

A Mathematical Model of Simultaneous Heat and Moisture Transfer during Drying of Potato

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ABSTRACT

A mathematical model of simultaneous heat and moisture transfer is proposed for the prediction of moisture and temperature distributions during drying in a slab-shaped solid. The model took into account the effect of moisture-solid interaction at the drying surface by means of sorption isotherms of food. Non-constant physical and thermal properties were also incorporated in the model. The model was applied to the air drying of potatoes. A finite difference method (Crank-Nicolson) was used in the solution of simultaneous heat and moisture transfer equations at different times during drying. When the experimental results were compared with those obtained from the finite difference method, good agreement was found.

NOTATION

A, B and C	Constants
$a_{\rm w}$	Water activity
\ddot{C}_{air}	Concentration of water vapour in the air (kg/m^3)
$C_{\rm surface}$	Concentration of water vapour at surface of the sample (kg/m^3)
$C_{\rm p}$	Specific heat of material $(J/kg ^{\circ}K)$
C_{p}^{r}	Specific heat of air $(J/kg \circ K)$
d^{r}	A characteristic dimension (m)
D	Diffusion coefficient of water (m^2/s)
D_0	Constant (m^2/s)
h	Heat transfer coefficient $(J/m^2 s ^{\circ}K)$
k	Thermal conductivity of air $(J/ms \ ^{\circ}K)$
Κ	Thermal conductivity of material $(J/m s ^{\circ}K)$
K _m	Mass transfer coefficient (m/s)

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L	Thickness of the sample (m)
Μ	Moisture content (kg water/kg solid)
m_1	Liquid flux $(kg/m^2 s)$
Nu	Nusselt number, dimensionless
Pr	Prandtl number, dimensionless $(C'_{\rm p} \mu/{\rm k})$
P _{sat}	Saturated water vapour pressure at specific temperature (kPa)
P_{v}^{out}	Vapour pressure at the surface of sample at specific temperature (kPa)
Q	Energy of activation (J/mol)
Ŕ	Gas constant (kg/mol °K)
Re	Reynolds number $(dV \rho/\mu)$
Т	Temperature (°K)
t	Time (s)
V	Velocity of air (m/s)
x	The space dimension in the direction of thickness (m)
θ	Temperature (°C)
λ	Heat of evaporation of water (J/kg)
μ	Viscosity of air (kg/m s)
ρ	Density of air (kg/m^3)
$\rho_{\rm b}$	Density of material (kg/m^3)
ρ_1	Density of liquid (kg/m^3)

INTRODUCTION

Simultaneous heat and moisture transfer take place during drying. Many physical, chemical and nutritional changes occur in foods during the drying process. Many of these changes are functions of temperature, moisture content and time. Therefore, undesirable effects could be better controlled, if temperature and moisture distributions in foods as a function of drying time could be accurately predicted (Mishkin et al., 1983). Mathematical models are very useful in the design and analysis of simultaneous heat and moisture transfer processes. Drying models can be divided into three major groups: (i) those involving empirical equations valid for specific processes; (ii) those based on basic heatand mass-diffusion models forming systems of simultaneous equations, and (iii) a group of more comprehensive models that associate energy, mass and momentum transport equations with all the thermodynamically interactive fluxes (Havakawa & Furuta, 1988). Rossen and Havakawa (1977) reviewed drving models encountered in the literature. Most of the drving models have simplifying assumptions, for example, shrinkage may not be considered; changes in physical and thermal properties during drying are not taken into account. Usually, the reason for these assumptions is to make the model equations simpler and thus easier to solve. However, these assumptions and the models based on them are not valid for foods in all moisture ranges and drying conditions. Balaban and Pigott (1986) presented data on shrinkage of fish muscle during drying. They concluded that the shrinkage of fish muscle along the dimension parallel to the muscle fibres is significantly different from that perpendicular to the fibres during air drying. Wang and Brennan (1995) studied

the shrinkage effect on the density, porosity and dimensional changes during potato drying. It was found that the density of potato at a given moisture content decreased with increasing drying temperature. The volume shrinkage of potato during drying decreased almost linearly with moisture content. The percentage changes in thickness, length and width during drying decreased linearly with increasing moisture content. Thermal properties, such as thermal conductivity and specific heat, are functions of moisture and temperature (Wang & Brennan, 1992a, b). Since most foods are hygroscopic in nature, one should consider how tightly they bind water, i.e. moisture-solid interaction during drying (Wang & Brennan, 1992c).

Therefore, the objectives of this work were: to develop a drying model which takes into account the effect of the water activity of the food on surface mass transfer; to include the variable physical and thermal properties into the model equations; to provide a method of solving the resulting equations; and to compare the temperature and moisture content values predicted with and without consideration of shrinkage, and to compare the predicted values with those obtained experimentally.

MATHEMATICAL MODEL

Berger and Pei's (1973) drying model equations were modified in this work. The models were used by Balaban (1989) for the drying of fish, but no information has been found as to the drying of vegetables.

The following assumptions are made in this work: (1) shrinkage is not negligible; (2) physical and thermal properties are assumed to be functions of local moisture content and temperature; (3) water vapour flux in the food during drying is assumed to be negligible; (4) the external heat and mass transfer is assumed to be proportional, respectively, to the temperature and the vapour pressure difference between the surface of the drying solid and the external drying media; (5) the samples are assumed to be continuous infinite rectangular slabs.

Assuming that Fick's law is applicable to mass-transfer mechanisms, the liquid flux, m_1 towards the surface of the drying material is:

$$m_1 = D\rho_1 \frac{\partial M}{\partial x} \tag{1}$$

A mass balance over a small volume element of unit cross-sectional area and thickness, dx, leads to the differential equation for the liquid transfer,

$$\rho_1 \frac{\partial M}{\partial t} = \frac{\partial}{\partial x} \left(D \rho_1 \frac{\partial M}{\partial x} \right) \tag{2}$$

Assuming that heat is transferred only by conduction through the solid, i.e. the heat flux,

$$q = -K \frac{\partial T}{\partial x} \tag{3}$$

a heat balance leads to (based on Fourier's law):

$$\rho_{\rm b} C_{\rm p} \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(K \frac{\partial T}{\partial x} \right) \tag{4}$$

The boundary condition at x = 0 (surface) for eqn (2) is

$$K_{\rm m}(C_{\rm surface} - C_{\rm air}) = D\rho_1 \frac{\partial M}{\partial x}$$
(5)

which is the mathematical statement of the fact that the liquid flux to the surface must be equal to the vapour flux away from the surface into the external drying media.

Similarly, the corresponding boundary condition for eqn (4) becomes:

$$h(T_{\rm air} - T_{\rm surface}) = \lambda_{\rm v} D \rho_1 \frac{\partial M}{\partial x} - K \frac{\partial T}{\partial x}$$
(6)

where $\lambda_v D\rho_1(\partial M/\partial x)$ denotes the amount of heat required to evaporate the liquid flux at the surface.

For boundary conditions at x=L it is assumed that no mass and heat is transferred across that surface of the drying material. Thus:

$$\frac{\partial M}{\partial x} = 0$$
 and $\frac{\partial T}{\partial x} = 0$, at $x = L$ (7)

The application of these equations to the drying of vegetables was tested in the present study.

MATERIALS AND METHODS

The temperature profile inside the samples was measured by thermocouples. Six nickel-chromel/constantan thermocouples (E-type) with a diameter of 0.04 mm were inserted into six stainless needles with an external diameter of 0.22 mm. The junction of the thermocouple was located right at the tip of the needle. The arrangement of six thermocouples located at different depths in the sample is shown in Fig. 1. The thermocouples (1-6) were used to monitor the temperature profile developed in the sample during drying from the surface of the sample to the bottom. The analogue signals generated by the thermocouples were converted to a digital signal by using an AD5744 analogical to digital converter (V-microcomputer Ltd). An amplifier was also used to enlarge the voltages generated by the thermocouples. The temperatures inside the sample were recorded.

To obtain the moisture distribution experimentally in the sample during drying, the samples of thickness 10 mm, length 45 mm and width 20 mm were dried for various times in an experimental hot-air dryer (Wang & Brennan, 1992c) at air temperatures of 40, 50, 60 and 70°C, flowing parallel to the evaporating surface at a rate of 4.0 m/s. Three partly dried samples were taken





Fig. 1. Locations of thermocouples in the sample in a vertical plane to determine the temperature distribution during drying.



Fig. 2. Arrangement of thermocouples in a sample in a horizontal plane 4.0 mm below the surface of the sample.

at various times and cut with stainless steel cork borer. The cylindrical-shaped samples of diameter 9.0 mm were sliced into ten horizontal slices of equal depth in which the moisture content was measured by drying the slices in a vacuum oven at 70°C for 24 h. In order to demonstrate that heat transfer occurs only in the direction perpendicular to the flow of drying air, the temperatures in a sample in a horizontal plane 4.0 mm below the surface were monitored. The arrangement of thermocouples in the sample is shown in Fig. 2.

Parameters used in the model

As mentioned before, there are various physical properties and transport parameters that are required during the solution of the differential equations describing the drying model. A summary of those properties that were experimentally determined is given below. Also, other relationships that were found from the literature are cited where used.

Mass transfer coefficient (K_m)

This parameter is used in the boundary condition in eqn (5), where the water reaching the surface is evaporated and is then carried away by the air stream. Burnnett and Myers (1962) gave the following relationship for K_m for turbulent flow over a flat plate parallel to the surface:

$$K_{\rm m} = 8.90 \times 10^{-3} \ V({\rm Re})^{-0.2} C_{\rm surface} / (C_{\rm surface} - C_{\rm air})$$
(8)

The value of C_{surface} was taken from the a_{w} relationship of Wang and Brennan (1991), assuming that the liquid and water vapour are in equilbrium at the surface.

Heat transfer coefficient (h)

The heat transfer coefficient (h) is used in the boundary condition for eqn (6). Earle (1983) gave the following relationship for h:

$$Nu = \frac{hd}{k} = 0.23 (Re)^{0.8} Pr^{0.4}$$
(9)

Thermal conductivity (K)

The thermal conductivity (K) of potato is used in both the boundary condition for eqn (6) and the differential eqn (4). The relationship given below was found experimentally (Wang & Brennan, 1992*a*). It was found that *K* was a function of moisture content, while temperature had little effect on thermal conductivity in the range studied. The relationship is of the form:

$$K = a + b \log(M) \tag{10}$$

Specific heat (C_p)

The specific heat (C_p) of potato is used in the differential eqn (4). The relationship given below was found experimentally (Wang & Brennan, 1992b). Specific heat was found to be a function of both temperature and moisture content. The relationship is of the form:

$$C_{\rm p} = 4.184 \times 10^3 (0.406 + 0.00146 \,\theta + 0.203 \,M - 0.0249 \,M^2) \tag{11}$$

Water activity (a_w)

The water activity (a_w) relationship is used in the boundary condition for eqn (5), where the diffusion of water comes to the surface and evaporates. It is assumed that, at the surface, the vapour pressure of water is than givet by the a_w relationship in equilibrium with the liquid water content at the surface:

$$a_{\rm w} = \frac{P_{\rm v}}{P_{\rm sat}} \tag{12}$$

Brooker (1967) related the P_{sat} of water vapour to the temperature, T (°K), in the form:

$$P_{\text{sat}} = \exp(53.53 - 6834.27/T - 5.169 \ln T)$$
(13)

The following relationship according to Brooker (1967) was used:

$$C_{\text{surface}} = 2.166 \, \frac{P_{\text{v}}}{T} \tag{14}$$

The a_w relationship given below was determined experimentally (Wang & Brennan, 1991). The Oswin (1946) relationship was used because the equation is suitable for representing the relationship between water activity and equilibrium moisture content of potato in the range of water activity 0–88%.

$$M = A[a_{\rm w}/(1 - a_{\rm w})]^B \tag{15}$$

Effective moisture diffusion coefficient (D)

The effective moisture diffusion coefficient (D) is used in the boundary conditions of eqns (5) and (6) and in the differential eqn (2). The relationship given below was determined experimentally (Wang & Brennan, 1991).

$$D = D_0 \exp\left(-\frac{Q}{RT}\right) \tag{16}$$

Density $(\rho_{\rm b})$ and thickness of sample (L)

The density (ρ_b) is used in the differential eqn (4). The relationship given below was determined experimentally (Wang & Brennan, 1993).

$$\rho_{\rm b} = [A + B \exp(CM^2)] \times 10^3 \tag{17}$$

The thickness of sample is used in the differential equations. The relationship given below was obtained experimentally (Wang & Brennan, 1993).

$$L = (A + BM) \times 10^{-3} \tag{18}$$

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The solution of the equations

As drying is a simultaneous heat and mass transfer process, correlation of moisture and temperature changes in the product involves solution of coupled differential equations. If the parameters involved in eqns (2), (4), (5) and (6), such as thermal and physical properties, are functions of moisture content or temperature, the equations are not linear and must be solved through numerical methods. A finite difference method (Crank-Nicolson) was used to solve the simultaneous heat and mass transfer equations.

The thickness of sample was divided into N finite difference points or nodes. Node 1 was the surface, and node N was the bottom of the sample. At each node, the general heat and mass transfer equations apply. In addition, at node 1 and at node N, the boundary conditions for the surface and for the bottom apply, respectively. Space derivatives $\partial/\partial x$ and $\partial^2/\partial x^2$ were approximated by algebraic expressions in time.

According to Crank and Nicolson (1947) the derivative $\partial^2 M/\partial x^2$ at node *i* could be represented as follows:

$$\frac{\partial^2 M}{\partial x^2} = \left[\frac{1}{2} \left(\frac{M_{i-1;n+1} - 2M_{i;n+1} + M_{i+1;n+1}}{(\Delta x)^2} + \frac{M_{i-1;n} - 2M_{i;n} + M_{i+1;n}}{(\Delta x)^2} \right) \right]$$
(19)

where $M_{i,n}$ represents the moisture content inside the sample at point *i* after *n* s drying. Similarly, the derivative $\partial^2 T/\partial x^2$ at node *i* could be obtained.

The computer program used to solve the above differential eqns (2), (4), (5), (6) and (7) was written in Fortran 77. At each time step during the solution, the values M_i and T_i were calculated at each node. After each time step, the thickness of the sample was adjusted using eqn (18). Figure 3 shows the flow diagram of the computational algorithm.

The input to the mathematical models is: drying air velocity, relative humidity and viscosity; initial sample dimensions; shrinkage as a function of moisture content; density as a function of moisture content; thermal conductivity and specific heat as functions of moisture content and temperature; diffusion coefficient of water as a function of temperature; heat and mass transfer coefficients at the surface as a function of air properties; moisture sorption isotherms of potato; number of nodes.

RESULTS AND DISCUSSION

Figure 4 represents the temperature profiles in a horizontal plane 4.0 mm below the surface (thermocouples located at various points, 1, 2, 3 and 4, see Fig. 2) at a drying air temperature of 60°C. It was found that there were no significant



Fig. 3. Flow diagram of the computational algorithm.



Fig. 4. Experimental temperature profiles in a horizontal plane below the surface of the sample 4.0 mm at drying air temperature of 60°C.



Fig. 5. The predicted moisture distribution in the sample as functions of both position and drying time at 60°C when shrinkage is considered. Number on each curve is the drying time in hours.

differences between the temperatures at the four points at 5% level. This may indicate that the heat is transferred only in the vertical direction, i.e. from the surface to the bottom of the sample.

Figure 5 shows the moisture distribution predicted, by using the equations, in the sample as function of both position and drying time during drying in air at 60°C. The shrinkage effect was taken into account during the calculation of moisture contents in the sample. Curve 1 refers to the moisture distribution from the surface to the bottom of the sample after 1 h drying. Curve 2 indicates the moisture distribution after 2 h drying and curve 3 after 3 h drying and so on. It can be seen that the surface moisture content during drying decreases sharply with the increase in drying time. The moisture content gradients from surface to bottom of the sample decrease with the increase in drying time while the moisture content near the bottom of the sample decreases steadily with drying time.

The experimental and predicted moisture contents in the potato in a drying air temperature of 60°C after 6 h drying are shown in Fig. 6. It was found that there was a close correlation between experimental and predicted values when the shrinkage is considered (at 5% level).

The drying model also predicted the temperature distribution in the sample during drying. Figure 7 represents the predicted temperature distribution in the sample during drying at 60°C. Shrinkage was considered during the calculation



Fig. 6. The experimental and predicted moisture content values in the potato at a drying air temperature of 60°C after 6 h drying when shrinkage is considered.



Fig. 7. The predicted temperature distribution in the sample as function of both position and drying time at 60°C when shrinkage is considered. Number on each curve is the drying time in hours.



Fig. 8. The experimental and predicted temperature values in the potato at a drying air temperature of 40°C after 6 h drying when shrinkage is considered.



Fig. 9. The comparison of predicted moisture content values with and without consideration of shrinkage effect at 60°C after 6 h drying.

of temperatures in the sample. The temperature at every location rose with the increase in the drying time. The temperature gradients between surface and bottom decreased with the increase in the drying time.

Figure 8 shows the predicted temperature values in the potato compared with experimental temperature values in air at 40°C after 6 h drying. It can be seen that a reasonably good agreement exists between experimental and predicted values when shrinkage is taken into account.

Figure 9 shows the comparison of predicted moisture content values after 6 h drying at 60°C with and without incorporating the shrinkage effect in the model. It can be seen that the difference between the two sets of data is significant as drying progresses. It was found that the predicted moisture content values were



Fig. 10. The comparison of predicted temperature values with and without consideration of shrinkage effect at 60°C after 6 h drying.

higher when the shrinkage effect is not taken into account in the model than when it is. This result was confirmed by a T-test at 5% level. The thickness of sample decreases due to shrinkage, therefore the internal moisture has less distance to diffuse through and reaches the surface faster. The model which does not take into account shrinkage predicts higher moisture contents at any given time.

Figure 10 represents the comparison of temperature gradients after 6 h drying at 60°C with and without the shrinkage element in the model. Again, there is a significant difference between the two sets of data. The model without the shrinkage component predicts lower temperatures at a specific point at any given time as compared with the model which includes this component. The rate of heat transfer inside the sample is increased because of the decrease in thickness of the sample. The difference between the two cases (with and without taking the shrinkage effect into account) rapidly increases with increasing drying time.

Some published work indicates that shrinkage does not affect drying behaviour (Jason, 1958). However, it was found from this work that shrinkage has an effect on the drying behaviour of potato. A more reasonable prediction of moisture content and temperature distributions was obtained when the shrinkage effect was considered. Therefore, it may be concluded that shrinkage is not a negligible factor during the calculation of moisture content and temperature values in the sample during drying.

The model described in this study produced reasonable results in prediction of moisture and temperature distribution in a rectangle or slab of potato at different times during drying. The model could be expanded to two or three dimensions and could be solved by other numerical methods, such as finite element methods. In this case, the solution for the model would be more complicated. Other coordinate systems, such as cylindrical or spherical coordinates depending upon the materials, could be considered, for example, when considering carrots and peas, respectively. Limitations of this model include: (1) the assumption of water vapour flux is negligible, and (2) no account was taken of the dependence of the effective moisture diffusion coefficient on moisture content. These two factors merit further consideration.

The model developed in this study could be useful for prediction of the drying times, i.e. the end-point of a drying process. The optimum residual moisture content of the final product is important for the stability of the product during storage. Preservation involves the removal of enough water to prevent microbiological deterioration or the growth of organisms that could give rise to food poisoning. The moisture content for most foods must be lowered to below 50% on a wet weight basis before protection is afforded against micro-organisms (Labuza, 1972). Therefore, from this study, it is suggested that the safe moisture content be based on the local moisture content gradients inside the product which is subjected to drying, the moisture content at some points in the product may exceed the safe moisture content. As a result, microbiological deterioration might take place in a particular portion of the piece before the equilibrium moisture content has been reached.

The prediction of moisture and temperature distribution could also be used in the study of chemical reactions that occur during drying processes, as such reactions generally depend on both temperature and moisture content (Labuza, 1972). It is well known that reactions between food components are often accelerated during dehydration leading to reduction in organoleptic quality and nutritional value. Labuza (1972) has also suggested the use of computers to predict the extent of deteriorative reactions which occur during food dehydration. He pointed out that (1) deterioration data as a function of moisture content and temperature; and (2) a knowledge of the food's moisture temperature distribution as a function of drying time are important in the determination of the extent of deterioration.

One of the irreversible changes that accompanies the dehydration of a food product is non-enzymatic browning, which can lead to loss of protein biological value. The prediction of the extent of non-enzymatic browning that occurs in food dehydration should be based upon local rather than average conditions of moisture content and temperature in the sample. When Aguilera *et al.* (1975) investigated this they found that there was a sizeable difference in the extent of non-enzymatic browning between values using average and local moisture contents. Therefore, it is important to know the moisture and temperature distribution in the sample during drying when predicting the extent of non-enzymatic browning.

CONCLUSIONS

The mathematical model developed enables the prediction of moisture content and temperature at different depths in the slab of potato at different drying times with reasonable accuracy. It was concluded that shrinkage had an influence on the drying behaviour of potato and should be taken into account in predictive models.

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