Greek Letters

| $\alpha$ | $=$ jet angle with radius in a horizontal plane |
| :--- | :--- |
| $\beta$ | $=$ jet angle with wall in a vertical plane |
| $\beta_{o}$ | $=$ temperature coefficient of volume expansion at $T_{o}$ |
| $\epsilon_{o}$ | $=$ electrostatic permittivity |
| $\theta$ | $=$ circumferential angle |
| $\mu$ | $=$ gas viscosity |
| $\mu_{r r,}, \mu_{\theta \theta}$, etc. $=$ components of eddy viscosity tensor |  |
| $\rho$ | $=$ density of suspending gas |
| $\rho_{d s}$ | $=$ density of dispersed solids |
| $\rho_{o}$ | $=$ characteristic average gas density |
| $\rho_{s}$ | $=$ density of solid particle |
| $\sigma_{r r}, \sigma_{\theta \theta}$, etc. = components of Reynolds stress tensor |  |

## Subscripts

$c \quad=$ characteristic value
$f \quad=$ relative to the gas velocity
$n \quad=$ nozzle
$r=$ radial component
$s \quad=$ solid particle
$z \quad=$ axial component
$\theta=$ circumferential component

## Superscripts and Symbols

* = dimensionless value
- $=$ temporal or ensemble average
$0 \quad=$ order of magnitude given in parentheses


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# Aeration and Mixing in Deep Tank Fermentation Systems 

Oxygen transfer characteristics for three sizes of equipment, from 76 mm to 7.6 m in diameter and for liquid depths from 4 to 21 m , correlate well and permit scaling of fermentation systems in size. Simple orifice inlets for gas flow, uniformly distributed over a tank bottom, effect rapid mixing and permit very high oxygen demands to be met. Supersaturation provides dissolved gases for the flotation and solids separation as an inherent processing procedure. Nearly uniform bulk liquid composition and a linear decrease of gas phase composition with depth permit a proper, integrated value of the oxygen saturation driving force to be employed. Results provide the design basis for large scale aeration systems which offer potential savings in both capital and energy.

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Large fermentation systems, including wastewater treatment processes, have been handicapped by the characteristics of a high oxygen demand and need for good

[^0]mixing, and in some cases by restrictions imposed by a shortage of land. Needed for the design of deep tanks which can meet these requirements is the performance of aeration systems as to type and spacing or aeration devices and predictable mass transfer characteristics under high hydrostatic heads. The trade currently considers deep tanks to be those of 8 to 9 m in height, but greater
depths are advantageous, and this study provides performance data for columns and tanks from 4 to 21 m in liquid depth.

The practical limiting liquid depths at which oxygen transfer processes become economically unfavorable are also of interest. The study covers oxygen transfer characteristics of a 76 mm diameter column to 21 m depth, a 1.8 m tank with 13 m of liquid, and a 7.6 m diameter tank at 10 m height. Transfer characteristics are com-
pared at unsteady state to water and at steady state to copper catalyzed sulfite solutions and a fermenting activated sludge mixed liquor.

No previous work has been reported describing mass transfer performance at the depths indicated or comparing transfer characteristics for an extreme range of column tank diameters, and type and number of aeration units. Scale-up of fermentation systems in size must be based on these design factors.

## CONCLUSIONS AND SIGNIFICANCE

Correlations relate the transfer factor ( $K_{L} a$ ) to ( $Q / S$ ), the ratio of air input rate to tank cross-section area, with liquid depth as the parameter. For the steady state transfer of oxygen to sulfite solution, the characteristics of a 76 mm diameter column with one air inlet orifice closely matched those for a 1.8 m diameter tank having four air inlet nozzles. Equations describing transfer were similar. Agreement over a wide range of variables, including geometry and depths to 21 m , permits performance to be assessed in a narrow column and applied to tank scale operations with confidence.

Unsteady-state transfer for oxygen to water in three sizes of vessels, $76 \mathrm{~mm}, 1.8 \mathrm{~m}$, and 7.6 m diameters, the latter with sixty-nine orifices, showed little effect of liquid depth. Also, performance for all three situations correlated with $(Q / S)$ by a single line which matches the results for steady state transfer at larger depths of 13 to 16 m . The difference between steady and unsteady state transfer is attributed to upwelling of water having a higher dissolved oxygen content for the unsteady state condition. Steady state transfer to sulfite solution is considered to more closely represent the characteristics of a fermenting medium where the dissolved oxygen content, though not zero, is constant. Use of the unsteady state correlation represents the more conservative situation.

Analysis of the deep tank situation is simplified because of the rapid mixing which occurs when air inlets are uniformly distributed over the bottom of the tank. A nearly completely mixed condition results as shown by a variation of less than 0.6 p.p.m. dissolved oxygen at 13 m depth. This permits the use of an average bulk composition for unsteady state analysis. Further, the variation in gas phase oxygen composition is shown to be essentially linear with depth for the sulfite situation, where the bulk liquid composition is zero and the transfer rate is rapid. These conditions permit the use of an integrated value of the mean concentration difference of oxygen over the depth as the proper driving force for transfer. Literature descriptions for deep tank situations have employed other methods which are not strictly correct.

The coefficient for the effect of temperature on ( $K_{\mathrm{L}} a$ ) was confirmed for bubble aeration as being $2 \% /{ }^{\circ} \mathrm{C}$.

The spacing of air inlet orifices is not a critical factor over a considerable range. The use of sixty-nine and thirty-seven nozzles for the same total air input rate in the 7.6 m tank at the different spacings did not change the transfer rate. The use of one and four nozzles in the 1.8 m tank showed similar characteristics, with those for one orifice being a little lower. These spacings, ranging from 0.8 to 1.8 m , and the very narrow $76-\mathrm{mm}$ column, gave comparable results.

The mixing action occurring in a wide tank results in the phenomenon of supersaturation of the liquid at the upper levels and unsaturation for the lower levels with respect to the total pressure. Oxygen and nitrogen gases show similar degrees of supersaturation in water which increases more rapidly than the depth. The percent above normal saturation is given by $\% \mathrm{SS}=1.50(h)^{1.35}$ which results from upwelling of the higher concentration liquid from the lower depths without sufficient time for complete desorption. Local circulation of the liquid is probably also a factor. At a 13 m depth, the percent supersaturation is nearly $50 \%$, and at 22 m it is $100 \%$ or twice the normal saturation value at the surface of the liquid. In a fermenting medium the oxygen is consumed but the nitrogen remains dissolved at the elevated levels. The nitrogen and other dissolved gases present can be employed to effect solids separation by flotation without the necessity of dissolving additional gas. The combination of the deep tank and flotation thus effects an additional economy in energy use.

A high percentage of the oxygen admitted is transferred in a deep tank system at the elevated pressures, and reductions in transfer rates from high temperatures and presence of dissolved materials have less effect relative to that for shallow basins. Economies of both capital and energy are indicated to be advantages of the deep tank approach, which requires no mechanical agitation with attendant maintenance requirements.

## ASSESSMENT OF DEEP TANK PERFORMANCE

In order to develop suitable equations to describe deep tank aeration processes, it is necessary to have information concerning the concentration distributions of oxygen in the gas and liquid phases. Gas phase compositions for transfer to water having a constant oxygen concentration, which is zero when sulfite solutions are used, are shown in Figure 1, where the distribution is linear with liquid depth. The situation is similar for transfer
to a biologically active system at a constant dissolved oxygen level. The near linearity arises because oxygen input by air is only one fifth of the gas phase and thus highly diluted, and the nitrogen content is constant. Also, only part of the gas phase oxygen content is depleted during rise. The gas phase composition cannot be strictly linear with height because of the decreasing hydrostatic head during rise and the decreasing driving force, and the depletion of oxygen in the gas phase. However, little


Fig. 1. Oxygen content of gas phase for a deep tank.
variation was observed up to 13 m in the 1.8 m diameter tank. The amount of oxygen depletion is less with transfer to water only, and the same linearity of gas composition is considered to hold.

A variation of liquid concentration with depth also affects the transfer rate and the analysis for a determination of transfer factors. In a wide tank, the rising columns of air induce an upflow of liquid near the gas phase and a downflow of bulk liquid which produces a mixing action. Figure 2 indicates the time to attain a uniform distribution of tank contents from an original temperature depth gradient. Only about 1.5 min are required to effect a uniform distribution, even though the rise of the bubbles at roughly $\mathrm{l} \mathrm{ft} / \mathrm{s}$ takes 40 s to reach the surface in this case.

In making unsteady state transfer studies it was also observed that the bulk liquid composition varied less than 0.6 p.p.m. in oxygen content from top to bottom. Although it might be expected that the much higher partial pressure of oxygen at the lower tank levels would result in a substantial variation of concentration with depth, such was not the case as shown in Figure 2. This results from liquid adjacent to the bubbles, which has


Fig. 3. Effect of temperature on ( $K_{L} a$ ) for the 1.8 m tank.
a high dissolved oxygen content, being carried upward with mixing and desorption occurring at the upper levels. As explained later, this accounts for the differences in transfer to liquid with a zero or constant oxygen content compared to that undergoing an increasing content. The concentration lines of Figure 2 also indicate that rapid mixing occurs, and a nearly complete mixed condition results in wide tanks having uniformly spaced air outlets on the tank bottom.
Observations through a manhole window near the tank bottom showed that some small bubbles remain stationary or move downward, giving evidence of circulation. Liquid dyes introduced near the manhole plate allowed visual observation and confirmed downflow of liquid in some tank regions. The dye was seen to rise rapidly when introduced in or near the rising bubble cloud and to flow down slowly where bubbles are absent. Motion pictures with an underwater camera showed the rising bubbles to spread and become relatively independent of each other.


Fig. 2. Oxygen and temperature distributions with time of aeration.
Use of the Stokes-Einstein equation for molecular diffusivity, and the surface renewal transfer model for the function of diffusivity, predicts that ( $K_{L} a$ ) increases with temperature at about $2 \% /{ }^{\circ} \mathrm{C}$ in the range of $0^{\circ}$ to $30^{\circ} \mathrm{C}$. The mass transfer factor ( $K_{L} a$ ) is customarily converted to $20^{\circ} \mathrm{C}$ to provide a reference situation, as

$$
\begin{equation*}
\left(K_{L} a\right)_{20}=\left(K_{L} a\right)_{T} \theta^{(20-T)} \tag{1}
\end{equation*}
$$

Figure 3 shows the variation of ( $K_{L} a$ ) with temperature at various air flow rates and a total liquid depth of 13 m in a 1.8 m diameter tank. The resulting parallel lines indicate a constant value of $\theta$ with air rate. The coefficient is obtained as the inverse logarithm of the common slope of the lines and has a value of 1.020 which agrees with the calculated value. All ( $K_{L} a$ ) values presented herein have been converted to $\left(K_{\mathrm{L}} a\right)_{20}$ using $\theta$ $=1.020$ and Equation (1). The value of the temperature coefficient $\theta$ for an impeller-sparger combination operation varies from 1.016 to 1.037 , and $\theta=1.020$ for bubble aeration (Eckenfelder and Ford, 1968).

Knowledge that the gas phase change with depth can be considered linear between inlet and outlet composi-
tions, and that the bulk liquid phase composition can be taken as constant at an average value, simplifies the transfer analysis and permits the development of simple models for the assessment of the transfer process.

For steady state transfer, such as to sulfite solutions with a zero dissolved oxygen content, a model was derived rigorously by Leber (1974) and reported by Urza and Jackson (1975) for minimal axial mixing:

$$
\begin{align*}
\frac{\left(y_{1}-y_{2}\right)}{\left(1-y_{1}\right)\left(1-y_{2}\right)} & +\ln \frac{y_{1}\left(1-y_{2}\right)}{y_{2}\left(1-y_{1}\right)} \\
= & \frac{6.99\left(K_{L} a\right) h^{\prime}\left(P_{b}+h / 2\right) \times 10^{-4}}{(Q / S)\left(1-y_{1}\right) H} \tag{2}
\end{align*}
$$

Use of Equation (2) requires the composition of the gas leaving the top of the tank which is readily obtained for steady state conditions. The equation may be rearranged to calculate ( $K_{L} a$ ); a correlation with ( $Q / S$ ), the superficial gas rise velocity, is indicated. In Equation (2), the saturation concentration driving force is related continuously to the gas phase composition at the total absolute pressure for the rising gas bubbles. Thus, this situation is capable of accurate evaluation subject to the assumption that ( $K_{L} a$ ) is either independent of depth or changes such that an average value may be employed.

No net nitrogen transfer occurs at steady state, which permits a calculation of the percent of the incoming oxygen transferred using oxygen and nitrogen balances to give

$$
\begin{equation*}
\% \mathrm{O}_{2} \text { transferred }=100\left(y_{1}-y_{2}\right) / y_{1}\left(1-y_{2}\right) \tag{3}
\end{equation*}
$$

Equation (3) neglects the water vapor content of the gas streams entering and leaving and is for zero carbon dioxide content in the gas at the top. A more rigorous form of the equation is employed for elevated temperatures and for the presence of carbon dioxide. Transfer factors for the unsteady state process can be developed for the situation of little mixing of column contents, which is the case for a narrow column. This requires analysis at increments of column height and knowledge of liquid compositions with depth and time. For this case, a constant ( $K_{L} a$ ) was demonstrated, except for end effects, in the 76 mm column for unsteady state transfer at heights to 15 m (Urza and Jackson, 1975).
The analysis of a tank with completely mixed contents is less complex where the bulk liquid composition is nearly uniform or varies little with depth, and an average composition suffices. The outlet gas composition is not readily measurable at unsteady state because of changing compositions with time. The rising liquid and mixing do not permit use of the incremental method to establish gas phase compositions, but the condition that the gas phase composition is linear with depth is employed.
( $K_{L} a$ ) values for unsteady state transfer to clean water can be evaluated from the rate equation for the liquid side controlled process:

$$
\begin{equation*}
(d C / d t)=\left(K_{L} a\right)\left(C_{s}-C\right)_{\text {mean }} \tag{4}
\end{equation*}
$$

The amount of oxygen gained by the liquid phase equals that lost by the gas phase and is used to determine the outlet gas concentration:

$$
\begin{equation*}
(d C / d t) V_{P}\left(10^{-6}\right)=Q\left(y_{1}-y_{2}\right) 32 / 22.4 \tag{5}
\end{equation*}
$$

The driving force of Equation (4) can be written as $\left(C_{s m}-C\right)=\left(C_{s}-C\right)_{\text {mean }}$, where the bulk liquid composition is taken as constant over the height of the tank. The integrated mean value of $C_{s}$ based on a linear gas phase composition and Henry's law constant at the operating temperature was determined and compared
to other averages (Shen, 1977):

$$
\begin{gather*}
C_{s m}=\frac{1}{H}\left[P_{b}\left(\frac{y_{1}+y_{2}}{2}\right)+\frac{h}{2}\left(\frac{2 y_{1}}{3}+\frac{y_{2}}{3}\right)\right] \\
C_{s a}=\frac{1}{H}\left[P_{b}\left(\frac{y_{1}+y_{2}}{2}\right)+\frac{h}{2}\left(y_{1}\right)\right]  \tag{6}\\
C_{s c}=\frac{1}{H}\left[P_{b}\left(\frac{y_{1}+y_{2}}{2}\right)+\frac{h}{2}\left(\frac{y_{1}+y_{2}}{2}\right)\right] \tag{8}
\end{gather*}
$$

Note that $C_{s m}=\left(y_{1} / H\right)\left(P_{b}+h / 2\right)=C_{s a}=C_{s c}$ for each of above for long term aeration ( $y_{1}=y_{2}$ ); as indicated below, water becomes supersaturated in tanks at values approaching but less than this calculated value of $C_{s m}$.

Equations (6), (7), and (8) are seen to differ only in the term which multiplies $h / 2$. Use of the three expressions in combination with Equations (4) and (5) to eliminate $y_{2}$ gives relationships from which ( $K_{L} a$ ) can be determined:
$\ln \left(B-C_{O}\right) /(B-C)=\left[\left(K_{L} a\right) /\left(1+B^{\prime} K_{L} a\right)\right] t$
$=($ Slope $) t$
Here, the slope is obtained from a plot as in Figure 8. Each expression for $\left(C_{s}-C\right)_{\text {mean }}$ gives the same value of $B=\left(P_{b}+h / 2\right) y_{1} / H$ but provides different values of $B^{\prime}=5.6110^{-6}(\rho V / H Q) X$, where

$$
\begin{gather*}
\text { for } \quad\left(C_{s m}\right), \quad X=\left(P_{b}+h / 3\right)  \tag{10}\\
\text { for } \quad\left(C_{s a}\right), \quad X=\left(P_{b}\right)  \tag{11}\\
\text { for } \quad\left(C_{s c}\right), \quad X=\left(P_{b}+h / 2\right) \tag{12}
\end{gather*}
$$

The simplicity of using the mean value for the saturation concentration indicates that arithmetic averages and middepth values, as have been employed for lesser liquid depths, are unnecessary. An example of transfer in the 1.8 m diameter tank for a depth of 13 m gives the values of $7.14,6.77$, and 7.43 corresponding to Equations (10), (11), and (12). The arithmetic average ( $C_{s a}$ ) gives a low transfer factor, and the middepth value ( $C_{s c}$ ) gives a high value. The above analysis assumes $Q$ constant, which is only approximately correct for unsteady transfer to water; the amount of oxygen transferred is not large and decreases as the bulk concentration increases.

Lister and Boon (1973) developed an equation for the mean saturation concentration and a corresponding equation for ( $K_{L} a$ ). However, a modified Henry's law constant was used which neglected the change of saturation value with liquid depth. Reported ( $\mathcal{K}_{\mathrm{L}} a$ ) values are lower than for the correct use of $C_{s m}$, and differences can be as high as $30 \%$ in some cases. The ( $K_{\mathrm{L}} a$ ) values determined were for dome diffusers in a square tank, 1.2 m on a side, and at depths to 7.5 m .

Schmit et al. (1976) report transfer factors for a tank 2 m wide and 7 m deep with lengths of 6 and 10 m and a diffuser positioned at the center of the tank. Dissolved oxygen probes were positioned at five points in the tank with only the average of the five readings reported; the variation with position was not given. Three different saturation values for computing ( $K_{L} a$ ) were considered: the middepth value, the calculated surface value from tables of oxygen solubility, and that attained upon long term aeration, with the last procedure used in the calculations. In the present tank studies, integrated mean values $C_{s m}$ were observed to be closely approximated by the long term saturation values.

For steady state transfer, the analysis is less complex, and the rate of oxygen transfer to the liquid is equated to the rate of oxygen transfer from the gas (Jackson and

Shen, 1976):

$$
\left(K_{L} a\right) V\left(C_{s m}-C\right)_{\rho}(10)^{-6}=\left(Q_{1} y_{1}-Q_{2} y_{2}\right) 32 / 22.4
$$

The nitrogen balance to eliminate $Q_{2}$ is

$$
\begin{equation*}
Q_{1}\left(1-y_{1}-y_{w 1}\right)=Q_{2}\left(1-y_{2}-y_{w 2}-y_{c}\right) \tag{14}
\end{equation*}
$$

For gas entering, $y_{w_{1}}$ can usually be taken as small and negligible, but at elevated tank temperatures, $y_{w 2}$ is a saturated value and must be considered. For fermenting systems where carbon dioxide is generated, $y_{c}$ must be included and can be measured along with $y_{2}$ or estimated from the oxygen consumed and converted to carbon dioxide. Conversely, when the correlation for $\left(K_{L} a\right)$ is known, $y_{2}$ and $Q_{2}$ can be determined from Equations (13) and (14).

## FACILITIES AND INSTRUMENTATION

The 76 mm diameter glass column has been described, Urza and Jackson (1975), with heights to 21 m , Jackson et al. (1975). A ceramic lined tank, 1.8 m in diameter by 14 m , was equipped at the bottom with a manhole and small windows for viewing and photographing and with two inlet ports for air or liquid input. The top of the tank permitted access to the center and perimeter positions. A water tight case was employed in combination with a movie camera for underwater pictures to observe the spread of the rising bubbles. Most of the performance evaluations were made with the 1.8 m tank; a limited amount of data was obtained using a tank of 7.6 m diameter and 10 m water depth.

The water for the tank studies was from wells which contained substantial amounts of dissolved oxygen; unsteady state transfer was observed by first depleting most of the oxygen by sulfite addition. Transfer in the 1.8 m diameter tank was also to sulfite solutions prepared by the addition of 136 kg of anhydrous sodium sulfite and 0.9 kg of $\mathrm{CuSO}_{4} \cdot 5 \mathrm{H}_{2} \mathrm{O}$, giving initial molarities of 0.04 for $\mathrm{SO}_{3}=$ and 0.00011 for copper.

A meter and pressure compensated dissolved oxygen probe (Yellow Springs Instrument Co.) were used for measurements at depths to 15 m . A Beckman paramagnetic oxygen analyzer provided compositions of the gas phase. Use of an inverted funnel permitted withdrawal of gas samples at various liquid depths without entrapment of liquid. A new instrument, the tensionometer, measured the partial pressure of dissolved gases in situ, D'Aoust et al. (1975). This instrument employs small diameter silastic tubing wrapped in a coil; dissolved gases diffuse to the tubing interior. A pressure transducer indicates the pressure at equilibrium conditions which is read through electrical circuitry to digital readout in millivolts. Calibration of the instrument is readily made by manometer, and it was found to be both reliable and stable in operation. The tensionometer measures the combined pressure of all the gases present which pass the membrane, in this case oxygen and nitrogen plus carbon dioxide if present. This permits a determination of the dissolved nitrogen by difference from simultaneous dissolved oxygen measurements.

Air input to deep tanks is most simply and probably best done by use of orifice outlets in the form of pipe nipples or drilled plugs. The type and number of the outlets employed are given in Table 1. Other types of aeration devices including venturi aerators of the kind used by Jackson (1964), ceramic diffusers as described by Lister and Boon (1973), and several types of airwater flow devices have been investigated as to performance under 13 m of liquid head in the 1.8 m diameter tank (Hoech, 1977). In terms of air capacity


Fig. 4. Steady state transfer in the 76 mm diameter column.


Fig. 5. Steady state transfer in the 1.8 m diameter tank.
per unit, energy efficiency, and simplicity of construction, the orifice is favored where such devices are uniformly distributed over the bottom of the tank.


Fig. 6. Unsteady state transfer factors in three vessel sizes.

## RESULTS

The use of ( $Q / S$ ) to correlate transfer characteristics is employed here for deep liquids as indicated by Equation (2). The term ( $Q / S$ ) provides a superficial gas rise velocity which is to be preferred to the more common ( $Q / V$ ) employed for shallow basins. Previous data reported by Urza and Jackson (1975) and Jackson et al. (1975) are correlated here as a function of ( $Q / S$ ) and ( $h$ ) for total water depths to 21 m , as in Figure 4, to give

$$
\begin{equation*}
\left(K_{L} a\right)_{20}=2.37(Q / S)^{1.07}(h)^{-0.45} \tag{15}
\end{equation*}
$$

Equation (15) is for steady state transfer and was obtained by use of sulfite and a copper catalyst. The two sizes of orifices employed did not show significant differences in the transfer characteristics produced.

A correlation for four liquid depths in the 1.8 m diameter tall tank for transfer at steady state is shown in Figure 5 to give

$$
\begin{equation*}
\left(K_{\mathrm{L}} a\right)_{20}=2.28(Q / S)^{1.14}(h)^{-0.55} \tag{16}
\end{equation*}
$$

It was not anticipated that transfer in the bubble column and the much wider diameter tank would be so similar, Equations (15) and (16), although Hughmark (1967) reports no wall effect on holdup for column diameters above 76 mm . Bubble size distributions produced must therefore be similar for the orifice inlets in the column and tank. The two orifices described in Figure 5 confirm the observation that orifice size is not a significant factor in bubble aeration. The work of Hoech (1977) did not show differences in transfer for a wide range of orifice sizes in a deep tank except for fine pore diffusers. Further, it was possible to simulate the bubble characteristics of the column and tank by employing a single hypodermic needle air inlet in a short column. Large bubbles were produced by the very small orifice, and the procedure was developed to predict the alpha factor for transfer when systems other than water are used.

Wall effects in the narrow column do not appear to be significant until very high ( $Q / S$ ) values are employed when slugging appears, as shown by Jackson et al.
(1975). Thus, the use of the tall column to predict large tank performance for scaling in size is indicated to be a useful and reliable procedure.


Fig. 7. Percent over saturation value at liquid surface.
Results of unsteady state transfer to water in three sizes of equipment are shown in Figure 6. The correlation obtained is

$$
\begin{equation*}
\left(K_{L} a\right)_{20}=0.53(Q / S)^{1.15} \tag{17}
\end{equation*}
$$

It is significant that the narrow column, the 1.8 m tank, and a large 7.6 m diameter tank show similar transfer characteristics, with the number of air orifices varying from one to four to sixty-nine for the three vessel sizes. Variation of the transfer factor with depth was small, with a slight negative exponent probable for $h$. It is also observed that the line in Figure 6 overlies that for the 12.9 m depth in Figure 5 and is similar to that for the 16.4 m depth in Figure 4. Thus, unsteady and steady state transfer factors are about the same for the larger depths, but whether this holds for depths greater than those indicated is not known. In any case, deep tank applications for fermentation or wastewater treatment are conducted at steady state conditions where a dissolved oxygen content is maintained at or near a constant value, and steady state conditions prevail.

Figure 6 permits the inference that very wide diameter tanks with air inlets uniformly distributed over the bottom of the tank will show the transfer characteristics at steady state indicated in Figures 4 and 5.

Long term aeration in wide tanks using distributed gas inlets leads to dissolved gas concentrations at the upper tank levels which are supersaturated with respect to the total pressures existing. Figure 7 shows the degrees of supersaturation of oxygen and nitrogen in clean water observed at various total liquid depths for a variety of temperatures and air input rates. The correlating equation is

$$
\begin{equation*}
\text { Percent supersaturation }=1.50(h)^{1.35} \tag{18}
\end{equation*}
$$

The degree of supersaturation, which is $100 \%$ at 22 m

$$
(h=10 \mathrm{~m})
$$

| Number of nozzles | $C_{s i}, \mathrm{mg} / L$ | $C_{0}, \mathrm{mg} / L$ | Q/S, $\mathrm{hr}^{-1}$ | Slope, $\mathrm{hr}^{-1}$ | T, ${ }^{\circ} \mathrm{C}$ | $\left(K_{L} a\right)_{T}, \mathrm{hr}^{-1}$ | $\left(K_{L} a\right)_{20}, \mathrm{hr}^{-1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 69 | 20.2 | 1.1 | 3.7 | 1.85 | 12.8 | 2.08 | 2.39 |
| 69 | 20.2 | 4.5 | 5.9 | 3.60 | 12.8 | 3.48 | 4.00 |
| 37 | 20.2 | 6.4 | 5.7 | 2.93 | 12.8 | 3.31 | 3.80 |

or twice the equilibrium saturation for the barometric pressure at the liquid surface, is seen to increase more rapidly than the depth. Oxygen and nitrogen are both gases of very low solubility, and although nitrogen has about half the solubility of oxygen, it is present in air at four times the oxygen concentration. Thus, nitrogen dissolves at about twice the concentration of oxygen at steady conditions, and the degree of supersaturation does not appear to be a function of solubility.

The relationship given by Equation (18) and Figure 7 appears to be general for bubble aeration systems. Included are two data points considered reliable from the work of Fujii and Fukuda (1974). Some data reported by Schmit et al. (1976) at depths of 5 to 6.4 m , calculated to percent supersaturation, lie near but below the line of Figure 7.

The supersaturation phenomenon observed is of interest when we consider dissolved gas flotation for solids separation. High levels of dissolved gases are inherent in deep tank gas-liquid aeration, where mixing is relatively rapid and additional gas need not be dissolved to effect flotation separation. Also, the dissipation of supersaturation may be required where settling of solids is employed. The gases do not normally come out of solution unless subjected to a rapid pressure drop and then may not completely attain equilibrium (Jackson and Shen, 1976).
Performance information for the 7.6 m diameter tank, though limited, confirms the above observations and scale-up considerations. Figure 8 is typical of the unsteady state procedures employed and the method of analysis using Equation (9). Table 2 gives results for three conditions of operation at unsteady state. The transfer factors agree with those for the column and smaller tank as indicated in Figure 6. The table also indicates that using about half the number of nozzles at the same air rate did not change performance.

## DISCUSSION FOR DESIGN AND APPLICATION

The use of four orifices for air discharge in the 1.8 m diameter tank and thirty-seven or sixty-nine in the 7.6 m diameter tank gave gas flows per orifice ranging from 2 to $13 \mathrm{~m}^{3} / \mathrm{hr}$. Many aeration devices operate in the range of 2 to $4 \mathrm{~m}^{3} / \mathrm{hr} /$ unit discharging. The use of the orifice permits very high gas flows per unit with effective oxygen transfer. Hoech (1977) ran a single nozzle in the 1.8 m tank at air input rates up to $34 \mathrm{~m}^{3} / \mathrm{hr}$ with transfer characteristics similar to those of Figure 6 as to slope but lying about $15 \%$ lower. Visual observations at air input velocities to about $60 \mathrm{~m}^{3} / \mathrm{hr}$ did not show evidence of slugging. Presumably, exceptionally high air input rates per orifice are possible without loss of transfer efficiency. Thus, oxygen can be supplied at very high rates per unit from a simple orifice, and high demand can be met by employing an appropriate number of air inlets. Limitations as to gas flow per unit and unit spacing were not encountered in the present study and those of Hoech, and air inlet designs are possible over a broad range of hole sizes, spacings, and air flow rates.


Fig. 8. Data for unsteady transfer, 7.6 m tank.
At steady state, as for a fixed dissolved oxygen content in a fermenting liquid for cell growth (single-cell protein production) or for wastewater treatment (activated sludge process), the air input requirements can be greatly reduced by use of the deep tank. For transfer to clean water, roughly $3 \%$ oxygen is transferred per meter of liquid depth, as pointed out by Jackson et al. (1975). The data obtained in the present deep tanks confirm this, one example of which is provided by Figure 1. Equation (3) applied to the top composition for an air input rate of $61 \mathrm{M}^{3} / \mathrm{hr}$ gives a transfer efficiency of $44 \%$ for the depth of 13 m . Previous indications in the tall column were that the percentage oxygen transferred began to drop off at 20 m , but additional heights still provide increased oxygen utilization. The presence of a finite dissolved oxygen content, dissolved substances such as surfactants, or an elevated processing temperature will reduce the transfer rate but relatively less so for deep tanks in comparison to shallow basins.

An optimization of total fixed and operating costs by Edwards et al. (1975) gave minimum costs at about 12 m depths, but with only small increases in total costs for greater depths. The example was for treatment of a pulp mill sulfite waste. The analysis showed savings in both capital and energy in comparison to an existing lagoon system treating the same wastewater.
Figures 4 and 5 show decreasing transfer factors with increasing total liquid depths. The effects of the increasing hydrostatic head and the longer rise times of the bubbles with depth are complex. For a given air input volume, the bubbles produced initially tend to be smaller with pressure increases, although not in proportion to the
pressure. During rise the bubbles both increase in size from reduced pressures and decrease in size from depletion of oxygen. Thus, the ( $a$ ) term for the transfer area of a distribution of bubble sizes is not readily predictable. Breakup is a factor initially, and coalescence may be a minor factor. This is indicated in Figure 4, where the data for the 4.1 m depth do not fit the correlation and ( $K_{L} a$ ) values are less, indicating an end effect. After an initial height of about 3 m , the bubbles have broken into smaller stable sizes and appear to be relatively unchanged thereafter. Visual and photographic observations in the tall column show that little coalescence of the bubbles occurs, and some very small bubbles exist along with a bulk size range of about 3 to 6 mm for the widest dimensions of the oscillating, elliptical shapes. It is apparent that most of the bubbles are in the mobiity range in contrast to smaller sizes acting as rigid spheres.
The effect of bubble size on the transfer coefficient $k_{L}$ is reflected through the rise velocity and bubble diameter by dimensionless numbers as quoted by Valentin (1967):

$$
\left(k_{L} d / D\right)=1.13(\mu / \rho D)^{1 / 2}(d U \rho / \mu)^{1 / 2}
$$

which reduces to

$$
\begin{equation*}
k_{L}=1.13(U D / d)^{1 / 2} \tag{19}
\end{equation*}
$$

Bubble rise velocities are nearly constant for the size range of 2 to 6 mm , and any variation in ( $k_{L}$ ) is primarily dependent upon bubble diameter. As the bubble expands from the reduced hydrostatic head more than it is depleted by oxygen transfer, the transfer coefficient decreases according to Equation (19). Thus, the transfer factor ( $K_{L} a$ ) is a complex function of initial bubble formation and size changes during rise; the result is a net decrease in value with increasing total liquid depth as indicated by Equations (15) and (16).

As to transfer rate, the effects of hydrostatic head on bubble formation and expansion, the effect of increasing bubble size which decreases ( $k_{L}$ ), the depletion of oxygen in the bubbles during rise which results in smaller bubbles and lower driving forces, and any coalescence or breakup of the bubbles all lead to a complex situation for evaluation. However, the net result is the observation that the percent oxygen transferred is close to $3 \% / \mathrm{m}$ of depth. Performance for a specific situation can be predicted by Equation (13), (14), and (16).

The effect of dissolved and suspended substances on transfer is needed. This is obtained by use of a transfer factor in Equation (13) which is for the liquid substrate employed, rather than clean water, as given by

$$
\begin{equation*}
\left(K_{L} a\right)_{\text {liquid }}=(\text { Alpha })\left(K_{L} a\right)_{\text {water }} \tag{20}
\end{equation*}
$$

The factor (alpha) normally has a value less than unity and is commonly about 0.80 to 0.85 for the activated sludge treatment of sewage wastes. Some industrial wastes show lower alpha values ranging from 0.5 to 0.7 , usually obtained only from actual experience. A simple arrangement to predict alpha values for bubble aeration systems, where the biological process is simulated in the laboratory, is described by Hoech (1977) in which he employs a section of glass pipe with a single hypodermic needle air inlet. Operation at unsteady state compares transfer to the mixed liquor of a fermentation medium to that for clean water. Values for the 7.6 m tanks were 0.63 for a pulp mill waste, Jackson and Shen (1976) with confirmation by Euhus (1977) in a tall column and by Hoech (1977) in the laboratory apparatus.

Numerous discussions appear in the literature as to the relative merits and discrepancies of steady and unsteady state processes for evaluating oxygen transfer
systems. As shown by Figures 4 to 6, depth is an important factor present in steady state transfer but less so for the unsteady state case. Also, the choice of catalyst for the sulfite oxidation and the purity of the sulfite are important factors. The desired situation for the use of sulfite is a chemical reaction sufficiently fast that the bulk composition of dissolved oxygen is maintained at a zero level but not so fast that the physical absorption rate is changed from that for water. Danckwerts (1970) indicated that these conditions will be met if the square root of the product of the reaction rate constant, the saturation concentration, and the diffusivity of oxygen is much less than the transfer coefficient for physical absorption. That this condition can be met or even calculated is difficult to assess, but with a copper catalyst it appears to be met approximately.

Urza and Jackson (1975) showed, for a tall bubble column, that the unsteady state transfer factor to water was within $10 \%$ of that for steady state transfer to sulfite solution. Jackson and Collins (1964), using a large size venturi aerator, obtained values for steady state transfer when aerating continuously to sulfite depletion, similar to those following for the unsteady state condition. Recently, Euhus (1977) in a flow column showed that transfer to water and sulfite were quite similar for a depth of 17 m . These investigations each employed copper as the catalyst. Euhus also showed that a cobalt catalyst at the same concentration and transfer conditions as for copper catalyst gave higher transfer factors by a factor of 5 . The transfer factor was also quite dependent on liquid flow rates which determine the contact time of the bubbles in the column.

The discrepancies in the literature in comparing sulfite and water as to oxygen transfer characteristics can result from many factors: choice of catalyst (cobalt should be avoided), method of aeration (bubble formation), presence of dissolved substances and impurities, difficulty of separating effects on ( $k_{L}$ ) and (a), relative liquid-gas movement (circulation) especially at unsteady state, difficulties of analysis for sulfite depletion at low concentrations, and liquid depth. At unsteady state, the absorp-tion-desorption of nitrogen has not normally been considered and can affect the gas phase composition during rise; for steady state, the net nitrogen transfer is zero, though it may absorb at the bottom and desorb at upper levels, and the changes in gas phase composition are small. It is believed that the present work has avoided many of these problems and that the sulfite method employed is a valid one upon which design for fermentation systems can be based.

Figure 6 for transfer to water shows little effect of liquid depth in contrast to Figures 4 and 5 for sulfite, where ( $K_{L} a$ ) decreases with increasing depth. Also, at the larger depths, ( $K_{L} a$ ) values are similar for the two processes. The only difference between the two situations, aside from the effect of dissolved salts which appears to be minor when sulfite is used (successive additions of sulfite as depleted and converted to sulfate did not change transfer rates), is the upwelling of liquid with the bubbles. For sulfite, circulation of a bulk liquid of zero concentration does not affect the transfer rate or the driving force. In contrast, at unsteady state, water from the bottom, having a high dissolved oxygen content from the high driving force, travels upward with the bubbles. The accompanying liquid reduces the actual concentration difference from the saturated interface because, at a given depth, the bulk liquid composition is delayed from attaining the liquid composition at the outer position of the liquid film adjacent to the bubbles. This mechanism is by eddy transport which is minimized by the presence of the sulfite but is present
for clean water. This postulate of a difference in mechanisms is not the same as that of a reduction of the molecular diffusion distance by chemical reaction. Many test situations for the latter involve a degree of turbulence greater than that for the rising bubbles in a deep liquid.

In any case, fermentation systems employ steady and constant concentrations of dissolved oxygen when operated on a continuous flow basis, and the absorption and consumption of oxygen by the active cells is similar to the situation for sulfite except that the dissolved oxygen level is finite and not zero. Performance data for transfer to water provide a more conservative approach to design, but the sulfite data permit detailed calculations, and in the event of rather deep tanks, the two situations give similar transfer characteristics. Therefore, the question of sulfite vs. clean water for performance evaluation appears moot for use of liquids at appreciable depths.

The data presented concerning rates of oxygen transfer to wide, deep tanks and the mixing necessary to keep the microbial cells in suspension permit applications of this method of conducting fermentations. An additional consideration is the supersaturation in the liquid from a deep tank withdrawn for sludge separation. A rapid pressure change causes desorption of dissolved gases to occur and microbial cells to float. Separation can be effected by flotation as an alternate to gravity settling which requires much longer times and more complex processing, especially if the dissolved gas must be dissipated. The high oxygen transfer efficiencies from long bubble rise times, the absence of mechanical action to effect mixing and oxygen transfer, the use of dissolved gas developed inherently in the deep tank for solids separation, and the ease of process control combine to reduce energy and capital costs. The performance expected has been demonstrated for the treatment of $2650 \mathrm{~m}^{3} /$ day of a whitewater from a refiner newsprint mill, Jackson and Shen (1976) and Jackson (1976).

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## NOTATION

$a \quad=$ interfacial transfer area per unit tank volume, $\mathrm{m}^{2} / \mathrm{m}^{3}$
$B, B^{\prime}=$ groupings of variables as defined by Equation (9)
$C \quad=$ average bulk concentration of oxygen in tank liquid at any position and time, $\mathrm{mg} / \mathrm{l}$.
$C_{s}=$ interfacial concentration of oxygen in liquid at any tank position and time, mg/l; $C_{s m}$, mean value over the liquid depth, $C_{s a}$, average of top and bottom values, $C_{\text {sc }}$, at middepth
$d \quad=$ characteristic bubble dimension, m
$D=$ liquid phase diffusivity, $\mathrm{m}^{2} / \mathrm{hr}$
$h$ = total liquid depth, no gas flow or holdup, $m$
$h^{\prime} \quad=$ total liquid depth with gas flow, m
$H=$ Henry's law constant, m of water ( $\mathrm{mg} / \mathrm{L}$ )
$K_{L}, k_{\mathrm{L}}=$ transfer coefficient, overall and liquid phase, $\mathrm{m} / \mathrm{hr}$
$P \quad=$ total pressure at a point in the tank, $m$ of water
$P_{b}=$ barometric pressure, m of water
$Q=$ gas flow at $0^{\circ} \mathrm{C}$ and 1 atm pressure, $\mathrm{m}^{3} / \mathrm{hr}$
$\mathrm{S}=$ cross-sectional area of tank, $\mathrm{m}^{2}$
$t=$ time, hr
$T=$ temperature, ${ }^{\circ} \mathrm{C}$
$U=$ bubble rise velocity, $\mathrm{m} / \mathrm{s}$
$V=$ total liquid volume in tank, $\mathrm{m}^{3}$
$y=$ gas stream composition at any tank position and time, for oxygen if no letter designation, mole fraction
$z \quad=$ distance from bottom of tank, m
$\mu \quad=$ liquid phase viscosity, $\mathrm{kg} / \mathrm{m}-\mathrm{s}$
$\rho \quad=$ density of water, $\mathrm{kg} / \mathrm{m}^{3}$
$\theta=$ temperature coefficient in Equation (1)

## Subscripts

$1=$ bottom of tank
$2=$ surface of liquid
$c \quad=$ carbon dioxide
$\omega=$ water vapor

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