsilon(z,h)$, unlike $\varepsilon_{\mathrm{o}}(z)$, depends on the crack depth h and the depth of the point on the wall, $z \leqslant h$. Because of the additional evaporation from the crack walls, $\varepsilon(z,h) \geqslant \varepsilon_{\mathrm{o}}(z)$ for $z \leqslant h$. At the same vertical elevation, below the crack tip ($z > h$), the linear shrinkage coincides with.

Subsidence of drying soil along a vertical elevation not containing a vertical crack (i.e. in the soil matrix), $S_{\mathrm{o}}(z)$ is determined as a function of soil depth, z

$$S_{\mathrm{o}}(z) = \int_{z}^{z_{\mathrm{m}}} \varepsilon_{\mathrm{o}}(z')\,\mathrm{d}z'. \tag{2}$$

Accounting for the additional linear shrinkage of the thin soil layer at the vertical-crack wall (Fig. 1), in a vertical section including a vertical crack of depth h, the potential subsidence, $S(z,h)$ at soil depth z is

$$S(z,h) = \begin{cases} \int_z^h \varepsilon(z',h)\,\mathrm{d}z' + \int_h^{z_m} \varepsilon_{\mathrm{o}}(z')\,\mathrm{d}z' & z \leqslant h \leqslant z_{\mathrm{m}}, \\ S_{\mathrm{o}}(z) & h \leqslant z \leqslant z_{\mathrm{m}}. \end{cases} \tag{3}$$

When integrating $\varepsilon(z,h)$ along the vertical-crack wall, the deflection of the latter is neglected (Fig. 1). It is clear that $S(z,h) > S_{\mathrm{o}}(z)$ for $z < h$. But in any horizontal cross-section, the total soil matrix area is many times larger than the total

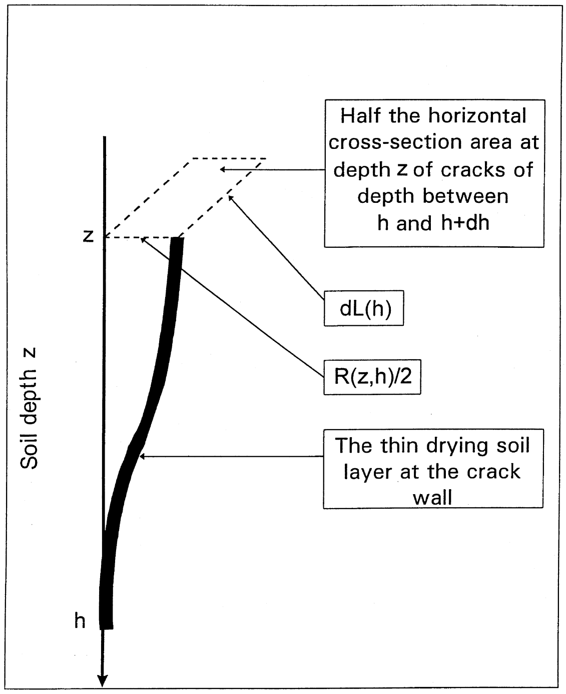

Fig. 1. Sketch of vertical crack.

cross-section area of vertical cracks. So moist soil matrix prevents from full realization of the subsidence $S(z,h)$ and therefore it is referred to as "potential" subsidence and the difference

$$\Delta S(z,h) \equiv S(z,h) - S_o(z) = \begin{cases} \int_z^h \left(\varepsilon(z',h) - \varepsilon_o(z')\right) dz' & z \leqslant h \leqslant z_m, \\ 0 & h < z \leqslant z_m, \end{cases} \tag{4}$$

which is referred to as the *potential relative subsidence*, at depth z, of a vertical section containing a vertical crack of depth h. The average inhomogeneity of soil subsidence can be characterized by the potential relative subsidence, at the walls of vertical cracks, at soil depth z, averaged for all depths h of the vertical cracks, $z \leqslant h \leqslant z_m$ that are possible for a given $z \geqslant 0$. This average value is given by

$$\overline{\Delta S}(z) = \int_z^{z_m} \Delta S(z,h)\, dW(z,h), \tag{5}$$

where $dW(z,h)$ is a weight factor in the averaging. The term $\overline{\Delta S}(z)$ will be referred to as the *mean potential relative subsidence* (MPRS).

2.2. Analytical approach

To calculate $\overline{\Delta S}(z)$, expression for the weighting factor, $dW(z,h)$ is needed together with the linear shrinkage, $\varepsilon(z,h)$ at the wall of a crack of depth h, at soil depth z. Data of linear shrinkage, $\varepsilon_o(z)$ and surface shrinkage, $\delta_o(z)$ of the soil matrix are assumed to be known.

2.2.1. The weight factor

Physically, the weighting factor, $dW(z,h)$ is the fraction of vertical cracks of depths between h and $h + dh$ (where $h \geqslant z$) of all vertical cracks of depths $\geqslant z$, or else the probability that the tip of a vertical crack of depth $h \geqslant z$ is in a depth interval dh in the range $z \leqslant h \leqslant z_m$. The expression for this probability can be obtained. The total horizontal cross-section area of vertical cracks at depth z (per unit area) is by definition equal to the horizontal surface shrinkage of the soil matrix, $\delta_o(z)$. The contribution to the total area $\delta_o(z)$ corresponding to cracks of depths between h and $h + dh$ (where $h \geqslant z$) is equal to $R(z,h) \cdot dL(h)$ (Fig. 1). Here $R(z,h)$ is the width of cracks of depth h at soil depth z ($h \geqslant z$) [6,7] and $dL(h)$ is the specific length of traces of cracks of depths between h and $h + dh$ on any horizontal plane above the depth h [6,7]. Thus, the weighting factor $dW(z,h)$ in averaging $\Delta S(z,h)$ on crack depths h, is

$$dW(z,h) = R(z,h)\, dL(h)/\delta_o(z). \tag{6}$$

Inserting Eq. (6) into Eq. (5), it gives

$$\overline{\Delta S}(z) = \frac{1}{\delta_o(z)} \int_z^{z_m} \Delta S(z,h)\, R(z,h)\, dL(h). \tag{7}$$

The functions $R(z,h)$, $L(h)$, and $\delta_o(z)$ are calculated from the curve describing the variation of the linear shrinkage with depth, $\varepsilon_o(z)$, the maximum crack depth, z_m, and the thickness of intensive-cracking layer, z_o [6,7].

2.2.2. Linear shrinkage at crack wall

The linear shrinkage, $\varepsilon(z,h)$ can be obtained as an approximation from physical considerations. The water content at the crack wall close to the soil surface, in the depth range $0 \leqslant z <\cong R(0,h)$, should be practically constant and coincide with that at the soil surface. This is supported by data [8,9]. Hence, it may also be assumed for the linear shrinkage (Fig. 2)

$$\varepsilon(z,h) = \varepsilon_o(0), \quad 0 \leqslant z <\cong R(0,h). \tag{8}$$

For larger depths, $R(0,h) < z \leqslant h$ the water content at the crack wall is still smaller than in the soil matrix, at the same depth; it increases as the crack tip is approached to the water content of the soil matrix at depth h. These notions are also supported by the data [8,9]. This implies from Eq. (8) that the linear shrinkage on the crack wall, $\varepsilon(z,h)$ (Fig. 2) is larger than the linear shrinkage in the soil matrix, $\varepsilon_o(z)$. It falls sharply with increasing depth in the vicinity of depth $z = R(0,h)$ and then decreases to merge with the "depth–linear shrinkage" curve of the matrix, $\varepsilon_o(z)$ at depth h. Thus, the functions $\varepsilon(z,h)$ and $\varepsilon_o(z)$ are equal at the corresponding points z' and z'' (Fig. 2) in the ranges given by Eq. (9)

$$\varepsilon(z',h) = \varepsilon_o(z''), \quad R(0,h) \leqslant z' \leqslant h, \ \ 0 \leqslant z'' \leqslant h. \tag{9}$$

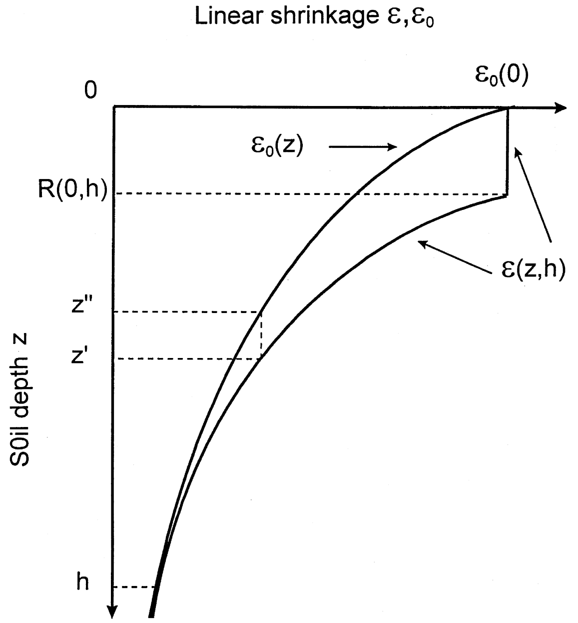

Fig. 2. Sketch of linear shrinkage, $\varepsilon(z,h)$ on the wall of a crack of depth h and $\varepsilon_o(z)$ of the soil matrix.

This equality is used to approximate the linear shrinkage at the crack wall, $\varepsilon(z,h)$, reducing the problem to finding a relation between points z' and z''. Based on the data [7,8], it can be assumed that the depth range $0 \leqslant z'' \leqslant h$ in the soil matrix is transformed to a depth range $R(0,h) \leqslant z' \leqslant h$ at crack wall by a proportional compression. Now,

$$(z' - R(0,h))/(h - R(0,h)) = z''/h, \quad R(0,h) \leqslant z' \leqslant h \tag{10}$$

and from this

$$z'' = \frac{h}{(h - R(0,h))}(z' - R(0,h)), \quad R(0,h) \leqslant z' \leqslant h. \tag{11}$$

Hence, the following simple approximation, based on Eqs. (8), (9) and (11), is used (Fig. 2)

$$\varepsilon(z,h) = \begin{cases} \varepsilon_o(0) & 0 \leqslant z \leqslant R(0,h), \\ \varepsilon_o\left(\frac{h}{(h-R(0,h))}(z - R(0,h))\right) & R(0,h) \leqslant z \leqslant h. \end{cases} \tag{12}$$

The approximation also follows

$$\left.\frac{\partial \varepsilon(z,h)}{\partial z}\right|_{z=h} \neq \left.\frac{\mathrm{d}\varepsilon_o(z)}{\mathrm{d}z}\right|_{z=h}. \tag{13}$$

Finally, it is necessary to consider $\varepsilon(z,h)$ when, for a given soil, the water content at the crack wall at depth $z = R(0,h)$ turns out to be lower than the shrinkage limit in the corresponding shrinkage curve [10]. In such a case, the linear shrinkage at the crack wall is practically constant and equal to $\varepsilon_o(0)$ at a somewhat deeper range $R(0,h) < z \leqslant \leqslant R(0,h) + \Delta R$ corresponding to a zone of zero shrinkage on the shrinkage curve [10]. As the water content reaches the shrinkage limit value at the depth $R(0,h) + \Delta R$, and larger, the linear shrinkage on the crack wall decreases. In this case, the depth–linear shrinkage curve of the soil matrix, $\varepsilon_o(z)$ should, unlike the curve in Fig. 2, exhibit practically constant values of shrinkage, $\varepsilon_o(0)$ near the soil surface in the range

$$0 < z \leqslant \Delta R. \tag{14}$$

If $\Delta R \neq 0$ $R(0,h)$ in Eqs. (8)–(12) should be replaced by $R(0,h) + \Delta R$. It means also that for cracks of depths $z \leqslant R(0,h) + \Delta R$ the depth–linear shrinkage curve of their walls, $\varepsilon(z,h)$ will be constant and equal to $\varepsilon_o(0)$.

2.3. Characteristics of horizontal cracks

Increase or decrease of MRPS within a certain depth range implies a trend of the thin drying soil layers at the vertical-crack walls to expand or to contract, respectively. However, in both cases the major soil volume between the vertical cracks, which is not subject to these changes, opposes such a trend. As a consequence, drying layers near the walls are under compression at depths where $\mathrm{d}\overline{\Delta S}(z)/\mathrm{d}z > 0$ and under tension where $\mathrm{d}\overline{\Delta S}(z)/\mathrm{d}z < 0$. Horizontal cracks initiate as ruptures in stretched sections of the drying layers near the walls of vertical cracks where $\mathrm{d}\overline{\Delta S}(z)/\mathrm{d}z < 0$. When these ruptures are formed it may be considered that the soil matrix between the vertical cracks is not deformed. So, the specific total width of these ruptures at the vertical-crack walls (per unit length of vertical profile) is equal to $-\mathrm{d}\overline{\Delta S}(z)/\mathrm{d}z$. Evaporation from the walls of these initial horizontal cracks enhances their growth, coalescence, and widening. Consider the statistical homogeneity of horizontal cracking development (i.e. similarity, on the average, of all vertical sections at a given depth). The specific volume of horizontal cracks (per unit volume of soil), or the horizontal-crack porosity, $v_{\mathrm{h}}(z)$ is equal to the specific total width of the ruptures of vertical cracks

$$v_{\mathrm{h}}(z)=\begin{cases}-\frac{\mathrm{d}\overline{\Delta S}(z)}{\mathrm{d}z} & \text{if } \frac{\mathrm{d}\overline{\Delta S}(z)}{\mathrm{d}z}<0,\\ 0 & \text{if } \frac{\mathrm{d}\overline{\Delta S}(z)}{\mathrm{d}z}\geqslant 0.\end{cases} \tag{15}$$

The expression for $v_{\mathrm{h}}(z)$ can be defined more precisely by subtracting from the right part of Eq. (15) the volume at intersections with vertical cracks, that has already been included in their volume. This is done by introducing a multiplier, $1-\delta_{\mathrm{o}}(z)$ accounting for the horizontal surface shrinkage at depth z. The horizontal surface shrinkage, $\delta_{\mathrm{o}}(z)$ coincides with the specific area of the horizontal cross-section of the vertical cracks at depth z. Then

$$v_{\mathrm{h}}(z)=\begin{cases}-\frac{\mathrm{d}\overline{\Delta S}(z)}{\mathrm{d}z}\cdot(1-\delta_{\mathrm{o}}(z)) & \text{if } \frac{\mathrm{d}\overline{\Delta S}(z)}{\mathrm{d}z}<0,\\ 0 & \text{if } \frac{\mathrm{d}\overline{\Delta S}(z)}{\mathrm{d}z}\geqslant 0.\end{cases} \tag{16}$$

The cumulative volume of horizontal cracks (upwards from depth z_{m}), is

$$V_{\mathrm{h}}(z)=\int_{z}^{z_{\mathrm{m}}} v_{\mathrm{h}}(z')\,\mathrm{d}z',\quad 0\leqslant z\leqslant z_{\mathrm{m}}. \tag{17}$$

The specific volume of horizontal cracks, in the form of Eq. (15), may also be written as

$$v_{\mathrm{h}}(z)=\frac{R_{\mathrm{h}}(z)}{d(z)}, \tag{18}$$

where $R_{\mathrm{h}}(z)$ is the mean width of horizontal cracks at depth z; and $d(z)$ the mean spacing between intersections of horizontal cracks with a straight vertical line at depth z [6,7]. According to Eqs. (15) and (18), it is found that

$$R_{\mathrm{h}}(z)=\begin{cases}-\frac{\mathrm{d}\overline{\Delta S}(z)}{\mathrm{d}z}\cdot d(z) & \text{if } \frac{\mathrm{d}\overline{\Delta S}(z)}{\mathrm{d}z}<0,\\ 0 & \text{if } \frac{\mathrm{d}\overline{\Delta S}(z)}{\mathrm{d}z}\geqslant 0.\end{cases} \tag{19}$$

3. Model prediction and validation

Model prediction of the characteristics of horizontal cracks in swelling soils as functions of depth is based on published data of depth – linear shrinkage curves for 17 field cases. They consist of eight cases [11], six cases [12], three cases [13] and two lysimeter cases from Ref. [5]. Used also are values of the parameters of crack network, z_{m} and z_{o}, and the relation, $z_{\mathrm{o}}\cong 0.1z_{\mathrm{m}}$ which were defined and calculated with these data [6,7]. The 17 depth – linear shrinkage curves of field cases are only for the macro-shrinkage depth range, $z<z_{\mathrm{s}}$, where z_{s} is an experimental depth measured by a flexible wire of a given diameter, D (1.5, 2, and 3 mm in Refs. [12,13,11], respectively). The linear shrinkage of the soil matrix, $\varepsilon_{\mathrm{o}}(z)$ in the depth range of micro-shrinkage ($z_{\mathrm{s}}\leqslant z\leqslant z_{\mathrm{m}}$) can be estimated. It is assumed experimentally that there is no shrinkage below the depth z_{s}, measured by a flexible wire [11]. However, actual shrinkage of the horizontal surface area at this depth, $\delta_{\mathrm{o}}(z_{\mathrm{s}})$ can be estimated by

$$\delta_{\mathrm{o}}(z_{\mathrm{s}})\cong L(z_{\mathrm{s}})\cdot D \tag{20}$$

where $L(z_{\mathrm{s}})$ is the specific length of crack traces in a horizontal cross-section at depth z_{s}. $L(z_{\mathrm{s}})$ can be estimated using the values of $z_{\mathrm{m}}, z_{\mathrm{o}}, z_{\mathrm{s}}$ [6,7]. To estimate the corresponding linear shrinkage at that depth, $\varepsilon_{\mathrm{o}}(z_{\mathrm{s}})$ Eqs. (1) and (20) can be used. Accounting for the very small value of $\varepsilon_{\mathrm{o}}(z_{\mathrm{s}})$ (preliminary estimates showed $\varepsilon_{\mathrm{o}}(z_{\mathrm{s}})\cong 1\text{–}4\times 10^{-3}$) the dependence of ε_{o} on z in the micro-shrinkage depth range may be described by the simple linear approximation

$$\varepsilon_{\mathrm{o}}(z)=\varepsilon_{\mathrm{o}}(z_{\mathrm{s}})\frac{(z_{\mathrm{m}}-z)}{(z_{\mathrm{m}}-z_{\mathrm{s}})},\quad z_{\mathrm{s}}\leqslant z\leqslant z_{\mathrm{m}}. \tag{21}$$

To estimate the character of horizontal cracks in the macro-shrinkage depth range ($z<z_{\mathrm{s}}$), it is reasonable to neglect the contributions of MPRS, $\overline{\Delta S}(z)$ of the micro-shrinkage depth range ($z\geqslant z_{\mathrm{s}}$). Within this approximation, Eqs. (16), (17) and (19) can be replaced, respectively, by

$$v_{\mathrm{h}}(z)=\begin{cases}-\frac{\mathrm{d}\overline{\Delta S}(z)}{\mathrm{d}z}\cdot(1-\delta_{\mathrm{o}}(z)) & 0\leqslant z\leqslant z_{\mathrm{s}} \text{ and } \frac{\mathrm{d}\overline{\Delta S}(z)}{\mathrm{d}z}<0,\\ 0 & z_{\mathrm{s}}\leqslant z\leqslant z_{\mathrm{m}} \text{ or } \frac{\mathrm{d}\overline{\Delta S}(z)}{\mathrm{d}z}\geqslant 0,\end{cases} \tag{22}$$

$$V_h(z) = \begin{cases} \int_z^{z_m} v_h(z')\,dz' & 0 \leqslant z \leqslant z_s, \\ 0 & z_s \leqslant z \leqslant z_m, \end{cases} \tag{23}$$

$$R_h(z) = \begin{cases} -\frac{d\overline{\Delta S}(z)}{dz} \cdot d(z) & 0 \leqslant z \leqslant z_s \text{ and } \frac{d\overline{\Delta S}(z)}{dz} < 0, \\ 0 & z_s \leqslant z \leqslant z_m \text{ or } \frac{d\overline{\Delta S}(z)}{dz} \geqslant 0. \end{cases} \tag{24}$$

The model was used to predict the MPRS, $\overline{\Delta S}(z)$, the specific volume, $v_h(z)$, the cumulative volume, $V_h(z)$, and the mean width, $R_h(z)$ of horizontal cracks. Quantitative data from direct measurements of the four characteristics of horizontal cracks in swelling soils are not available. The following was used for validation of the model predicted horizontal-crack characteristics as functions of soil depth:

- data on subsidence at the soil surface [14];
- data on the total vertical-crack volume in the macro-shrinkage depth range, $z < z_s$ [11–13,5] and corresponding specific volume, cumulative volume, and mean width of vertical cracks as functions of soil depth [6,7] (the weight factor of Eq. (6) was used in averaging width of vertical cracks at a given soil depth);
- data on specific volume and mean width of vertical cracks in the micro-shrinkage depth range from Ref. [15];
- qualitative physical considerations of relative magnitudes of similar characteristics of horizontal and vertical cracks; and
- two photographs of a vertical soil cross-section in the macro-shrinkage depth range from Ref. [16].

4. Results and discussion

4.1. The mean potential relative subsidence (MPRS)

Variation of MPRS with depth was calculated for the nineteen cases mentioned above. The predicted curves were of a similar nature (see Fig. 3 as an example). Generally the predicted MPRS decreases with depth increase (possibly passing through a maximum near the soil surface) from the first few millimeters (the maximum of MPRS value at the soil surface for all the nineteen cases is ≈5 mm) to zero (see as an example Fig. 3). Most of the change takes place in the macro-shrinkage depth range $z < z_s$ (Fig. 3). This is expected since in the micro-shrinkage depth range, the width of vertical cracks, the evaporation from their walls, and the vertical shrinkage of thin layers at their walls are much smaller in comparison with these in the macro-shrinkage depth range.

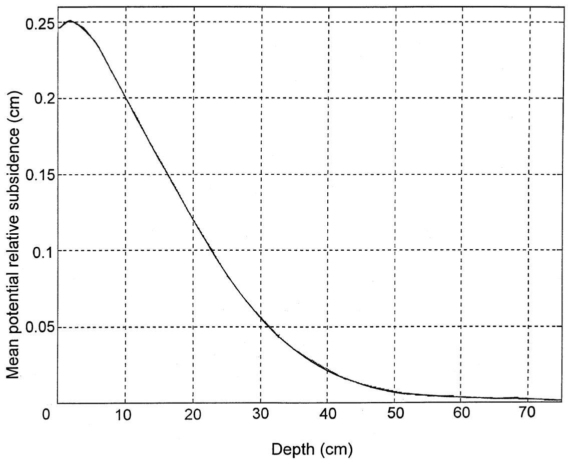

Fig. 3. Mean potential relative subsidence (MPRS) as a function of depth, predicted for profile O4H [11] ($z_s = 75$ cm [11] and $z_o = 36$ cm, $z_m = 290$ cm [6,7]).

Table 1 gives the predicted MPRS values, $\overline{\Delta S}(0)$ which are compared with experimental values of subsidence at the soil surface from measurements of Ref. [14], Fig. 6 in the Zevulon Valley in Israel on six dates. The estimates of MPRS at the soil surface are appreciably smaller than the experimental subsidence, by, at least, 1 order of magnitude. This result is in accordance with the following physical consideration. Most of vertical cracks reaching to any given depth have very small width and correspondingly very small evaporation from their walls at the depth. So the usual subsidence at any depth z must appreciably surpass the MPRS.

4.2. Horizontal-crack porosity, mean width, and cumulative volume

Figs. 4–6 show that in most cases the porosity, the mean width, and the cumulative volume of

Table 1
Model predictions of MPRS values and experimental values of subsidence at the soil surface in the Zevulon Valley, Israel on six dates from [12,14]

Type	Subsidence at the soil surface, cm					
	May 30	June 20	July 16	Aug. 26	Sept. 19	Oct. 21
Model – MPRS at walls of vertical cracks	0.02	0.05	0.08	0.12	0.18	0.19
Experiment – subsidence	0.2	0.6	1.2	3.3	3.8	4.9

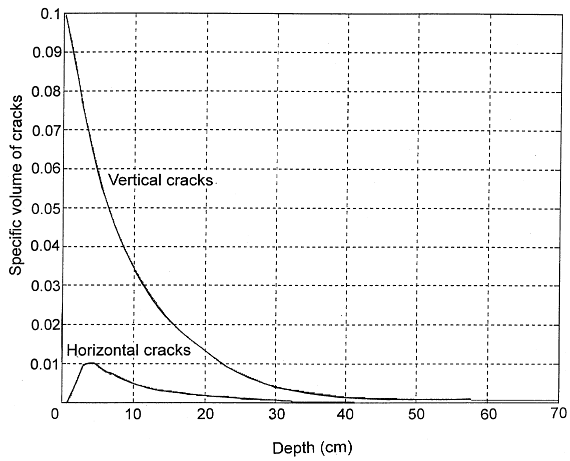

Fig. 4. Model prediction of horizontal-crack porosity (specific volume of cracks) as a function of depth (conditions of Zevulon Valley, Israel, on August 26: $z_s = 70$ cm, $z_m = 225$ cm [12,14]) and the vertical-crack porosity (the same conditions, $z_o = 21.4$ cm [6,7]).

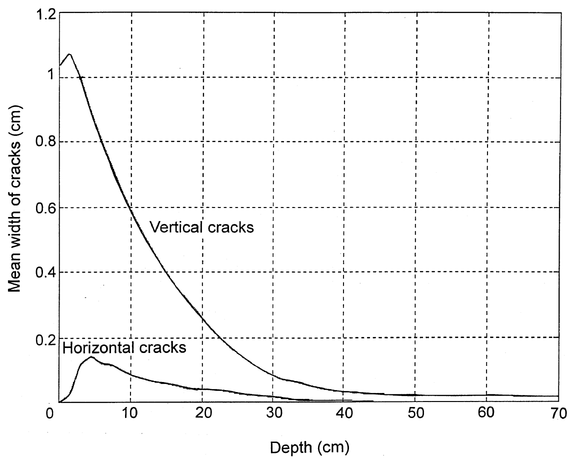

Fig. 5. Mean width of horizontal and vertical cracks as a function of depth (conditions as in Fig. 4).

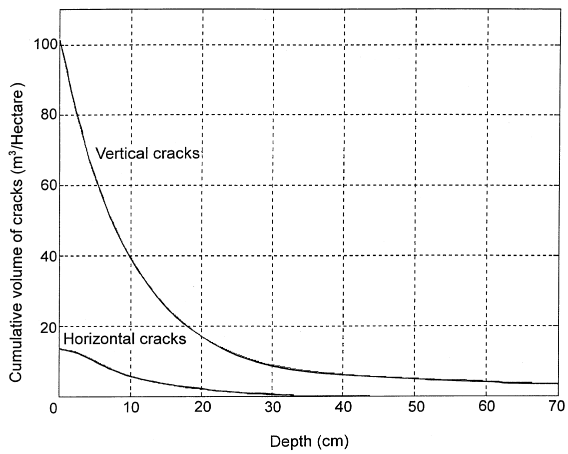

Fig. 6. Cumulative volume of horizontal and vertical cracks as a function of depth (conditions as in Fig. 4).

horizontal cracks are smaller in the macro-shrinkage depth range in comparison with those of vertical cracks by, at least, 1 order of magnitude. It means that not only MPRS, $\overline{\Delta S}(z)$ is of small values, but also the absolute values of derivative of MPRS are small enough (see Eqs. (16), (17) and (19)). According to the data [15] in the micro-shrinkage depth range (1.5 – 5.5 m), the porosity and width of vertical cracks are in the ranges of 2×10^{-3} to 3×10^{-5} and 43–1 μm, respectively. Comparison of data [15] shows also relatively small values of the estimated porosity and mean width for horizontal cracks in the micro-shrinkage depth range, $z_s \leqslant z \leqslant z_m$.

In a number of cases, comparison between porosity and mean width of horizontal and vertical cracks shows that there is a depth interval, in the macro-shrinkage depth range, where their values are of the same order of magnitude (as an example

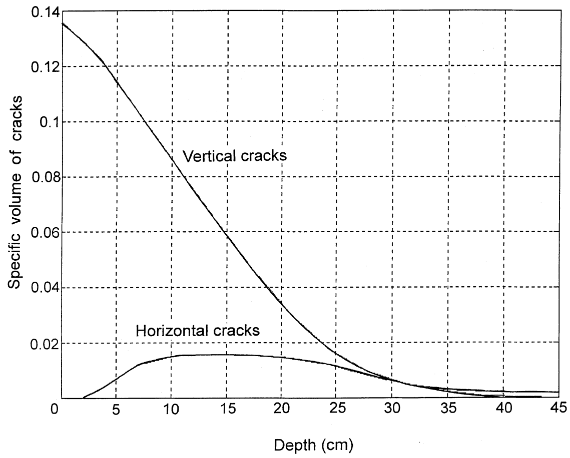

Fig. 7. Model prediction of horizontal-crack porosity (specific volume of cracks) as a function of depth for profile GTO3 from Ref. [11] and the vertical-crack porosity for the same profile ($z_s = 45$ cm [11], $z_o = 44$ cm, $z_m = 524$ cm, [6,7]).

see Figs. 7 and 8 where this depth interval is 15 – 35 cm). It means that, in spite of its small values, MPRS, $\overline{\Delta S}(z)$ decreases with depth sharply enough, so that the absolute values of its derivative are large enough (see Eqs. (16), (17) and (19)). This result agrees qualitatively with the observations in Ref. [17] of the ratio of horizontal to vertical hydraulic conductivities in a soil.

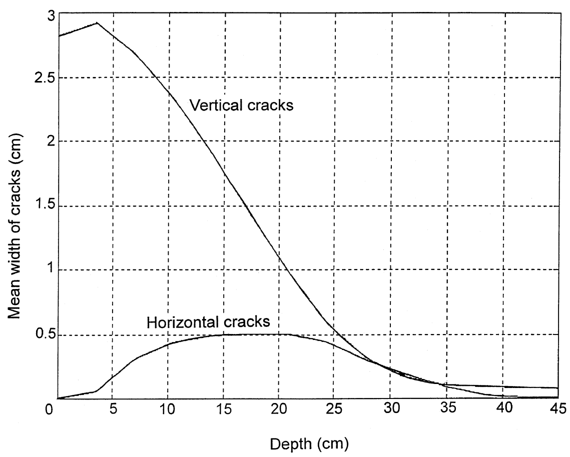

Fig. 8. Comparison between the mean widths of horizontal and vertical cracks (conditions as in Fig. 7).

4.3. Nontensile near-surface layer

In all the cases analysed, it was found that near the soil surface there is a layer of thickness of up to ≈ 3 cm where the MPRS increases or, at least, does not decrease (Fig. 3). It means that in this layer the soil at vertical-crack walls is, on the average, compressed in the vertical direction (or, at least, not under a tensile stress) and horizontal cracks do not appear (see curves for horizontal cracks in Figs. 4 and 7 near the soil surface). The two photographs (Figs. 9 and 10) of a vertical soil cross-section [16] are in qualitative accordance with this result.

4.4. The maximum in the horizontal-crack porosity

The horizontal-crack porosity has a broad maximum in the layer, $0 \leqslant z \leqslant z_s$ (Figs. 4 and 7). This also agrees qualitatively with the two photographs [16] (Figs. 9 and 10).

4.5. The maximum of the mean width of horizontal cracks

A maximum for the mean width of horizontal cracks has been also predicted by the model (Figs. 5 and 8) and is also in qualitative agreement with the two photographs [16] (Figs. 9 and 10).

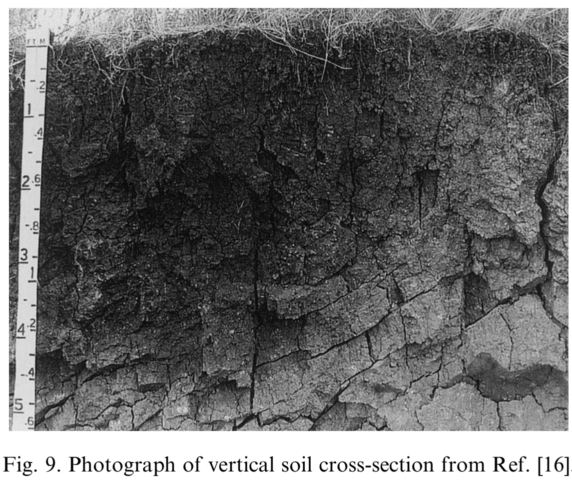

Fig. 9. Photograph of vertical soil cross-section from Ref. [16].

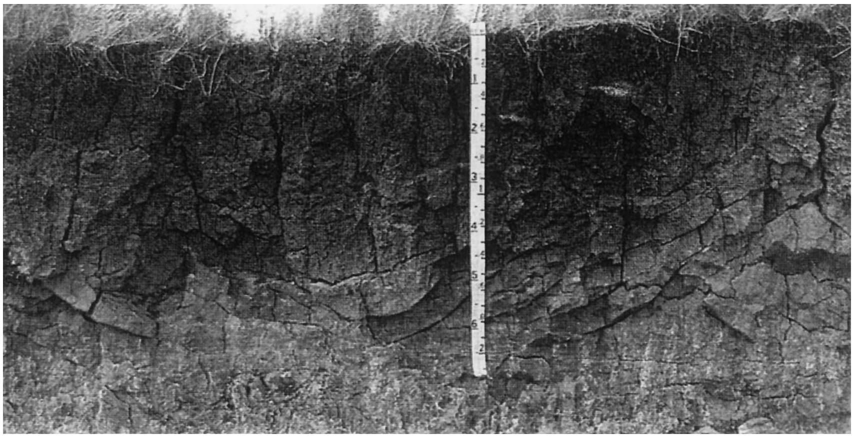

Fig. 10. Photograph of vertical soil cross-section from Ref. [16].

4.6. The development of the horizontal cracks during drying

Estimates of porosity, cumulative volume, and mean width of horizontal cracks at different times during drying, based on the data [12,13,5], show (see as an example Fig. 11) that at any given depth these parameters increase simultaneously with those of vertical cracks (Fig. 12). This result is natural. When the vertical cracks are widening, evaporation from their walls increases at any given depth, causing stronger shrinkage of thin layers at the walls of the cracks.

5. Summary and conclusion

Horizontally oriented cracks in swelling soils influence water and solute transport. A model for predicting their mean width and volume as functions of soil depth is a prerequisite for constructing a model of hydraulic properties of the whole crack network in a swelling soil.

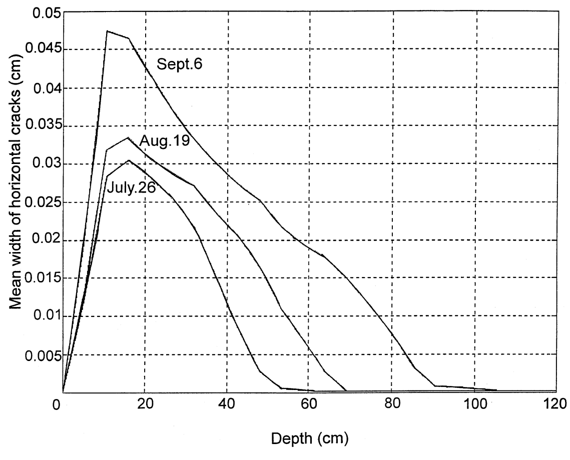

Fig. 11. Model predicted mean widths of horizontal cracks as functions of soil depth on various dates (conditions of Saskatchewan, Canada from Ref. [13], $z_s = 80$ cm [13], $z_o = 76$ cm, $z_m = 800$ cm, [6,7]).

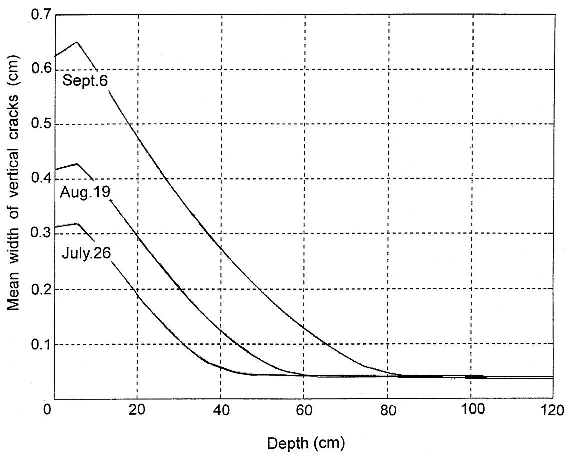

Fig. 12. Predicted mean widths of vertical cracks as functions of depth (conditions as in Fig. 11).

The major idea of this work is the appearance of horizontal cracks as a result of inhomogeneous soil subsidence caused by the more rapid drying, and shrinkage, of thin soil layers at vertical-crack walls. Compared to the shrinkage of the moist soil matrix, at the same depth, horizontal cracks originate as ruptures in the stretched sections of drying layers of the walls of vertical cracks.

The term MPRS, $\overline{\Delta S}(z)$ was introduced to quantify the average inhomogeneity of soil subsidence, depending on soil depth (Eqs. (4) and (7)). Negative values of the derivative, $\mathrm{d}\overline{\Delta S}(z)/\mathrm{d}z$ indicate a depth range where stretched sections occur, on the average, at vertical-crack walls. The absolute values of the derivative, in this depth range, determine the specific volume of horizontal cracks (horizontal-crack porosity), their cumulative volume, and their mean width as functions of soil depth (Eqs. (16), (17) and (19)).

To calculate the function $\overline{\Delta S}(z)$, two parameters are needed to determine the geometry of crack networks in clay soils [6,7], namely, the maximum crack depth z_{m} and the thickness of an intensive-cracking layer z_{o}. Also needs is measured data for depth–linear shrinkage of the soil matrix, $\varepsilon_{\mathrm{o}}(z)$ (or water content plus a shrinkage curve) and the linear shrinkage at the wall of a vertical-crack of depth h, $\varepsilon(z,h)$. An approach is proposed to estimate $\varepsilon(z,h)$, if $\varepsilon_{\mathrm{o}}(z)$ is known, from the experimental data [8,9].

Model predictions of the horizontal-crack characteristics, based on depth–linear shrinkage curves for 19 experimental cases [11–13,5] and estimates of z_{m} and z_{o} for these cases [6,7], were obtained. In 17 cases, the depth–linear shrinkage curve are for a depth range $0 \leqslant z \leqslant z_{\mathrm{s}}$, where z_{s} is an experimental thickness of a shrinkage layer obtained from measurements with a flexible wire of a finite given diameter D. A simple linear approximation of the depth–linear shrinkage curve, based on values of $z_{\mathrm{m}}, z_{\mathrm{o}}, z_{\mathrm{s}}$, and the diameter D, was used for the depth range $z_{\mathrm{s}} \leqslant z \leqslant z_{\mathrm{m}}$ (Eq. (21)). Since quantitative data on horizontal crack characteristics are not available, the predictions were compared with published data on similar characteristics of vertical cracks and subsidence at the soil surface. Accounting for a number of physical considerations, the comparison showed satisfactory agreement in the whole range of soil depths, $0 \leqslant z \leqslant z_{\mathrm{m}}$. Some qualitative results of predicted horizontal-crack characteristics are also in agreement with published observations. They include the existence:

- of a nontensile upper soil layer of a depth up to $\approx$3 cm;
- of broad maximum of the horizontal-crack porosity and mean width in the layer $0 \leqslant z \leqslant z_{\mathrm{s}}$;
- of a depth range (at least, in some cases) where the mean width and porosity of horizontal and vertical cracks are of the same order of magnitude.

It may be concluded that the comparison of the model predictions with available data, observations, and physical considerations verifies its feasibility.

Acknowledgements

The research is supported in part by the Israel Ministry of Science and Arts and the BARD Foundation.

References

[1] J.J.B. Bronswijk, Modeling of water balance, cracking and subsidence of clay soils, J. Hydrol. 97 (1988) 199–212.
[2] J.J.B. Bronswijk, Prediction of actual cracking and subsidence in clay soils, Soil Sci. 148 (1989) 87–93.
[3] J.J.B. Bronswijk, Shrinkage geometry of a heavy clay soil at various stresses, Soil Sci. Soc. Am. J. 54 (1990) 1500–1502.
[4] J.J.B. Bronswijk, Relation between vertical soil movements and water-content changes in cracking clays, Soil Sci. Soc. Am. J. 55 (1991) 1220–1226.
[5] J.J.B. Bronswijk, Drying, cracking, and subsidence of a clay soil in a lysimeter, Soil Sci. 152 (2) (1991) 92–99.
[6] V.Y. Chertkov, I. Ravina, Model of fracture network geometry in swelling clay soils, in: A.L. Yarin (Ed.), Proceedings of The 26th Israel Conference on Mechanical Engineering, Haifa, 21 & 22 May 1996, pp. 523–525.
[7] V.Y. Chertkov, I. Ravina, Modeling the crack network of swelling clay soils, Soil Sci. Soc. Am. J. 62 (1998) 1162–1171.
[8] H.M. Selim, D. Kirkham, Soil temperature and water content changes during drying as influenced by cracks: a laboratory experiment, Soil Sci. Soc. Am. Proc. 34 (1970) 565–569.

[9] J.T. Ritchie, J.E. Adams, Field measurement of evaporation from soil shrinkage cracks, Soil Sci. Soc. Am. Proc. 38 (1974) 131–134.
[10] D. Hillel, Fundamentals of Soil Physics, Academic Press, New York, 1980, pp. 350 & 351.
[11] A. Zein el Abedine, G.H. Robinson, A study on cracking in some vertisols of the Sudan, Geoderma 5 (1971) 229–241.
[12] D.H. Yaalon, D. Kalmar, Extent and dynamics of cracking in a heavy clay soil with xeric moisture regime, in: J. Bouma, P.A.C. Raats (Eds.), Proc. ISSS Symp. on Water and Solute Movement in Heavy Clay Soils, ILRI, Wageningen, The Netherlands, 1984, pp. 45–48.
[13] G.S. Dasog, D.F. Aston, A.R. Mermuth, E. De Jong, Shrink-swell potential and cracking in clay soils of Saskatchewan, Can. J. Soil Sci. 68 (1988) 251–260.
[14] D.H. Yaalon, D. Kalmar, Vertical movement in an undisturbed soil: continuous measurement of swelling and shrinkage with a sensitive apparatus, Geoderma 8 (1972) 231–240.
[15] L.D. McKay, J.A. Cherry, R.W. Gillham, Field experiments in a fractured clay till. 1. Hydraulic conductivity and fracture aperture, Water Resour. Res. 29 (1993) 1149–1162.
[16] L.P. Wilding, R. Puentes (Eds.), Vertisols: their distribution, properties, classification and management, Technical monograph No.18, Soil Management Support Services, Texas A & M University Printing Center, College Station, Texas 77843, 1988.
[17] J. Bouma, J. Loveday, Characterizing soil water regimes in swelling clay soils, in: L.P. Wilding, R. Puentes (Eds.), Vertisols: Their Distribution, Properties, Classification and Management, Technical Monograph No. 18, Soil Management Support Services, Texas A & M University Printing Center, College Station, Texas 77843, 1988, pp. 83–96.