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PREDICTING WETTING FRONT ADVANCE IN SOILS USING SIMPLE LABORATORY DERIVED HYDRAULIC PARAMETERS

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Abstract—An approximate simulation of wetting front advance or leaching time is often the necessary and sufficient information required to optimize irrigation strategies or wastewater land treatment systems. For such simulations, we show that only three physically comprehensible and easily measured parameters are necessary, namely the saturated hydraulic conductivity, and the slope and the intercept of the soil–water potential plotted against the natural logarithm to the soil–water content. We derived the new 3-parameter simulation model from a fully linear flow model based on the Richards equation with the additional assumption of zero conductivity at zero soil–water content. A simple analytical solution for water infiltration during saturation at the soil surface, based on the 3-parameter model, gave results that compared well with measured wetting front advances in three Danish loamy and coarse sands and, also, with semi-analytical and numerical solutions from literature. Less agreement between predicted and measured wetting front advance was observed during infiltration into an initially dry forest soil due to extreme soil–water repellency. The presented derivation procedure assuming linear water flow seems promising for obtaining simple, approximate analytical solutions for unsaturated flow problems. Copyright © 1996 Elsevier Science Ltd

INTRODUCTION

Rapid, approximate prediction of the wetting front advance and, thus, the time for the wetting front to reach a certain depth is often needed in cases of water or wastewater infiltration in soils. Yamaguchi *et al.* (1990, 1994, 1995) showed the wastewater application rate and the subsequent leaching time to be critical in obtaining optimal nitrification/denitrification in rapid infiltration land treatment systems for renovation of municipal wastewater. During crop irrigation with reclaimed municipal wastewater, an estimate of the time for the wetting front to reach the lower part of the plant root zone is important when deciding the irrigation schedule (Asano, 1985; Smith *et al.*, 1985). Prediction of the mean residence time for the water in the unsaturated zone below a sanitary or chemical landfill coupled with simple models for the attenuation of leachate pollutants to the soil–aquifer material makes it possible to estimate the degree of protection afforded to adjacent groundwater reservoirs (Larsen *et al.*, 1992; Robinson, 1992). In the cases mentioned, data for the unsaturated hydraulic conductivity are typically not available.

Recently, we have presented numerically simple but accurate simulation models for transport processes in soil systems (Moldrup *et al.*, 1989, 1992, 1993). But, the numerical simplicity has not reduced the comprehensive hydraulic parameter requirements for this kind of models. However, only an approximate prediction of soil–water movement and leaching time is needed in many water and sewage infiltration studies. In these cases, the time consuming measurement of the unsaturated hydraulic conductivity as function of the soil–water content is not justified. Several studies have proposed models for predicting the unsaturated hydraulic conductivity from the soil–water characteristic (retention) curve, but these models contain parameters such as pore tortuosity and pore connection terms which are difficult to assess quantitatively (van Genuchten and Nielsen, 1985; Nielsen *et al.*, 1986).

In this study, we take a different approach with the objective to obtain a model with minimum parameter requirement for approximate prediction of wetting front movement. We adopt a fully linear flow model (Philip, 1973; Warrick, 1975; Govindaraju *et al.*, 1992) and show that with an additional assumption, only three physically comprehensible and easily measured parameters are needed

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to obtain an approximate unsaturated flow calculation. We test the 3-parameter model against measured infiltration data and semi-analytical and numerical solutions.

THE THREE PARAMETER, LINEAR FLOW MODEL

The usual way to model the movement of water in unsaturated soil is by numerical solution of the non-linear flow equation by Richards (1931). The Richards equation for one-dimensional, unsaturated water flow in homogeneous soils is

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left[K(\theta)\frac{\partial(\Psi-z)}{\partial z}\right] \qquad (1)$$

where θ is the soil–water content (cm^3 cm^{-3}), Ψ is the soil–water potential (cm), K(θ) is the unsaturated hydraulic conductivity (cm min^{-1}) at a soil–water content equal to θ, t is the time (min), and z is the soil depth (positive downward). Assuming

$$\frac{K}{K_s}=e^{\alpha\Psi}\text{ and }\frac{\partial(\frac{K}{K_s})}{\partial\theta}=\beta \qquad (2)$$

the linear form of equation (1) becomes (e.g. Govindaraju *et al.*, 1992)

$$\frac{\partial K}{\partial t}=D\frac{\partial^2 K}{\partial z^2}-V\frac{\partial K}{\partial z} \qquad (3)$$
$$D=\frac{\beta K_s}{\alpha}$$
$$V=\beta K_s$$

where K_s is the saturated hydraulic conductivity (cm min^{-1}), β (assumed constant) is the slope of the relative hydraulic conductivity (K/K_s) vs the soil–water content (θ), and α (cm^{-1}, assumed constant) is the slope of the unsaturated hydraulic conductivity (K) vs the soil–water potential (Ψ) in a ln-normal coordinate system.

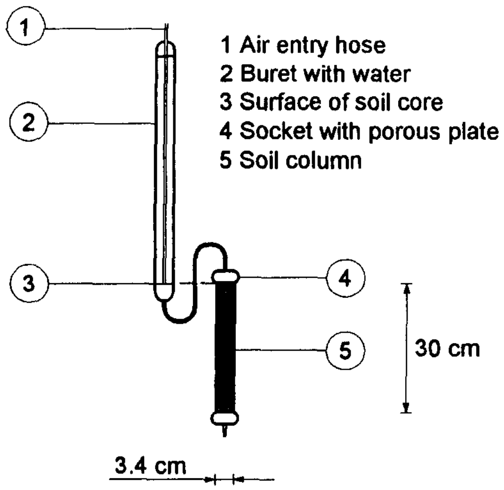

Fig. 1. Illustration in principle of laboratory set-up for vertical infiltration experiments.

Equation (3) is mathematically identical to the convection–dispersion equation (CDE) for solute transport in streams or soils. Hence, the numerous available analytical solutions to the CDE (e.g. van Genuchten and Alves, 1982) can be used for unsaturated flow predictions if equation (2) are valid. For soils where the assumptions of linear relationships between K/K_s and θ, and between K (ln scale) and Ψ are reasonably valid, or in case of large uncertainty in the hydraulic parameters (large soil heterogeneity), equation (3) seems a convenient choice for obtaining a rapid, approximate flow prediction.

If we furthermore assume zero conductivity at zero soil–water content, i.e. $K(\theta=0)=0$, equation (2) yield

$$\frac{K}{K_s}=e^{\alpha\Psi}\text{ and }\frac{K}{K_s}=\beta\theta \qquad (4)$$

Combining equation (4), we obtain

$$\Psi=C_1+C_2\ln\theta$$
$$C_1=\frac{1}{\alpha}\ln\beta \qquad (5)$$
$$C_2=\frac{1}{\alpha}$$

where the constants C_1 and C_2 are the intercept and the slope, respectively, of the soil–water potential plotted against the natural logarithm to the soil-water content. Inserting equation (5) in equation (3) gives

$$\frac{\partial K}{\partial t}=K_sC_2\exp\left(\frac{C_1}{C_2}\right)\frac{\partial^2 K}{\partial z^2}-K_s\exp\left(\frac{C_1}{C_2}\right)\frac{\partial K}{\partial z} \qquad (6)$$

Thus, only 3 parameters, namely C_1, C_2 and K_s, need to be known *a priori* to carry out a calculation based on the linear flow model [equation (3)] under the additional assumption of $K(\theta=0)$ equal to zero.

We note that a mathematically analogous parameter to the solute dispersivity (ratio of hydrodynamic dispersion coefficient to pore-water velocity; Nielsen *et al.*, 1985) in the convection-dispersion equation for solute transport can now be defined for water transport in unsaturated soils. From equations (3) and (5) we obtain for the 3-parameter flow model

$$\lambda_w=\frac{D}{V}=C_2 \qquad (7)$$

where λ_w is labelled the soil–water dispersivity (cm) and is equal to the slope of the Ψ-ln(θ) curve.

EXAMPLE OF MODEL USE: CONSTANT HEAD INFILTRATION INTO FOUR LOAMY AND SANDY SOILS

Use of the 3-parameter model is illustrated on data from constant head (water saturation at the soil surface), vertical infiltration experiments on four different soil types. The four soils used were from three research fields belonging to the Danish Ministry

Table 1. Physical characteristics of the soils used, including soil–water repellency given as Water Drop Penetration Time (WDPT), bulk density of packed soil (ρ_b), initial (θ_0) and final (θ_s) soil–water contents during the infiltrations experiments, and the saturated hydraulic conductivity (K_s) of the packed soil columns

	Clay/silt/sand (%)	Organic matter (%)	WDPT (s)	ρ_b (g/cm^3)	θ_0 (cm^3)	θ_s (cm^3)	K_s (cm/min)
Jyndevad	4/7/89	2.3	<5	1.5	0.08	0.42	0.30
Foulum	8/25/67	2.5	Not measured	1.5	0.10	0.41	0.082
Hornum	6/21/73	3.1	1250 ± 400	1.4	0.06	0.41	0.045
Poulstrup	4/13/83	4.1	>3600	1.2	0.02	0.50	
Exp. 1							0.025
Exp. 2							0.028
Yolo light clay[a]	31/45/24	1.1	Not measured	1.3	0.24	0.50	0.000738

[a] Data from Moore (1939).

of Agriculture (Jyndevad, Foulum, and Hornum) sampled at 0–15 cm soil depth, and one mixed hardwood forest soil (Poulstrup) sampled at 10–20 cm soil depth. All soil samples were stored in the dark at 2°C until the analyses were made. Before the infiltration experiments, the soil was air dried, passed through a 2-mm sieve, and mixed thoroughly to obtain a homogeneous mixture. The soil–water repellency for the Jyndevad, Hornum and Poulstrup soils was measured as Water Drop Penetration Time (WDPT) according to the procedure of Letey (1969) and Wessel (1988) but using well-defined water drops of approx. 0.05 ml each and 10 repetitions for each measurement. Soil texture data were measured according to Klute (1986). The infiltration apparatus presented by Moldrup *et al.* (1994a) but modified to constant head infiltration was used for the infiltration experiments, see Fig. 1. The soil was packed to constant bulk density in plexiglass columns (4.0-cm o.d., 3.4-cm i.d., 20-cm length). The physical characteristics of the four soils together with the bulk densities of the packed soils and the initial and final water contents (θ_0 and θ_s) during the infiltration experiments are given in Table 1. As the shape of the curve describing measured wetting front advance as function of time for the forest soil (Poulstrup) was very different (linear) compared to the three agricultural soils, a second infiltration experiment was carried out for the Poulstrup soil to confirm the phenomenon (Table 1).

The soil–water characteristic curves between $0 > \Psi > -200$ cm H_2O measured on soil from the same sampling sites and at the same bulk densities as used in the infiltration experiments were taken from Jacobsen (1989, Jyndevad, Foulum, and Hornum) and Kruse *et al.* (1995, Poulstrup). Based on the soil–water characteristic curves it was assumed that data points below -200 cm H_2O would correspond to fairly immobile water as a sharp drop in the soil–water retention curves was observed. For the Jyndevad, Foulum and Hornum soils, approximately the same values of saturated hydraulic conductivity were obtained in the present study and in Jacobsen (1989), e.g. 0.045 vs 0.044 cm min^{-1} for the Hornum soil. Estimation of the parameters C_1 and C_2 from the

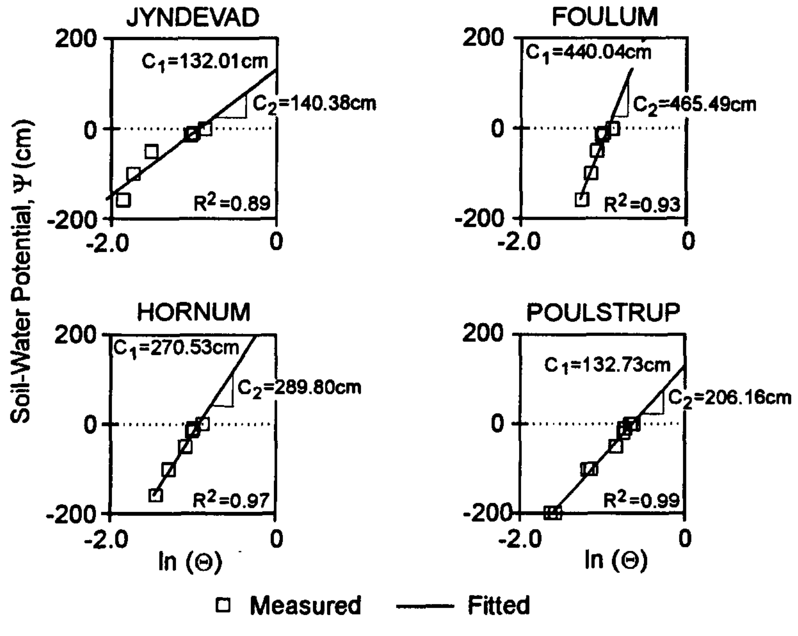

Fig. 2. Soil–water characteristic curves for the four Danish, loamy and sandy soils fitted to $\Psi = C_1 + C_2 \ln \theta$.

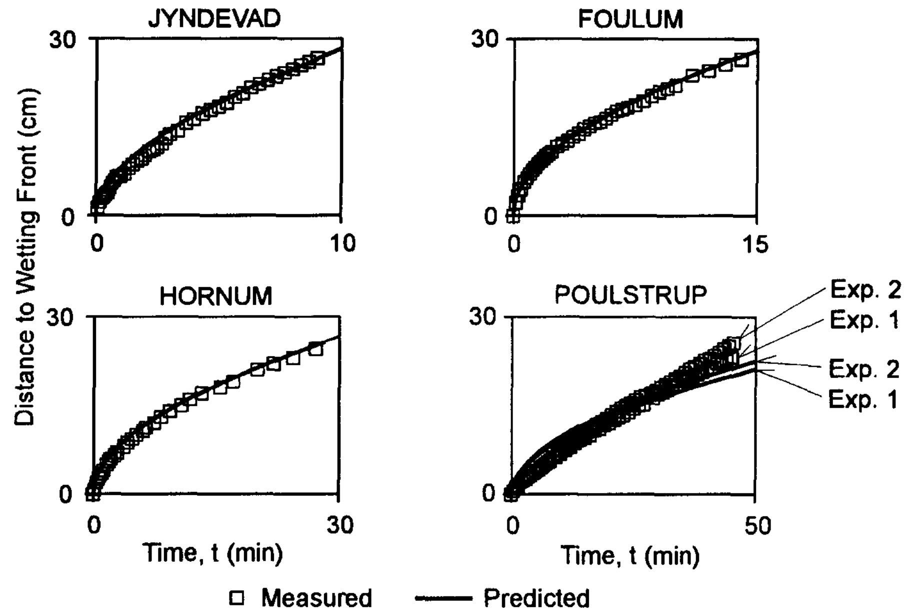

Fig. 3. Comparison between predicted [3-parameter flow model, equations (9) and (10)] and measured wetting front advance during constant head infiltration into four, initially dry loamy and sandy soils.

soil–water characteristic curves together with the corresponding regression coefficients are shown in Fig. 2. In general, the assumed model for the soil–water characteristic curve [equation (5)] gave a good description of the data (R_2 between 0.89 and 0.99, Fig. 2).

We assume that $K(\theta_0) = 0$ because of the initially low water content. For the initial and boundary conditions of the infiltration experiment

$$\begin{array}{lll} K = 0 & z \geqslant 0 & t = 0 \\ K = K_s & z = 0 & t > 0 \\ K = 0 & z \to \infty & t \geqslant 0 \end{array} \quad (8)$$

and recalling that $\theta = (1/\beta)\ (K/K_s)$ cf. equation (4), the analytical solution of Lapidus and Amundson [1952; also cf. equation (3) of Yamaguchi *et al.* (1989)] to the CDE can be slightly rephrased to give the analytical solution to equation (6) in form of the soil–water content as function of depth and time

$$\frac{\theta}{\theta_s}(z,t) = \frac{1}{2}\operatorname{erfc}\left(\frac{z - Vt}{\sqrt{4Dt}}\right) + \frac{1}{2}\exp\left(\frac{Vz}{D}\right)\operatorname{erfc}\left(\frac{z + Vt}{\sqrt{4Dt}}\right) \quad (9)$$

$$D = C_2 \exp\left(\frac{C_1}{C_2}\right)K_s$$

$$V = \exp\left(\frac{C_1}{C_2}\right)K_s$$

where erfc is the complementary error function. The erfc terms can easily be calculated using the accurate series approximations of van Genuchten (1985).

We define the position of the wetting front to correspond to the soil–water content where

$$\frac{\theta - \theta_0}{\theta_s - \theta_0} = \frac{1}{2} \quad (10)$$

Figure 3 shows the predicted [equations (9) and (10)] and measured wetting front advance in the four soils. It is seen that the measured and predicted curves compare well. However, in disagreement with the shape of the predicted curve for infiltration into the Poulstrup soil, an almost linear relationship between the measured distance to the wetting front and time was observed (Fig. 3). This is probably due to the extreme water repellency of the Poulstrup soil (WDPT > 3600 sec, Table 1).

A linear relationship between accumulated infiltration and time and, also, a reduction in the rate of water infiltration can be expected for extreme water repellent soils (WDPT > 3600 s) due to the hydrophobic properties exhibited when the soils are dry (Wallis *et al.*, 1990). Both phenomena are evident for the Poulstrup soil, Fig. 3, as the infiltration curve is more linear and it takes more time for the same amount of water to infiltrate into the Poulstrup soil compared to the other three coarse-textured soils. As the Richards equation (and therefore the 3-parameter model) does not take into account hydrophobic interactions, this explains the deviations between the shapes of the predicted and measured curves for the Poulstrup soil.

Normally, forest soils are more water repellent compared to agricultural soils because of the amount, composition and nature of the organic matter of the soils (Wallis and Horne, 1992). This agrees with the WDPT measurements in Table 1. For the three less organic and less water repellent agricultural soils (Jyndevad, Hornum, and presumingly also Foulum) the agreement between measured and predicted wetting front advance is excellent. Thus, the simple analytical solution to the 3-parameter flow model [equations (9) and (10)] seems to provide a rapid and fairly accurate prediction of wetting front advance in coarse-textured soils, these soil types being typical of most infiltration and wastewater land treatment systems (Asano, 1985).

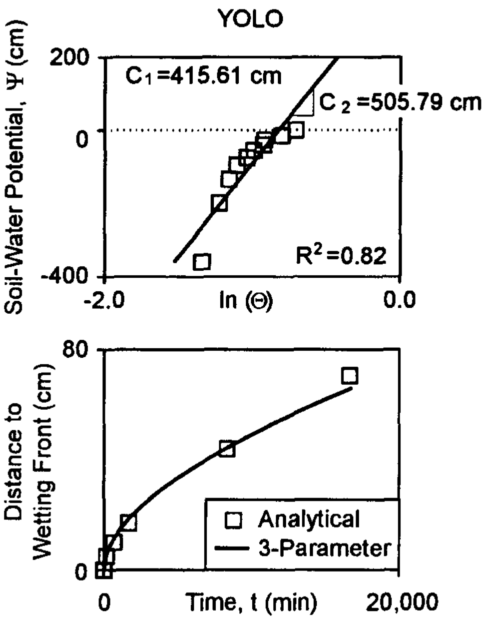

Fig. 4. Soil–water characteristic curve for Yolo light clay fitted to $\Psi = C_1 + C_2 \ln \theta$, and comparison between the 3-parameter flow model and the semi-analytical solution of Philip (1957) for predicting wetting front advance into Yolo light clay.

TEST AGAINST SEMI-ANALYTICAL SOLUTION AND NUMERICAL MODEL FOR INFILTRATION INTO A CLAYEY SOIL

This test is for the classical example of infiltration into Yolo light clay (e.g. Moore, 1939; Philip, 1957; Haverkamp *et al.*, 1977; Moldrup *et al.*, 1989), given the initial and boundary conditions of equation (8). The soil texture data (Table 1) and the soil–water characteristic curve were taken from Moore (1939). Soil–water characteristic data for $0 > \Psi > -350$ cm H_2O were used. As no sharp drop in the soil–water characteristic curve was observed for the Yolo soil, we chose -350 cm as minimum value of Ψ corresponding to the definition of mobile soil–water suggested by Addiscott and Whitmore (1991). Fitting equation (5) to the soil–water characteristic curve to obtain the parameters C_1 and C_2, and the comparison between the 3-parameter model [equations (9) and (10)] and the semi-analytical solution to equation (1) by Philip (1957) for predicting wetting front advance is shown in Fig. 4. A good agreement between the 3-parameter model and the semi-analytical solution is seen (Fig. 4). The semi-analytical solution is a Taylor series containing integrals that are mathematically involved to solve and thus the simple 3-parameter solution [equation (9)] provides a useful alternative.

The soil–water characteristic curve of a clayey soil is typically more non-linear in the high soil–water content interval compared to a sandy soil (Haverkamp *et al.*, 1977). This is the main reason for the lower regression obtained when fitting equation (5) to the Yolo soil–water characteristic curve ($R^2 = 0.82$, Fig. 4) compared to the four loamy and sandy soils ($R^2 = 0.89$–0.99, Fig. 2). We note that omitting the data point at $\Psi = -350$ cm H_2O for the Yolo soil would not change the C_1, C_2 or R^2 values significantly, i.e. including this additional data point compared to the four sandy soils does not contribute the lower R^2 value. In spite of the low R^2 value, the 3-parameter model prediction of the wetting front advance is still sufficient for most purposes (Fig. 4).

To evaluate not only the wetting front advance but also the soil–water content profiles obtained by the 3-parameter model, we tested the 3-parameter model against a numerical solution to the Richard's equation [equation (1)] in the above case of infiltration into Yolo light clay. The numerical comparison solution was the accurate Moving Mean Slope (MMS) model of Moldrup *et al.* (1989, 1993) using the expressions for the measured hydraulic curves (soil–water characteristic and unsaturated hydraulic conductivity curves) given in Haverkamp *et al.* (1977). The soil–water content profiles at two different times during infiltration into Yolo light clay predicted by the 3-parameter and the MMS models, respectively, are shown in Fig. 5.

The prediction of the mean position of the wetting front (defined as the intercept with the dotted line in Fig. 5) by the 3-parameter model is fairly accurate. However, it is obvious that the assumption of linearity in the hydraulic curves [equations (4) and (5)] inherent in the 3-parameter model results in an approximately linear soil–water content profile, i.e. causes an erroneously large spreading of the computed soil–water content curve (Fig. 5). This is in agreement with the analyses of the linearized flow equation [equation (3)] by Philip (1973). Thus, the 3-parameter flow model should be used only for prediction of wetting front advance and mean leaching time but not for predicting the actual soil–water dynamics within the soil such as the spreading of the soil–water content or soil–water potential curves.

To show the sensitivity of the 3-parameter model to the parameters D and V, calculations of wetting front advance in Yolo light clay using different combinations of D and V values are shown in Fig. 6. Using the same value of D as in Fig. 4 but with V values ranging between 0.01 and 2.0 times the value used in Fig. 4, only small differences in the predicted

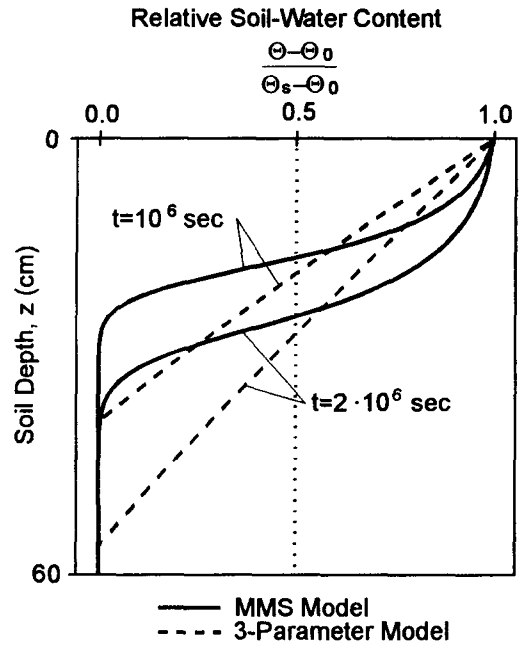

Fig. 5. Comparison between the 3-parameter flow model and the numerical Moving Mean Slope (MMS) model for predicting soil–water content profiles during infiltration into Yolo light clay.

wetting front advance are seen. However, large differences occur when varying the D values in a similar way (Fig. 6). This is due to the relationship $D = C_2 \cdot V$ [cf. equation (7)], i.e. the pronounced influence of the slope (C_2) of the Ψ versus $\ln(\theta)$ curve on the prediction by the 3-parameter flow model is obvious.

The large model sensitivity to the C_2 value [equal to the soil–water dispersivity, λ_w, cf. equation (7)] makes it important to accurately estimate the slope of the soil–water characteristic curve within the soil–water potential interval considered, i.e. to have sufficient measurement points within the high water-content range. Subsequently, the higher the R^2 value obtained when fitting equation (5) to the soil–water characteristic curve, the better the assumptions of the 3-parameter model are fulfilled, and the more precise the prediction of the wetting front as function of time.

CONCLUSIONS

As equation (9) illustrates, many well-known analytical solutions to the convection–dispersion equation for solute transport in streams or soils can easily be rephrased to give the wetting front movement. However, for variable boundary conditions the new, fully linearized flow model [equation (6)] needs to be solved by conventional numerical methods [with the exception of input expressed as step functions cf. Warrick (1975)]. Correction terms to remove numerical dispersion and criteria for avoiding numerical instability when using an ordinary finite-difference scheme to solve this type of linear transport equation were given by Moldrup *et al.* (1994b).

Govindaraju *et al.* (1992) found that solving the linear flow model [equation (3)] compared to solving the Richards equation typically gave deviations in predicted, unsaturated hydraulic conductivity of 0–20% for constant head and constant flux infiltration and of 0–50% for redistribution (drainage). Hence, we suggest the linear model concept be used only for water and sewage infiltration and not for drainage calculations. In case of sewage infiltration, the sometimes high sodium content and the organic particles can cause deterioration of soil structure and clogging of soil pores, and thereby reduced K_s and infiltrability with time (Asano, 1991). Thus, K_s and soil–water characteristic measurements from the time a soil-aquifer treatment plant was established is not necessarily representative for the actual situation.

The 3-parameter model seems able to give a simple, approximate estimate of wetting front advance in unsaturated soils for which only $\Psi(\theta)$ and K_s need be known *a priori*. Although the mean position of the wetting front is fairly accurately predicted (e.g. Fig. 3), the strict assumptions [equations (4) and (5)] inherent in the 3-parameter flow model will typically cause erroneously large spreading of the computed soil–water content profiles. Therefore, the 3-parameter model should be used only for prediction

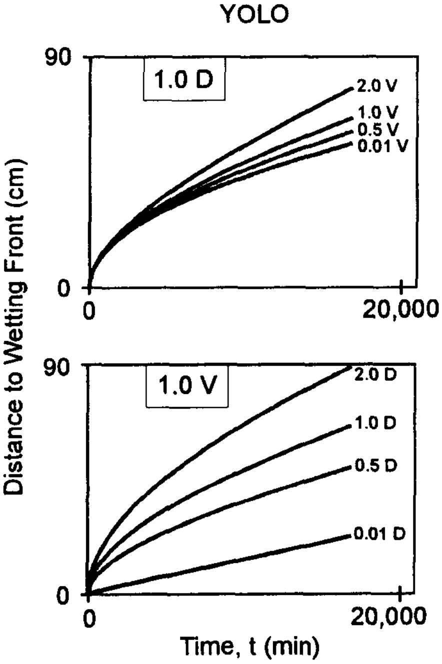

Fig. 6. Sensitivity of the 3-parameter flow model [equations (9) and (10)] to the parameters V and D. Case: Yolo light clay, $1.0\,D = 0.849\,\text{cm}^2\,\text{min}^{-1}$ and $1.0\,V = 0.00168\,\text{cm}\,\text{min}^{-1}$.

of the mean leaching time and the position of the wetting front.

The presented derivation procedure assuming linear water flow and zero hydraulic conductivity at zero soil–water content seems promising for obtaining simple, approximate analytical solutions for various water and wastewater infiltration problems.

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