PERGAMON

International Journal of Heat and Mass Transfer 42 (1999) 3103–3112

International Journal of HEAT and MASS TRANSFER

Natural convection over a non-isothermal vertical plate

M. Havet*, D. Blay

ENSMA, Laboratoire d'Etudes Thermiques, B.P. 109, FUTUROSCOPE, 86960 Cedex, France

Received 27 September 1998; received in revised form 28 November 1998

Abstract

In many applications, convective heat transfer is coupled with conductive and radiative heat transfer which generate temperature gradients along the walls and may greatly affect natural convective heat transfer. In this paper, the influence of the non-uniformity of wall temperature on the heat transfer by natural convection along a vertical plate having a linearly distributed temperature (characterized by the slope S) is pointed out. Heat transfer correlations giving Nusselt number vs Rayleigh number and S for both laminar and turbulent flows are obtained. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

In many processes, natural convective heat transfer along walls is coupled with conductive and radiative heat transfer. This is typically the case in radiant heating of enclosures. Radiant heating generates temperature gradients along the walls; the resulting natural convection heat transfer may be greatly affected by these heterogeneous wall temperatures. By the way, classical correlations of natural convection along an isothermal vertical wall cannot be used without a lack of precision.

Since Schmidt and Beckmann experiments in 1930 [1], the study of natural convection along a vertical flat plate has been greatly investigated due to its extensive applications in engineering like electronic cooling equipment, building application or crystal growth processes. Different boundary conditions have been studied, works on the subject have mostly dealt with constant wall temperature or uniform heat flux, the case of an isothermal wall immersed in a stratified medium has received great attention too. Gebhart et al. [2] made a complete synthesis of these studies, but concerning natural convective heat transfer along a non-isothermal wall immersed in an isothermal medium, literature is rather poor.

When the environment is isothermal and stagnant, a coordinate transformation allows to simplify the steady laminar boundary-layer equations from partial differential equations to ordinary differential equations. Sparrow and Gregg [3] gave a solution of this system in the case of exponential and power law temperature distributions. They showed the influence of the wall temperature distribution on heat transfer but they did not find a simple relation to express it. Afterwards, Yang et al. [4] found a new similarity solution of the boundary-layer equations for more complicated wall temperature distributions. Their conclusions were similar to those of Sparrow and Gregg: heat transfer is greatly affected by the wall temperature distribution. They showed that the local temperature difference between the wall and the medium (ΔT) is not sufficient to determine the local heat flux. Classical correlations

* Corresponding author. ENITIAA, Département Génie des procédés Alimentaires, Rue de la géraudière, BP 82225, 44322 Nantes Cedex, France.

0017-9310/99/$ - see front matter
PII: S0017-9310(98)00374-3

Nomenclature

g	gravitational force
Gr	Grashof number $(=g\beta\Delta Tx^3/\nu^2)$
l	Van Driest length
Nu_x	local Nusselt number
$\overline{Nu}$	average Nusselt number
Pr	Prandtl number
Pr_t	turbulent Prandtl number
Ra	Rayleigh number $(=g\beta\Delta Tx^3Pr/\nu^2)$
S	Slope of the wall temperature linear distribution
T	temperature
ΔT	characteristic temperature difference
u, v	velocity components
x, y	space coordinates

Greek symbols

α	thermal diffusivity
α_t	turbulent thermal diffusivity
β	volumetric coefficient of thermal expansion
Γ	function of transition from the laminar to the turbulent regime
δ	boundary layer thickness
ρ	density
ν	kinematic viscosity
ν_t	eddy viscosity

Subscripts

e	external zone of the boundary layer
i	internal zone of the boundary layer
iso	isothermal configuration
L	length of the wall
m	mean
max	maximum
x	location
tr	transition
∞	location away from the wall
0	location at the leading edge $(x=0)$

for a uniform value of ΔT along the wall cannot be used in such problems. Further, Semenov [5] derived the system of ordinary differential equations for all possible distributions of the wall and environment temperature leading to a similarity solution of the steady state boundary-layer equations. He chose the temperature difference at the leading edge of the wall as the characteristic temperature difference.

Similarity transformations are limited to few specific cases, so research has been investigated in order to expand available solutions to include problems with nonsimilar surface conditions. Series expansion methods were used by Yang et al. [6] in the case of exponential and sinusoidal wall temperature distribution. Kao et al. [7] developed methods of local similarity and local non-similarity for cases with non-uniform wall temperature. Other researchers conducted investigations using various methods and techniques to handle with particular configurations [8,9]. Recent papers concerning free convection in porous media used these methods to handle with power law, exponential or sinusoidal wall temperature [10,11].

These methods cannot be applied with total confidence to problems of arbitrary wall temperature distribution. From this consideration, Lee and Yovanovitch [12] developed a new approximate method to study the boundary-layer flow over a flat plate having an arbitrary surface temperature variation. They carried out the linearization of the conservation equations by introducing an effective velocity which characterizes the boundary-layer flow induced by the buoyant force. Their model is accurate even if it needs an empirical correlation to be closed.

Another alternative to study natural convection over a flat plate with an arbitrary temperature distribution is the use of numerical methods which are the most versatile for handling general boundary conditions [11,13]. Moreover, these numerical methods, either finite-difference, finite-volume or finite-element methods, can handle with turbulent boundary-layer flows.

The objective of the present work is to determine the influence of the non-uniformity of wall temperature on heat transfer by turbulent natural convection over a vertical flat plate with a linearly varying temperature distribution. This problem has not yet been studied in turbulent regime. The non-uniformity of wall temperature is defined by the slope of the temperature profile (S). We developed a numerical model based on a finite-volume formulation to solve boundary-layer equations. Turbulence was taken into account via an algebraic model. A complete parametric study was carried out both in laminar and turbulent regimes and for different slopes of the wall temperature profile. This numerical approach is of great interest for two main reasons. On one hand, it gives a better understanding on convective flow and heat transfer. On the other hand, it allows to provide easy-to-use correlations $Nu=f(Ra, S)$ in order to predict heat transfer.

2. Problem statement

The geometry and the coordinate system of the present problem are shown in Figs. 1 and 2, where a vertical plate is depicted with a linear temperature distribution. The plate is immersed in a quiet fluid which is assumed to be maintained at a uniform temperature. The velocity and temperature fields in two-dimensional, steady-state turbulent natural convective flow may be described by the set of usual boundary-layer equations, given as:

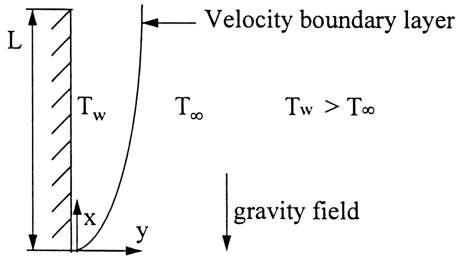

Fig. 1. Geometry and coordinate system.

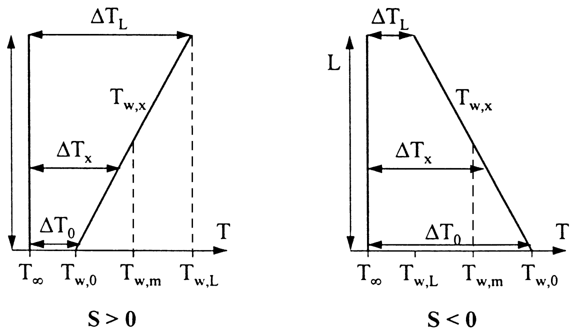

Fig. 2. Wall and ambient temperature profiles according to S.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial uu}{\partial x}+\frac{\partial vu}{\partial y}=\frac{\partial}{\partial y}\left((\nu+\nu_t)\frac{\partial u}{\partial y}\right)+g\beta(T-T_\infty) \tag{2}$$

$$\frac{\partial uT}{\partial x}+\frac{\partial vT}{\partial y}=\frac{\partial}{\partial y}\left((\alpha+\alpha_t)\frac{\partial T}{\partial y}\right) \tag{3}$$

with boundary conditions:

$y=0$: $u=0$; $v=0$; $T=T_w(x)$

$y\longrightarrow\infty$: $u\longrightarrow 0$; $v\longrightarrow 0$; $T\longrightarrow T_\infty$

$x=0$: $u=0$; $v=0$; $T=T_\infty$

$x=L$: $\frac{\partial u}{\partial x}=0$; $\frac{\partial v}{\partial x}=0$; $\frac{\partial T}{\partial x}=0$.

3. Numerical procedure

The discretization of the boundary-layer equations is based on the finite-volume formulation. We used the parabolic characteristic of these equations to solve them step by step. At the station x, the flow results from downstream conditions, u-momentum and energy conservation equations are solved using the TDMA algorithm [14]. Continuity equation is then solved to obtain the transverse velocity v.

4. Turbulence modelling

The eddy-viscosity model is used to model the Reynolds stress and the turbulent heat flux. We used an algebraic model for the turbulent viscosity. Algebraic models were developed to describe the turbulence in a forced-convection boundary-layer. The most famous are those of Cebeci-Smith [15] and Baldwin-Lomax [16]. According to Spalding and Patankar [17], those models are accurate if the variation of the mixed length with the wall distance is expressed by Van Driest's equation. New algebraic models were developed in the past decade and were satisfactorily used for solving forced convection boundary layers [18,19]. Natural-convective boundary layer flows retained less attention. We selected the model of Cebeci-Khattab [20] which is derived from Cebeci-Smith's one and the model of Mason–Seban [21]. A comparison of these models was made by the present authors [22]. The model of Mason–Seban was chosen for the present study. It is characterized by the contribution of the effective viscosity and the turbulent Prandtl number which are considered separately for the internal and external zones of the boundary-layer. For the internal zone

$$\nu_t = l_{\mathrm{i}}^2 \left| \frac{\partial u}{\partial y} \right| \Gamma_{\mathrm{tr}}$$

$$Pr_{\mathrm{t}} = 0.85$$

$$l_{\mathrm{i}} = 0.41y[1 - \exp(-y^+/26)]$$

and for the external zone

$$\nu_t = l_{\mathrm{e}}^2 \left| \frac{\partial u}{\partial y} \right| \Gamma_{\mathrm{tr}}$$

$$Pr_t = 0.50$$

$$le = 0.09\delta$$

with:

$$y^+ = \frac{yu_\tau}{\nu}$$

$$u_\tau = \sqrt{\nu\left(\frac{\partial u}{\partial y}\right)_w}$$

and δ is the distance to the wall where $u = 0.05u_{\max}$.

The limit between the internal and the external zone is chosen such as the turbulent viscosity is continuous at this point. The function Γ_{tr} describes the transition from the laminar regime ($\Gamma_{\mathrm{tr}} = 0$) to the turbulent state ($\Gamma_{\mathrm{tr}} = 1$). Expressions of this function exist for forced-convection boundary-layers [23] but the lack of data for the transition regime for natural-convection boundary-layer led us to consider a linear variation of Γ_{tr} between 0 and 1 which seems to be a good approximation [24]. According to Doan-Kim-Son [25], the transition zone extends from $Gr_x = 2 \times 10^9$ to $Gr_x = 10^{10}$.

5. Model accuracy

Using our model, we calculated the laminar natural convection boundary layer in the case of an isothermal vertical plate immersed in a stagnant isothermal med-

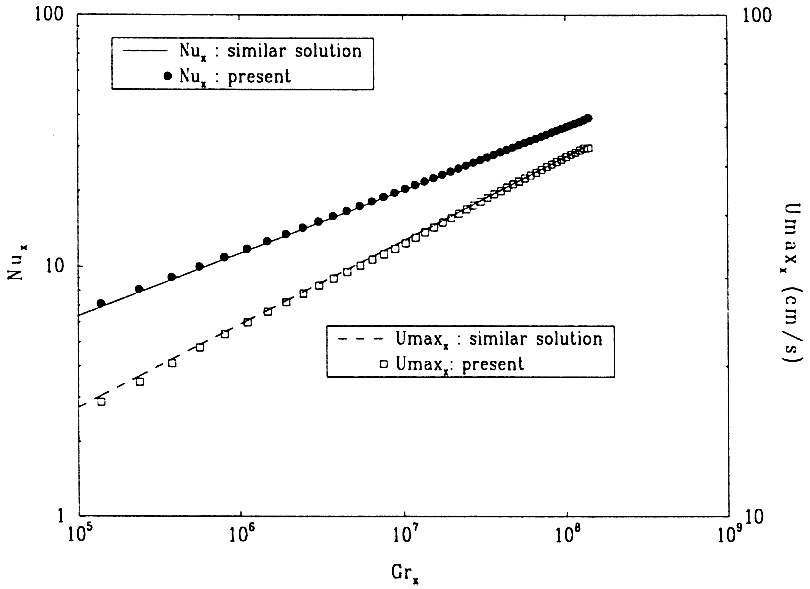

Fig. 3. Comparison of predicted Nusselt number and maximum upward velocity with similarity solutions in laminar regime.

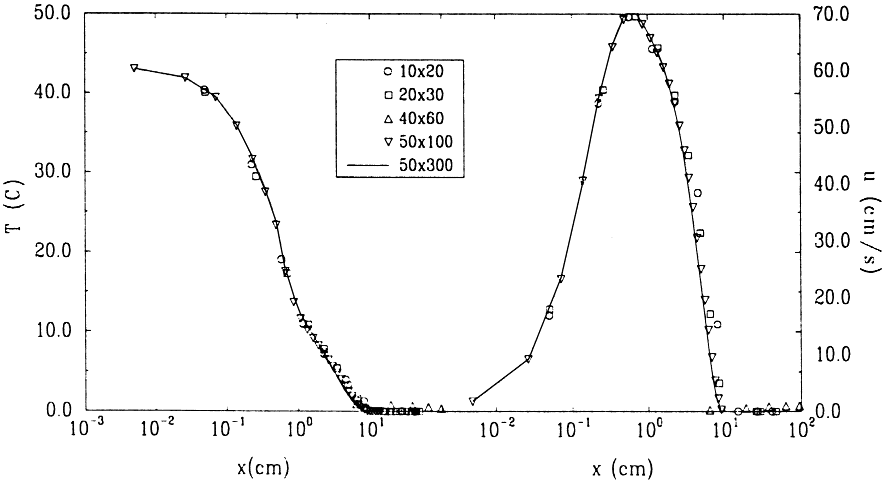

Fig. 4. Grid independence demonstration: temperature and u-velocity profiles.

ium. The comparison with similar solutions [26] is given in Fig. 3. Similar solutions are the following for $Pr = 0.72$ (case of air).

$$Nu_x = 0.357 Gr_x^{0.25} \tag{4}$$

$$u_{\max,x} = \frac{\nu\sqrt{Gr_x}}{2x}. \tag{5}$$

In laminar regime, whatever the local Grashof number is, the model agrees very well with similar solutions for both heat transfer and maximal longitudinal velocity. In turbulent regime, grid dependence was first checked. Uniform grid spacing was employed in the longitudinal direction (x) but non-uniform grid spacing was used in the transversal direction (y) where at least one grid point was located inside the viscous sublayer. Fig. 4 shows the temperature and u-velocity profiles for five different grids at $Gr_x = 4.5 \times 10^{10}$. Further results were obtained using a 40×60 mesh grid.

In turbulent regime, our results were compared with the experimental data of Doan-Kim-Son [25] and Kato et al. [27] for $Gr_x = 4.5 \times 10^{10}$. Results show that the algebraic turbulence model overestimates the maximum velocity and underestimates the heat transfer by nearly 15% (Table 1). Such results mean that the model predicts a too weak turbulent diffusion. Despite this, Figs. 5 and 6 show that the numerical profiles of velocities and temperatures are in a good agreement with the experimental measurements. Moreover, Fig. 7 shows that the slope of our numerical results (the correlation in Fig. 7) is the same as those of experimental results: so the discrepancy is not dependent on Grashof number.

Table 1
Nu_x and u-velocity ($Gr_x = 4.5 \times 10^{10}$)

Mesh x–y	20–10	30–30	60–40	Kato's experiments
Nu_x	291.8	280.6	279.1	329.0
$U_{\max}$ (cm/s)	67.59	67.63	67.50	59.0

6. Parametric study

Our main objective was to determine the influence of the non uniform wall temperature distribution on heat transfer. We focused on the case of a linearly wall temperature distribution characterized by the slope S. Both negative and positive S values were investigated

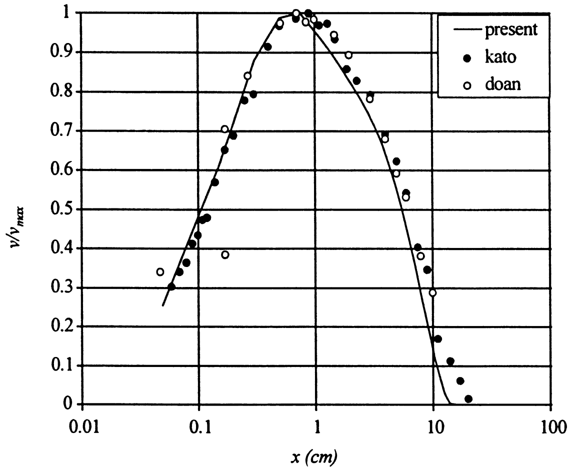

Fig. 5. Comparison of predicted u-velocity profiles with experimental results in turbulent regime ($Gr_x = 4.5 \times 10^{10}$, $S = 0$).

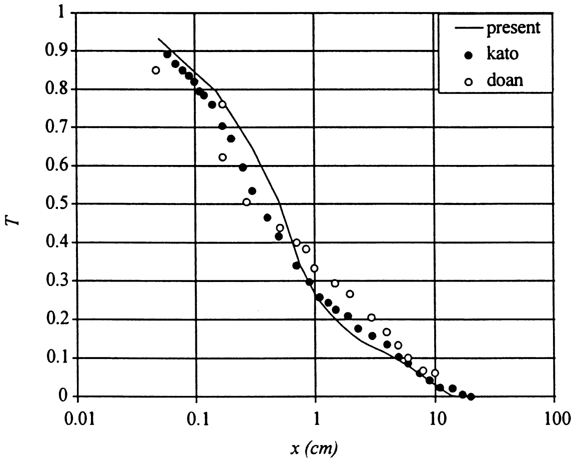

Fig. 6. Comparison of predicted temperature profiles with experimental results in turbulent regime ($Gr_x = 4.5 \times 10^{10}$, $S = 0$).

(Fig. 2). We determined the influence of this parameter on velocity and temperature profiles as well as on the Nusselt number for a large range of Rayleigh number. Nusselt and Rayleigh numbers were based on the mean temperature difference $\Delta T = (T_{w,m} - T_\infty)$.

6.1. Velocity and temperature profiles

We first studied the influence of the wall temperature distribution on the u-velocity profiles (Figs. 8 and 10) and on the temperature profiles (Figs. 9 and 11) in laminar regime ($Ra = 10^6$) and in turbulent regime

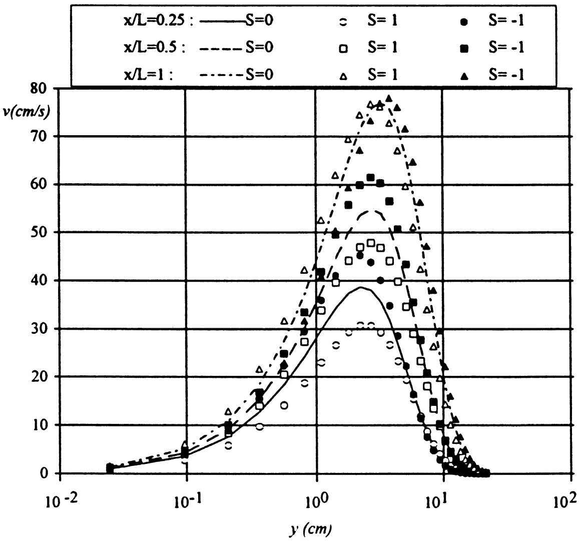

Fig. 8. Velocity profiles for different S and x-location ($Ra = 10^6$).

($Ra = 10^{10}$). For different wall temperature distributions ($S = -1$, 0, 1), we focused on the influence of S at three locations (quarter, mid-height and top of the wall).

In laminar regime ($Ra = 10^6$), Fig. 8 presents the u-velocity profiles at these three locations ($x/L = 0.25$, $x/L = 0.5$ and $x/L = 1$). As expected, the higher is the location, the greater is the velocity: in the isothermal case ($S = 0$), the maximum value of the u-velocity is respectively about 38, 54 and 77 cm s^{-1}. The influence of

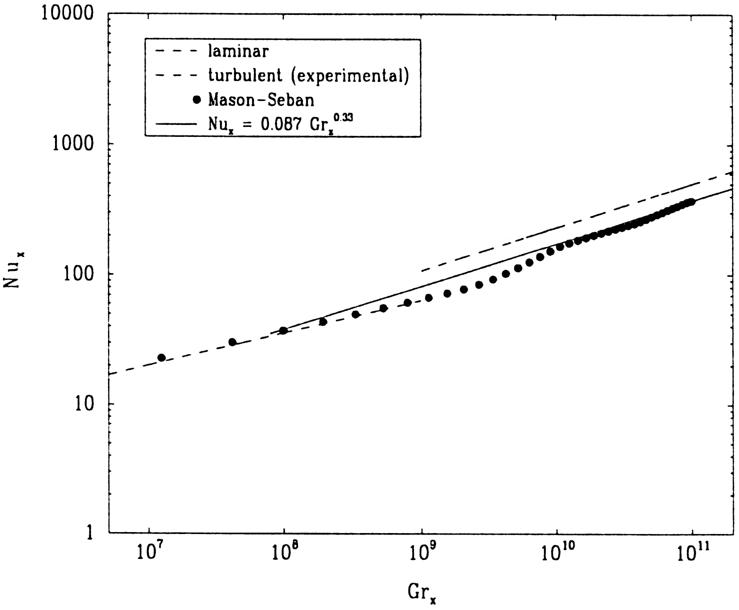

Fig. 7. Comparison of predicted Nusselt number with experimental results in turbulent regime ($S = 0$).

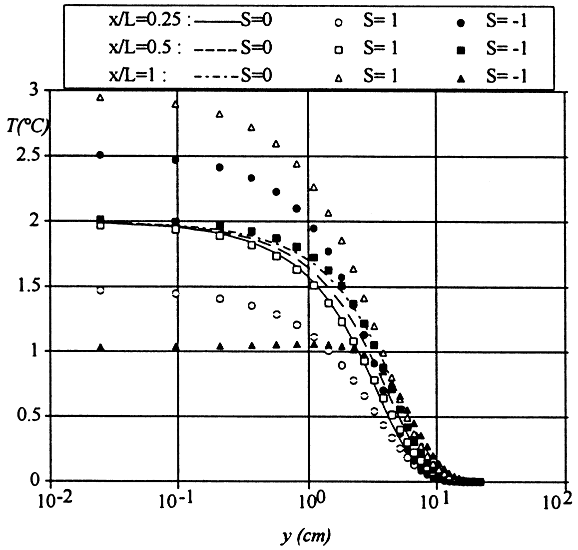

Fig. 9. Temperature profiles for different S and x-location ($Ra=10^6$).

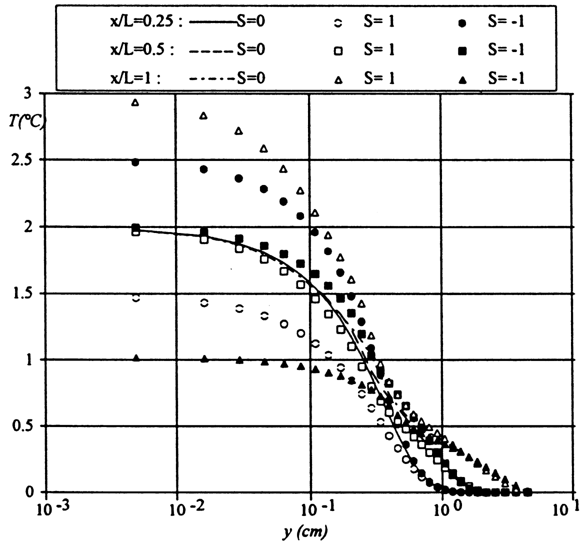

Fig. 11. Temperature profiles for different S and x-location ($Ra=10^{10}$).

the wall temperature distribution can be observed: in the lower part of the wall ($x/L=0.25$), the u-velocity is greater for a negative value of S than for the isothermal configuration ($S=0$); it is lower for a positive value of S. Fig. 9 presents the corresponding temperature profiles. In the lower part of the wall ($x/L=0.25$), the wall temperature is greater for a negative value of S than for the isothermal configuration ($S=0$); it is lower for a positive value of S. The largest temperature difference ($T_{w,x}-T_\infty$) is obtained in the case of $S=-1$, and consequently, the buoyancy force is more important in this case than in the cases $S=0$ and $S=1$. The previous remarks on the u-velocity profiles confirm this result, velocities are the lowest for $S=1$ and the greatest for $S=-1$.

At mid-height ($x/L=0.5$), the temperature profiles are not dependent on S, but u-velocity profiles depend on the wall temperature slope. Velocities are larger for negative than for positive values of S. This result is due to the fact that the u-velocity at a location x depends on the cumulative effects of the buoyancy force from the leading edge of the wall to this location.

At the top of the wall ($x/L=1$), the u-velocity pro-

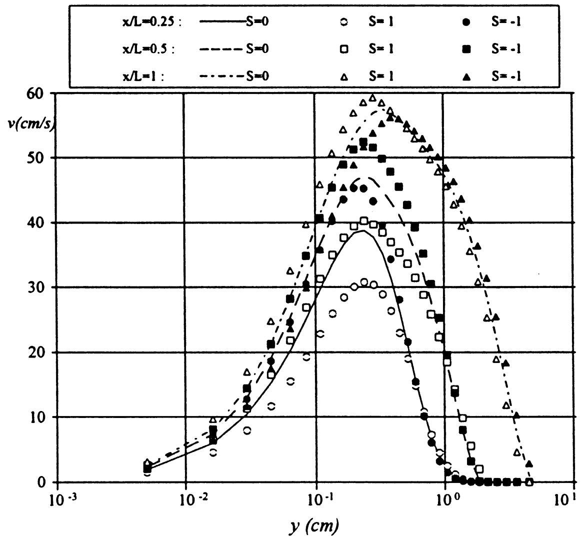

Fig. 10. Velocity profiles for different S and x-location ($Ra=10^{10}$).

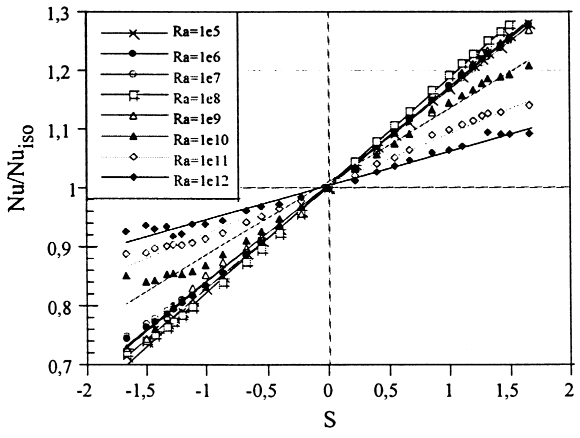

Fig. 12. Influence of S on the heat transfer.

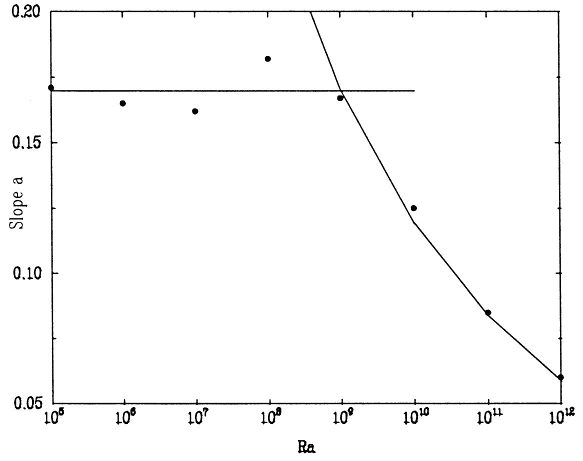

Fig. 13. Slope of the linear regression.

files are similar for $S=-1$, 0 and 1, due to identical cumulative buoyant effects. The buoyancy force integrated along the plate is equal to $\rho_0 g\beta L(T_{w,m}-T_\infty)$.

In turbulent regime ($Ra=10^{10}$), identical phenomena are observed (Figs. 10 and 11).

These results show that if the mean temperature difference $\Delta T=(T_{w,m}-T_\infty)$ is chosen as the characteristic temperature difference rather than the temperature difference at the leading edge of the wall, the buoyancy force is not dependent on S, and both Rayleigh and Nusselt numbers are based on this characteristic temperature difference [5].

6.2. Heat transfer

We studied the influence of the wall temperature distribution on the ratio of Nusselt numbers: $\frac{\overline{Nu}}{\overline{Nu}_{iso}}$ where $\overline{Nu}$ is the average Nusselt number for the present configuration, and $\overline{Nu}_{iso}$ is the average Nusselt number for the isothermal configuration at the same Rayleigh number. The use of this ratio minimizes the systematic error due to the turbulence model which, as seen before, underestimates the Nusselt number by nearly 15%. This choice also offers the advantage to be independent of Prandtl number. Results in Fig. 12 show that the higher S, the larger the heat transfer. The influence of S is rather important since heat transfer is affected by 20% for $|S|=1$. It is also worth noting that this effect is more important for laminar than for turbulent regime.

$$\frac{\overline{Nu}}{\overline{Nu}_{iso}}$$

appears to be a linear function of S:

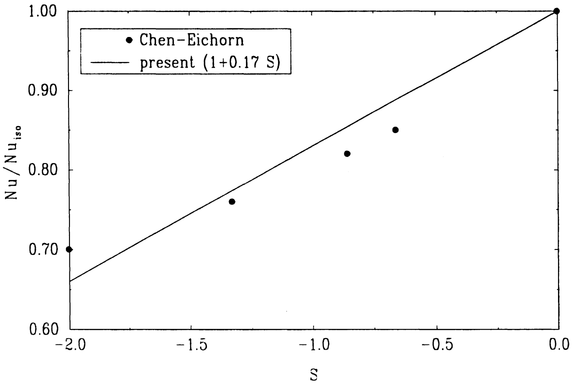

Fig. 14. Comparison with Chen and Eichorn experiments.

$$\frac{\overline{Nu}}{\overline{Nu}_{iso}}=1+aS$$

where a is a function of Rayleigh number as shown in Fig. 13.

In laminar regime, a can be considered as constant but it depends on Rayleigh number in turbulent regime. In these conditions, the influence of the wall temperature distribution on the heat transfer can be finally given by the following equations:

$$Ra\leq 2\times 10^9:\quad \frac{\overline{Nu}}{\overline{Nu}_{iso}}=1+0.17S \tag{6}$$

$$Ra\succ 2\times 10^9:\quad \frac{\overline{Nu}}{\overline{Nu}_{iso}}=1+4.15Ra^{-0.15}S. \tag{7}$$

6.3. Comparison with experimental results

As discussed earlier, Yang et al. [4] showed that heat transfer is affected in the same way by the wall temperature distribution and the medium temperature distribution. However, literature is poor concerning the influence of the wall temperature distribution. The lack of experimental results in such a configuration led us to compare our numerical results to experimental results obtained in the case of an isothermal plate immersed in a linearly stratified medium. The symmetry of both configurations allows this comparison. Chen and Eichorn [28] experimentally studied the case of natural convective heat transfer along a vertical isothermal surface immersed in a stratified medium. This is the only experimental work available in the literature; Angirasa and Srinivasan [29] numerically studied this case and their results agree with experimental results of Chen and Eichorn.

Our numerical results obtained in the case of a non isothermal plate immersed in an isothermal fluid were compared to their measurements in the case defined by: $Gr = 10^6$ and $Pr = 6$. Results in Fig. 14 show a good agreement (maximum relative error < 6%).

7. Conclusion

A numerical study of a natural convection flow along a non isothermal vertical plate immersed in an isothermal fluid was carried out. The wall temperature was characterized by a linear distribution with a slope S.

The following conclusions were obtained. First, we showed that buoyancy forces are locally affected by S; but total buoyancy forces based on mid-height temperature difference are not dependent on the slope of the temperature profile. Second, heat transfer is greatly influenced by the wall temperature distribution. This influence which is more important in laminar regime than in turbulent regime is given by the following equations:

$$Ra \leq 2 \times 10^9: \quad \frac{\overline{Nu}}{\overline{Nu}_{\text{iso}}} = 1 + 0.17S$$

$$Ra \succ 2 \times 10^9: \quad \frac{\overline{Nu}}{\overline{Nu}_{\text{iso}}} = 1 + 4.15Ra^{-0.15}S$$

where $\overline{Nu}$ is the average Nusselt number for the non isothermal case, and $\overline{Ni}_{\text{iso}}$ is the average Nusselt number for the isothermal configuration at the same Rayleigh number. In the case of a laminar flow, these numerical results were compared to Chen and Eichorn experimental results with a good agreement. So these equations can be used to easily predict the heat transfer along a non-isothermal vertical plate.

References

[1] E. Schmidt, W. Beckmann, Das temperatur und geschwendigkeitsfeld vor einer warme abgebenden senkrechten platte bei naturliche convection, Technishe Mechanik und Thermodynamik (1930) 465–472.

[2] B. Gebhart, Y. Jaluria, R. Mahajan, B. Sammakia, Buoyancy Induced-flows and Transport, Hemisphere, New York, 1988.

[3] M. Sparrow, J.L. Gregg, Similar solutions for free convection from a nonisothermal vertical plate, Transactions of the ASME 80 (1958) 379–386.

[4] K.T. Yang, J.L. Novotny, Y.S. Cheng, Laminar free convection from a nonisothermal vertical plate immersed in a temperature stratified medium, Transactions of the ASME 80 (15) (1972) 1097–1109.

[5] V.I. Semenov, Similar problems of steady-state laminar free convection on a vertical plate, Heat Transfer—Sov. Research 16 (1984) 67–85.

[6] J. Yang, D.R. Jeng, K.J. De Witt, Laminar free convection from a vertical plate with nonuniform surface conditions, Numerical Heat Transfer 5 (1982) 165–184.

[7] T.T. Kao, G.A. Domoto, H.G. Elrod, Free convection along a non isothermal vertical flat plate, ASME Journal of Heat Transfer 99 (1977) 72–78.

[8] P.C. Filho, L. Goldstein, Free convection from a vertical plate with non-uniform temperature distribution, in: National Heat Transfer Conference, Houston, U.S.A., Vol. 2, ASME, 1988, pp. 213–218.

[9] V.S. Burak, S.V. Volkov, O.G. Martynenko, P.P. Khramstsov, I.A. Shikh, Heat transfer by natural convection and hydrodynamics on a vertical plate with a discontinuity in the heat flux, Journal of Engineering Physics and Thermophysics 68 (2) (1995) 153–156.

[10] H.S. Kou, D.K. Huang, Some transformations for natural convection on a vertical flat plate embedded in porous media with prescribed wall temperature, International Communications in Heat and Mass Transfer 23 (2) (1996) 273–286.

[11] T.Y. Na, I. Pop, A note to the solution of Cheng–Minkowycz equation arising in free convection flow in porous media, International Communications in Heat and Mass Transfer 23 (5) (1996) 697–703.

[12] S. Lee, M.M. Yovanovitch, Linearization method for buoyancy induced flow over a nonisothermal vertical plate, Journal of Thermophysics and Heat Transfer 7 (1) (1993) 158–164.

[13] J.H. Merkin, I. Pop, Conjugate free convection on a vertical surface, International Journal of Heat and Mass Transfer 39 (7) (1996) 1527–1534.

[14] S.V. Patankar, Numerical Heat Transfer and Fluid Flow, Hemisphere Publishing Corp, Washington, DC, 1980.

[15] T. Cebeci, A.M.O. Smith, Analysis of Turbulent Boundary Layers, Academic Press, New York, 1974.

[16] B. Baldwin, H. Lomax, Thin layer approximation and algebraic model for separated turbulent flows, AIAA paper 78-257, 1978.

[17] D.B. Spalding, S.V. Patankar, Heat and Mass Transfer in Boundary Layers, Morgan Grambion Press, London, 1967.

[18] D.A. Johnson, L.S. King, A mathematically simple turbulence closure model for attached and separated turbulent boundary layers, AIAA Journal 23 (11) (1985).

[19] M.S. Olmcen, R.L. Simpson, Evaluation of algebraic eddy-viscosity models in three dimensional turbulent boundary layers flows, AIAA Journal 31 (9) (1993).

[20] T. Cebeci, A. Khattab, Prediction of turbulent free convective heat transfer from a vertical flat plate, Journal of Heat Transfer 97 (1975) 469–471.

[21] H.B. Mason, R.A. Seban, Numerical predictions for turbulent free convection from vertical surfaces, International Journal of Heat and Mass Transfer 17 (1974) 1329–1336.

[22] M. Havet, Etude du couplage thermique par conduction–convection–rayonnement dans une enceinte en présence d'une source radiative, Application au chauffage

de halls industriels par panneaux radiants, PhD thesis, University of Poitiers, 1995.
[23] J.M. Price, J.E. Harris, Computer program for solving compressible nonsimilar-boundary-layer equations for laminar, transitional or turbulent flows of a perfect gas, Langley report 1-8037, NASA, 1972.
[24] A.K. Kulkarni, S.L. Chou, Turbulent natural convection flow on a heated vertical wall immersed in a stratified atmosphere, ASME Journal of Heat Transfer 111 (1989) 368–384.
[25] Doan-Kim-Son, Contribution à l'étude de la zone de transition et de la zone de turbulence établie dans un écoulement de convection naturelle sur une plaque plane verticale isotherme, PhD thesis, University of Poitiers, 1977.
[26] S. Kakac, R.K. Shah, W. Aung, Handbook of Single Phase Convective Heat Transfer, John Wiley and Sons, New York, 1987.
[27] S. Kato, S. Murakami, R. Yoshie, Experimental and numerical study on natural convection with strong density variation along a heated vertical plate, in: Ninth Symposium on Turbulent Shear Flow, Kyoto, August, 1993.
[28] C.C. Chen, R. Eichorn, Natural convection from a vertical surface to a thermally stratified fluid, Journal of Heat Transfer (1976) 446–451.
[29] D. Angirasa, J. Srinivasan, Natural convection heat transfer from an isothermal vertical surface to a stable thermally stratified fluid, ASME Journal of Heat Transfer (1992) 917–923.