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AN ANALYSIS OF THE EFFECT OF THE VERTICAL FISSURING IN MOLE-DRAINED SOILS ON DRAIN PERFORMANCES

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ABSTRACT

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Numerical solutions of the seepage equation of groundwater flow were used in an analysis of the effect on drain performance of the herring-bone pattern of vertical fissuring, with fractures fanning out from the central slit, in mole-drained soils. Drain performances were assessed from values of the dimensionless parameter $W_m = 2E_m/qD^2$, where E_m is the 'seepage potential' at the position of maximum water-table height when the steady rainfall is q and the drain spacing is $2D$. W_m decreased with increase in the length of the fractures and, to a lesser extent, with decrease in the spacing of them, showing that the fracturing enables a mole-drainage system to cope with higher rainfall rates and to produce more rapid water-table drawdowns.

INTRODUCTION

An important feature of mole-drained soils is the vertical fissuring produced by the mole plough in the vicinity of the drain channels. As the channel is formed by the bullet of the mole plough, the leg-blade, to which the bullet is attached, makes a vertical slit in the soil above. From this slit fractures open up at regular intervals on either side, making an acute angle, approximately equal to 45°, and pointing in the direction from which the mole plough travelled, thus producing the typical herring-bone system of vertical fissures described by Nicholson (1942, p. 99).

The slit produced by the leg-blade persists for many seasons and remains an obvious feature whenever the ground dries out. The fractures are not as wide as the slit but nevertheless can be seen easily, particularly immediately after the mole draining. Godwin et al. (1981) examined the fissuring after mole draining in many soils at various soil-water conditions. They observed that the fractures were between 0.09 and 0.36 m long and between 5 and 60 mm wide, repeated every 0.10 to 0.18 m, and that they extended down to the depth of the mole drains.

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The fissuring has always been considered to play an important role in the drainage of heavy soils. It is often assumed that the main purpose of mole drainage is to induce easier removal of surface water via the fissures, and that the role of the mole drains as sinks for groundwater control is of secondary importance. However, the experiments of Leeds-Harrison et al. (1982) with mole drains formed in the usual way with fissuring and with mole drains specially formed without fissuring show that mole drains do act as effective sinks in the groundwater zone. This paper examines theoretically, using seepage analysis (Youngs, 1965; 1966; 1980), the importance of the extent of vertical fissuring on the drain performance of mole-drained soils.

ANALYSIS OF GROUNDWATER FLOW

The herring-bone pattern of vertical fissures, described by Nicholson (1942, p. 99) and in more detail by Godwin et al. (1981), that are produced to the depth of the drain channels during the mole-draining operation, is shown in plan view in Fig. 1. The width of these fissures allows water to be conducted with little resistance compared with that moving in the soil itself. Thus the fissures, being connected to the mole channel, may be assumed to act as sinks in the groundwater region in a similar way to vertically faced ditch drains when there is a water table above drain level.

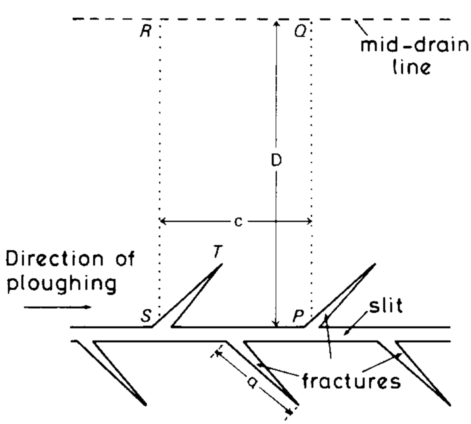

Fig. 1. Plan view of the vertical fissuring produced during mole draining.

The flow of water to the mole drains via the fissures may be found from the hydraulic head distribution in the groundwater zone, obtained by solving numerically Laplace's equation in three dimensions by the finite difference or finite element method. However, if there is negligible flow below drain level, the horizontal seepage to drains in the groundwater region below the water table is described more simply in two dimensions by the seepage equation (Youngs, 1965; 1966; 1980):

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = -q(x,y) \qquad (1)$$

where $q(x,y)$ is the flux, measured positive downwards, through the water table at the position (x,y) and E is the 'seepage potential' defined by:

$$E = \int_0^H K(z)\,\{h(x,y,z) - z\}\,\mathrm{d}z \tag{2}$$

where $K(z)$ is the hydraulic conductivity assumed to vary with height z, $h(x,y,z)$ is the hydraulic head at the point (x,y,z) in the groundwater zone, and H is the height of the water table at (x,y) above the datum level which is taken to be drain level. The components of the horizontal seepage Q_x and Q_y in the x- and y-directions are given by:

$$Q_x = -\frac{\partial E}{\partial x}, \qquad Q_y = -\frac{\partial E}{\partial y} \tag{3}$$

The boundary conditions to be applied to Eq. (1) are that $E = 0$ at the fissures, assuming water is removed via the drain channels as fast as it enters thus maintaining the fissures empty, and that the gradient of E normal to the watershed is zero. In using this analysis it is assumed that the scale of natural fissuring which is important for water flow through clay soils, is small compared with that of the fissures created by the mole draining operation so that a hydraulic conductivity of the bulk soil can be used.

The maximum water-table height H_m that is maintained by a steady-state rainfall rate q in drained lands, occurs at the position where E is a maximum, E_m. It can be argued (Youngs, 1965; 1980) that the maximum water-table height must lie between the bounds given by:

$$\int_0^{H_m} K(z)\;(H_m - z)\;\mathrm{d}z > E_m > \int_0^{H_m} K(z)\;(H_m - z)\;\mathrm{d}z - \int_0^{H_m} K(z)\;\left[\int_z^{H_m} \{q/K(z)\}\;\mathrm{d}z\right]\;\mathrm{d}z \tag{4}$$

Thus if E_m is found by solving Eq. (1) with the given boundary conditions, bounds for H_m can be calculated. When $K(z)$ increases with height, the bounds are found to be close.

For an installation of parallel drain channels $E = E_m$ everywhere on the watershed that lies midway between drains. With the herring-bone pattern of drainage fissures that occur with mole draining, the watershed still lies midway between drain lines if the fractures in adjacent drain lines are opposite each other. The value of E rises and falls along the mid-line with the repetition of the fractures, although the difference between maximum and minimum values is very small indeed unless the fractures extend near the watershed. The maximum value E_m occurs on the watershed at a position dependent on the fracture length and spacing. If the fractures are not opposite each other, the watershed does not follow the mid-line between drains but zigzags about it. However, the deviation from the mid-line is negligible for the fracture lengths produced in practice in mole-drained lands, and it will be assumed here that the mid-line is the watershed in all situations. Thus in the analysis of the mole-drainage problem, it is necessary only to consider the region PQRS shown in Fig. 1. Values of E in this region are repeated throughout the whole region with reflection in the line QR midway between drains and with values of E on PQ being the same as on SR.

The region PQRS is redrawn in Fig. 2 in which the boundary values of the drainage problem to be applied to Eq. (1) are shown. It is assumed that the drain line is in the x-direction. The boundary values are:

$$\left.\begin{aligned} &E = 0, \quad 0 \leqslant x \leqslant c, \quad y = 0 \\ &E = 0, \quad x = y, \quad 0 \leqslant \sqrt{x^2 + y^2} \leqslant a \\ &\frac{\partial E}{\partial y} = 0, \quad 0 \leqslant x \leqslant c, \quad y = D \\ &E(0, y) = E(c, y) \end{aligned}\right\} \tag{5}$$

where c is the repetition distance of the fractures, a the length of the fractures, and $2D$ is the spacing between the central slits of the mole drainage system.

The maximum value of E_m can be found by solving Eq. (1) subject to the conditions (5). Bounds for the maximum water-table height can then be determined from Ineq. (4).

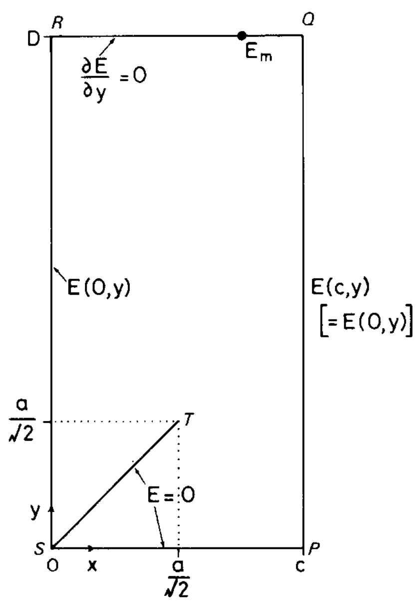

Fig. 2. Seepage region considered in the analysis.

NUMERICAL SOLUTION OF THE SEEPAGE EQUATION

Equation (1) can be rewritten in the form:

$$\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} = -2 \tag{6}$$

in terms of the dimensionless variables $W = 2E/qD^2$, $X = x/D$ and $Y = y/D$. The boundary values (5) become:

$$\left.\begin{array}{l} W = 0, \quad 0 \leqslant X \leqslant c/D, \quad Y = 0 \\ W = 0, \quad X = Y, \quad 0 \leqslant \sqrt{X^2 + Y^2} \leqslant a/D \\ \dfrac{\partial W}{\partial Y} = 0, \quad 0 \leqslant X \leqslant c/D, \quad Y = 1 \\ W(0, Y) = W(c/D, Y) \end{array}\right\} \tag{7}$$

In solving Eq. (6) numerically, the region PQRS in the (X, Y) space is divided into a grid of $m \times n$ nodes with an interval distance h in both the X- and Y-directions so that $h = c/mD = 1/n$. The finite difference form of Eq. (6) for the values of W at the nodes is:

$$W_{i,j+1} + W_{i,j-1} + W_{i+1,j} - 4\, W_{i,j} = -2h^2 \tag{8}$$

When the values of W at the nodes are not in agreement with Eq. (8), a better estimate $W'_{i,j}$ of $W_{i,j}$ is given by:

$$W'_{i,j} = W_{i,j} + \omega\,(W_{i,j+1} + W_{i,j-1} + W_{i+1,j} + W_{i-1,j} - 4\, W_{i,j} + 2h^2) \tag{9}$$

using the successive overrelaxation method (for example, see Roache, 1972, pp. 117—119), where ω is a factor between 1 and 2 that improves the convergence of the iteration procedure. For rectangular regions ω may be calculated, but in our case the fracture ST where $W = 0$, cutting into the region PQRS, does not permit a theoretical estimation of ω. In working out solutions for the drainage problems, $\omega = 1.6$ gave satisfactory convergence with $|(W'_{i,j} - W_{i,j})/W'_{i,j}| < 10^{-6}$ after 1000 iterations with a grid of between 400 and 800 nodes and with h lying between 0.01 and 0.05. With larger values of ω it was found that the process diverged.

Using an initial trial solution for W corresponding to the drainage situation of a single slit without the fracture ST:

$$W_{r,s} = hs(2 - hs) \tag{10}$$

where $W_{r,s}$ is the value of W at any node (r, s). Equation (9) was applied to each node in turn, beginning with the nodes closest to the boundary SP at $Y = 0$ and working outwards towards the watershed boundary QR at $Y = 1$. The nodes were thus recalculated in the following order:

$$\begin{array}{l} W_{0,1},\ W_{1,1},\ W_{2,1},\ \ldots,\ W_{m-1,1} \\ W_{0,2},\ W_{1,2},\ W_{2,2},\ \ldots,\ W_{m-1,2} \\ \cdot \\ \cdot \\ \cdot \\ W_{0,n},\ W_{1,n},\ W_{2,n},\ \ldots,\ W_{m-1,n} \end{array} \qquad (11)$$

taking note of the conditions at the boundaries imposed by the repetition of the region PQRS, namely, $W_{0,s} = W_{m,s}$ and $W_{-1,s} = W_{m-1,s}$ for the calculation of the nodes along RS, and $W_{r,n+1} = W_{r,n-1}$ for the nodes along QR. When all the nodes had been calculated, the iteration procedure began again at the node $W_{0,1}$ until the values of W at all the nodes changed insignificantly on applying Eq. (9). The values of W at the nodes then were a numerical solution of Eq. (6) subject to the conditions (7).

EFFECT OF FRACTURE LENGTH AND SPACING ON DRAIN PERFORMANCE

The dimensionless parameter W_m $(= 2E_m/qD^2)$ is related to the shape factor A of a drainage system, introduced by Youngs (1980) in connection with drainage areas of different geometries, by the equation $W_m = 2A/D^2$. For parallel ditch drains dug to an impermeable floor and maintained empty, $W_m = 1$. W_m may be regarded as a measure of drain performance since it combines the maximum water-table height and hydraulic conductivity in the variable E_m, the steady rainfall rate q and the half drain spacing D. For the same water-table height in a given soil, the bounds of E_m given by Ineq. (4) are independent of the geometry of the drainage installation. Thus, if W_m varies on account of different geometries, q or D or both q and D can change to keep E_m constant and therefore to maintain the same water-table height in the soil.

Numerical solutions of Eq. (6) with boundary conditions (7) for W were found for a series of fracture lengths a between zero and $0.6D$ for values of the fracture spacing c of $0.1D$, $0.2D$, $0.3D$, $0.4D$ and $0.5D$. Values of W_m, the maximum value of W in the region PQRS on the watershed QR, obtained in these solutions, are shown in Fig. 3 plotted as a function of the dimensionless fracture length a/D for the values of dimensionless fracture spacing c/D used in the calculations. It is seen that W_m decreases considerably as a/D increases from zero to 0.6 but increases only slightly as c/D increases from zero to 0.5 for all values of a/D.

The variation of the parameter W over the seepage region is shown in Fig. 4 for fracture lengths of $0.56D$ and $0.28D$ when the repetition distance of the fractures is $0.4D$. Also included in Fig. 4 is the situation for drainage to parallel slits with no fractures when W is given by:

$$W = Y(2 - Y) \qquad (12)$$

The positions of the maximum values W_m are shown in Fig. 4. However, along the watershed, the value of W changes little and is to all intents and purposes constant for a given fracture length and spacing, indicating that the water-table height midway between drain lines varies negligibly in spite of the fractures penetrating into the region from the slits. This is found to be the case even for fractures penetrating half-way towards the mid-drain line when the variation in W is less than one part in 10^4.

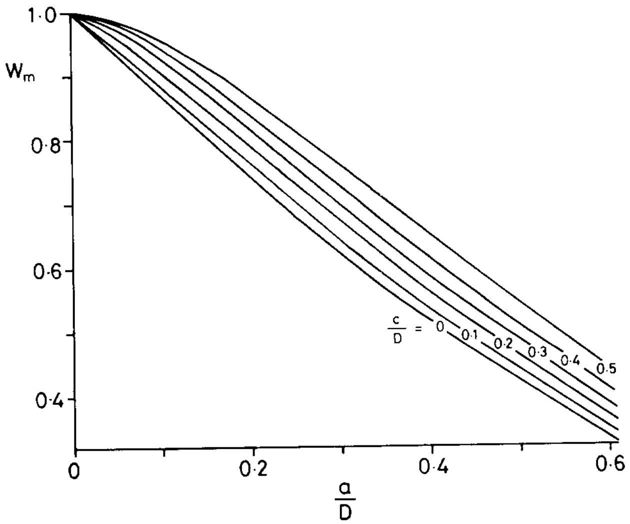

Fig. 3. Variation of W_m (= $2E_m/qD^2$) with dimensionless fracture length a/D for various dimensionless fracture spacings c/D.

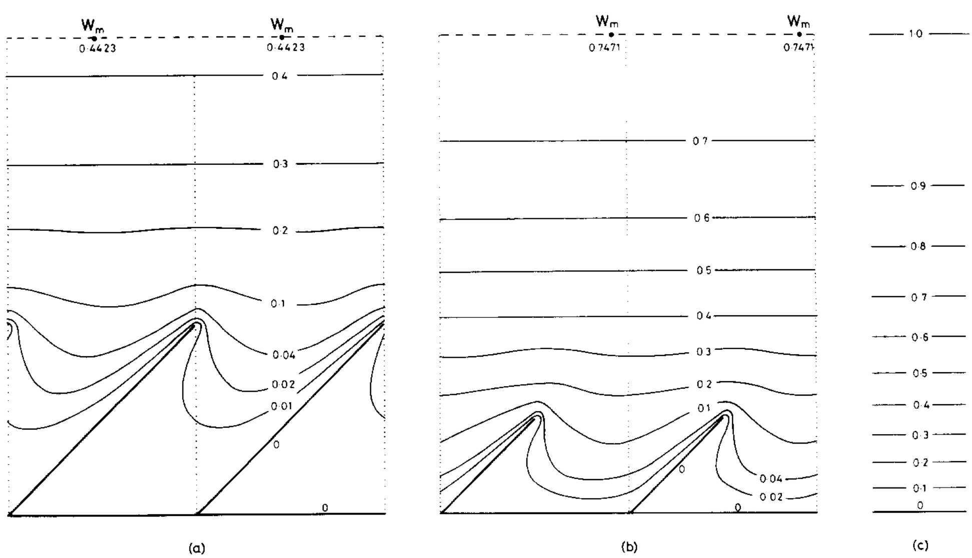

Fig. 4. Variation of W over the seepage region for a fracture spacing c of $0.4D$ when the fracture length a is: (a) $0.56D$; and (b) $0.28D$. The variation of W for drainage to parallel slits with no fractures is shown in (c).

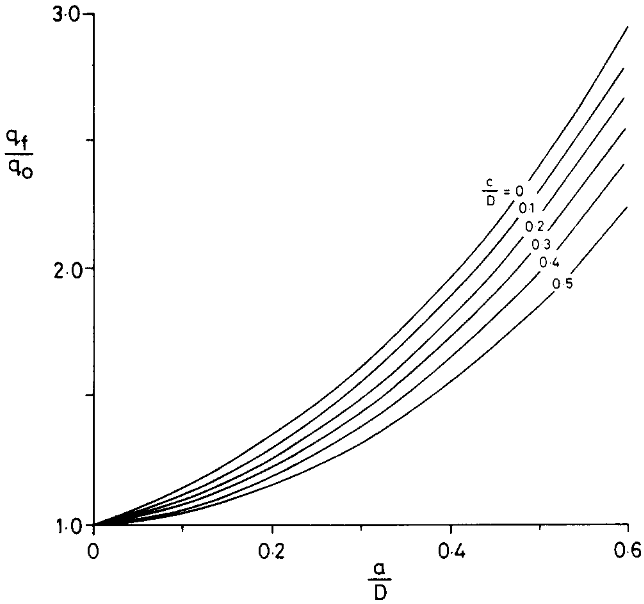

Fig. 5. Variation of the relative rainfall acceptance rate q_f/q_o with dimensionless fracture length a/D for various dimensionless fracture spacings c/D.

If q_o is the steady rainfall rate that maintains the water-table height at height H_m when there is no fracturing but just a central slit drainage channel, then the rainfall q_f that will maintain the water-table height at H_m when there is fracturing is given by:

$$q_f = 2E_m/W_m D^2 = q_o/W_m \tag{13}$$

$W_m < 1.0$ and, as seen in Fig. 3, is very dependent on the fracture length and slightly dependent on fracture spacing. The relative rainfall acceptance rate q_f/q_o is plotted in Fig. 5 as a function of the fractional length a/D for the range of fractional fracture spacings c/D for which calculations were performed. It is seen that the rainfall acceptance rate is considerably increased when the fractures penetrate some distance towards the mid-drain line.

The spacing of a system of mole drains required to maintain the water-table height at a given level with a given steady state rainfall rate is wider the greater the extent of fracturing. If $2D_f$ is the spacing of a mole drainage system with fractures, for the water-table height to be maintained at a height corresponding to a given E_m-value with a steady state rainfall rate q,

$$E_m = qW_m D_f^2/2 = qD_s^2/2 \tag{14}$$

where $2D_s$ is the spacing of an equivalent parallel drainage system without fractures required to maintain the same water-table height with the rainfall q. Thus D_s, the equivalent parallel drain half-spacing, is given by:

$$D_s = D_f/\sqrt{W_m} \tag{15}$$

When there is continuous rainfall at a high rate on poorly conducting soils, ponding of water on the soil surface cannot be prevented even with the most intensive drainage system. In such situations the rate of fall of the water table after the cessation of rainfall is a more realistic indication of the adequacy of a given drainage installation than the steady state rainfall rate required to maintain the water table at a given level. Assuming that the non-steady state falling water-table situation can be approximated by a succession of steady states, we can write:

$$q = -S\,\frac{\mathrm{d}H_\mathrm{m}}{\mathrm{d}t} \tag{16}$$

for the average flux q through the water table at time t, where S is the specific yield. Thus, writing $q = 2E_\mathrm{m}/W_\mathrm{m}D^2$, Eq. (14) can be integrated to give:

$$t = W_\mathrm{m} \int_{H_\mathrm{o}}^{H_t} (SD^2/2E_\mathrm{m})\,\mathrm{d}H_\mathrm{m} \tag{17}$$

for the time t for the maximum water-table height to fall from H_o to H_t. The integral in Eq. (17) depends only on the water-table height and soil properties and not on the length and spacing of the fractures. The time t is proportional therefore just to the value of W_m and hence is reduced with more extensive fracturing. Thus, writing t_f for the time t for the given water-table drawndown when there are fractures and t_o for the time when there is drainage only to a central slit drain, we have:

$$t_\mathrm{f} = W_\mathrm{m} t_\mathrm{o} \tag{18}$$

Since $W_\mathrm{m} < 1.0$, $t_\mathrm{f} < t_\mathrm{o}$. Equation (18) gives the reduction in time of the water-table drawdown as a result of the presence of fractures.

DISCUSSION

The physical model of the mechanism of groundwater movement into mole drains assumes that the water level in the vertical fissures is maintained at drain level. With no back pressure in the mole-drain channel, this is the case for the wide central slit. However, the fractures that fan out from this slit taper away from the mole drain channel, thereby increasing the resistance to flow of water and hence giving rise to a higher water level in them. Thus the value of E must increase on the fracture ST in Fig. 2 towards T, and the assumption that $E = 0$ on ST is an approximation. However, when the hydraulic conductivity of the soil increases with height, as is usually the case, the increase in E at the fracture is small compared with E_m and thus there is little effect on the value of W_m which is a measure of drain performance. Even for a soil of uniform hydraulic conductivity, a rise in water level in the whole of the system of fissures would need to be 0.316 H_m for there to be a 10% increase in the value of W_m.

For drains drawn 2 m apart, the most extensive of the patterns of fractures observed by Godwin et al. (1981) produces a reduction of the value of W_m from 1.0 for no fracturing to a value of about 0.5. Thus from Eq. (13) such a system of fractures has the effect of being able to accept twice the rainfall rate required to maintain the water table at the same height above a drainage system without fractures. In particular it is able to accept twice the rainfall rate without surface ponding. It follows that for the same water-table height the drain outflow would be doubled. Also, from Eq. (18) the time for a given water-table drawdown would be halved. The experimental work of Leeds-Harrison et al. (1982) confirms these theoretical predictions concerning the effect of the vertical fissuring on groundwater flow. The outflow from the drains of the plots that contained mole drains with normal fracturing was greater than that for the plots with mole drains drawn without fracturing. Also the time that the drains flowed for the former was less than for the latter, indicating a decrease in the time for the water-table drawn-down. Thus, both theory and experiment show that the herring-bone pattern of vertical fissures in mole-drained soils plays an important role in the groundwater flow as well as in the removal of surface water.

The graphs presented in Fig. 3 show the importance of fracture length in mole-drained lands. Godwin et al. (1981) observed that the fracture length increased as the soil dried out, while experiments in a soil tank at Silsoe College by Dr. P. Leeds-Harrison (personal communication, 1983) have demonstrated that it is greater the wider the leg-blade of the mole-plough. Galvin (1983) noted the production of wider and longer fractures with the wider leg of a gravel mole-drainage machine than with an ordinary mole plough, and considered that the production of such wider vertical cracks is important for the drainage of impermeable soils that are subject to creep failure. He drew attention to the need for the re-design of mole-drainage machines if a wider leg is to be used and for further investigations in order to reconcile conflicting demands of wider cracking and mole stability. Other factors to be considered are the greater drawbar pull and energy requirements for the production of wider cracks that occur in drier soils and with wider leg-blades on the mole plough. Consideration of all these factors would lead to optimum fracturing for the best drainage of mole drained soils.

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