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Use of kinematic wave theory to model irrigation on cracking soil

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Abstract Border irrigation in Australia is characterised by excessive run-off and variable water use efficiencies. This paper discusses an analytical irrigation model that, because of its simplicity, represents a tool for improving water management in border irrigation. Infiltration data on four replicate 75-m^2 plots were obtained from one irrigation of perennial pasture on a duplex, red-brown earth. Infiltration was characterised by high initial and low final rates. Kostiakov, modified Kostiakov, Horton, Philip and linear infiltration functions were fitted to the data. The linear function fitted the data well, and the parameters have physical interpretation. Consequently, an analytical kinematic wave model of border irrigation that incorporates the linear infiltration function is discussed. The analytical model was compared to more complex numerical models of border irrigation. The analytical model does not require a computer and performed sufficiently well to have application for border irrigation management in the field.

Introduction

The adverse impacts of irrigated agriculture, namely salinity, sodicity, waterlogging, acidity and algal blooms, are well documented and are in part a consequence of poor irrigation management. In Australia, irrigated pastures occupy 0.7 million hectares and are mostly border (also referred to as surface, bay, border-check and border strip) irrigated. The management of water on border-irrigated pastures is largely by trial and error, so that border irrigation is characterised by excessive run-off and variable water use efficiencies. Significant improvements in border irrigation management must be achieved if irrigation of pastures is to remain viable agricultural land use.

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To achieve improvements in irrigation efficiency at border scale, a simple, user-friendly tool for predicting irrigation discharge and duration a priori is needed; to date, no such tool exists. Maximising border irrigation performance depends on matching irrigation discharge and duration to the physical dimensions, surface roughness and infiltration characteristics of the irrigation border. Consequently, an irrigation farmer must be able to estimate values for each independent parameter prior to irrigation.

In this paper, a simple linear infiltration function is shown to be appropriate for describing infiltration into a duplex, red-brown earth. The two parameters in this function have physical significance, allowing their independent estimation in the field. An analytical solution of the kinematic wave equations that incorporates the linear infiltration function is discussed and its value for improving border irrigation management is considered.

Infiltration

The performance of border irrigation depends on the infiltration characteristic of the soil, the surface roughness and dimensions of the irrigation border and the irrigation discharge and duration. Together, these factors determine advance and recession rates of irrigation water down a border (and hence the uniformity of irrigation), as well as infiltrated depth. The infiltration characteristic of the soil is the most difficult of these factors to quantify, and may have large spatial and temporal variability.

Kostiakov (1932) proposed one of the earliest infiltration models. It was empirically derived, simple and gave good prediction over short time spans. Shortcomings of the model are that the infiltration rate approaches zero with increasing opportunity time, and the parameters have no physical interpretation and therefore can only be obtained from experimental data. A constant rate term was included in the modified Kostiakov equation to correct the problem of zero final infiltration rate (USDA 1979). Horton (1940) developed an empirical exponential equation incorporat-

Table 1 Infiltration equations (Z, cumulative depth of infiltration; t, opportunity time; a, A, f_0, k, S, β and γ, empirically fitted parameters)

Equation name	Equation
Kostiakov	$Z = kt^a$
Modified Kostiakov	$Z = kt^a + f_0 t$
Philip	$Z = St^{1/2} + At$
Horton	$Z = \gamma(1 - e^{-\beta t}) + f_0 t$

Table 2 Textural analysis of Lemnos loam at the experimental site (ρ_B Bulk density)

Depth (m)	C. Sand (%)	F. Sand (%)	Silt (%)	Clay (%)	ρ_B (g/cm^3)
0.1	7	30	31	29	1.5
0.2	4	29	29	38	1.7
0.6	1	18	24	56	1.7
1.1	1	24	24	50	1.7
1.4	1	23	22	54	1.6

ing an initial and a final infiltration rate. A physically based infiltration equation was derived by Philip (1957) from the first two terms of the infinite series solution of Richards' equation. The Kostiakov, modified Kostiakov, Horton, and Philip equations are given in Table 1. Although numerous other attempts have been made to model infiltration, these equations remain the most widely used.

The linear infiltration function

Collis-George (1977) presented a linear infiltration equation in which cumulative infiltration was expressed as a constant plus the final infiltration rate by time. Other researchers have suggested the applicability of this equation for describing infiltration into cracking soils (Evans et al. 1990; Maheshwari and Jayawardane 1992; Mitchell and van Genuchten 1993). In the linear infiltration model (Eq. (1)), cumulative infiltration, Z, is expressed as the depth of water rapidly infiltrating into cracks and being sorbed through crack walls, Z_{CR}, plus the depth of water which infiltrates at rate i_f over time t.

$$Z = Z_{CR} + i_f t \qquad (1)$$

Infiltration data collection

An experiment was conducted to collect infiltration data for a red-brown earth classified as a Lemnos loam (Skene and Poutsma 1962) or *Natric xeralf* (SMSS 1983), in the Shepparton Irrigation Region, Victoria, Australia (36°26′S, 145°15′E, 113 m above mean sea level). Lemnos loam typifies border-irrigated soils in the Shepparton Irrigation Region. Falling head infiltration was measured on four replicate 75-m^2 plots, (plots A, B, C and D) for one irrigation. A summary of the textural characteristics of Lemnos loam appears in Table 2. The clay fractions are dominated by illite and kaolinite, with a small amount of smectite (Rengasamy 1983). Although defined as a duplex soil, (Northcote et al. 1975) as opposed to a cracking clay soil, Lemnos loam exhibits pronounced shrinkage upon drying, resulting in the formation of cracks. Three years prior to the trial, the plots were sown with a mixture of white clover (*Trifolium repens*), strawberry clover (*T. fragiferum*), perennial ryegrass (*Lolium perenne*) and paspalum (*Paspalum dilatatum*), which are representative of the perennial-legume-based pastures of the region.

The trial plots were flood irrigated when cumulative class A pan evaporation minus rainfall reached 50 mm. Irrigation water salinity was 0.1 dS/m. To eliminate lateral water loss, plastic skirts had been inserted around the perimeter of the plots to a depth of 0.5 m. A known volume of water (ca. 5000 l) was applied rapidly (within 30 s) to obtain early time data to ascertain the effect of soil cracks on infiltration. Ponded depths were recorded both manually, and automatically using capacitive water level sensors and data recorders logging continuously. Due to very high initial infiltration rates, coupled with disturbances in the surface water profile during the first 60 s of irrigation, the initial ponded depth could not be measured. It was therefore calculated by volume balance, using plot profiles from a 1-m grid survey of the plots. Since the volume of water remaining on the surface at any time was known, the volume infiltrating was determined as the initial ponded depth less the remaining depth. The effect of soil swelling on the volume balance was not significant and was neglected in the calculations.

Infiltration results

Cumulative infiltration as a function of time is presented in Fig. 1. The infiltrated depths within the first 60 s were 18.0, 24.1, 22.5 and 25.0 mm for plots A, B, C and D, respectively, representing an average of 49% of the total irrigation. Final infiltration rates (determined after 1 h) were 2.9, 1.8, 2.1, and 2.5 mm/h for trials A, B, C and D, respectively.

Infiltration modelling and discussion

The Kostiakov, modified Kostiakov, Philip, Horton and linear equations were fitted to the infiltration data from plots A, B, C and D, using Genstat 5, Release 3.1 (Lawes Agricultural Trust, Rothamsted Experimental Station, (Harpenden, UK)). Table 3 shows the means of the fitted parameters and their coefficients of variation for the four trial plots. A comparison of goodness of fit of the functions was made visually (e. g. Fig. 2) and using the coefficients of determination (Table 3). For simplicity, only one set of infiltration data, trial D, is shown in Fig. 2. Although the infiltration data are serially correlated, the coefficients of determination serve as a useful method of comparison of fit.

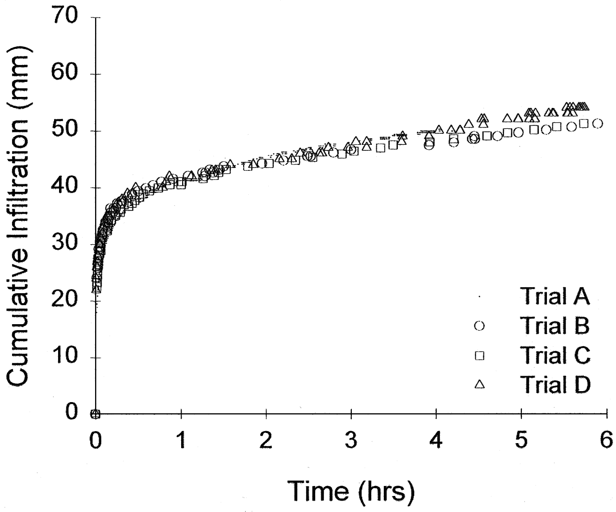

Fig. 1 Cumulative infiltration versus time for four replicate trials (*A*, *B*, *C* and *D*) of one irrigation of 75-m^2 perennial pasture plots

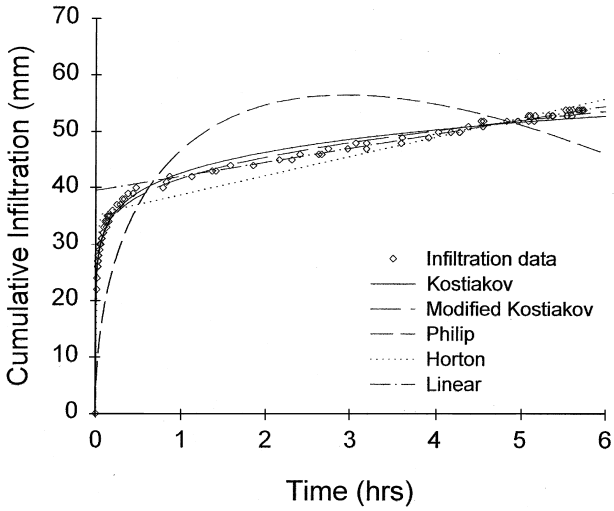

Fig. 2 Best-fit Kostiakov, modified Kostiakov, Philip, Horton and linear infiltration functions for the cumulative infiltration data of trial D

Table 3 Means and coefficients of variation (*CV*) and determination (r^2) for the four trials of the fitted infiltration parameters (*CV*s are of the best-fitting parameters of the four replicate irrigation trials; r^2s are for the best-fit infiltration functions for trials A, B, C and D)

Equation	Parameter	Mean	CV (%)	r^2
Kostiakov	k	41.6 mm/min^a	1.7	1.00
	a	0.124	11.3	
Modified Kostiakov	k	41.7 m/min^a	1.4	1.00
	a	0.127	18.0	
	f_0	−0.170 mm/min	344.6	
Philip	S	67.4 mm/min$^{1/2}$	3.3	0.95
	A	−21.2 mm/min	6.0	
Horton	γ	35.9 min	1.6	1.00
	β	42.3 min^{-1}	45.5	
	f_0	3.31 mm	10.8	
Linear	Z_{CR}	35.9 mm	2.3	0.99
	i_f	2.35 mm/h	17.1	

The Kostiakov and modified Kostiakov equations produced fits of similar accuracy for the 6 h of irrigation. With an increased opportunity time, the modified Kostiakov equation should give a superior fit, as a result of the final infiltration rate term, f_0. However, for the duration considered here, f_0 is not significantly different from zero, hence the increased complexity of the extra parameter is not warranted for these data. The Philip equation gave a relatively poor prediction of infiltration, and may be considered unsuitable for modelling infiltration into cracking soils. Although not as good as the Kostiakov or modified Kostiakov, the Horton equation predicted infiltration reasonably well. It simulated well the rapid crack filling and low final rates, but required three parameters, while the Kostiakov equation required only two. Additionally, the dimensionless exponent, β, is subject to large variability between trials, (coefficient of variation = 46% for these data) making applicability for field conditions dubious. The linear infiltration model was fitted for opportunity time after 1 h in order to retain the physical significance of the final infiltration rate, i_f, in the equation, and the goodness of fit was then determined using the whole data set. Since the final rate was fitted for times greater than 1 h, the linear model gave a very good fit for this period. It did, however, tend to overpredict infiltration during the early part of the infiltration phase. In reality, the errors introduced by assuming that crack-filling occurs instantaneously, followed by a constant infiltration rate, are likely to be less than the errors associated with field estimation of the infiltration parameters.

In summary, the Kostiakov function provides the best fit of the data (Table 3, Fig. 2). The linear function, whilst giving satisfactory prediction, has the advantage that the parameters have physical interpretation. The linear function offers an additional advantage over the Kostiakov function, in that variations in infiltration due to changes in antecedent water content may be readily accounted for in the crack fill term, Z_{CR}. The relative size of Z_{CR} is highly dependent on antecedent water content in the soil (J. B. Prendergast, unpublished data), being larger at lower antecedent soil water contents. The significance of the linear infiltration function in modelling of border irrigation is discussed below.

Border irrigation modelling

Models that describe the unsteady, gradually and spatially varied flow problem of border irrigation have been available for many years. These models generally use various approximations of the Saint-Venant, or complete dynamic wave equations, and are, from the most to the least complex and with progressively more assumptions, the zero-inertia, kinematic wave and volume balance models. Walker and Skogerboe (1987) comprehensively summarised the methods of solution of these equations. In their complete form, the Saint-Venant equations have no known analytical solution and therefore require numerical methods.

Kinematic wave theory has become widely accepted as suitable for modelling irrigation of sloped, free-draining borders. The kinematic wave equations, like the full Saint-Venant equations, generally require numerical methods and extensive computation for solution.

Modelling border irrigation without resorting to digital computers offers many advantages in terms of simplicity, reliability, speed and cost. If a model is to find acceptance for real-time control of irrigation, it must be simple, accurate, robust and inexpensive. Consequently, attempts have been made to simulate border irrigation analytically. On steep borders with an infiltration function of simple form, irrigation advance alone can be predicted via the Lewis-Milne equation, by assuming that ponded depth is not a function of time. Philip and Farrell (1964) developed the general solution to the Lewis-Milne equation, but their model did not predict advance rate satisfactorily. Singh et al. (1990) produced an improved Lewis-Milne equation for the advance phase of border irrigation, verifying the work of Singh and Prasad (1983). Their solution does not, however, include the storage and recession phases of border irrigation.

Volume balance techniques offer an alternative method for analytical simulation of border irrigation. A major limitation of volume balance methods is that direct consideration of the effect of surface roughness is not possible, and some arbitrary water profile shape must be assumed. Hart et al. (1968) developed a volume balance model that could be solved graphically. Yu and Singh (1989) presented a more recent volume balance model that assumed parabolic surface and subsurface flow profiles on advance, and for recession used an iterative adaptation of the Strelkoff (1977) model. Their model gave good results on irrigation borders up to 100 m in length, but due to the assumptions of parabolic profile shape the model may be less accurate for longer border lengths; the advent of laser grading has meant that irrigation borders may exceed 1 km in length, limiting the usefulness of the model.

Cunge and Woolhiser (1975), Sherman and Singh (1978) and Turbak and Morel-Seytoux (1988) developed analytical kinematic wave models either by averaging total infiltration over the whole duration of irrigation to obtain a constant infiltration rate, or by assuming infiltration to be a constant. Depending on the true shape of the infiltration function, assuming an average infiltration is likely to result in significant error when predicting irrigation advance and recession. Weir (1983) presented exact, numerical and graphical kinematic models. His exact solution used a linear infiltration function with kinematic theory and applied volume balance to calculate advance. More recently, Mailhol (1992) presented a Laplace transform solution of the Lewis-Milne equation, using a linear infiltration function for furrow irrigation. An analytical solution of the kinematic wave equations, incorporating the linear infiltration function (Eq. (1)) is presented below.

Analytical model development from kinematic theory, using the linear infiltration function

The Saint-Venant equations describe the conservation of mass, momentum and/or energy for water flowing across a soil surface. In the derivation of the Saint-Venant equations, it is assumed that the flow of water is unsteady and spatially varied, bed slopes are relatively small and that the water is flowing along a prismatic channel. The Saint-Venant equations consist of a continuity equation (Eq. (2)) and a momentum equation (Eq. (3)), as

$$\frac{\partial y}{\partial t} + \frac{\partial q}{\partial x} + \frac{\partial Z}{\partial t} = 0 \tag{2}$$

and

$$S_{\mathrm{f}} = S_0 - \frac{\partial y}{\partial x} - \frac{v}{g}\frac{\partial v}{\partial x} - \frac{1}{g}\frac{\partial v}{\partial t} \tag{3}$$

where x and t are the space and time dimensions, respectively, q, y and v are the discharge rate per unit width, depth and velocity, respectively, g is the acceleration due to gravity, and S_{f} and S_0 are the friction and bed slopes, respectively. Kinematic solutions assume that dynamic terms in the momentum equation are negligible, meaning that bed slope and friction slope are equal. The momentum equation is then normalised by a steady uniform discharge, approximated by a uniform flow formula, the general form of which is

$$q = \alpha y^m \tag{4}$$

where α and m are empirically fitted parameters. If the Manning equation is used, then $\alpha = 1/n \cdot S_0{}^{1/2}$, and $m = 5/3$, where n is the Manning roughness parameter. The boundary conditions for the whole of the irrigation cycle may be summarised as

$$\begin{aligned} &y(x, 0) = 0 \\ &y(0, t) = y_0 \quad \text{for } t < t_{\mathrm{co}} \\ &y(0, t) = 0 \quad \text{for } t \geq t_{\mathrm{co}} . \end{aligned}$$

Advance

Substitution of the linear infiltration equation (Eq. (1)) and the uniform flow formula (Eq. (4)) into the continuity equation (Eq. (2)) yields

$$\frac{\partial y}{\partial t} + m\alpha y^{(m-1)}\frac{\partial y}{\partial x} = -i_{\mathrm{f}} \tag{5}$$

Because $\partial Z/\partial t$ is equal to a constant, i_{f}, it is possible to derive an analytical solution to Eq. (5). One such solution is obtained via the characteristic equations

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -i_{\mathrm{f}} \tag{6}$$

and

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-i_\mathrm{f}}{\alpha\, m\, y^{m-1}} \tag{7}$$

Integrating the characteristic equations gives

$$y = -i_\mathrm{f}\, t + F\,(\alpha\, y^m + i_\mathrm{f}\, x) \tag{8}$$

where F is a free function. At the advance, y is finite and continuity gives

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\alpha\, y^m}{(y + Z_\mathrm{CR})} \tag{9}$$

Integrating Eq. (7) subject to the second boundary condition above gives

$$y = \left(y_0^m - \frac{i_\mathrm{f}}{\alpha}\, x \right)^{\frac{1}{m}} \tag{10}$$

Substituting Eq. (10) into Eq. (9) and integrating, given the boundary condition that at $x=0$, $t=0$, yields an equation for the advancing front

$$t = \frac{m\, y_0}{i_\mathrm{f}} \left[1 - \left(1 - \frac{i_\mathrm{f}}{q_0}\, x \right)^{\frac{1}{m}} \right] - \frac{Z_\mathrm{CR}}{i_\mathrm{f}} \ln\left(1 - \frac{i_\mathrm{f}}{q_0}\, x \right) \tag{11}$$

Eq. (11) is essentially the same as that derived by Weir (1983) by applying volume balance between $t=0$ and $t=t$. The maximum distance water can advance on an infinitely long border is q_0/i_f m. For the case when $Z_\mathrm{CR}=0$, Eq. (11) is equivalent to those derived by Cunge and Woolhiser (1975), Sherman and Singh (1978) and Turbak and Morel-Seytoux (1988).

Depletion

At $t=t_\mathrm{co}$, the flow of water onto the bay is discontinued, and a kinematic shock of velocity, $v=\mathrm{d}q/\mathrm{d}y$, travels down the bay. From Eq. (4) and Eq. (10)

$$v = \alpha\, n \left(y_0^m - \frac{i_\mathrm{f}}{\alpha}\, x \right)^{\frac{m-1}{m}} = \frac{\mathrm{d}x}{\mathrm{d}t} \tag{12}$$

Solving Eq. (12), with the boundary condition that $x=0$ when $t=t_\mathrm{co}$, yields the equation for the depletion characteristic

$$t = t_\mathrm{co} + \frac{y_0}{i_\mathrm{f}} \left[1 - \left(1 - \frac{i_\mathrm{f}}{q_0}\, x \right)^{\frac{1}{m}} \right] \tag{13}$$

Equating Eq. (11) and Eq. (13) gives the distance down the irrigation bay at which the depletion characteristic meets the advance front, beyond which point Eq. (11) is no longer valid. No generalised analytical solution has yet been developed to describe advance in this region, although Weir (1985) developed some analytic approximations accurate to within 2%.

Recession

When the flow of water onto the bay is discontinued at $t=t_\mathrm{co}$, characteristics with $0 < y_\gamma < y_0$ originate from the upstream boundary. Then, from Eq. (8)

$$y_\gamma = -i_\mathrm{f}\, t_\mathrm{co} + F(\alpha\, y_\gamma^m) \tag{14}$$

The equation for recession is found by substitution, along the characteristic on which $y_\gamma = 0$, yielding

$$t = t_\mathrm{co} + \frac{y_0}{i_\mathrm{f}} \left(\frac{i_\mathrm{f}}{q_0}\, x \right)^{\frac{1}{m}} \tag{15}$$

Equation (15) is essentially the same as that derived by Cunge and Woolhiser (1975) for the case where infiltration rate was constant, since the crack-fill component, Z_CR, of Eq. (1) is satisfied during advance. The intake opportunity time at any point down the border is the difference between advance, (Eq. (11)), and recession (Eq. (15)). Infiltrated depth is determined by substituting the opportunity time at that point into the infiltration equation (Eq. (1)). Infiltrated volume per unit width, z, is obtained by integrating between Eq. (15) and Eq. (11), and substituting in Eq. (1) to obtain

$$\begin{aligned} Z_\mathrm{CR}\, x + i_\mathrm{f} \Biggl(t_\mathrm{co}\, x + \frac{m\, y_0}{i_\mathrm{f}} \Biggl\{ & \frac{q_0}{i_\mathrm{f}\,(m+1)} \Biggl(\left(\frac{i_\mathrm{f}}{q_0}\, x \right)^{\frac{m+1}{m}} \\ & - m \left[\left(1 - \frac{i_\mathrm{f}}{q_0}\, x \right)^{\frac{m+1}{m}} - 1 \right] \Biggr) - x \Biggr\} \\ & - \frac{q_0\, Z_\mathrm{CR}}{i_\mathrm{f}^2} \left[\left(1 - \frac{i_\mathrm{f}}{q_0}\, x \right) \cdot \left(\ln\left(1 - \frac{i_\mathrm{f}}{q_0}\, x \right) - 1 \right) + 1 \right] \Biggr) \end{aligned} \tag{16}$$

Analytical irrigation model testing

Maheshwari and McMahon (1993) tested six irrigation models against data collected from 67 border irrigation events at five locations in south-eastern Australia: Mount Derrimut, Mitchells, Kerang, Shepparton and Griffith. They found the Walker model (W. R. Walker and F. Gichuki, unpublished report) best for predicting advance and the Strelkoff model (Strelkoff 1985) best for predicting recession. Therefore, the Walker and Strelkoff models have been chosen to assess the performance of the analytical irrigation model presented above (i.e. Eq. (11) and Eq. (15) for predicting advance and recession, respectively). A more recent version of the Strelkoff model (Strelkoff 1990) was used in the analyses.

Model data

Field data for model comparison presented by Maheshwari and McMahon (1993) are given in Table 4. Where a range of data was given, the average of maximum and minimum values has been used in the model tests.

Maheshwari and McMahon (1993) described infiltration by the modified Kostiakov equation at all five experimental locations. Since a linear function is required for the analytical irrigation model, linear functions were fitted to infiltration data generated using the modified Kostiakov equation parameters of Maheshwari and McMahon (1993), using Genstat 5, Release 3.1 (Lawes Agricultural Trust, Rothamsted Experimental Station). The modified Kostiakov equation parameters and the fitted linear infiltration function (Eq. (1)) parameters are presented in Table 5.

Advance results and discussion

Irrigation advance, as predicted analytically and by the Strelkoff and Walker models for the data of Maheshwari and McMahon (1993), appears in Fig. 3. Also shown in Fig. 3 is the depletion characteristic of the analytical solution (Eq. (13)). The Strelkoff and Walker models were both run in kinematic wave (KW), zero inertia (ZI) and hydrodynamic (HD) modes.

The Strelkoff model did not provide a solution for one KW and three HD simulations. Otherwise, all models predicted similar advance trajectories. For the Walker model, advance in HD mode was identical to that in ZI mode, but somewhat slower in KW mode for all five sites.

Although Eq. (11) holds only until the depletion characteristic reaches the advancing front, in the first instance Eq. (11) was considered to hold until surface water had either infiltrated or run off. This assumption did not affect simulations for Mount Derrimut, Kerang or Griffith, since depletion did not reach the advancing front before the end of the bay. For Mitchells and Shepparton, where the depletion characteristic intersected the advance, the above assumption overestimated the advance rate for advance beyond the point of intersection, and the unrealistic discontinuity at the point of maximum advance. Theoretically, the assumption should also have overestimated cumulative infiltration and, therefore, underestimated the maximum advance distance. However, for Mitchells and Shepparton, the point of maximum advance predicted by the analytical model was greater than that predicted by the other models. The situation appears to result from the faster recession predicted by the analytical model (discussed below) contributing to a smaller overall infiltration opportunity time. The analytic approximations of Weir (1985) for this region would tend to exacerbate the difference between the predictions of the analytical and other models.

Recession results and discussion

Irrigation recession for the data of Maheshwari and McMahon (1993) as predicted analytically and by the Strelkoff and Walker models appears in Fig. 3. While advance trajectories were similar for all models, recession trajectories varied between models and between modes of solution. Recession in analytical and Walker KW solutions began, somewhat unrealistically but in accordance with kinematic theory, at cut-off; Strelkoff KW recession began closer to the ZI and HD solutions. As with advance, the Strelkoff model did not provide a solution for one KW and three HD simulations. For the Strelkoff KW simulations that did provide a solution, recession was typically instantaneous, indicating that the Strelkoff model did not perform satisfactorily in KW mode.

Table 4 Field data used in model comparison (after Maheshwari and McMahon 1993)

Site	$L \times W$ (m) × (m)	Slope ($m \cdot m^{-1}$)	q_0 ($L \cdot s^{-1} \cdot m^{-1}$)	t_{co} (min)	Manning's n
Mount Derrimut	50 × 4.5	0.0011–0.0021	1.92–3.30	25–50	0.15–0.38
Mitchells	400 × 40	0.0005–0.0013	1.77–2.11	202–333	0.25–0.47
Kerang	240 × 30	0.0006–0.0012	1.32–3.80	88–275	0.15–0.51
Shepparton	360 × 60	0.0019–0.0027	1.78–1.95	162–226	0.33–0.50
Griffith	85 × 10	0.0002–0.0009	0.61–1.11	117–268	0.04–0.21

Table 5 Infiltration data used in model comparison

Site	Average modified Kostiakov parameters (Maheshwari and McMahon 1993)			Fitted linear parameters	
	k ($mm \cdot s^{-a}$)	a	f_0 ($mm \cdot min^{-1}$)	Z_{CR} (mm)	i_f ($mm \cdot h^{-1}$)
Mount Derrimut	32.7	0.02	0.02	38.0	1.6
Mitchells	34.8	0.04	0.09	47.0	6.5
Kerang	54.6	0.02	0.01	63.4	1.3
Shepparton	10.6	0.18	0.05	40.8	7.6
Griffith	55.8	0.01	0.01	60.1	0.9

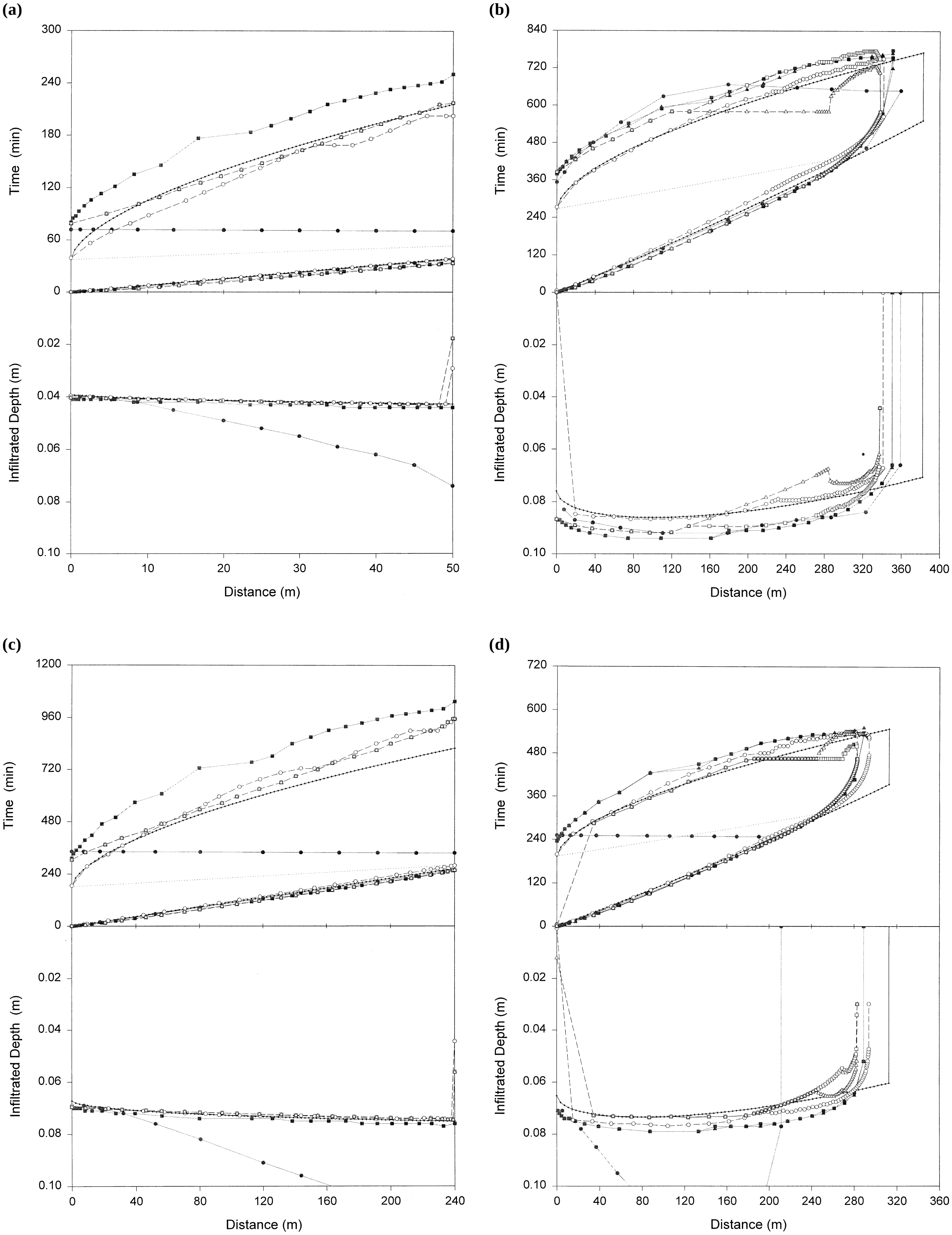

Fig. 3 Legend next page

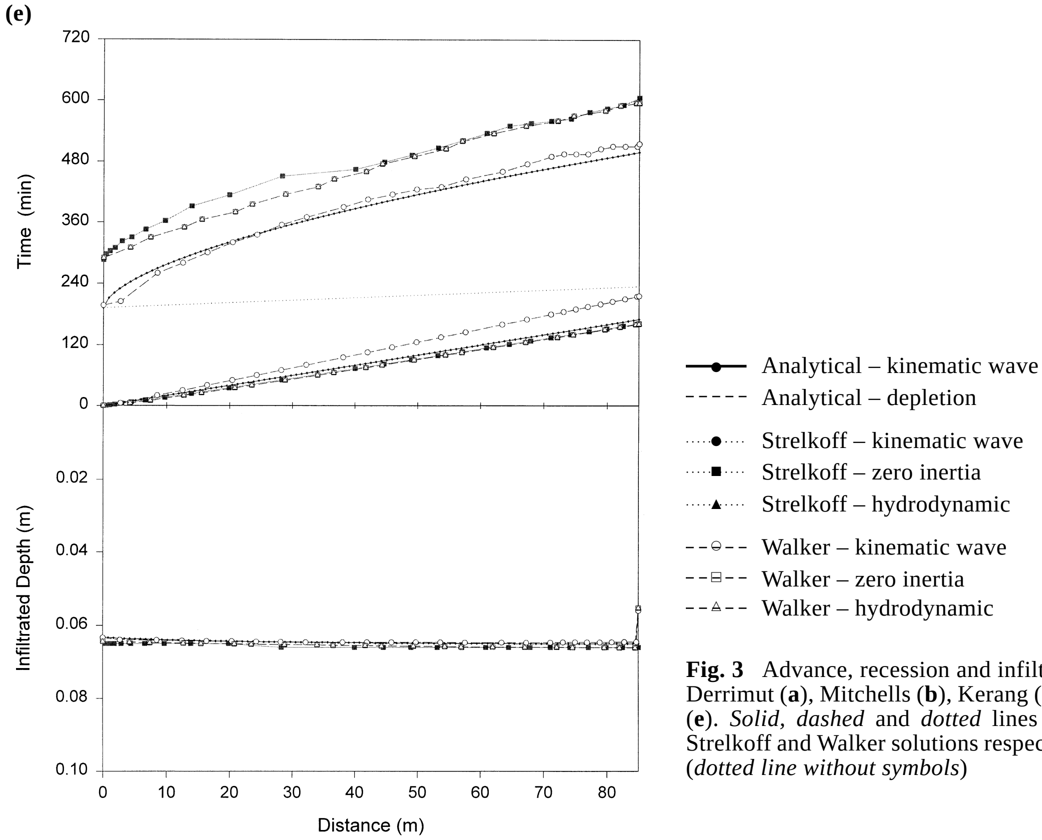

Fig. 3 Advance, recession and infiltration for irrigations at Mount Derrimut (**a**), Mitchells (**b**), Kerang (**c**), Shepparton (**d**) and Griffith (**e**). *Solid, dashed* and *dotted* lines correspond to the analytical, Strelkoff and Walker solutions respectively; depletion is also shown (*dotted line without symbols*)

The Walker model also produced instances of instantaneous recession over some distance, but unlike the Strelkoff model, these occurred in HD and ZI modes. The Walker HD and ZI predictions of recession were typically identical, except where instantaneous recession caused the two to diverge. Recession for the analytical model was usually closest to that of the Walker model in KW mode.

Recession was slowest in the Strelkoff ZI model in all simulations, and usually fastest in the analytical model. Although Maheshwari and McMahon (1993) found the Strelkoff model best for predicting recession, the model "did not show recession times accurately" (T. Strelkoff, personal communication). The true recession time in the Strelkoff model should actually be later than that shown, since static or vertical recession depth (the depth remaining at a point when flow velocities at a point drop virtually to zero) was considered in the model solution to infiltrate instantaneously. The analytical solution does not consider a static or vertical recession component (i.e. $y_\gamma=0$ to obtain Eq. (15)), and consequently predicts recession sooner than the other models. A static or vertical recession parameter (either as a function of bay length or as a constant over the length of the bay) could be simply included in Eq. (15), should field testing indicate that it is warranted. By introducing such a parameter, the cumulative infiltration opportunity time would increase, and the overestimation of advance for Mitchells and Shepparton would be corrected.

Predicted infiltration results and discussion

Infiltrated depths for the data of Maheshwari and McMahon (1993) as predicted analytically and by the Strelkoff and Walker models appear in Fig. 3. Despite the differences in predicted opportunity times between the models and modes of solution, predictions of infiltration were similar. This is partly because relatively low final infiltration rates of the soils being considered (0.9 – 7.6 mm/h) make infiltration fairly insensitive to variations in opportunity time. The assumption that upstream head drops immediately to 0 to t_{co} means that recession in KW simulations is faster than in other modes of solution, resulting in reduced infiltration opportunity time, and consequently reduced infiltration. The empirical parameter to account for vertical recession, discussed above, would improve the situation.

Advance, run-off and volume balance

A comparison of either irrigation run-off as a percentage of volume applied, or advance distance as a percentage of border length if run-off did not occur (Table 6), provides the most comprehensive assessment of model performance, since it takes into account errors in calculating volume applied, and in predicting both advance and recession. The Walker model underestimated the volume applied (irrigation discharge by duration) in all simulations and by as much as 5%. It was also subject to errors in predicting

Table 6 Comparison of either irrigation run-off as a percentage of applied volume, or advance distance as a percentage of border length if run-off did not occur, for the analytical, Strelkoff and Walker irrigation models. No run-off occurred at Mitchells and Shepparton (*n. r.* the model did not provide a result)

Site	Analytical model (%)	Strelkoff model			Walker model		
		KW	ZI	HD	KW	ZI	HD
Mount Derrimut	65	54	64	n. r.	68	64	64
Mitchells	96	90	88	88	85	85	85
Kerang	37	19	36	n. r.	32	37	37
Shepparton	87	59	80	80	82	79	79
Griffith	45	n. r.	44	n. r.	20	44	44

infiltrated volume (in some instances infiltrated volumes were many times larger than the volume applied), despite providing reasonable prediction of infiltrated depth and percentage run-off.

For the simulations with run-off (Mount Derrimut, Kerang and Griffith), the analytical model predicted run-off volumes within 2% of those predicted by the Walker model in ZI and HD modes, the Strelkoff model in ZI mode and, for the simulations that provided a solution, the Strelkoff model in HD mode (Table 6). For the simulations without run-off (Mitchells and Shepparton), the analytical model did not perform as well, predicting advance distances within 10% of those predicted by the Walker and Strelkoff models in ZI and HD modes (Table 6).

Overall, the analytical model appears to be better suited to heavier (lower final infiltration rate) soils. As already discussed, improved analytical model prediction would be achieved by incorporating an empirical parameter to delay recession. The effect of this would be greatest on the lighter soils (Mitchells and Shepparton). Field experimentation is required to determine suitable parameter values.

Conclusions

The linear infiltration function (Eq. (1)) is suitable for describing infiltration into soils that exhibit shrinkage and cracking upon drying. Equation (1) has two main advantages over the more usual infiltration functions.

The first advantage is that the two parameters, i_f and Z_{CR}, have physical interpretation. This allows their estimation in the field, without the need to perform infiltration tests. The final infiltration rate, i_f, relates to the soil particle size distribution, with an inverse relationship between clay content and i_f. The extent of spatial and temporal variation in i_f requires further field investigation: standard ring infiltrometer techniques should be adequate to determine this variability. The crack-fill component, Z_{CR}, exhibits an inverse relationship with antecedent water content, which in turn exhibits a direct relationship with cumulative evaporation less rainfall ($E-R$) since the previous irrigation. Further field investigation is also required to determine the exact form of the '$(E-R)-Z_{CR}$' relationships for various soil types. Investigations of both i_f and Z_{CR} form the basis of a subsequent paper.

The second advantage that the linear function possesses over most other infiltration functions is that the rate of change of infiltration over time, $\partial Z/\partial t$, is a constant. This attribute enables derivation of an analytical solution to the kinematic wave equations without the need for further simplifying assumptions, as shown above.

The analytical model, incorporating the linear infiltration function, performed sufficiently well in the preliminary tests described above to warrant further investigation. Field testing of the model, including sensitivity analysis of the input parameters, forms part of a subsequent study. The fact that the analytical solution does not require a computer, so is not subject to numerical instabilities, means that it has the potential for incorporation in electronic irrigation timing devices and, ultimately, in whole-farm irrigation automation systems.

The irrigation data from south-eastern Australia used in the model comparisons covered a wide range of border lengths (50 – 400 m), widths (4.5 – 60 m) and slopes (0.055 –0.23%), irrigation discharges (8.6 – 111.9 l/s) and durations (37.5 – 267.5 min) and surface roughness (Manning's *n*, 0.14 – 0.41). Soil types and infiltration characteristics also varied considerably between sites: crack-fill, 38.0 –63.4 mm; final rate, 0.9 – 6.5 mm/h. As the majority of surface irrigation in Australia and throughout the world is performed on soils that exhibit crack development on drying, the applicability of such an analytical irrigation model is wide ranging.

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