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A boundary element method for analysis of contaminant transport in fractured and non-fractured porous media

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Abstract

This paper presents a boundary element method for modelling contaminant transport of a contaminant species in porous media which consists of both fractured and non-fractured material. In the fractured material, blocks of solid matrix comprising the parent soil material are assumed to be separated by discontinuities in the form of fissures, fractures or joints. The method developed in this paper utilises a double porosity model to deal with contaminant transport in the presence of the discontinuities and solid blocks. The underlying assumption of a double porosity model is that the solid matrix blocks, said to have the primary porosity, constitute one continuum while the network of discontinuities, the secondary porosity, form the other continuum. Flux transfer between the two interacting continua is represented by a source/sink term in the governing equation. When there are no discontinuities, the material reduces to a single continuum with one porosity and so the formulation is able to deal with deposits which consist of both fissured and non-fissured material. This approach is used to develop a boundary element technique which can enable a wide range of problems of practical interest to be analysed. © 1998 Elsevier Science Ltd.. All rights reserved.

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1. Introduction

In many practical situations involving contaminant transport in soil and rock, the properties of the porous media are known to vary widely. It is therefore conceivable that the permeability and dispersivity values between any two locations within short distances in a problem domain may differ by several orders of magnitudes, this could be due to either the significantly different properties of the parent materials or it could also be due to the presence of (or lack of) discontinuities e.g. joints, fractures and fissures within the parent materials. In the case of landfills for example, the integrity of clay layers may be compromised by the presence of desiccation and shrinkage cracks which can affect the effectiveness of the barrier adversely. It has also been reported that in clayey tills where many landfill or waste sources are to be situated structural defects are found to occur widely (e.g. [6,17,8]). The fact that material non-homogeneity and discontinuities are present quite regularly in the porous media and that they can have significant impact on the contaminant transport process suggest that it is both useful and necessary to develop appropriate models which take account of these effects.

In an earlier paper, Leo and Booker [10] have presented a boundary element method for analysing contaminant transport in fractured porous media in which the sets of fracture planes are assumed to be orthogonal to each other. Leo and Booker [11,12] have also presented a boundary element method for the analysis of non-homogeneous non-fractured porous media. In this paper, it will be shown that both techniques can be combined to develop a more general boundary element technique which may be used to analyse sites consisting of zones of a number of different materials which may be either fissured or non-fissured. While Rowe and Booker [16] have shown that a finite layer method can be formulated for a system consisting of both fractured and unfractured uniform layers of soil, proving to be a valuable tool in many cases, however the boundary element method developed here is more general and so is able to deal with more complex geometries in practice which cannot be covered by the finite layer method. For example, a landfill which is embedded in a soil deposit must be treated as a surface source when using the finite layer method but the embedment is easily modelled using the boundary element method.

2. Fractured and non-fractured porous media

The theories relating to a double porosity model of contaminant transport in fractured material have been discussed in detail elsewhere previously (e.g. [1,4,7,10,14,15,18,21]).

In the model proposed by Rowe and Booker [14,15], the fracture planes are assumed to be orthogonal to each other. Fig. 1 shows a schematic diagram of the fracture system where the fracture spacings are denoted by $2H_1$, $2H_2$, $2H_3$ and the effective apertures are shown as $2h_1$, $2h_2$, $2h_3$. Set 1 of the fractures is parallel to the $\tilde{y}\tilde{z}$ plane, set 2 to the $\tilde{z}\tilde{x}$ plane and set 3 to the $\tilde{x}\tilde{y}$ plane and where $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ are a set of local axes.

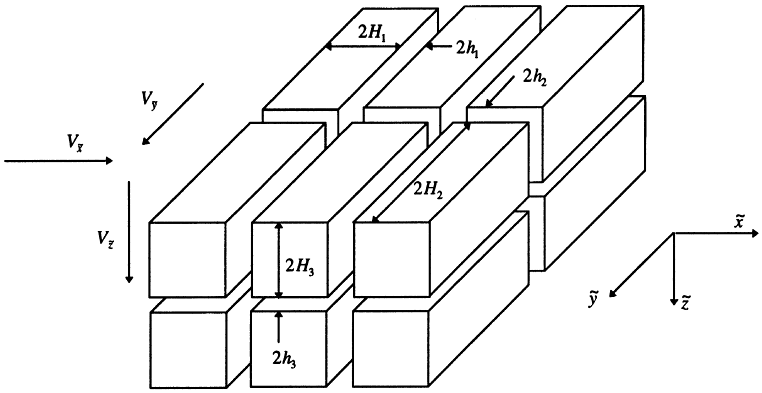

Fig. 1. Fracture system in porous media.

For a homogeneous fractured porous media where the groundwater advection is temporally steady and spatially uniform, the equation of contaminant transport in the local co-ordinate $(\tilde{x}, \tilde{y}, \tilde{z})$ axes is given by

$$\nabla(\mathbf{D}_{\mathrm{a}}\nabla c) - \mathbf{V}_{\mathrm{a}}\nabla c = (n_{\mathrm{f}} + \beta_{\mathrm{f}}K_{\mathrm{f}})(\frac{\partial c}{\partial t} + \gamma_{\mathrm{f}}^{*}c) + q \tag{1}$$

where, $\mathbf{D_a}$, 'effective' tensor of hydrodynamic dispersion. The components of the 'effective' tensor are given by Leo and Booker [10]:

$$D_{a\tilde{x}\tilde{x}} = (D_0 + \alpha_{\mathrm{L}} V_{\tilde{x}})\left(\frac{h_2}{H_2} + \frac{h_3}{H_3}\right),$$

$$D_{a\tilde{y}\tilde{y}} = (D_0 + \alpha_{\mathrm{L}} V_{\tilde{y}})\left(\frac{h_3}{H_3} + \frac{h_1}{H_1}\right),$$

$$D_{a\tilde{z}\tilde{z}} = (D_0 + \alpha_{\mathrm{L}} V_{\tilde{z}})\left(\frac{h_1}{H_1} + \frac{h_2}{H_2}\right),$$

$D_{aij} = 0$ when $i \neq j$ and i, j spans the index set $(\tilde{x}, \tilde{y}, \tilde{z})$, D_0, coefficient of molecular diffusion, α_{L}, longitudinal dispersivity in the fractures, V_i, component of the seepage velocity in the direction of ith co-ordinate axis, $\nabla = (\partial/\partial\tilde{x}, \partial/\partial\tilde{y}, \partial/\partial\tilde{z})^{\mathrm{T}}$, $\mathbf{V}_a$, vector of the components of the Darcy velocity, $n_{\mathrm{f}} = h_1/H_1 + h_2/H_2 + h_3/H_3$ is the fracture porosity$\beta_{\mathrm{f}} = 1/H_1 + 1/H_2 + 1/H_3$ represents the surface area per unit volume, K_{f}, linear distribution coefficient defined as the mass of contaminant sorbed per unit area of surface divided by the concentration of the contaminant, $\gamma_{\mathrm{f}}^{*} = \gamma_{\mathrm{f}}/R_{\mathrm{f}}$, γ_{f} sum of the first order radioactive and biodegradation constants, $R_{\mathrm{f}} = (1 + \beta_{\mathrm{f}}K_{\mathrm{f}}/n_{\mathrm{f}})$, the retardation coefficient due to adsorption on the surface walls of the fractures.

The quantity q is the rate at which the contaminant migrates into the matrix per unit volume of the fracture-matrix system and it can be conceptualised as follows in a simple experiment in which it is assumed that fracture-wall adsorption, contaminant decay and biodegradation and other sources and sinks are not present. Suppose that the background concentration is identically zero everywhere initially and the concentration of the fracture set is increased instantaneously at $t=0$ to c_{f0} and held constant, then the quantity of contaminant entering the matrix will be proportional to the fracture concentration but will vary with time so that,

$$q(t) = c_{f0}\eta(t), \tag{2}$$

where $\eta(t)$ can be determined by monitoring the amount of contaminant which must be added to maintain the fracture concentration constant. If the fracture concentration varies with time and it is assumed that the process is a linear hereditary one then the Boltzmann superposition principle can be used to show that,

$$q(t) = c_{f0}\eta(t) + \int_0^t \eta(t-\tau)\frac{\partial c_f(\tau)}{\partial \tau}\,d\tau. \tag{3a}$$

This relationship can be conveniently expressed in terms of a Laplace transform (Eq. (4)) and it is found

$$\overline{q} = s\overline{c}_f\overline{\eta}. \tag{3b}$$

A theoretical value of the quantity η can be derived by assuming that the solid matrix acts as storage blocks diffused into by contaminant from the surrounding fractures. It may be seen to represent the rate at which contaminant diffuses into the matrix for a unit increase of the contaminant concentration in the surrounding fractures. The simplicity of Eq. (3b) as well as a number of other reasons which have been elaborated ([1,14,9]) make it desirable to formulate the equations in Laplace transform space. Expressions for the Laplace transform of q and η have been derived previously ([1,14,9]).

Taking the Laplace transform

$$\overline{c} = \int_0^\infty e^{-st}c(x,y,z,t)\,dt \tag{4}$$

of Eq. (1), it is found that,

$$\nabla(\mathbf{D}_a\nabla\overline{C}) - \mathbf{V}_a\nabla\overline{C} = (n_f + \beta_f K_f)(s+\gamma_f^*)\left(\overline{C} - \frac{c_0}{s+\gamma_f^*}\right) + \overline{q} \tag{5}$$

and following Rowe and Booker [14,15], the Laplace transform of q (the form of the expression below applies more generally for a case with fracture-wall adsorption, contaminant decay and/or biodegradation) is found to be,

$$\overline{q} = (s+\gamma_m^*)\overline{\eta}\left(\overline{c} - c_0 s + \gamma_f^*\right) \tag{6}$$

where, $\gamma_{\mathrm{m}}^{*}=\gamma_{\mathrm{m}}/R_{\mathrm{m}}$, γ_{m}, the sum of the decay and biodegradation constants for the contaminant species in the solid matrix, $\mathrm{R}_{\mathrm{m}}=1+\rho_{\mathrm{m}}K_{\mathrm{m}}/n_{\mathrm{m}}$ is the retardation coefficient of the matrix, n_{m}, the porosity of the solid matrix, K_{m}, the distribution coefficient of the solid matrix.

It is perhaps worth pointing out at this stage that the behaviour of a non-fractured material can be recovered from that of a fractured material merely by setting q to zero and then replacing the parameters for the fractured material by the corresponding parameters for the non-fractured material, this leads to the equations [11,12]

$$\nabla(\mathbf{D}_{\mathrm{a}}\nabla\overline{c})-\mathbf{V}_{\mathrm{a}}\nabla\overline{c}=(n+\rho K_{\mathrm{d}})(s+\gamma^{*})\left(\overline{c}-\frac{c_0}{s+\gamma^{*}}\right), \tag{7}$$

where, $\mathbf{D}_{\mathrm{a}}$, 'effective' hydrodynamic dispersion tensor of the contaminant in the non-fractured material. The coefficients of the 'effective' hydrodynamic dispersion tensor for an isotropic material are defined as the product of the porosity and the normal hydrodynamic dispersion tensor

$$D_{akl}=n\left[(D_0+\alpha_{\mathrm{T}}V)\delta_{kl}+(\alpha_{\mathrm{L}}-\alpha_{\mathrm{T}})\frac{V_kV_l}{V}\right],$$

V, magnitude of the seepage velocity, α_{T}, coefficient of tranversal dispersivity, n, porosity of the non-fractured material, $\mathbf{V}_{\mathrm{a}}$, Darcy velocity vector in the non-fractured material, ρ, dry density of the non-fractured material, $\mathbf{K}_{\mathrm{d}}$, linear distribution coefficient of the sorbed contaminant onto the solid grains of the non-fractured material $\gamma^{*}=\frac{\gamma}{R_{\mathrm{d}}}$, γ, the sum of decay and biodegradation coefficients of the contaminant in the non-fractured material, $R_{\mathrm{d}}=1+\rho K_{\mathrm{d}}/n$, the retardation coefficient in the non-fractured material.

It can be observed that the general form of Eq. (5)Eq. (7) remains identical thus enabling a boundary element method to be developed which can accommodate both fractured and non-fractured materials.

3. Governing equations for zoned porous media

For prismatic landfills which are commonly found in practice, it will suffice to consider only plane conditions. The attention in this paper will therefore be restricted to developing a boundary element method for plane analysis only. The porous media will usually consists of zones of several different materials, some of which may be fractured while others are not. Each zone will be assumed to consist of fairly homogeneous material. In each zone j, the Laplace transform equation of contaminant transport for a single species in local $(\tilde{x},\tilde{z})$ space is given by,

$$\nabla(\mathbf{D}_{\mathrm{a}j}\nabla\overline{c})-\mathbf{V}_{\mathrm{a}j}\cdot\nabla\overline{c}=\theta_j\left(\overline{c}-\frac{c_{0j}}{\Lambda_j}\right), \tag{8}$$

where for definiteness, a subscript j has been added to each of the quantities defining the properties in zone j. For a non-fractured zone it follows from Eq. (7) that:

$$\theta_j = (n_j + \rho_j K_{dj})(s + \gamma_j^*), \tag{9a}$$

$$\Lambda_j = s + \gamma_j^*, \tag{9b}$$

and for a fractured zone it follows from Eq. (5)Eq. (6) that:

$$\theta_j = n_{fj} R_{fj}(s + \gamma_{fj}^*) + \overline{\eta}_j(s + \gamma_{mj}^*), \tag{10a}$$

$$R_{fj} = 1 + \frac{\beta_{fj} K_{fj}}{n_{fj}}, \tag{10b}$$

$$\Lambda_j = s + \gamma_{fj}^*, \tag{10c}$$

The local $(\tilde{x}, \tilde{z})$ co-ordinates are related to the global (x,z) co-ordinates as follows,

$$\begin{aligned} \tilde{x} &= x \cos \theta_j + z \sin \theta_j, \\ \tilde{y} &= -x \sin \theta_j + z \cos \theta_j, \end{aligned} \tag{11}$$

where θ_j is the angle between the local and global co-ordinate system. Thus Eq. (11) can be used to perform a co-ordinate transform between the global and local co-ordinate systems.

If the local co-ordinate system is chosen so that the local axes are parallel to the principal directions of $\mathbf{D}_{aj}$, then Eq. (8) reduces to,

$$D_{a\tilde{x}\tilde{x}j} \frac{\partial^2 \overline{c}}{\partial \tilde{x}^2} + D_{a\tilde{z}\tilde{z}j} \frac{\partial^2 \overline{c}}{\partial \tilde{z}^2} - V_{a\tilde{x}j} \frac{\partial \overline{c}}{\partial \tilde{x}} - V_{a\tilde{z}j} \frac{\partial \overline{c}}{\partial \tilde{z}} = \theta_j \left(\overline{c} - \frac{c_{0j}}{\Lambda_j} \right) \tag{12a}$$

and the normal component of the mass flux on the boundary is given by,

$$\overline{f}_{\tilde{n}} = V_{a\tilde{n}j}\overline{c} - (\mathbf{D}_{aj} \nabla \overline{c})\mathbf{I}_j \tag{12b}$$

where, $V_{a\tilde{n}j}$, component of Darcy velocity normal to the boundary of zone j, $\mathbf{I}_j$, unit normal vector to the boundary of zone j. Now suppose that,

$$\overline{c}_p = \sigma_j \tag{13}$$

is a particular solution of Eq. (12a)Eq. (12b). The simplest and commonest case will be when the initial concentration in the zone c_{0j} is spatially constant, then it is easily found that $\overline{c}_p = c_{0j}/\Lambda_j$. It follows from Eq. (13) that,

$$\Delta \overline{c} = \overline{c} - \sigma_j \tag{14}$$

satisfies the equation,

$$D_{a\tilde{x}\tilde{x}j} \frac{\partial^2 \Delta\overline{c}}{\partial \tilde{x}^2} + D_{a\tilde{z}\tilde{z}j} \frac{\partial^2 \Delta\overline{c}}{\partial \tilde{z}^2} - V_{a\tilde{x}j} \frac{\partial \Delta\overline{c}}{\partial \tilde{x}} - V_{a\tilde{z}j} \frac{\partial \Delta\overline{c}}{\partial \tilde{z}} = \Theta_j \Delta\overline{c} \tag{15a}$$

and the normal component of the mass flux, $\overline{f}_{\tilde{n}\sigma j}$ on the boundary, due to σ_j is,

$$\bar{f}_{\tilde{n}\sigma j} = V_{a\tilde{n}j}\sigma_j - (\mathbf{D}_{aj}\nabla\sigma_j)\mathrm{I}_j$$

so that,

$$\rightarrow_{\tilde{n}} = \bar{f}_{\tilde{n}} - \bar{f}_{\tilde{n}\sigma j}. \tag{15b}$$

It is possible to reduce Eq. (15a) to the more mathematically convenient modified Helmholtz equation by using a second series of co-ordinate transforms [9] as follows:

$$\tilde{x} = u_j X, \tag{16a}$$

$$\tilde{z} = w_j Z, \tag{16b}$$

$$V_{aXj} = w_j V_{a\tilde{x}j}, \tag{16c}$$

$$V_{aZj} = u_j V_{a\tilde{z}j}, \tag{16d}$$

$$\Delta\bar{f}_{\mathrm{N}} = (w_j l_{\hat{x}j} L_{Xj} + u_j l_{zj} L_{Zj})\Delta\bar{f}_{\tilde{n}}, \tag{16e}$$

where $l_{\tilde{x}j}$, $l_{\tilde{z}j}$ and L_{Xj}, L_{Zj} are the direction cosines of the normals on the boundary of zone j in the $(\tilde{x}, \tilde{z})$ and (X,Z) spaces, respectively, and:

$$u_j = \left(\frac{D_{a\tilde{x}\tilde{x}j}}{D_{aj}}\right)^{1/2}, \tag{17a}$$

$$w_j = \left(\frac{D_{a\tilde{z}\tilde{z}j}}{D_{aj}}\right)^{1/2}, \tag{17b}$$

$$D_{aj} = (D_{a\tilde{x}\tilde{x}j} D_{a\tilde{z}\tilde{z}j})^{1/2}. \tag{17c}$$

Furthermore, introducing the change of variable:

$$\Delta\bar{c} = \Delta\bar{c}^{*}\mathrm{e}^{(\varpi_j X + \lambda_j Z)}, \tag{18a}$$

$$\Delta\bar{f} = \Delta\bar{f}^{*}\mathrm{e}^{(\varpi_j X + \lambda_j Z)}, \tag{18b}$$

where,

$$\omega_j = \frac{V_{aXj}}{2D_{aj}}, \tag{19a}$$

$$\lambda_j = \frac{V_{aZj}}{2D_{aj}}, \tag{19b}$$

it is found that Eq. (15a) can be reduced to the familiar modified Helmholtz equation,

$$D_{aj}\nabla^2\Delta\bar{c}^{*} = S_j\Delta\bar{c}^{*} \tag{20a}$$

where,

$$S_j = \Theta_j + D_{aj}(\omega_j^2 + \lambda_j^2). \tag{20b}$$

Thus proceeding formally in the manner outlined in Leo and Booker [11,12], Eq. (20a) can be used to formulate the boundary integral equation,

$$\varepsilon(\mathbf{r}_0)\Delta\bar{c}^*(\mathbf{r}_0) = \int_j (\Delta\bar{c}^*\Delta\bar{f}_\mathrm{N} - \Delta\bar{c}\Delta f_\mathrm{N}^*)\,\mathrm{d} \tag{21}$$

where,

$$\varepsilon(\mathbf{r}_0) = \begin{cases} 1 & \text{if } \mathbf{r}_0 \text{ is within the domain of zone } j, \\ 0 & \text{if } \mathbf{r}_0 \text{ is outside the domain of zone } j, \\ \frac{1}{2} & \text{if } \mathbf{r}_0 \text{ lies on a smooth boundary of zone j or its} \\ \multicolumn{2}{l}{\text{value is the subtended angle } \div 2\pi \text{ if the boundary is not smooth,}} \end{cases}$$

$\mathbf{r}_0$, position vector of the point of disturbance, Γ^j, transformed boundary of zone j in the (X,Z) space, $\Delta\bar{c} = 1/2\pi D_{aj}\, K_0\left(\sqrt{S_j/D_{aj}}R\right)$, the fundamental solution of Eq. (20a), K_0 , modified Bessel function of the second kind of order zero. $\Delta\bar{f}_\mathrm{N} = V_{a\mathrm{N}j}/2\ \Delta\bar{c} - D_{aj}\ \partial\Delta\bar{c}/\partial N$, $V_{a\mathrm{N}j}$, the normal component of the Darcy velocity on the boundary of zone j in (X,Z) space, $R = \left[(X - X_0)^2 + (Z - Z_0)^2\right]^{1/2}$, $X_0, Z_0 =$ co-ordinates of the point of disturbance.

A boundary element approximation of the boundary integral equation (21) can be formulated using conventional techniques widely described in literature (e.g. [5,2,11,12] and takes the form of,

$$\mathbf{H}_j^*\Delta\bar{\mathbf{c}}^* = \mathbf{G}_j^*\Delta\bar{\mathbf{f}}_\mathrm{N}^*, \tag{22}$$

where $\mathbf{H}_j{}^*$, $\mathbf{G}_j{}^*$ are the fully populated influence matrices, $\Delta\bar{\mathbf{c}}^*$, $\Delta\mathbf{f}_\mathrm{N}^*$ are the vectors of nodal values on the boundary elements of zone j. If the co-ordinate transformations in Eq. (11) and the change in variables given in Eqs. (14), (15b), (18a) and (18b) were reversed, it is found that Eq. (22) can be expressed in natural co-ordinates and in terms of the nodal values of the variables $\bar{c}, \bar{f}_\mathrm{N}$ as,

$$\mathbf{H}_j\bar{\mathbf{c}} = \mathbf{G}_j\bar{\mathbf{f}}_\mathrm{N} + \mathbf{B}_j. \tag{23}$$

The vector $\mathbf{B}_j$ arose due to the presence of the non-zero initial concentration in the zone. Using Eq. (23) the contributions from all the zones are then assembled to form a global system,

$$\mathbf{H}\bar{\mathbf{c}} = \mathbf{G}\bar{\mathbf{f}}_\mathrm{N} + \mathbf{B}, \tag{24}$$

where $\mathbf{H}$, $\mathbf{G}$ and $\mathbf{B}$ are the global influence matrices and vector. It is noted that the influence matrices in the global system are not fully populated. Eq. (24) is used to solve for the unknown concentration and normal mass flux on the external boundaries and on the interfaces between the zones. Once these values are known the value of the concentration can be solved at any internal point in the domain.

It should however be observed that the solution found by applying these equations will be in the Laplace transform domain and thus has to be inverted to obtain the solution in the time domain. This is done by performing a numerical inversion of the Laplace transform using the algorithm by Talbot [19].

4. Application

4.1. Verification

An example of a test problem for verifying the correctness of the codes is given here. It compares the semi-analytical solutions of the contaminant distribution from a long elliptical source in the (x,z) space (shown in the inset of Fig. 2) against the numerical solutions obtained by the boundary element codes. The geometry of the given elliptical source is a special case in that when it is transformed spatially, the resulting transformation yields a cylindrical source where the coefficients of hydrodynamic dispersion will be isotropic. Hence it is possible to develop semi-analytic solutions based on an approach given by Rahman and Booker [13]. The elliptic source is assumed to have an x-intercept of 7.071 m and a z-intercept of 3.536 m. Within the source, the initial concentration is given as 1000 mg/l and the diffusion coefficient of the contaminant within the source is very large relative to the surrounding soil. It may be seen that in this case, the concentration within the source will remain spatially uniform but will however diminish with time since the mass of contaminant within the source is finite. The components of the Darcy velocities are

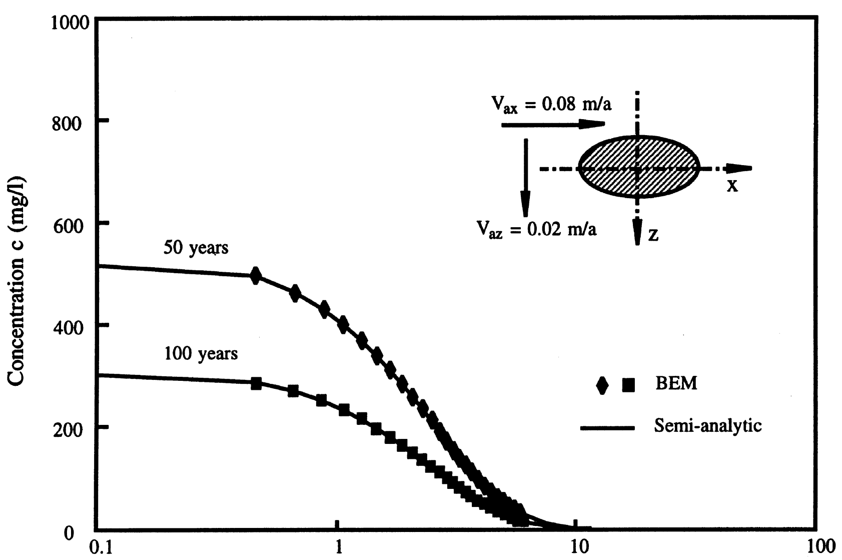

Fig. 2. Test problem 1 – comparison of BEM and semi-analytic solutions in fractured porous media.

assumed to be 0.08 and 0.02 m/a in the x and z directions, respectively. The surrounding porous media is assumed to be fractured, the fracture and matrix properties are given in Table 1 where the fractures spacings H_1,H_2,H_3 and the aperture opening sizes h_1, h_2, h_3 are taken to be along the x,y,z directions, respectively.

Solutions were found in the boundary element method using a total of 320 constant-valued boundary elements. As shown in Fig. 2, the agreement between the semi-analytical solutions and the boundary element solutions are found to be very good.

4.2. Illustrative Examples

Two illustrative examples are presented to demonstrate the boundary element method described herein. To demonstrate the effects of the presence of fractures in the porous media, the first illustrative example considers two cases. In the first case a 20 m $\times$ 5 m rectangular repository containing a single species of contaminant with an initial concentration of 1000 mg/l is situated in surrounding porous medium which is fractured. For the second case, the same repository is assumed to be founded in intact (non-fractured) porous medium. In both cases, there is a downward Darcy velocity of 0.004 m/yr. Two sets of vertical fracture planes (aperture 60 μm, spacing 0.2 m) are present in the intact material of the fractured material and the longitudinal dispersivity in the fractures is 1 m.

The effective diffusion coefficient in the intact material of the fractured porous medium is 0.0003 m^2/yr. Fig. 3 gives the distribution of the contaminant at 1000 years for the fractured porous medium. One of the distinguishing features in the contour plots is the significantly one-dimensional nature of the contaminant migration in the fractured medium, generally along the direction of the advection velocity, because of the highly anisotropic values of the coefficient of hydrodynamic dispersion used in the simulation. Compared with Fig. 4, which is for the unfractured case, the contaminant migration in the fractured material is much faster along the direction of the advection. This was in spite of a higher value of effective diffusion (0.004 m^2/yr) used for the intact material in the unfractured case.

Table 1
Test problem 1 – properties of fractured material

Properties of fractures								
D_0 (m^2/yr)	α_L(m)	$\beta_f K_f$	h_1 (m)	h_2 (m)	h_3 (m)	H_1 (m)	H_2 (m)	H_3 (m)
0.02	1.0	0	10^{-3}	0.1	10^{-3}	0.1	10^{-3}	0.1

Properties of matrix		
D_m(m^2/yr)	n_m	$\rho_m K_m$
0.02	0.1	0

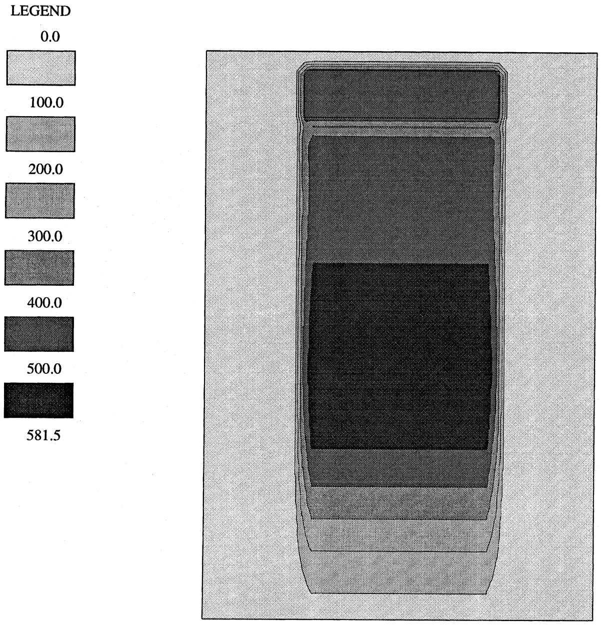

Fig. 3. Contaminant distribution for fractured porous medium at 1000 years.

A second example is presented to demonstrate the use of the boundary element method to model the effects of a proposed single cell landfill. The dimensions of the landfill are shown in Fig. 5 and it is located in an 8 m thick fractured material underlain by a 2 m thick confined aquifer. The physical properties of the landfill, liner and aquifer are presented in Table 2 while the properties of the fractured material are identical to those in Table 1. Initially, the background concentration in the porous media is assumed to be zero everywhere and in the landfill, the contaminant concentration is specified as 1000 mg/l.

Two design options have been considered viz. (a) no liner is used (b) a 1 m thick clay liner with adsorptive properties is incorporated in the landfill design. The flow regime is assumed to be directed downwards from the base of the landfill towards the aquifer below due to the presence of a 1 m head difference. In the aquifer, the flow is assumed to be predominantly in the horizontal direction.

4.3. *Design case (a)*

In this case, no liner is used in the landfill design. The fractured material is taken to have a hydraulic conductivity value of 1.78×10^{-7} cm/s thus giving a Darcy velocity in the underlying fractured material of approximately 0.014 m/yr. Using a total of 276 boundary elements, the boundary element solutions are presented in

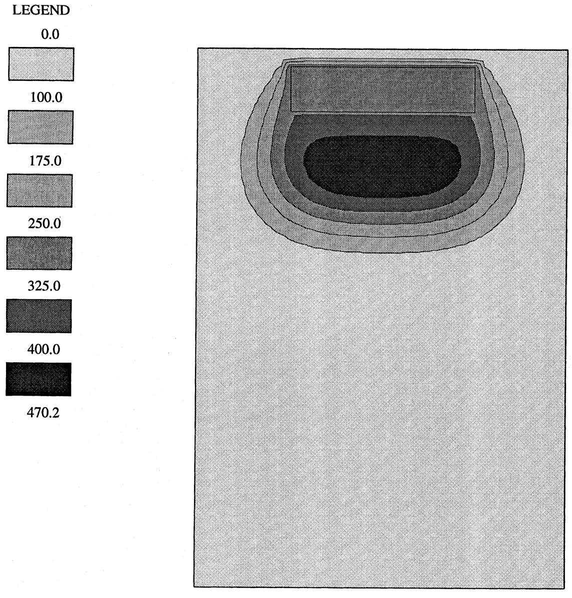

Fig. 4. Contaminant distribution for non-fractured porous medium at 1000 years.

Figs. 6–8 showing the contaminant distribution at 10, 50 and 100 years after the start of landfilling. Within 50 years, it is shown that the contaminant plume will have reached and contaminated the underlying aquifer. Given a Darcy velocity of 0.014 m/yr, the true (or seepage) velocity within the fractures will be 0.7 m/yr (since the plane velocity in the z-direction is $h_1/H_1 + h_2/H_2 = 2 \times 10^{-2}$). Thus, if diffusion into the solid matrix and mechanical dispersion in the fractures were ignored, it is interesting to find that the plug flow would contaminate the aquifer in less than 10 years. However as the contaminant contours in Fig. 6 show, the plume (defined as a

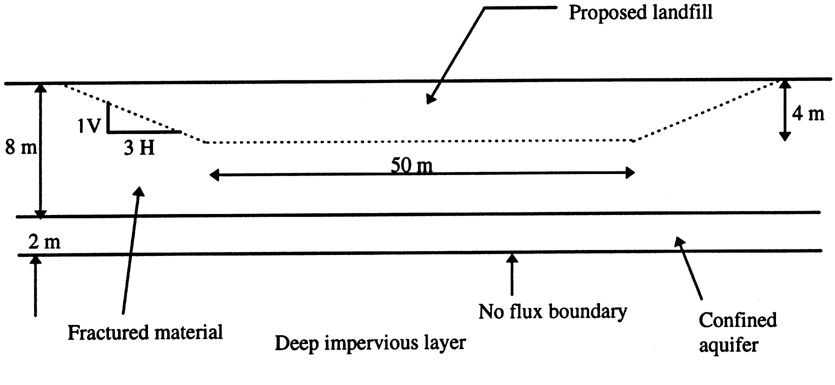

Fig. 5. Profile beneath proposed landfill.

Table 2
Properties of non-fractured zones

Zone	D_{oj} (m^2/yr)	α_{Tj} (m)	α_{Lj} (m)	n_j	$\rho_j K_{dj}$	V_{axj} (m/yr)	V_{azj} (m/yr)
Landfill	1.0	0	0	1.0	0	0	0
Liner (Case b)	0.005	0.01	0.1	0.4	5	0	0.004
Aquifer	1.0	0.2	0.2	2.0	0.35	1.4	0

contour of 100 mg/l) would not have reached the aquifer at the elapsed time of 10 years. This is largely due to the attenuation of the concentration profile arising from the diffusion of the contaminant into the solid matrix.

4.4. Design case (b)

Several options may be considered for improving the performance of the landfill design. The common methods include using a more impervious clay layer as a liner below the base of the landfill. In this design case, a 1 m thick clay liner with sorptive properties ($\rho_j K_{dj} = 5$) has been incorporated in the design. The composite liner-fractured soil will result in a Darcy velocity of about 0.004 m/a, assuming that the hydraulic conductivity of the liner is one-tenth the hydraulic conductivity of the fractured soil (i.e. 1.78×10^{-8} cm/s). Using a total of 346 constant-valued boundary elements, solutions have been obtained for this design case again for 10, 50 and 100 years after start of landfilling. Unlike the previous case, it is observed that contaminant distribution is now much more contained within the landfill after an elapsed time of 10 years as shown in Fig. 9. It would seem that the clay liner with its adsorptive properties has successfully been able to prevent the contaminant plume

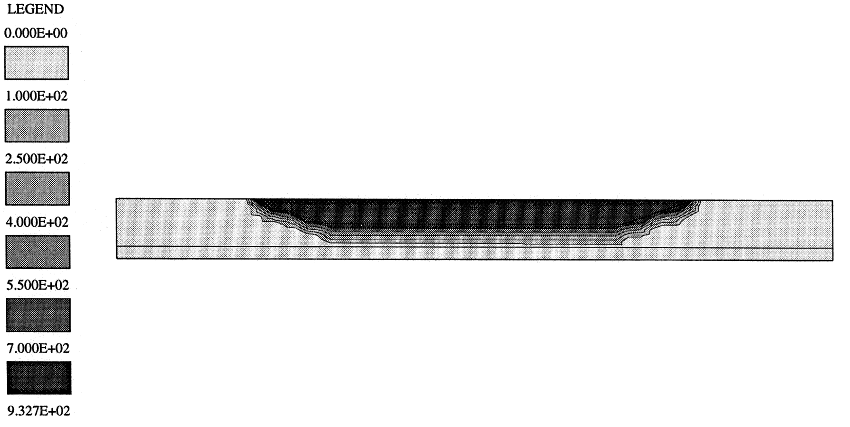

Fig. 6. Design case (a) – contaminant distribution at 10 years.

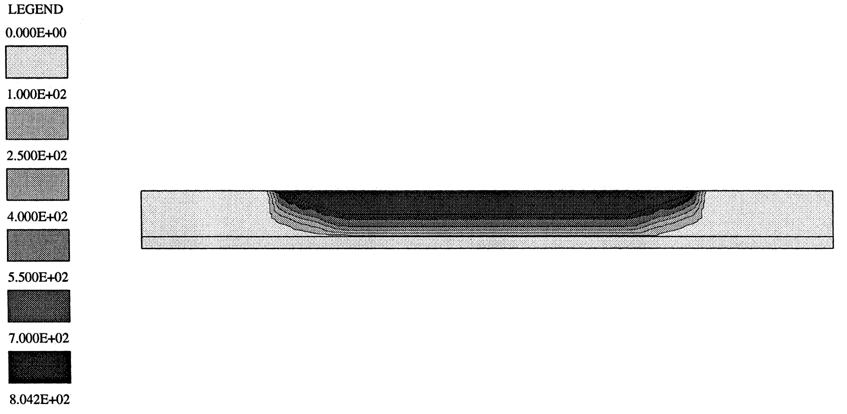

Fig. 7. Design case (a) – contaminant! distribution at 50 years.

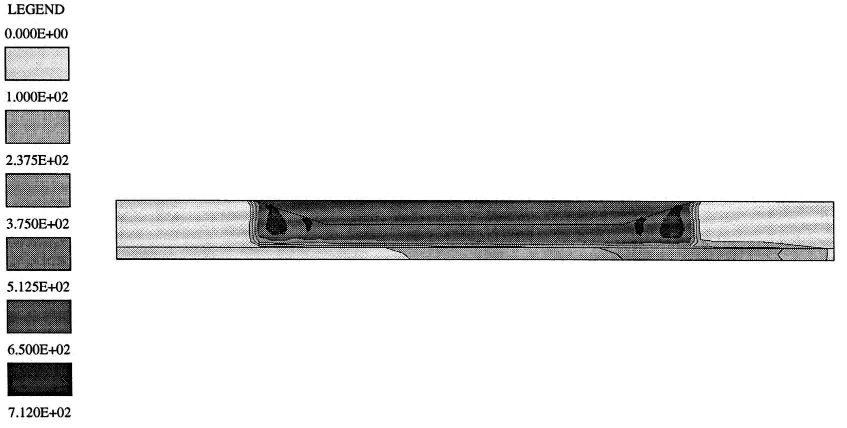

Fig. 8. Design case (a) – contaminant distribution at 100 years.

from breaking through into the aquifer at the elapsed time of 50 and 100 years (Figs. 10 and 11). The results suggest that the clay liner has considerably improved the performance of the landfill.

5. Conclusion

A boundary element method has been presented for analysing contaminant transport in porous media which may or may not be fractured. To deal with the

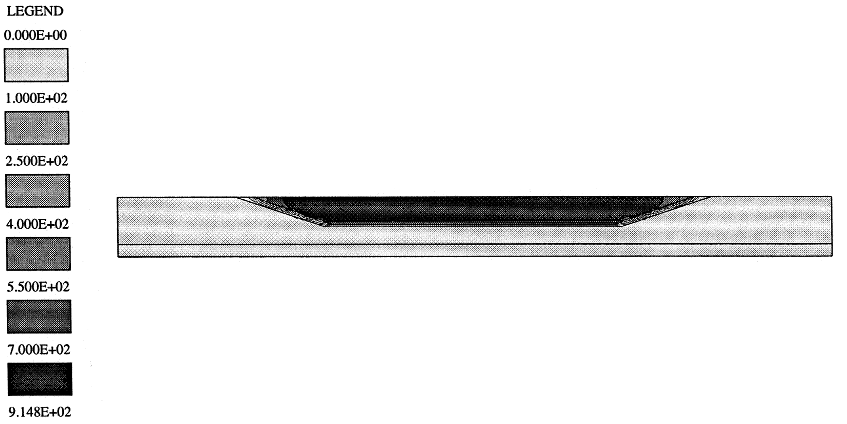

Fig. 9. Design case (b) – contaminant distribution at 10 years.

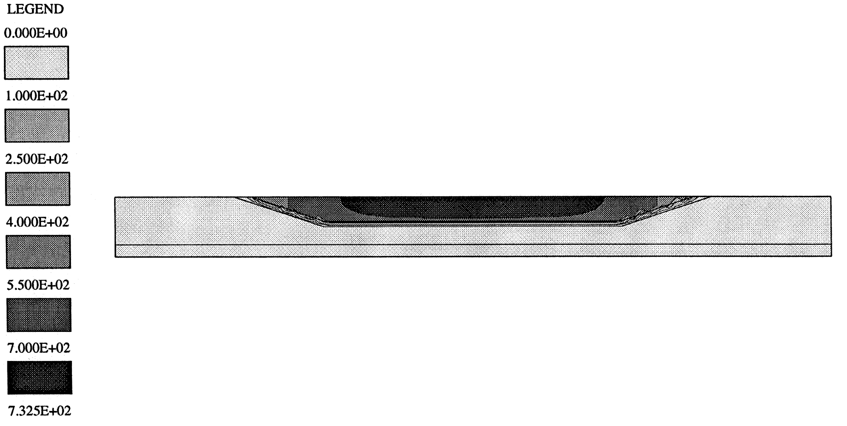

Fig. 10. Design case (b) – contaminant distribution at 50 years.

fractured material, a double porosity model has been used. A Laplace transform and a series of co-ordinate transforms are then used to formulate a boundary integral equation from which the system of algebraic boundary element equations can be obtained. As an illustrative example, the boundary element method developed has been used to help assess the design of a landfill.

For further reading please see references [3,20].

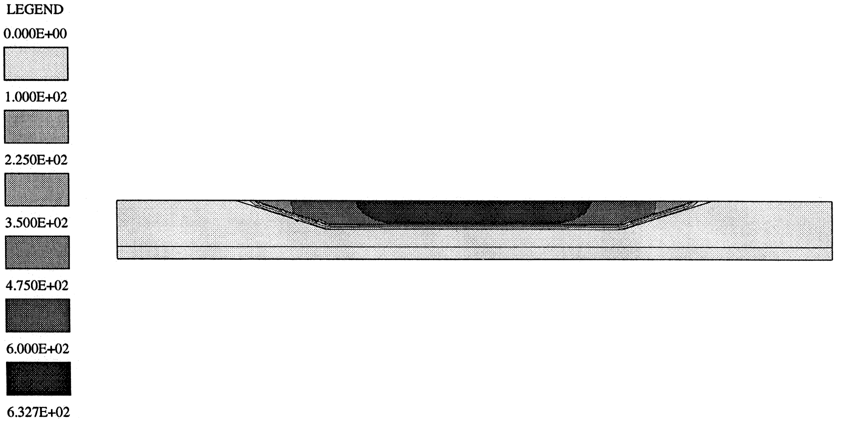

Fig. 11. Design case (b) – contaminant distribution at 100 years.

References

[1] Barker JA. Block-geometry functions characterizing transport in densely fissured media. Journal of Hydrology 1985;77:263–79

[2] Barnerjee PK, Butterfield R. Boundary element methods in engineering science. London: McGraw-Hill, 1981

[3] Bear J. Hydraulics of groundwater. New York: McGraw-Hill, 1979

[4] Bibbly R. Mass transport of solutes in dual-porosity media. Water Resources Research 1981;17(4):1075–81

[5] Brebbia CA, Walker S. Boundary element techniques in engineering. London: Newnes-Butterworths, 1980

[6] Cherry JA. Hydrogeologic contaminant behaviour in fractured and unfractured clayey deposits in Canada. In: Kobus HE, Kinzelbach W, Balkema A, editors. Contaminant Transport in Groundwater. Netherdams: Rotterdam, 1989:11–20

[7] Dykhuizen RC. A new coupling term for dual-porosity models. Water Resources Research 1990;26:351–6

[8] Jorgensen PR, Fredericia J. Migration of nutrients, pesticides and heavy metals in fractured clayey tills. Geotechnique 1992;V42:67–77

[9] Leo CJ. Boundary element analysis of contaminant transport in porous media. Ph. D. Thesis, Australia: Unversity of Sydney, 1994

[10] Leo CJ, Booker JR. Boundary element analysis of contaminant transport in fractured porous media. International Journal for Numerical and Analytical Methods in Geomechanics 1993;17(7):471–92

[11] Leo CJ, Booker JR. A boundary element method for analysis of contaminant transport in porous media I: Homogeneous porous media. International Journal for Numerical and Analytical Methods in Geomechanics 1998a (in print)

[12] Leo CJ, Booker JR. A boundary element method for analysis of contaminant transport in porous media II: Non homogeneous porous media. International Journal for Numerical and Analytical Methods in Geomechanics 1998b (in print)

[13] Rahman MS, Booker JR, Pollutant migration from deeply buried repositories. International Journal for Numerical and Analytical Methods in Geomechanics 1989;13:37–51

[14] Rowe RK, Booker JR. A semi-analytic model for contaminant migration in a regular two- or three-dimensional fractured network: conservative contaminants. International Journal for Numerical and Analytical Methods in Geomechanics 1989;13:531–50
[15] Rowe RK, Booker JR. Contaminant migration in a regular two- or three-dimensional fractured network: reactive contaminants. International Journal for Numerical and Analytical Methods in Geomechanics 1990;14:401–25
[16] Rowe RK, Booker JR. Modelling of two-dimensional contaminant migration in a layered and fractured zone beneath landfills. Canadian Geotechnical Journal 1991;V28:338–52
[17] Rudolph DL, Cherry J, Farvolden R. Groundwater flow and solute transport in fractured lacustrine clay near Mexico City. Water Resources Res. 1991;V9(5):1332–56
[18] Sudicky EA. The Laplace transform Galerkin technique for efficient time-continuous solution of solute transport in double-porosity media. Geoderma 1990;46:209–32
[19] Talbot A. The accurate numerical integration of Laplace transforms. Journal Inst. Mathematical Applications 1979;23:97–120
[20] Tang DH, Frind EO, Sudicky EA. Contaminant transport in fractured porous media: Analytical solution for a single fracture. Water Resources Research 1981;17(3):555–64
[21] Zimmerman RW, Chen G, Hadgu T, Bodvarson GS. A numerical dual-porosity model with semi-analytical treatment of fracture/matrix. Water Resources Research 1993;29(7):2127–37