SIMULATION OF CONTINUOUS DRYING PROCESSES BY INTEGRAL EQUATIONS

T. VIRÁG, G. GY. HALÁSZ[†] and J. B. ZHELEV[‡]

Research Institute for Technical Chemistry of the Hungarian Academy of Sciences, H-8201 Pf. 125, Veszprem, Hungary

(First received 29 December 1987; accepted in revised form 12 October 1988)

Abstract—A model of industrial drying under varying external conditions is presented. The starting point was the integral equation

$$\rho(z,\tau) = K(z,0)I_a + \int_0^L K(z,y) \left\{ f[\rho(y,\tau)] - \frac{\mathrm{d}}{\mathrm{d}\tau} \rho(y,\tau) \right\} \mathrm{d}y$$

which can be considered as the general mathematical model of a hydrodynamically linear chemical apparatus. A detailed and verified model of heat and mass transfer processes in a one-dimensional steadystate tunnel dryer is presented. A computer program, based on an integral equation approach without essential changes, is applied to the simulation of continuous dryers. The calculations prove that the developed model is suitable for the simulation of heat and mass transfer processes occurring in the dryer.

INTRODUCTION

Drying is a widely used operation in the chemical industry. A mathematical description of drying is useful for both sizing the apparatus and the simulation of the processes which is often necessary for the control and automation of a technological process. For the solution of these types of problems engineers may either use simple analytical expressions (Keey, 1972; Sazhin, 1984; Pakowski and Mujumdar, 1987) and apply principles which were elaborated for the design and automation of a chemical apparatus using their own or others' experimental data, or may describe the operation of the apparatus by a system of differential equations containing the equations of motion, balances and sources with the appropriate boundary conditions.

The simple analytical expressions are not satisfactory for the determination of drying operational units and processes (Romankov, 1987). The other method, based on the solution of differential equations, has obvious advantages when a dryer with a given structure has to be analysed. Problems generally arise in the description of the flow, because the influence of the presence of a solid phase and evaporation on stress and transfer coefficients is not known (Keey, 1972). Simplifying assumptions are therefore introduced and their validity should be checked experimentally in every case. In the other case, even if one has the momentum, energy and mass balance equation sets representing a real drying process, a joint numerical solution of these is impossible even at the present advanced level of mathematics and computing. That is why, instead of momentum balance equations, various uniformly by integral equations, and it can be considered as a general form of the classical mixing models. The advantage of the algorithm and program elabor-

ated on this basis is that they can be used relatively easily for modelling dryers of different types under simple physical conditions (Német *et al.*, 1987; Virág *et al.*, 1987).

mixing models are generally used (Pakowski and

(1979) and Virág and Halász (1989) handles the al-

gebraic, ordinary and partial differential equations

The mathematical model worked out by Virág

Mujumdar, 1987; Sazhin, 1984).

Moreover the successive improvement of the applied drying model, and the more and more precise description of hydrodynamical conditions, may be done in the frame of the same equation and numerical solution.

This article deals with the modelling of a onedimensional steady-state dryer by demonstrating the effectiveness of the integral equation approach. Attention will be paid to the simultaneous solution of the flow (mixing) and balance problems.

LITERATURE

The drying process consists of three simultaneous processes: conduction of heat and moisture in the bulk of the solid, heat and mass transfer through the solid-gas interface, and convection and conduction in the gas phase. The drying rate is limited by the rate of the slowest of these processes. In such a sense internally or externally controlled (balance) processes can be distinguished (Sazhin, 1984; Pakowski and Mujumdar, 1987). The balance problem can be solved by the use of Dalton's and Newton's laws. For the calculation of the transfer coefficients there are a lot of dimensionless equations which are based on the results of numerous experiments (Sherwood *et al.*, 1975).

[†]To whom correspondence should be addressed.

[‡]Department of Chemical Engineering, Technical University Budapest, Budapest, Hungary.

For the solution of the internal problem many empirical and numerical procedures are available (Keey, 1972; Romankov; 1987, Szentgyörgyi and Örvös, 1985). Constant external conditions are usually supposed throughout the calculation. In a real dryer this postulation is not valid due to the movement of the product and the flow of the drying gas as can be illustrated by a simple counter-current dryer. In the studied tunnel dryer the drying process can be decomposed into the following three periods (Fig. 1):

(1) The material to be dried enters the apparatus at a relatively low temperature and interacts with the warmer output drying gas of high relative humidity. As a result the gas is cooled down and its relative humidity is increased. The temperature of the material is lower than the dew-point temperature of the gas and part of the humidity is condensed. This is accompanied by an increase in the temperature of the material and its surface is wetted by the condensed steam.

(2) Inside the dryer the temperature of the material is appropriately high and the partial pressure of the surface water is higher than that of the steam in the drying gas. Therefore the surface water starts to evaporate, and the temperature changes only slightly, being about the wet-bulb temperature.

(3) The evaporation front penetrates inside the material to be dried and it is heated up.

These three periods are similar to the well-known heating, constant-rate and falling-rate periods (Keey, 1972; Luikov, 1968; Sazhin, 1984) occurring in the case of drying under constant external conditions. In a counter-current dryer, however, the decrease in the drying rate is not inevitable in the third period (it is nearly constant in the present example), as the loss of vapour partial pressure above the material to be dried (or the increase in internal resistance to heat and mass transfer) is compensated by the decrease in the humidity and increase in the temperature of the gas around it. Moreover the constant drying rate period emerges in those cases as well, when the humidity of the input material is below the critical value, because of the usual condensation at the input.

To investigate these effects one is concerned with the simultaneous solution of the flow (mixing) and balance problems. For this the following fundamental equations are used (Luikov, 1968; Sazhin, 1984):

Newton's law describing the energy transfer:

$$J_e = \alpha S(t_g - t_m). \tag{1}$$

Dalton's law for the mass transfer:

$$J_m = \frac{\beta}{R_v T_g} S(p_m - p_g). \tag{2}$$

The equations of state:

(1) the specific internal energy of the material to be dried:

$$u = c_m^* t_m + x_m c_w t_m \tag{3}$$

(2) the specific enthalpy of the drying medium:

$$h = c_g t_g + x_g c_v t_g + r x_g. \tag{4}$$

The balance equations:

(1) the mass balance:

$$-G_m \frac{\mathrm{d}x_m}{\mathrm{d}\tau} = G_g \frac{\mathrm{d}x_g}{\mathrm{d}\tau} = J_m \tag{5}$$

(2) considering the heat transfer through the walls of the dryer:

$$J_d = k_d S_d (t_g - t_{ch}) \tag{6}$$



Fig. 1. Drying periods in the tunnel dryer.

1530

(3) the energy balances:

$$G_m \frac{\mathrm{d}u}{\mathrm{d}\tau} = J_e - r J_m \tag{7}$$

and

$$-G_g \frac{\mathrm{d}h}{\mathrm{d}\tau} = J_e - r J_m + J_d. \tag{8}$$

The heat and mass transfer coefficients were calculated from the dimensionless equations. In the case of simultaneous heat and mass transfer for flat surface solid-gas systems the following equation was suggested by Sherwood *et al.* (1975) for the calculation of the transfer coefficient:

$$j_D = j_H = 0.037 R e^{0.2} \quad (8000 < Re < 300,000) \quad (9)$$

or, the way it is usually written in the literature on drying:

$$Sh = \frac{\beta l}{D} = 0.037 Sc^{0.33} Re^{0.8} \frac{P}{P - p_g}$$
(10)

and

$$Nu = \frac{\alpha l}{\lambda} = 0.037 Pr^{0.33} Re^{0.8}.$$
 (11)

In the case of the simulation of a tunnel dryer these equations were used to calculate the α and β heat and mass transfer coefficients.

Beside these equations the following relationships will be used:

The connection between the water partial pressure of vapour in the drying gas and its humidity:

$$p_g = P \frac{x_g}{Ml + x_g}.$$
 (12)

When air is used for drying Ml = 18.016/29.964 = 0.6220.

The saturation pressure was estimated by Antoine's equation (Reid *et al.*, 1977):

$$p_m = \exp[23.196 - 3816.44/(T_m - 46.13)].$$
 (13)

As is known, Newton's [eq. (1)] and Dalton's law [eq. (2)] can only be applied for stationary processes, i.e. for the period with a constant drying rate, and they do not apply in this form for the third period with a decreasing drying rate (Luikov, 1968). After some modifications according to Lebedev (1955) (Luikov, 1968) it can be applied in this period. According to the experimental data of Lebedev it is satisfactory to estimate the transfer coefficient from the following empirical equation:

$$\frac{\alpha_2}{\alpha} = \left[\frac{X_m}{(X_m)_{\rm cr}}\right]^{\sigma}.$$
 (14)

Here σ does not depend on the method and circumstances of drying. For various materials the exponent takes different values. It characterizes the bond between the water and the material, and the difference between the actual and the geometric evaporation surface. The mathematical modelling of the flows in the dryer is based on the integral equation approach developed by Virág (Virág and Halász, 1989) for the uniform mathematical treatment of the unit operations of linear flow.

Instead of the well-known second-order partial differential equation

$$\frac{\partial}{\partial \tau}\rho(z,\tau) - D\frac{\partial^2}{\partial z^2}\rho(z,\tau) + w\frac{\partial}{\partial z}\rho(z,\tau) = f(z,\tau) \quad (15)$$

one starts from the integral equation

$$\rho(z,\tau) = K(z,0)I_o + \int_0^L K(z,y) \left\{ f[\rho(y,\tau)] - \frac{\mathrm{d}}{\mathrm{d}\tau}\rho(y,\tau) \right\} \mathrm{d}y$$
(16)

which can be considered as the general mathematical model of a linear one-dimensional apparatus.

The integral equation approach can only be applied if the following conditions are satisfied (Virág and Halász, 1988):

(1) The state of the system is uniquely represented by the density distribution of the characteristic extensive parameters.

(2) To every constant input of the system there belongs an asymptotically stable equilibrium state.

(3) The stream flow of the system is linear. In a usual representation of the differential equation this means that the coefficients in the momentum balance equations are independent of the density of the other extensive parameters. In other words this means that the velocity space is independent of the transfer processes. The fulfillment of this and the next criteria is necessary in the application of all mixing models.

(4) The system is time-invariant. This means that the velocity distribution and the transfer coefficients depend only on the change of time but not on its absolute value. This, however, does not mean that the processes should be steady-state.

The parameter K(z, y) in eq. (16) is the kernel function characteristic for the flow circumstances. From a mathematical point of view it is the Green function of the boundary-value problem (15).

According to the above-mentioned conditions an asymptotically stable stationary state corresponds to every constant input of the system. In this stationary state:

$$\rho(z) = K(z, 0) I_o + \int_0^L K(z, y) f[\rho(y)] \, \mathrm{d}y.$$
 (17)

Let the $f[\rho(y)]$ input be particularly chosen as $\delta(y-z_o)$, a delta distribution centered at point z_o . This will describe the system where at site z_o a point-like source of unit intensity exists. On the basis of eq. (16)

$$\rho(z) = K(z, 0) I_o + \int_0^L K(z, y) \,\delta(y - z_o) \,\mathrm{d}y$$

= $\frac{I_o}{w} + K(z, z_o)$ (18)

i.e.

$$p(z) - K(z, 0)I_o = K(z, z_o).$$
(19)

Therefore the kernel function $K(z, z_o)$ characteristic for the flow circumstances of the system has a welldefined physical meaning: it will give the extensivedensity change at point z in the stationary state of the system when a source of unit intensity exists at point z_{e} . This fact can be used for a direct measurement of the kernel function.

If a special form of the kernel function K(z, y) is substituted into eq. (16) one of the usual mixing models is obtained (Virág and Halász, 1989). If, for example:

$$K(z, y) = \frac{1}{w} \quad (0 \le z, y \le L) \tag{20}$$

is substituted into eq. (16), the ordinary differential equation characteristic for the perfectly mixed system is obtained.

The kernel function for the plug-flow system is

$$K(z, y) = \begin{cases} \frac{1}{w} & \text{if } 0 \leq y \leq z \leq L\\ 0 & \text{if } 0 \leq z < y \leq L. \end{cases}$$
(21)

(This corresponds to a first-order partial differential equation.)

The kernel function of a diffusion model giving a second-order partial differential equation is

$$K(z, y) = \begin{cases} \frac{1}{w} & \text{if } 0 \leq y \leq z \leq L\\ \frac{1}{w} e^{\frac{Pe(z-y)}{L}} & \text{if } 0 \leq z < y \leq L. \end{cases}$$
(22)

The source function f(y) in eq. (16) represents real sources in homogeneous phases, and between heterogeneous phases it characterizes the transport of extensives.

FUNDAMENTAL POSTULATES

It is assumed that the drying process is uniquely characterized by the following extensive parameters: the mass of the material to be dried and its moisture content, the internal energy of the solid, the mass of the dry drying gas and its vapour content, and the enthalpy of the wet gas.

The sources will describe the mass and energy transfer between the material and the drying gas and the energy transfer between the gas and the wall of the dryer. When an extensive quantity input occurs not only at both ends of the dryer but at intermediate points too, these will be considered as sources at the appropriate sites.

During the drying the mass of bone-dry material and of absolutely dry drying gas can be considered constant. These substances are therefore looked upon as inert carriers and calculations are made from specific data.

Density changes in the absolutely dry part of the material as a function of temperature and moisture content (i.e. as a function of distance) can be neglected.

The flow linearity of the drying gas means that its flow rate and density are supposed to be constant along the length of the apparatus. This is the crudest physical approximation made to unify the mathematical model although a density change in the drying gas as a function of temperature is obvious along the dryer.

It is also postulated that the rate of energy and mass transfer is determined by the processes occurring in the boundary layer of the solid and gas phase. So a solution of the mentioned internal problem can be ianored.

As a consequence of this a lumped-parameter description is applied in the calculation-in other words, the temperature and moisture content inside the product is supposed to be constant and equal to the appropriate integral mean value.

It is supposed that the heat and mass transfer is described by eqs (1) and (2) where the heat and mass transfer coefficients can be calculated from the empirical equations for the Nusselt and Sherwood numbers.

SIMPLIFYING ASSUMPTIONS

The dryer will be considered to be one-dimensional, which means that the drying medium and the product flow parallel to the length of the apparatus, and in the planes perpendicular to it the temperature and the moisture content are equal and correspond to the integral mean values. This simplification is in fact not required by the applied mathematical theory but the modelling becomes easier.

The operation of most continuous dryers is steadystate so for the sake of simplicity of numerical procedures one will be concerned only with the steadystate drving.

The temperatures of the product carrier mechanism and that of the product are identical.

The vapour pressure over the product will be replaced by the saturation pressure corresponding to the temperature of the product. To making allowances for deviations occurring during the falling-rate period modifications (15) will be used.

In eq. (10) it is supposed that $P/(P - p_a) \approx 1$.

Further assumptions made in the simple illustrative example for modelling of the tunnel dryer are the following:

(1) Obviously, the product is not mixed and the kernel function of the movement is characterized by eq. (21). The mixing of the drying gas is described by the diffusion model (22).

(2) In the simulation the heat and mass transfer coefficients will be considered constant along the dryer.

Since the thermal parameters of the wall of the dryer are not known the heat transfer coefficient will be estimated as $k_d = 5 \times 10^{-3} \text{ kW/m^2/K}$.

For ceramic tiles the exponent in eq. (14), $\sigma = 2.9$, will be taken (Luikov, 1968).

THE SYSTEM OF INTEGRAL EQUATIONS

On the basis of the above assumptions the general integral eq. (16) will be simplified considerably. For density of the characteristic extensive parameters of the system the following integral equation can be written:

$$\rho_i(z) = \frac{I_{oi}}{w_i} + \int_0^L K_i(z, y) f_i[\rho(y)] \, \mathrm{d}y$$
(i = 1, 2, 3, 4, 5, 6). (23)

Divide both sides of eq. (23) by the density of the inert carrier $\rho_{ct}(z)$ at point z:

$$\frac{\rho_i(z)}{\rho_{ci}(z)} = \frac{I_{io}}{w_i} \frac{1}{\rho_{ci}(z)} + \int_0^L K_i(z, y) \frac{1}{\rho_{ci}(z)}$$
$$\times f_i[\rho(y)] dy.$$
(24)

Rearranging:

$$x_{i}(z) = x_{io} \frac{\rho_{ci}(0)}{\rho_{ci}(z)} + \int_{0}^{L} K_{i}(z, y) \frac{\rho_{ci}(y)}{\rho_{ci}(z)}$$

× F.[x(y)] dy (i = 1, 2, 3, 4). (25)

The number of the equations to be solved will be decreased from six to four because of the use of specific parameters. Due to the postulated constant value of the density of the carriers

$$\rho_{cm}(0) = \rho_{cm}(y) = \rho_{cm}(z) \tag{26}$$

$$\rho_{cg}(0) = \rho_{cg}(y) = \rho_{cg}(z). \tag{27}$$

Consequently, eq. (25) can be written as

$$x_i(z) = x_{io} + \int_0^L K_i(z, y) F_i[x(y)] \, dy \quad (i = 1, 2, 3, 4).$$
(28)

The energy density and humidity of the product and the drying media flow together and one can distinguish only two kernel functions: one for the flow of the material to be dried:

$$K_m(z, y) = K_1(z, y) = K_2(z, y)$$
 (29)

and another for the drying gas:

1

$$K_g(z, y) = K_3(z, y) = K_4(z, y).$$
 (30)

The interpretation of the sources on the basis of eqs (5), (7) and (8) is

$$F_1 := J_e / G_m - r (J_e / G_m)$$
(31)

$$F_2 := -J_m / G_m \tag{32}$$

$$F_{3} := -J_{e}/G_{g} - J_{d}/G_{g} + r(J_{m}/G_{g})$$
(33)

$$F_4 := -F_1(G_m/G_g). \tag{34}$$

In the energy balance [eq. (7)] used for the calculation of sources, the term describing the heating of the product carrier mechanism is omitted, because it was lumped into the internal-energy term for the sake of simplicity. According to the original assumptions the temperatures of the transport device and the product are identical, and for the heating up the following equation can be written:

$$(G_{cm}c_{cm}+G_mc_m)\frac{\mathrm{d}t_m}{\mathrm{d}\tau}=\frac{G_{cm}c_{cm}+G_mc_m}{G_m}G_m\frac{\mathrm{d}t_m}{\mathrm{d}\tau}.$$
 (35)

Therefore in eq. (3) the specific heat capacity is modified:

$$c_{m}^{*} = \frac{G_{cm}c_{cm} + G_{m}c_{m}}{G_{m}}.$$
 (36)

SIMULATION OF A CONTINUOUS TUNNEL DRYER

In the typology of integral equations the system of eqs (28) is called the Hammerstein-type of equations where the kernel and the source functions are separable, the source function is not linear in terms of the x variable, and the limits of integration are constant. For the solution of these equations a new and stable algorithm and program were worked out by Halász and Virág (1982).

For the illustration of the efficiency of the model and the program the heat and mass transfer processes were simulated in a dryer used in the ceramic industry. The dryer operates in the China Factory of Budapest and it is shown in Fig. 2. The size of the tunnel is $32 \times 1.14 \times 1.97$ m and it can be considered as onedimensional.

The product: tiles $(0.151 \times 0.151 \times 0.05 \text{ m})$, packed on carriages in 8×5 columns is moved through the tunnel counter-currently to the drying gas. Strictly



Fig. 2. Scheme of the tunnel dryer.

speaking the movement is not continuous—in 2 h one carriage of wet tile is introduced and one dry carriage leaves the tunnel—but the feed may be considered as quasi-continuous and the whole process as quasi-stationary.

The product is not mixed and its kernel function is represented by eq. (21). Mixing in the drying gas can be described by the diffusion model (22). The flow of the drying media in the tunnel is ensured in such a way that four input and discharge parts can be distinguished. *Pe* can be estimated as 8.

Dimensionless parameters occurring in eqs (10) and (11) were calculated for $t_g = 60^{\circ}$ C and $t_m = 100^{\circ}$ C. The air velocity, w, is 2 m/s and the typical dimension is the length of the tiles.

Physical parameters according to Nevenkin and Nachev (1982) are $P = 101 \times 10^3$ Pa, and x = 0.02 kg/kg (Table 1).

Longitudinal temperature and humidity distributions for the drying gas were measured in the dryer. Measuring holes on the wall and doors of the dryer were used for sampling. Humidity measurements were made by an ULTRAKUST Hygrophil 4455 hygrometer. It displays the wet- and dry-bulb temperatures as well as the value of the relative humidity. Using this information the absolute humidity was taken from Nevenkin and Nachev (1982). The relative humidities correspond best to the values taken at a pressure of 101 kPa. By means of installed instruments the operation of the dryer was measured for 76 h (Fig. 3).

The input data corresponding to the experiments were the following:

$S = 420 \text{ m}^2$	$t_{go} = 165^{\circ}\mathrm{C}$
$S_d = 100 \text{ m}^2$	$x_{go} = 0.007 \text{ kg/kg}$
$C_m^* = 1.5 \text{ kJ/kg/K}$	$W_g = 2000 \text{ kg/h}$
$t_{mo} = 17^{\circ}\mathrm{C}$	Pe = 8
$x_{mo} = 0.055 \text{ kg/kg}$	$\beta = 0.015 \text{ m/s}$
$x_{\rm cr} = 0.130 \rm kg/kg$	$\alpha = 0.014 \text{ kW/m}^2/^{\circ}\text{C}$
$W_m = 790 \text{ kg/h}$	$k_d = 5 \times 10^{-3} \text{ kW/m^2/°C}.$

The simulation program is very sensitive to the critical humidity value occurring in eq. (14). This depends not only on the properties of the material but also on the type of drying process. By using the experimental data this parameter was fitted by simulation experiments. Further calculations were made using the value obtained (0.130).

Table 1. Values of the dimensionless numbers and transfer coefficients

Number	Equation	Value at 60°C	Value at 100°C
Re	wlp/µ	16,064	14.541
Sc	$\mu/\rho/D$	0.553	0.553
Pr	μ/a	0.668	0.616
Sh	$0.037 \ Sc^{0.33} \ Re^{0.8}$	65.07	64.96
Nu	$0.037 Pr^{0.33} Re^{0.8}$	69.17	67.33
β	Sh D/l	0.0146	0.01795
α	Nu λ/1	0.0137	0.0150



Fig. 3. Temperature fluctuation of the gas in the dryer.

RESULTS AND DISCUSSION

Applying the above-described model numerical experiments were carried out for the simulation of the tunnel dryer. The goal was to investigate the application possibilities, to study the accuracy of the simulation, and the effect of the input parameters.

The calculated and measured values are compared in Fig. 4. For the measurement of the humidity distribution the measured value at the first point is much higher than the calculated one. It is very likely caused by the uncertainty of the 100°C humidity data. Figure 3 shows the 76-h temperature fluctuation of the dryer and the results of the simulation. As can be seen by the supposition of the steady-state mode of operation the mean value was obtained.

In order to study the effect of the input, data values were changed by 25% and numerical experiments were carried out.

Results indicate that these small changes in the transfer coefficients and the surface influence the distribution of the temperature and humidity only negligibly (Fig. 5). The deviation of Sh and Nu amounts to 0.1 and 2.7%, respectively, along the length of the dryer. Consequently, the postulation of constant values for the transfer coefficients will not cause a significant error. For other dryers this may not be true.



Fig. 4. Comparison of the calculated and measured values.



Fig. 5. Effect of the mass transfer coefficient on the temperature and humidity distribution.

No theoretical difficulty emerges if a change in the transfer coefficients is taken into consideration. In such a case, however, the program should also contain the empirical equations referring to the physical properties of the drying gas.

A change in the humidity of the drying media does not practically influence the temperature distribution; it merely leads to the translation of the humidity curve of the drying media and to a small change in the length of the first condensation period.

As can be expected, the mass flow rate strongly affects the advancement of the drying. If a large amount of drying gas is used periods (1) and (2) can be neglected. In the case of a low mass flow rate for the drying gas the condensation period will be longer, and a deep drying of the material and a decrease in the intensity of the drying will not occur (Fig. 6). A similar effect is caused by the mass flow rate of the product and the inlet temperature of the drying gas and the product.

The need to solve the internal problem is avoided by the use of the Lebedev correction (14). The choice of this model is not based on physical considerations. Its main advantage is its simplicity. When it is used in numerical experiments the results are satisfactory. It should be noted, however, that under changing external circumstances the "critical" water content occurring in eq. (14) cannot be defined in a consistent way. This parameter was determined by a fitting procedure. The change in this parameter (Fig. 7) influ-



Fig. 6. Effect of the mass flow rate of drying gas on the temperature and humidity distribution.



Fig. 7. Effect of the critical water content on the temperature and humidity distribution.

ences the course of the drying process heavily-a higher critical water content makes the distribution curve straighter and decreases the slope (i.e. the intensity of the drying). If, however, the input humidity is essentially higher than the critical value the program tends to be divergent because correction (14) gives a far too large mass transfer coefficient. In this case the program should be corrected in a way that no correction is applied over the critical water content. Other, more realistic models may also be used in a particular problem [if, for example, in eq. (13) the pressure change over the drying material is known as a function of the temperature and humidity]. For a stable numerical solution the derivatives of these relationships should be continuous according to the x and uvariables.

When certain materials (wood, ceramic) are dried the quality of the product is preserved only if the pressure and humidity gradients remain under certain limits. In such cases the solution of the internal problem is unavoidable. This problem can only be characterized by the use of the distributed parameters. The temperature and humidity distribution in the material can only be determined on this basis. Because, for a given product, the maximal value of the internal gradient corresponds to a maximal drying rate in the simulation one is not concerned with this problem. It is supposed that this maximal allowed value of the drying rate is either numerically (Szentgyörgyi and Örvös, 1985) or experimentally known for a given product. The purpose was to adjust and control the operation of the dryer by simulation in such a way that the drying intensity should be near but not exceed the maximum allowed value.

The basis of the integral equation modelling of drying is the kernel function describing the movement of drying gas and the material to be dried. The following two possibilities arise: in the case of a relatively simple motion (or in the case of new dryers) the kernel function is calculated analytically by the program. For this the mixing model of eqs (20)-(22) or other models are necessary. In the modelling of a tunnel dryer this possibility was chosen. The other alternative is based on the direct experimental determination of the kernel function. According to eqs (18), (19) and (28) this can be made on the basis of the following equation:

$$K_i(z, z_o) = x_i(z) - x_{io}.$$
 (38)

The dryer is divided into theoretical sections; each section is provided with a unit mass or energy source and the concentration or temperature changes correspondingly are measured in each section.

An important condition for using the proposed method is the hydrodynamical linearity. This means that the kernel function must be determined before solving the equations, or-in other words-the transfer processes influence the velocity distribution only negligibly. This condition is not fulfilled for large temperature changes, because of isobar volume

changes in the gas. In this case the kernel function must be determined iteratively.

The calculations prove that the developed hydrodynamically linear model is suitable for the simulation of the heat and mass transfer processes occurring in a dryer. In the modelling of a tunnel dryer the good agreement of the calculated and measured values proves the reality of the initial postulates. The numerical solution is found after five-seven iterations.

NOTATION

а	diffusivity of heat, m^2/s
с	isobar specific heat capacity,
	kJ/kg/K
c*	modified heat capacity, kJ/kg/K
D	mass diffusivity of air-steam mix-
	ture, m ² /s
D.	apparent diffusivity of axial mix-
a .x	ing. m^2/s
f	source density, $kg/m^3/s$ or kW/m^3
<i></i>	$\begin{pmatrix} 1 \end{pmatrix}$
F[x(y)] :=	$\left\{=\frac{1}{1-1}f\left\{\rho[x(y)]\right\}\right\}$ source of the
	$\langle \rho_c(y) \rangle$
	specific extensives, kg/s/kg carrier
0	or kW/kg carrier
G	mass of the absolute dry carrier, kg
$h = x_3$	specific enthalpy of the drying
	medium, kJ/kg dry air
I _o	specific input flow, kg/m ² /s or
	kW/m ²
Je	energy transfer flux, kW
J_m	mass transfer flux, kg/s
j _D	$[=(\beta/w)(p_g/P)Sc^{2/3}] \qquad (\text{Colburn's})$
	mass transfer coefficient)
j _н	$[=(\alpha/c_a\rho w)Pr^{2/3}]$ (Colburn's heat
	transfer coefficient)
K(.,.)	kernel function, s/m
k	heat transfer coefficient, $kW/m^2/K$
L	length of the dryer, m
1	typical dimension, m
Ml	ratio of the molar mass of the
	steam to the effective molar mass of
	the drying medium
$Nu (= \alpha l / \lambda)$	Nusselt number.
P	total pressure, Pa
р	partial pressure, Pa
$Pe(=wL/D_{er})$	Peclet number
Pr(=u/a)	Prandtl number
R	universal gas constant, kJ/kg/K
r	heat of evaporation, kJ/kg
$Re(=wlo/\mu)$	Reynolds number
S	surface of transfer. m^2
$\tilde{S}c (= \mu/a/D)$	Schmidt number
Sb (= Bl/D)	Sherwood number
T	absolute temperature. K
t	temperature. °C
$\frac{1}{u}(=\mathbf{r}_{1})$	specific internal energy of the prod-
- (- ~1)	uct. kJ/kg dry basis
w	mean velocity m/s
w	mass flow rate of the carrier enter-
**	ing the dryer kg/s
	mg me uryer, kg/s

1538	T. VIRÁG et al.		
x	humidity, moisture kg/kg dry basis	m solid material	
x_i	=x or the specific energy content,	o inlet	
•	kJ/kg dry basis	v vapour	
v	distance, m	w water	
z	distance, m		
		D P P P D P N C P C	
Greek letters			
α	heat transfer coefficient, $kW/m^2/K$	Halász, G. and Vírág, T., 1982, A numerical method for the	
β	mass transfer coefficient, m/s	177–179	
$\delta(y-z_o)$	$=\begin{cases} 1 & \text{if } y = z_o \\ 0 & \text{if } y = z_o \end{cases}$ delta distribution	Keey, R. B., 1972, Drying Principles and Practice. Pergamon Press, Oxford.	
λ	thermal conductivity, $kW/m/K$	Lebedev, P. D., 1955, Drying by Infra-red Rays (in Russian), pp. 132–135. Gosenergoizdat, Moscow.	
ρ	density, kg/m ³ or kJ/m ³	Luikov, A. V., 1968, Theory of Drying (in Russian). Energia,	
μ	dynamic viscosity, Pas	Moscow.	
σ	empirical exponent	ematical modelling of drying in spouted bed on the basis of	
τ	time, residence time, s	minimal energy consumption. J. appl. Chem. 59, 1920–1923.	
:-	equation of definition	Nevenkin, Sz. and Nachev, N., 1982, Thermodynamic Proper- ties of Moist Air (in Bulgarian). Technika, Sofia.	
Subscripts		Pakowski, Z. and Mujumdar, A. S., 1987, Basic process	
с	inert (absolutely dry) carrier	calculations in drying. In Handbook of Industrial Drying	
ch	referred to the channel	New York	
с т	carrier mechanism	Reid, R. C., Prausnitz, J. M. and Sherwood, T. K., 1977, The	
cr	critical	Properties of Gases and Liquids. McGraw-Hill, New York.	
d	referred to the wall of the dryer (to energy loss)	Romankov, P. G., 1987, Development of theory and practice of drying and adsorption processes (in Russian). TOHT 21, 35-43.	
g	gas	Sazhin, B. S., 1984, Fundamentals of Drying (in Russian).	
i	ith extensive property	Khimiya, Moscow.	
	i=1 for the internal energy of the product	Sherwood, K. T., Pigford, R. L. and Wilke, C. R., 1975, Mass Transfer. p. 230. McGraw-Hill, New York.	
	i=2 for the moisture content of the	Szentgyorgyi, S. and Orvos, M., 1985, An approximation	
	nroduct	content in the falling period of drving. Drving Technol. 3.	
	i=3 for the enthalow of the drying	399–419.	
	medium	Virág, T., 1979, A uniform treatment of the mathematical	
	i=4 for the humidity of the drying	models of a linear flow unit operation (in Hungarian). PhD	
	medium	Unesis, Hungarian Academy of Sciences, Budapest. Virág T and Halász G Gy 1989 Integral equation an-	
	i = 5 for the dry product	proach for simulation of heat and mass transfer processes.	
	i = 6 for the dry drying medium	Drying Technol. (in press).	
i	the ith section $(i-1, 2, M)$	Virág, T., Zhelev, J. B. and Halász, G. Gy., 1987	
1	modified according to Lebedev	Possibilities for modelling spray drying by integral equa- tions (in Russian). <i>Periodica Polytech</i> . 31 , 283–294.	