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A BOUNDARY ELEMENT METHOD FOR ANALYSIS OF CONTAMINANT TRANSPORT IN POROUS MEDIA—II: NON-HOMOGENEOUS POROUS MEDIA

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SUMMARY

In the second paper in the series, the boundary element method for analysing contaminant migration problems in homogeneous porous medium developed in the earlier paper by Leo and Booker[1] is extended to the non-homogeneous porous media. This extension enables potential application in practical design of landfills and in soils which cannot be idealized as being homogeneous. Conventional boundary element techniques are used to deal with non-homogeneity in that the non-homogeneous domain is approximated by a finite number of homogeneous zones. The method may be used, for instance, in assessing the effectiveness of linear configuration surrounding waste repositories. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: boundary element method; contaminant migration; mass transport; non-homogeneous porous media

1. INTRODUCTION

Present emphasis on better environmental safeguards has generally led to the introduction of more stringent regulations governing the controlled disposal and storage of waste. Therefore, in designing a waste facility, such as a landfill, it is often necessary to incorporate the use of a low permeability liner system to act as a barrier to limit the migration of the waste substances into the surrounding soil and groundwater. The characteristics of a linear barrier must be that it is

1. compatible with the stored substances,
2. of sufficient strength, and thickness to resist damage during installation and operation,
3. constructed of materials which will limit groundwater seepage and delay the migration of the contaminants from the waste disposal facility.

Many forms of liners are in use today and these include compacted clay layers, sand–bentonite mixtures, asphaltic sealants, synthetic membranes or a combination of these. The design of liners is a complex task and of great importance as the groundwater advection and contamination

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migration are relatively slow processes; thus the ramifications of the design will usually not be known for many years.

In addition to the engineered layers employed in the liner system, natural soil formation is often found to be either layered, in which case the bedding planes are nearly parallel, or consist of zones of soils, each with relatively homogeneous properties. Usually, if any of the underlying soil layer is composed of low permeability clay and the groundwater advection is small in magnitude, then molecular diffusion will be the dominant transport process. However, in a sandy aquifer the presence of comparatively high magnitude of groundwater advection normally leads to a predominance of advection and mechanical dispersion over molecular diffusion.

All the above factors give rise to non-homogeneity in the porous media in the context of contaminant migration problems. Given the inherent variability of the soil–liner system and in natural soil, a waste facility will in a number of practical cases be situated in non-homogeneous surroundings. It is important to take account of the non-homogeneity in the system, since it may otherwise lead to an inaccurate assessment of the impact of potential contamination.

In an earlier paper, Leo and Booker[1] developed a BEM method for homogeneous porous medium which cannot accommodate spatially non-homogeneous problems. The objective in this paper, the second in the series, is to address this limitation by extending the technique so that contaminant transport problems in non-homogeneous domains may also be analysed. In essence, the approach consists of dividing up the non-homogeneous domain into a finite number of homogeneous zones, each of these zones can then be treated separately and assembled together again at a later stage. This extension is an important one since it enables the BEM to be employed for many practical applications. The boundary element method requires relatively simple data input preparations but is quite capable of modeling complicated geometry. This method is therefore well suited for testing the adequacy of the liner systems surroundings the waste repository and for establishing the relative performances of different design options. The application of this technique is demonstrated by an illustrative example involving a hypothetical landfill.

2. GOVERNING EQUATIONS

A typical waste facility situated in two-dimensional non-homogeneous porous media which may be divided up into a finite number of homogeneous zones is shown schematically in Figure 1. In each zone j, the Laplace transform equation of contaminant transport for a single species of contaminant in the global (x, z) space is given by (see Reference 1),

$$(\mathbf{D}_{aj} \cdot \mathbf{\nabla}) \cdot \mathbf{\nabla} \bar{c} - \mathbf{V}_{aj} \mathbf{\nabla} \bar{c} = \Theta_j \left(\bar{c} - \frac{c_{0j}}{\Lambda_j} \right) \tag{1}$$

where for definiteness, a subscript j has been added to each of the quantities defining the properties of zone j, i.e. $\mathbf{D}_{aj} = n_j \mathbf{D}_j$ is the 'effective' hydrodynamic dispersion tensor of the contaminant species in zone j, n_j is the soil porosity of zone j, $\mathbf{D}_j$ is the hydrodynamic dispersion tensor of the contaminant species in zone j, $\mathbf{V}_{aj} = n_j \mathbf{V}_j$ is the vector of the components of the Darcy groundwater velocity in zone j, $\mathbf{V}_j$ is the vector of the true groundwater velocity in zone j and c_{0j} is the initial background concentration in zone j. $\bar{c}$ represents the Laplace transform concentration of the contaminant in the interstitial pore water and

$$\Theta_j = (n_j + \rho_j K_{dj})(s + \gamma_j^*)$$
$$\Lambda_j = s + \gamma_j^*$$

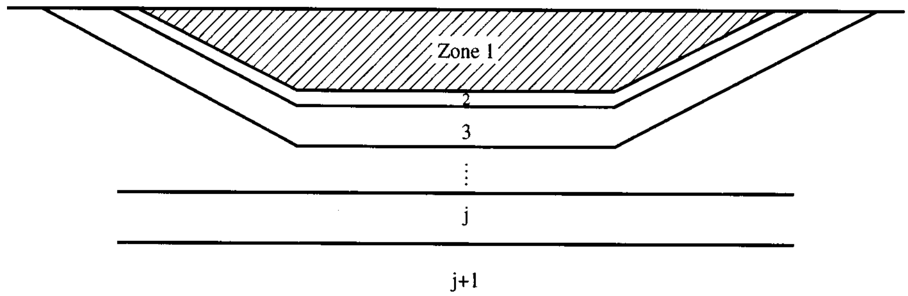

Figure 1. Schematic of a typical landfill

It is often mathematically convenient to re-write equation (1) based on the principal directions of the hydrodynamic dispersion tensor $\mathbf{D}_{aj}$. This may be accomplished by selecting a local zonal co-ordinate system $(\tilde{x}, \tilde{z})$ with reference axes which are parallel to the principal direction of $\mathbf{D}_{aj}$. The local co-ordinates and the global co-ordinates are then related as follows:

$$\tilde{x} = x\cos\theta_j + z\sin\theta_j \tag{2}$$

$$\tilde{z} = -x\sin\theta_j + z\cos\theta_j \tag{3}$$

where θ_j is the angle between the principal axes and the global co-ordinate system. Figure 2 shows the effect of a co-ordinate rotation on a point (x, z) in global co-ordinates. After rotation, the point is represented by $(\tilde{x}, \tilde{z})$ in the local co-ordinate space. The equation of contaminant transport in the local co-ordinate space of zone j may thus be written as

$$D_{a\tilde{x}\tilde{x}j}\frac{\partial^2\bar{c}}{\partial\tilde{x}^2} + D_{a\tilde{z}\tilde{z}j}\frac{\partial^2\bar{c}}{\partial\tilde{z}^2} - V_{a\tilde{x}j}\frac{\partial\bar{c}}{\partial\tilde{x}} - V_{a\tilde{z}j}\frac{\partial\bar{c}}{\partial\tilde{z}} = \Theta_j\left(\bar{c} - \frac{c_{0j}}{\Lambda_j}\right) \tag{4}$$

where $D_{a\tilde{x}\tilde{x}j}$, $D_{a\tilde{z}\tilde{z}j}$ and $V_{a\tilde{x}j}$, $V_{a\tilde{z}j}$, are the components of the 'effective' hydrodynamic dispersion and the Darcy velocity in the $\tilde{x}$, $\tilde{z}$ directions, respectively, and the normal component of the mass flux on the boundary is

$$\bar{f}_{\tilde{n}} = V_{a\tilde{n}j}\bar{c} - \left(D_{a\tilde{x}\tilde{x}j}\frac{\partial^2\bar{c}}{\partial\tilde{x}}\, l_{\tilde{x}j} + D_{a\tilde{x}\tilde{x}j}\frac{\partial^2\bar{c}}{\partial\tilde{z}}\, l_{\tilde{z}j}\right) \tag{5}$$

where $l_{\tilde{x}j}$, $l_{\tilde{z}j}$ are the direction cosines of the normal on the boundary of zone j in the local $\tilde{x}$–$\tilde{z}$ co-ordinate space. Now, let $\bar{c}_p = \sigma_j$ be a particular solution of equation (4) then $\Delta\bar{c} = \bar{c} - \sigma_j$ satisfies the equation

$$D_{a\tilde{x}\tilde{x}j}\frac{\partial^2\Delta\bar{c}}{\partial\tilde{x}^2} + D_{a\tilde{z}\tilde{z}j}\frac{\partial^2\Delta\bar{c}}{\partial\tilde{z}^2} - V_{a\tilde{x}j}\frac{\partial\Delta\bar{c}}{\partial\tilde{x}} - V_{a\tilde{z}j}\frac{\partial\Delta\bar{c}}{\partial\tilde{z}} = \Theta_j\Delta\bar{c} \tag{6}$$

and the normal component of the mass flux, $\bar{f}_{\tilde{n}\sigma j}$ on the boundary, due to the contributions of σ_j is

$$\bar{f}_{\tilde{n}\sigma j} = V_{a\tilde{n}j}\sigma_j - \left(D_{a\tilde{x}\tilde{x}j}\frac{\partial\sigma_j}{\partial\tilde{x}}\, l_{\tilde{x}j} + D_{a\tilde{z}\tilde{z}j}\frac{\partial\sigma_j}{\partial\tilde{z}}\, l_{\tilde{z}j}\right) \tag{7}$$

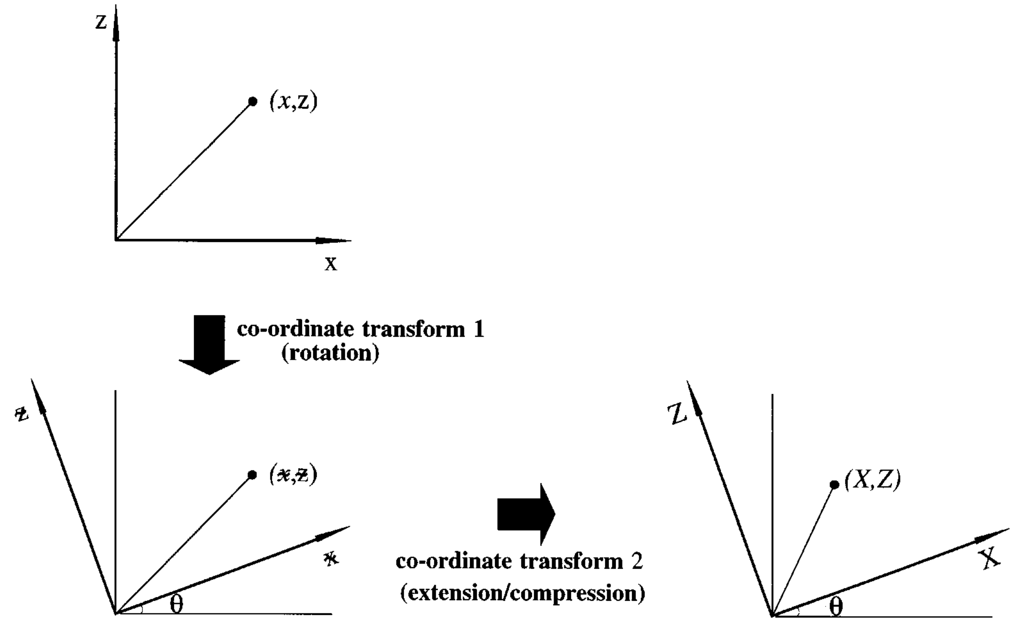

Figure 2. Co-ordinate transforms: (1) rotation; (2) extension/compression

so that

$$\Delta \bar{f}_{\tilde{n}} = \bar{f}_{\tilde{n}} - \bar{f}_{\tilde{n}\sigma j} \tag{8}$$

Assuming that spatially uniform groundwater flow occurs in each zone, but may vary in direction and magnitude from zone to zone (providing continuity conditions are satisfied), then the transformations described previously in Reference 1 may be utilized to simplify equation (6), viz. the co-ordinate transform,

$$\tilde{x} = u_j X \tag{9}$$

$$\tilde{z} = w_j Z \tag{10}$$

$$V_{aX_j} = w_j V_{a\tilde{x}j} \tag{11}$$

$$V_{aZ_j} = u_j V_{a\tilde{z}j} \tag{12}$$

$$\Delta \bar{f}_N = (w_j l_{\tilde{x}j} L_{Xj} + u_j l_{\tilde{z}j} L_{Zj}) \Delta \bar{f}_{\tilde{n}} \tag{13}$$

where L_{Xj}, L_{Zj} are the direction cosines of the normal on the boundary of zone j in the X–Z space, and

$$u_j = \left(\frac{D_{a\tilde{x}\tilde{x}j}}{D_{aj}} \right)^{1/2} \tag{14}$$

$$w_j = \left(\frac{D_{a\tilde{z}\tilde{z}j}}{D_{aj}}\right)^{1/2} \tag{15}$$

$$D_{aj} = (D_{a\tilde{x}\tilde{x}j} D_{a\tilde{z}\tilde{z}j})^{1/2} \tag{16}$$

As shown in Figure 2, a point $(\tilde{x}, \tilde{z})$ in local co-ordinates will be transformed to a point (X, Z) in the transformed space defined by equations (9) and (10). In addition, the mathematical transform given by

$$\Delta\bar{c} = \Delta\bar{c}^* \mathrm{e}^{(\varpi_j X + \lambda_j Z)} \tag{17}$$

$$\Delta\bar{f}_N = \Delta\bar{f}_N^* \mathrm{e}^{(\varpi_j X + \lambda_j Z)} \tag{18}$$

where

$$\varpi = \frac{V_{aXj}}{2D_{aj}}$$

$$\lambda_j = \frac{V_{aZj}}{2D_{aj}}$$

can be introduced to reduce equation (6) to the familiar modified Helmholtz equation

$$D_{aj}\nabla^2 \Delta\bar{c}^* = S_j \Delta\bar{c}^* \tag{19}$$

where

$$S_j = \Theta_j + D_{aj}(\varpi_j^2 + \lambda_j^2) \tag{20}$$

Thus, a boundary integral representation of equation (19) is

$$\varepsilon(\mathbf{r}_0)\Delta\bar{c}^*(\mathbf{r}_0) = \int_{\Gamma^j} (\Delta\bar{c}^* \Delta\bar{f}_N^{\#} - \Delta\bar{c}^{\#} \Delta\bar{f}_N^*)\, \mathrm{d}\Gamma \tag{21}$$

where

$$\varepsilon(\mathbf{r}_0) = \begin{cases} 1 & \text{if } \mathbf{r}_0 \text{ is within domain of zone } j \\ 0 & \text{if } \mathbf{r}_0 \text{ is outside domain of zone } j \\ \frac{1}{2} & \text{if } \mathbf{r}_0 \text{ lies on a smooth boundary of zone } j \text{ or the} \\ & \text{subtended angle } 2\pi \text{ if the boundary is not smooth} \end{cases}$$

$\mathbf{r}_0$ is the position vector of point of disturbance, Γ^j the transformed boundary of zone j in the X–Z plane, $\Delta\bar{c}^{\#} = (1/2\pi D_{aj})K_0(\sqrt{S_j/D_{aj}}\, r^{\#})$, $\Delta\bar{f}_N^{\#} = (V_{aNj}/2)\,\Delta\bar{c}^{\#} - D_{aj}\,(\partial\Delta\bar{c}^{\#}/\partial N)$, $V_{aNj} = V_{aXj}L_{Xj} + V_{aZj}L_{Zj}$ the normal component of the advection on the transformed boundary of zone j, $r^{\#} = [(X - X_0)^2 + (Z - Z_0)^2]^{1/2}$, and X_0, Z_0 the co-ordinates of the point of disturbance.

2.1. Boundary element approximation

If it is assumed that the boundary Γ^j of zone j is divided up into N_{ej} constant-valued elements with N_{ej} nodes, each at the midpoint of the boundary elements, the boundary element approximation of equation (21) thus leads to a system of simultaneous algebraic equations,

$$\mathbf{H}_j^* \Delta\bar{\mathbf{c}}^* = \mathbf{G}_j^* \Delta\bar{\mathbf{f}}_N^* \tag{22}$$

where $\Delta\bar{\mathbf{c}}^*$, $\Delta\bar{\mathbf{f}}_N^*$ are the $N_{ej}\times 1$ vectors of the nodal values of $\Delta\bar{c}^*$ and $\Delta\bar{f}_N^*$ on the boundary of zone j. As the matrices $\mathbf{H}_j^*$, $\mathbf{G}_j^*$ for zone j have to be subsequently assembled in a global system, together with the corresponding matrices of the other zones, it is convenient to express the values of concentration and normal flux with respect to their natural, untransformed co-ordinate space. This can be done using the relationships already established, viz.

$$\Delta\bar{c}^* = (\bar{c} - \sigma_j)\mathrm{e}^{-(\varpi_j X + \lambda_j Z)} \tag{23}$$

$$\Delta\bar{f}_N^* = (w_j l_{\tilde{x}j} L_{Xj} + u_j l_{\tilde{z}j} L_{Zj})(\bar{f}_{\tilde{n}} - \bar{f}_{\tilde{n}\sigma j})\mathrm{e}^{-(\varpi_j X + \lambda_j Z)} \tag{24}$$

Thus, when equations (23) and (24) are substituted into equation (22), it is found that

$$\mathbf{H}_j\bar{\mathbf{c}} = \mathbf{G}_j\bar{\mathbf{f}}_{\tilde{n}} + \mathbf{B}_j \tag{25}$$

where $\bar{\mathbf{c}}$, $\bar{\mathbf{f}}_{\tilde{n}}$ are the $N_{ej}\times 1$ vectors of the concentration $\bar{c}$ and the normal mass flux $\bar{f}_{\tilde{n}}$ on the boundary in the local natural co-ordinates and $\mathbf{B}_j$ is clearly a known $N_{ej}\times 1$ vector which arises from the particular solution $\bar{c}_p = \sigma_j$. Now since $\bar{c}$ and $\bar{f}_{\tilde{n}}$ remain invariant when the reference Cartesian co-ordinate axes are rotated through an angle θ_j (a result of applying equations (2) and (3)), it thus follows that

$$\mathbf{H}_j\bar{\mathbf{c}} = \mathbf{G}_j\bar{\mathbf{f}}_n + \mathbf{B}_j, \tag{26}$$

viz., in particular, $\bar{\mathbf{f}}_n = \bar{\mathbf{f}}_{\tilde{n}}$ where $\bar{\mathbf{f}}_n$ is the vector of the nodal values of mass flux normal to the boundary of the domain of zone j in the natural global co-ordinates.

3. FORMULATION OF GLOBAL MATRICES

The matrices $\mathbf{G}_j$ and $\mathbf{H}_j$ for each zone can now be assembled into global $\mathbf{G}$ and $\mathbf{H}$ matrices by invoking the compatibility of the concentrations and the contaminant fluxes at the interface boundaries of the zones (see e.g. Reference 2). This may be illustrated by considering a simple

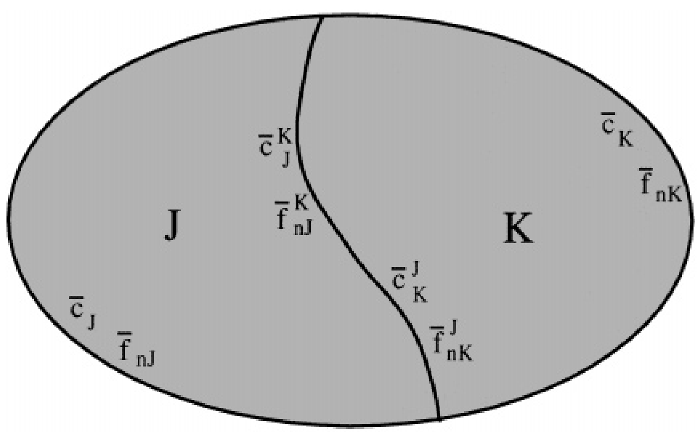

Figure 3. Adjoining zones J and K

example consisting of two zones, J and K, as shown in Figure 3. The following notations have been used:

1. $\bar{c}_J$, $\bar{\mathbf{f}}_{nJ}$, $\mathbf{B}_J$ denote the vectors of concentrations, normal fluxes and the part of $\mathbf{B}_j$, on the external boundaries of zone J, respectively,
2. $\bar{\mathbf{c}}_J^K$, $\bar{\mathbf{f}}_{nJ}^K$, $\mathbf{B}_J^K$ denote the vectors of concentrations, normal fluxes and the part of $\mathbf{B}_j$, on the J–K interface of zone J, respectively,
3. $\bar{\mathbf{c}}^K$, $\bar{\mathbf{f}}_{nK}$, $\mathbf{B}_K$ denote the vectors of concentrations, normal fluxes and the part of $\mathbf{B}_j$, on the external boundaries of zone K, respectively,
4. $\bar{\mathbf{c}}_K^J$, $\bar{\mathbf{f}}_{nJ}^J$, $\mathbf{B}_K^J$ denote the vectors of concentrations, normal fluxes and the part of $\mathbf{B}_j$, on the K–J interface of zone K, respectively.

In zone J, the system of equations in (26) can be written as

$$\begin{bmatrix} \mathbf{H}_J^{11} & \mathbf{H}_J^{12} \\ \mathbf{H}_J^{21} & \mathbf{H}_J^{22} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{c}}_J \\ \bar{\mathbf{c}}_J^K \end{bmatrix} = \begin{bmatrix} \mathbf{G}_J^{11} & \mathbf{G}_J^{12} \\ \mathbf{G}_J^{21} & \mathbf{G}_J^{22} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{f}}_{nJ} \\ \bar{\mathbf{f}}_{nJ}^K \end{bmatrix} + \begin{bmatrix} \mathbf{B}_J \\ \mathbf{B}_J^K \end{bmatrix} \tag{27}$$

and for zone K,

$$\begin{bmatrix} \mathbf{H}_K^{11} & \mathbf{H}_K^{12} \\ \mathbf{H}_K^{21} & \mathbf{H}_K^{22} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{c}}_K \\ \bar{\mathbf{c}}_K^J \end{bmatrix} = \begin{bmatrix} \mathbf{G}_K^{11} & \mathbf{G}_K^{12} \\ \mathbf{G}_K^{21} & \mathbf{G}_K^{22} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{f}}_{nK} \\ \bar{\mathbf{f}}_{nK}^J \end{bmatrix} + \begin{bmatrix} \mathbf{B}_K \\ \mathbf{B}_K^J \end{bmatrix} \tag{28}$$

The compatibility conditions are

$$\bar{\mathbf{c}}_J^K = \bar{\mathbf{c}}_K^J \tag{29}$$

$$\bar{\mathbf{f}}_{nJ}^K = -\bar{\mathbf{f}}_{nK}^J \tag{30}$$

The boundary value problem will be well posed if either the values of concentration or the normal mass flux are known on the external boundary. There will therefore be sufficient number of equations to solve for the unknown concentrations and normal mass fluxes on the external boundaries and the interfaces. To illustrate the procedure, suppose that the values of normal component of the mass flux are prescribed on the external boundaries of zone J and K. It follows from equations (27), (28) and the compatibility conditions (29) and (30) that

$$\begin{bmatrix} \mathbf{H}_J^{11} & \mathbf{H}_J^{12} & -\mathbf{G}_J^{12} & 0 \\ \mathbf{H}_J^{21} & \mathbf{H}_J^{22} & -\mathbf{G}_J^{22} & 0 \\ 0 & \mathbf{H}_K^{12} & \mathbf{G}_K^{12} & \mathbf{H}_K^{11} \\ 0 & \mathbf{H}_K^{22} & \mathbf{G}_K^{22} & \mathbf{H}_K^{21} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{c}}^J \\ \bar{\mathbf{c}}_J^K \\ \bar{\mathbf{f}}_{nJ}^K \\ \bar{\mathbf{c}}_K \end{bmatrix} = \begin{bmatrix} \mathbf{G}_J^{11}\bar{f}_{nJ} + B_J \\ \mathbf{G}_J^{21}\bar{f}_{nJ} + B_J^K \\ \mathbf{G}_K^{11}\bar{f}_{nK} + B_K^J \\ \mathbf{G}_K^{21}\bar{f}_{nK} + B_K \end{bmatrix} \tag{31}$$

The right-hand side of equation (31) is completely known, thus it can be utilized to find the unknown values of concentration on the external boundaries and the unknown values of mass flux and concentration on the interfaces. Once concentrations and the normal fluxes are completely known on the external boundaries and interfaces, the concentration $\bar{c}^*$ at any internal point within any zone may then be found, from the discretized form of equation (21) and using the relationship in equation (17). The real concentration in time domain is finally found by applying the inversion algorithm of Talbot.[3] Other forms of boundary conditions can be similarly accommodated as can situations in which there are more than one region.

4. APPLICATION

4.1. Verification

4.1.1. Test Problem 1. Two test problems which have been used for verifying the accuracy of the boundary integral equation solutions are presented here. The first test compares the boundary integral equation solutions for a one-dimensional situations with the semi-analytical results from Rowe and Booker.[4] The problem consists of a surface landfill underlain by two layers of 4 m deep clayey soil which is further underlain by a 2 m aquifer stratum (Figure 4). At the upper boundary, the landfill is assumed to contain a finite mass of leachate so that the concentration of the contaminant within it will diminish with time as contaminant is transported into the underlying soil. Beneath the aquifer is a deep layer of impervious soil and the contaminant flux across this boundary interface is assumed to be zero. A total of 180 boundary elements, with each zone consisting of about 60 elements, were used to model this problem. Shown in Figure 4 are the groundwater flow conditions and soil properties (in inset) as well as the contamination profile with depth z at 100 and 500 years. The effect of non-homogeneity in the domain is clearly evident, particularly in the concentration profile at 500 years, where a distinct kink appears at the interface of zones 1 and 2. The BEM is able to model this accurately as reflected in the good agreement between the numerical results and the semi-analytic solutions presented in Figure 4.

4.1.2. Test Problem 2. The second test problem involves a deeply buried cylindrical source located in infinite porous media (Figure 5) and is designed to test the numerical solutions of contaminant transport in a two-dimensional situation. The source is assumed to be surrounded by annular homogeneous isotropic soil layers, each of the surrounding annular layers is assumed

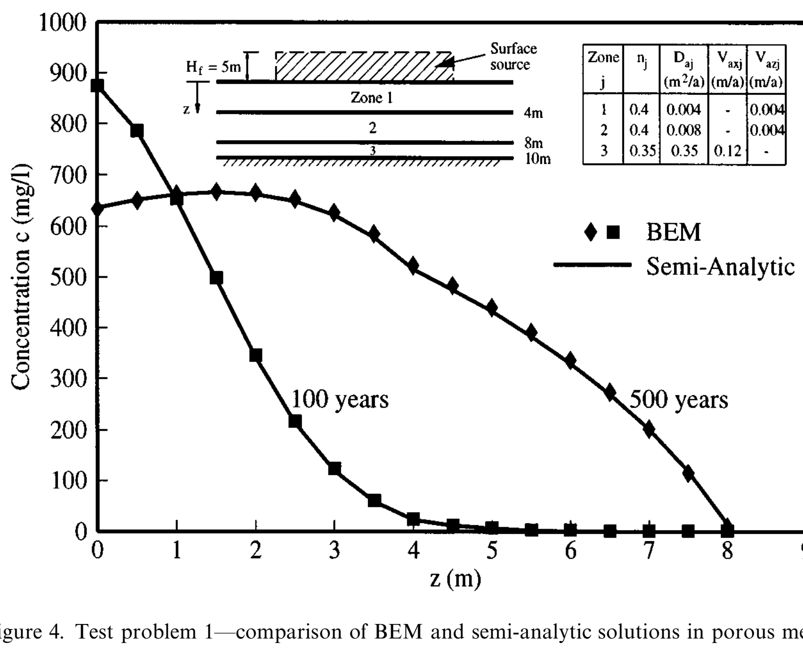

Figure 4. Test problem 1—comparison of BEM and semi-analytic solutions in porous media

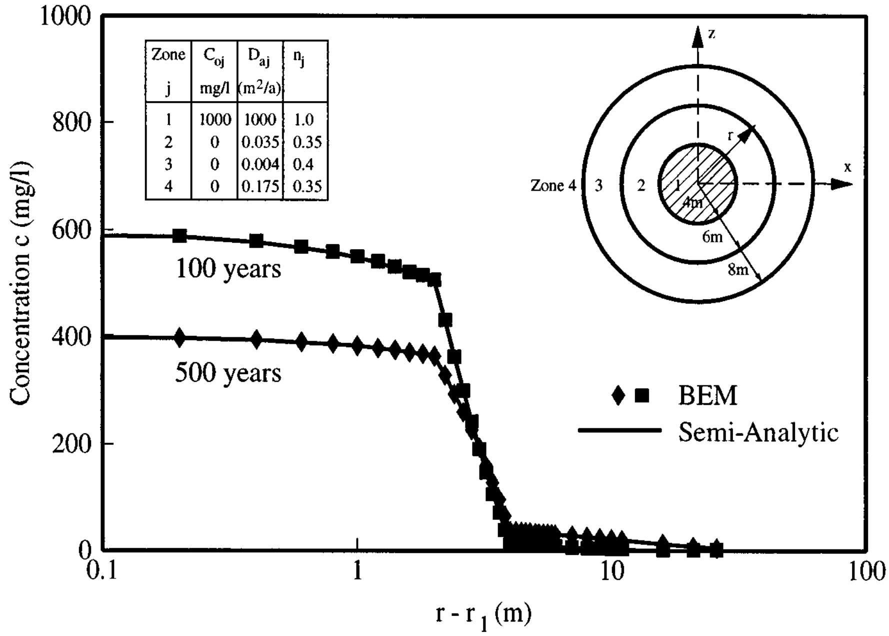

Figure 5. Test problem 2—comparison of BEM and semi-analytic solutions of deeply buried cylindrical repository in porous media (no advection)

to be made up of a different soil material which has geological properties shown in the inset to Figure 5. The concentration at the infinite boundary of the porous media is assumed to vanish. It is assumed that no advection is present. For this case, the discretization involves the use of a total of 200 elements and within each zone about 80 elements have been used. The contamination profile in the radial direction is shown at elapsed times of 100 and 500 years. Comparing the numerical solutions and the semi-analytic solutions developed in the paper by Leo and Booker[5] in Figure 5, it is apparent that there is an excellent agreement.

4.2. Illustrative example

In the following example, the boundary element technique developed in this paper will be used to assess the design of a hypothetical sanitary landfill which is proposed in a generally clayey stratum 8 m deep further underlain by a 2 m deep confined aquifer (shown in Figure 6). It will be assumed that beneath the aquifer is a deep layer of highly impermeable soil and that no flux exchange occurs between this layer and the aquifer. The proposed landfill is long, partially embedded in the stratum of clay, the base length is 50 m, the proposed depth of landfilling is 4 m and the side slopes are 3H : 1V. In the earlier paper,[6,1] a lumped-parameter model (e.g. Reference 7), has been used to simulate the concentrations in the waste repository. The basic assumption made in this kind of model is that the diffusion coefficient of the contaminant in the repository is very large so that the concentration within it will be spatially constant although diminishing with time.

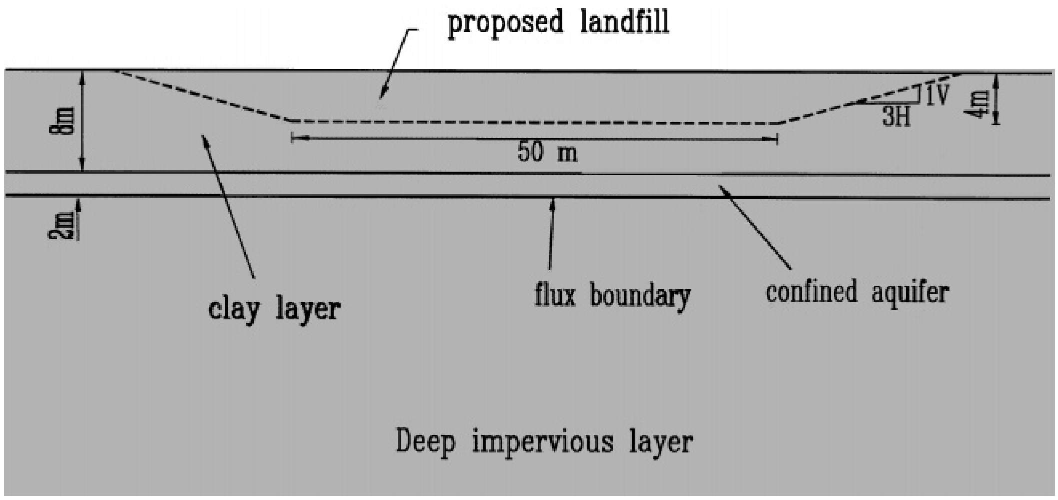

Figure 6. Soil profile at location of proposed landfill

In contrast, the more general boundary element method presented in this paper allows the use of any initial distribution of the contaminant as long as a particular solution corresponding to the initial distribution can be found. For the purpose of the illustration here, it will be assumed that the background distribution is initially a constant $c_0 = 1000$ mg/l in the landfill and zero elsewhere. The mass of the contaminant in the landfill is thus given by the product of the volume of the landfill, the porosity of the landfill (assumed = 1·0 in this case) and c_0. A finite diffusion value of 0·5 m^2/a has been assumed in the repository zone which is treated just like any other soil material but if the value of this diffusion coefficient is increased to a very large value it is found that the repository behaviour approaches that of a lumped-parameter model.

As previously mentioned, the migration of contaminants from the landfill into a natural groundwater regime is a potential environmental hazard which requires careful analysis. Assuming that environmental regulations pertaining to waste containment do not permit contaminant plume (defined as the contour of concentration 100 mg/l) to infiltrate the aquifer within 100 years, two design options will now be considered: (1) no liners used and (2) a composite linear system consisting of a leachate collection system with a geomembrane underlain by a 1 m layer of clay liner. The properties of the landfill, liner, soil, aquifer and the values of the groundwater flow velocities for the different zones are summarized in Table I. To be specific, the coefficients of hydrodynamic dispersion in the soil and liner have been evaluated using the equation

$$D_{kl} = (D_0 + \alpha_T V)\delta_{kl} + (\alpha_L - \alpha_T)\frac{V_k V_l}{V} \tag{32}$$

where D_0 is the isotropic coefficient of molecular diffusion, α_L, α_T are the longitudinal and transverse dispersivities, δ_{kl} is Kronecker's delta, V_k, V_l are the components of the average groundwater velocity, V is the magnitude of the average groundwater velocity and the set (k, l) range over the index set (x, z).

Table I. Illustrative example—properties of zones

Domain	D_{0j} (m^2/a)	α_{Tj} (m)	α_{Lj} (m)	n_j	$\rho_j K_{dj}$	V_{axj} (m/a)	V_{azj} (m/a)
Landfill	0·5	0	0	1·0	0	0	0
Liner (case (1b))	0·02	0·05	0·5	0·4	10	0	0·02
Clayey soil (aquitard)	0·05	0·1	1·0	0·4	0	0	0·02
Aquifer	1·0	1·0	10·0	0·35	0	2·8	0

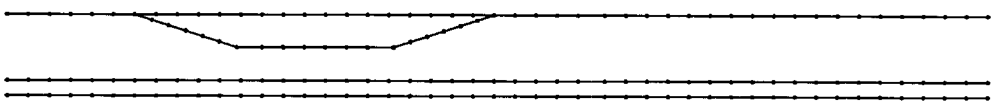

Figure 7. A typical schematic representation of boundary element mesh

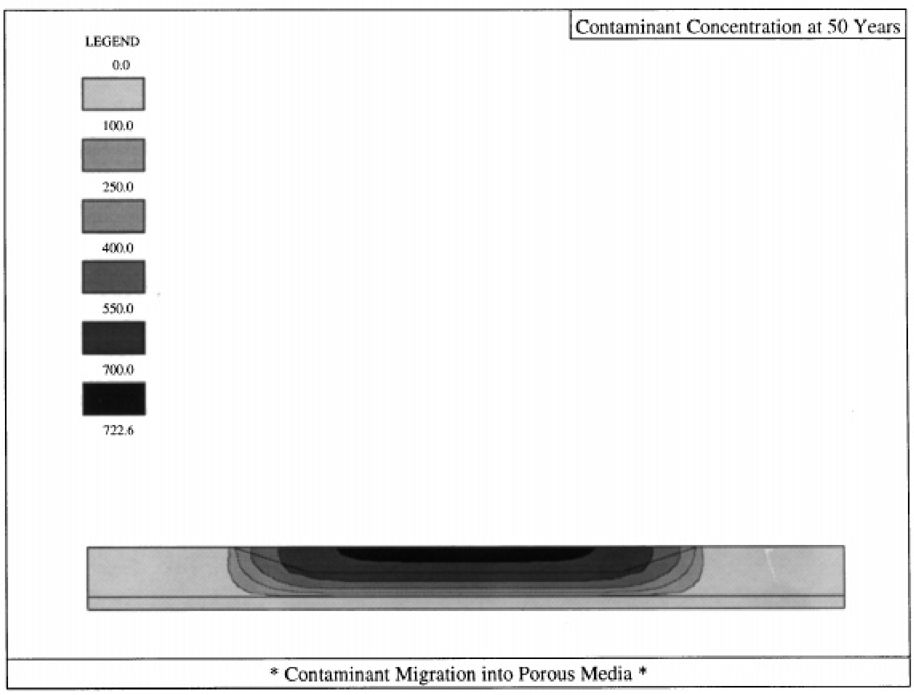

Figure 8. Design case (a)—contaminant distribution at 50 years

For the purpose of illustration, contamination contours have been presented for times at 50 and 100 years. The ground surface and the boundaries at infinity are specified as having zero concentration throughout the period of simulation. It has been assumed that a total head difference of about 2·5 m exist between the base of the landfill and the top of the aquifer, the hydraulic conductivity of the soil in the aquitard or the clay liner is of the order of 10^{-7} cm/s, thus

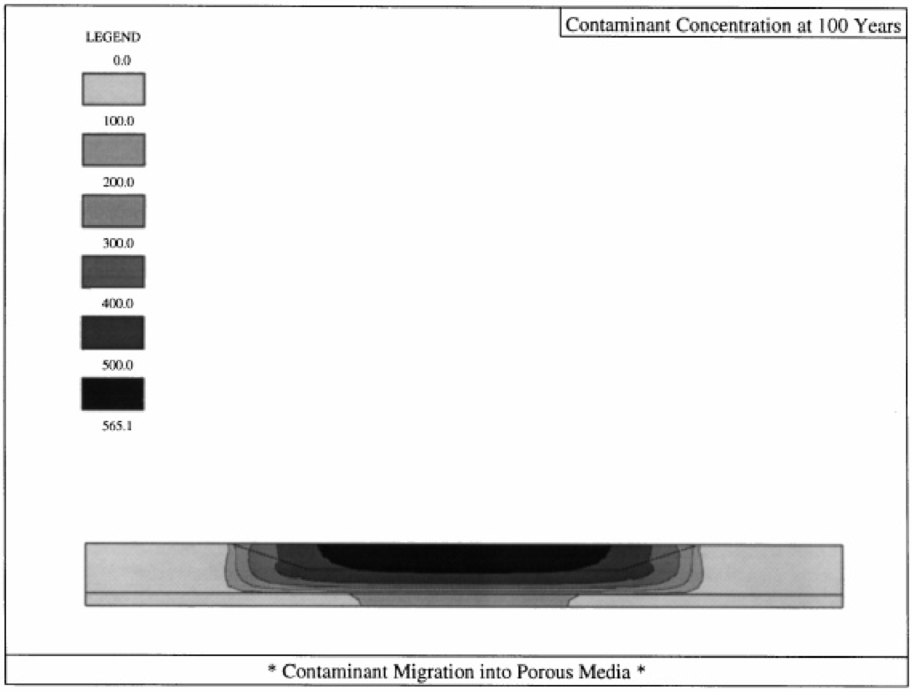

Figure 9. Design case (a)—contaminant distribution at 100 years

yielding a Darcy velocity of about 0·02 m/a in the z (downwards) direction. For simplicity, this value of Darcy velocity will be assumed to occur uniformly throughout the aquitard or the liner in both cases (a) and (b) below. The groundwater flow in the aquifer is in the horizontal direction.

4.2.1. Design case (*a*). A total of 276 boundary elements have been used to model case (a), a schematic representation of the mesh is shown in Figure 7. In this design option, no engineered liner is used, thus, the natural aquitard is the only 'barrier' situated between the landfill and the aquifer, and represents a relatively simple landfill design. This is a 'do-nothing' case where the natural soil is relied upon to provide sufficient barrier. However, it is observed that only 4 m separates the base of the landfill from the top of the aquifer. At 50 years (Figure 8) it is found that the contaminant plume is beginning to penetrate into the aquifer, and has clearly done so at 100 years (shown in Figure 9). Based on the analysis, the simulation results clearly indicate that the design will not be acceptable and points to a need for introducing some form of liner barriers as an impediment to contaminant transport.

4.2.2. Design case (*b*). It is presently quite common to see the use of geomembranes and a leachate collection system in many landfill design. A composite barrier consisting of a 2 mm thick geomembrane with a diffusivity value of 3×10^{-4} m^2/a, a 0·5 m drainage layer and a 1 m layer of clay liner completely surrounding the landfill is assumed to be employed in this case. The geomembrane has a value of diffusivity which is much lower than the value of 0·02 m^2/a found in the clay liner. The presence of punctures or holes in the geomembrane is assumed to be

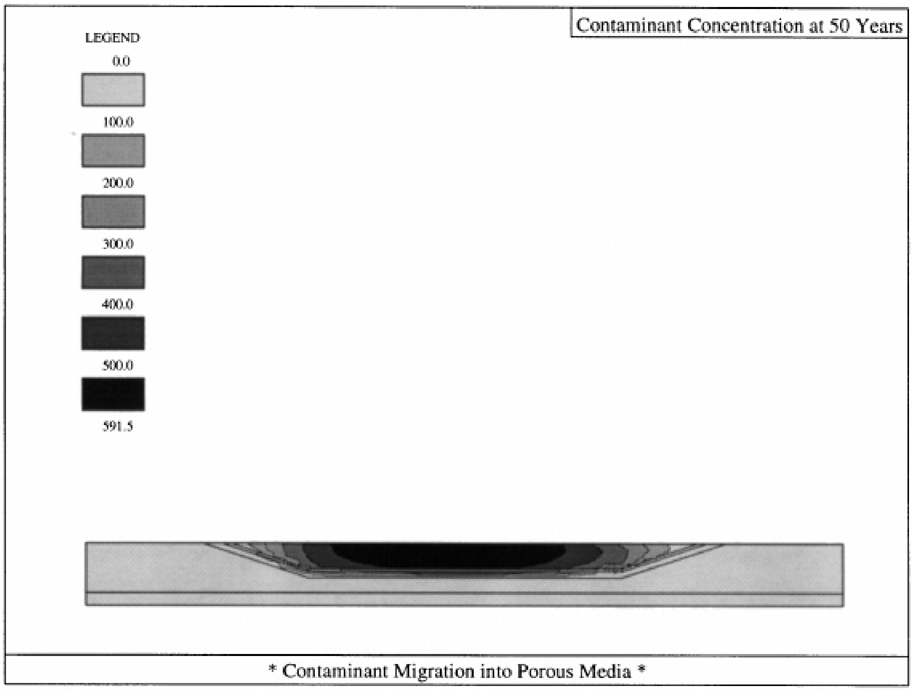

Figure 10. Design case (b)—contaminant distribution at 50 years

accommodated by the value of diffusivity. The function of the drainage layer (which is assumed to have properties similar to the aquifer) provided below the geomembrane is to remove as much as possible the mass of contaminant which has penetrated the geomembrane. A Darcy velocity of 1 m/a is assumed to flow along the entire drainage layer parallel to the geomembrane-drainage layer interface and towards a small sump located at one corner of the base of the landfill. The physical properties of the adsorptive clay liner situated beneath the drainage layer are shown in Table II. In this design, it is intended that the mass of contaminant which is not removed by the leachate collection system will be adsorped and delayed by the clay liner from contaminating the aquifer. A total of 424 boundary elements have been used to model this case. It is clearly seen that the contaminant plume has not managed to penetrate the liner barrier at elapsed time of 50 years (Figure 10). Finally, at 100 years (Figure 11), the peak concentration in the repository has diminished to about 405 mg/l but still no contaminant breakthrough of the clay liner has been observed. Based on this analysis, this case represents a more satisfactory design option than the former.

5. CONCLUSIONS

In this paper, a boundary element method for analysing contaminant transport within non-homogeneous soils has been developed. The method is capable of dealing with non-homogeneity in the porous media, thus liner barriers commonly used in landfill designs can be accommodated in the analysis. This method offers a versatile tool for evaluation of potential migration of

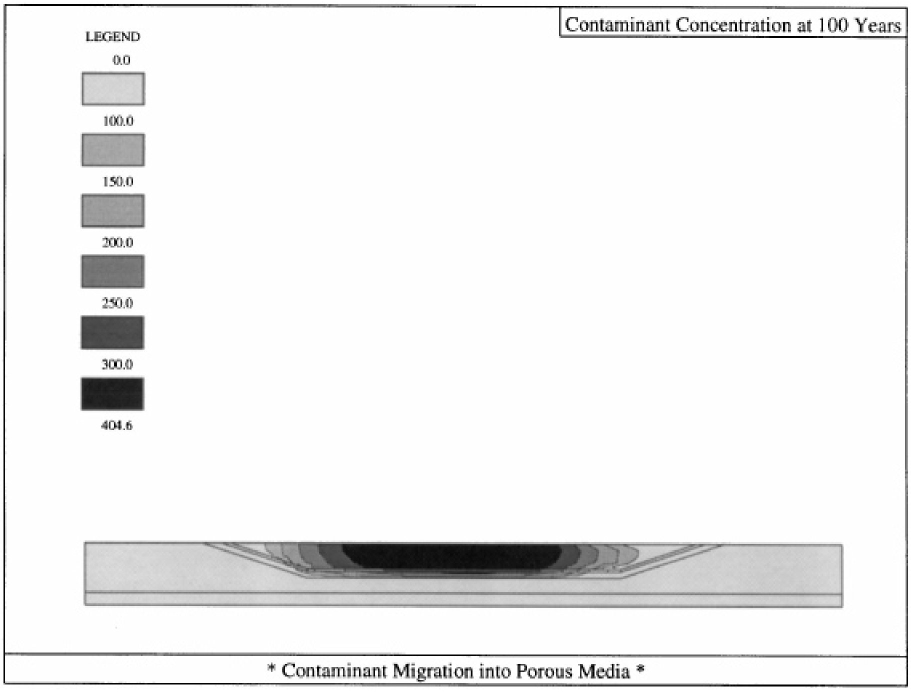

Figure 11. Design case (b)—contaminant distribution at 100 years

contaminants from waste repositories into a variety of underlying soil. Like other conventional boundary element methods, spatial non-homogeneity in the domain is accounted for by subdividing the porous media up into a finite number of relatively homogeneous zones of hydrological and geological properties. Providing not too many zones are involved, this method is found to be an efficient technique for solving contamination problems in porous media. When dealing with a highly variable domain needed to be approximated by a very large number of zones, then alternative FEM or FDM techniques would appear to be more efficient.

The numerical solutions have been verified against semi-analytic solutions for single-phase solute transport in non-homogeneous media. Comparisons with semi-analytic solutions have shown that even when using a relatively coarse spatial discretization, reasonable accuracy can be achieved. An illustrative example has been used to demonstrate the application of the method to analyse a practical problem of landfill design. It may be concluded that the illustrative demonstrates the effectiveness of the boundary element method presented in this paper in simulating potential contamination emanating from a waste repository into non-homogeneous porous media.

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