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Particle Deposition by Diffusion and Interception from Boundary Layer Flows

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An expression for the interception efficiency is derived for particles in boundary layer flows over single spheres and cylinders. The analysis is carried out by determining the point of closest approach of a streamline to the collector surface. Particles with radii smaller than this distance on outlying streamlines will not be collected. The interception efficiency is the ratio of the volumetric flow within this streamline to the total flow swept out by the collector. Deposition by interception can be considered a limiting case of the diffusion of particles of finite diameter. Using a well-known transformation, the interception efficiency can be combined with theoretically based correlations for convective diffusion to permit estimation of deposition efficiencies over the whole range in which both diffusion and interception are important.

NOMENCLATURE

a	collector radius
a_p	particle radius
D	particle diffusivity
d	collector diameter
d_p	particle diameter
F	dimensionless infinite series in Eq. (5)
f_i	a group of functions that appear in the solution to the boundary layer equations for cylinders; $f_i(0) = f_i'(0) = 0$
G	dimensionless infinite series in Eq. (6)
g_i	a group of functions that appear in the solution to the boundary layer equations for spheres; $g_i(0) = g_i'(0) = 0$
J	total deposition rate on a cylinder per unit length or total deposition rate on a sphere
k	particle transfer coefficient
n_∞	particle concentration far from collector surface
Pe	$= 2aU_\infty/D$, Peclet number
R	$= a_p/a$, interception parameter
Re	$= 2aU_\infty/\nu$, Reynolds number
U_∞	fluid velocity far from collector surface
u	component of fluid velocity in boundary layer parallel to collector
u_i	coefficients in the Blasius series for u
x	boundary layer coordinate parallel to collector surface
y	boundary layer coordinate perpendicular to collector surface
η	dimensionless boundary layer coordinate perpendicular to collector surface
η_R	collection efficiency
θ_c	angle at which streamlines approach collector surface most closely
μ	$= R \cdot \mathrm{Pe}^{1/3}\mathrm{Re}^{1/6}$
ν	kinematic viscosity
Ψ	stream function
Ψ_c	streamline that approaches collector surface at angle θ_c at a distance a_p

INTRODUCTION

The combined effects of diffusion and interception in the transport of small particles from low speed flows were analyzed by Friedlander (1967). He used a transforma-

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tion of variables to correlate the experimental data available at that time for filtration by fibrous filters at low Reynolds numbers. The same analysis was applied successfully by Lee and Liu (1982) to a large body of new data, which they obtained by careful experimental measurements.

An analysis of particle transport from high Reynolds number (boundary layer) flows was made by Fernandez de la Mora and Friedlander (1982). They showed that the dimensionless mass transfer coefficient for particle deposition from boundary layer flows is given by a function of a single dimensionless group:

$$kd_p/D = f(\mu), \qquad \mu = R \cdot \mathrm{Pe}^{1/3}\mathrm{Re}^{1/6}. \quad (1)$$

Using this result they were able to correlate data on particle and gas transport to blade-shaped roughness elements covering a surface. [The same result, Eq. (1), was given originally by Friedlander (1958) on intuitive grounds. He then used the analysis to correlate data on filtration efficiencies, but much of those data were taken at low Reynolds numbers, not boundary layer flows.]

The form of the function f in Eq. (1) is known for the diffusion and interceptions regions in their asymptotic limits. For $\mu \to 0$, diffusion controls, the dependence on d_p vanishes and $kd_p/D \sim \mu$. For $\mu \to \infty$, the dependence on D vanishes and $kd_p/D \sim \mu^3$. Thus, experimental data plotted with kd_p/D as a function of μ on a log–log plot should have a slope of unity for small values of μ and a slope of 3 for large values of μ.

In the analysis of Fernandez de la Mora and Friedlander (1982) the mass transfer coefficient was calculated for the forward stagnation point. In this note, the interception efficiency for cylinders and spheres is calculated for the entire flow up to the point where separation takes place. Deposition due to circulating eddies or turbulence behind the collector is not considered. The form of the general correlation including diffusion and interception is also discussed.

In deriving expressions for the collection efficiency by interception, the following assumptions are made:

1. The velocity distribution around the collector can be obtained from boundary layer theory for a particle-free flow.
2. The particle radius a_p is much smaller than the boundary layer thickness, which also means that $a_p \ll a$.
3. Particles follow the fluid streamlines; that is, diffusion and inertia can be neglected as well as particle motion due to the local velocity gradient.

BOUNDARY LAYER VELOCITY DISTRIBUTIONS

Velocity profiles in the boundary layers near cylinders and spheres based on the potential flow pressure profile are given by Schlichting (1955, pp. 133, 164). Our analysis is not limited to those profiles and can use other velocity distributions available in the literature. Near a cylinder, the velocity profile parallel to the surface is given by the following expression:

$$u = u_1 x f_1'(\eta) + 4u_3 x^3 f_3'(\eta) + 6u_5 x^5 f_5'(\eta) + 8u_7 x^7 f_7'(\eta) + \cdots, \quad (2)$$

where $\eta = y(u_1/\nu)^{1/2}$ and

$u_i = 2U_\infty[-1^{(i+3)/2}]/a^i i!$.

The velocity profile parallel to the surface of a sphere is given by a similar expression:

$$u = u_1 x g_1'(\eta) + 2u_3 x^3 g_3'(\eta) + 3u_5 x^5 g_5'(\eta) + 4u_7 x^7 g_7'(\eta) + \cdots, \quad (3)$$

where $\eta = y(2u_1/\nu)^{1/2}$ and

$u_i = \frac{3}{2}U_\infty[-1^{(i+3)/2}]a^i i!$.

[A typographical error in the numerical values of some coefficients in the velocity profile near a sphere occurs in the English version of *Boundary Layer Theory* (Schlicting, 1955, p. 226) up to and including the sixth edition. Some of the coefficients written $\frac{7}{3}$ should be $\frac{70}{3}$.] Since the particle size is much smaller than the momentum boundary layer thickness, only the velocity very near the collector surface need be considered. To obtain expressions for velocities near the surface, we expand f_i' and g_i' in Taylor series around $\eta = 0$ and retain the leading nonzero term.

$$f_i'(\eta) \simeq f_i'(0) + f_i''(0)\eta = f_i''(0)\eta, \tag{4a}$$

$$g_i'(\eta) \simeq g_i'(0) + g_i''(0)\eta = g_i''(0)\eta. \tag{4b}$$

Substituting Eq. (4a) into Eq. (2) gives the velocity profile in the region near a cylinder:

$$u = \left[u_1 x f_1''(0) + 4u_3 x^3 f_3''(0) + 6u_5 x^5 f_5''(0) + \cdots\right]\eta = U_\infty F(x/a)\eta. \tag{5}$$

Substituting Eq. (4b) into Eq. (3) gives the simplified velocity profile near a sphere:

$$u = \left[u_1 x g_1''(0) + 2u_3 x^3 g_3''(0) + 3u_5 x^5 g_5''(0) + \cdots\right]\eta = U_\infty G(x/a)\eta. \tag{6}$$

It is more convenient to use Eqs. (5) and (6) than Eqs. (2) and (3) since the x and y dependencies have been separated.

CALCULATION OF THE INTERCEPTION EFFICIENCY

The interception efficiency is calculated as follows. The angle of closest approach of the fluid streamlines to the collector surface, θ_c, is calculated (Figure 1). Next, one determines the value of the stream function that approaches the surface to within a particle radius at θ_c. That streamline is the critical streamline, Ψ_c. All streamlines between the surface and Ψ_c will deposit their particles at angles less than θ_c, and particles on the streamline Ψ_c will deposit at θ_c. Particles on streamlines outside Ψ_c will not deposit. The interception efficiency η_R can be obtained from the critical streamline value as a ratio of the flow from which particles are removed to the total flow.

Asymptotic expressions for stream functions very near the collector surface are obtained by integrating Eqs. (5) and (6). For cylinders

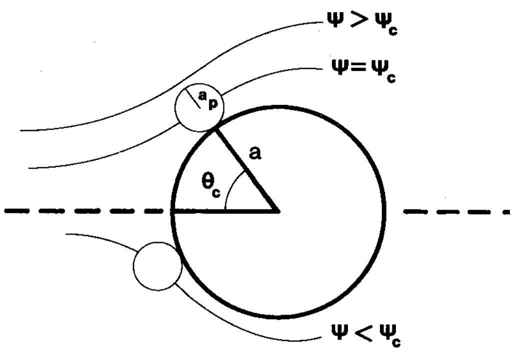


FIGURE 1. Deposition by interception. The angle of closest approach for a streamline in a boundary layer flow is $\theta_c = 57.8°$ for both cylinders and spheres.

$$\Psi = U_\infty(\nu/u_1)^{1/2} F(x/a)\eta^2/2, \tag{7}$$

and for spheres

$$\Psi = U_\infty(\nu/2u_1)^{1/2} G(x/a)\eta^2/2. \tag{8}$$

The derivation of the interception efficiency is shown for cylinders. To find the relationship of η to x/a on any given streamline, set $d\Psi$ equal to zero.

$$d\Psi = 0 = U_\infty(\nu/u_1)^{1/2}\left[F(x/a)\eta\, d\eta + F'(x/a)\tfrac{1}{2}\eta^2\, d(x/a)\right]. \tag{9}$$

Rearranging Eq. (9) gives

$$\frac{d\eta}{d(x/a)} = \frac{-F'(x/a)\eta/2}{F(x/a)}. \tag{10}$$

At the closest approach of the streamline to the collector surface, $d\eta/d(x/a) = 0$ and $d^2\eta/d(x/a)^2 > 0$. For the condition on the first derivative to be satisfied,

$$F'(x/a) = 0. \tag{11}$$

From Eq. (5), $F(x/a)$ is given by the series:

$$U_\infty F(x/a) = u_1 x f_1''(0) + 4u_3 x^3 f_3''(0) + 6u_5 x^5 f_5''(0) + 8u_7 x^7 f_7''(0) + \cdots. \tag{12}$$

Substituting the definitions of u_i and using the tables of Schlichting to calculate $f_i''(0)$ provides numerical values for the coefficients in Eq. (12) (Schlichting, 1955, p. 134). Retaining only the four leading terms of the series reduces Eq. (12) to a four-term polynomial in x/a:

$$F(x/a) \simeq 2.4652(x/a) - 0.9659(x/a)^3 + 0.1032(x/a)^5 - 0.0065(x/a)^7. \tag{13}$$

Differentiating Eq. (13) and solving $F'(x/a) = 0$ gives θ_c, the angle of closest approach of the streamlines to the cylinder surface:

$$\theta_c = (x/a)_c = 1.0095 \text{ rad} = 57.8°. \tag{14}$$

The critical value of Ψ can now be obtained by substituting $\theta_c = 1.0095$ and $y = a_p$ into Eq. (7):

$$\Psi_c = U_\infty (\nu/u_1)^{1/2} F(x/a = 1.0095) \times \tfrac{1}{2}\left[a_p (u_1/\nu)^{1/2}\right]^2. \tag{15}$$

For cylinders, $u_1 = 2U_\infty/a$. Making that substitution into Eq. (15) and evaluating F at $x/a = 1.0095$ gives Ψ_c:

$$\Psi_c = (\nu a U_\infty/2)^{1/2}(1.5962/2) R^2 (2aU_\infty/\nu). \tag{16}$$

The interception efficiency for cylinders is defined as the volumetric flow from which particles are removed per unit length divided by the total volumetric flow swept out by the cylinder per unit length. This definition and the critical stream function give the following expression for η_R:

$$\eta_R = \Psi_c/aU_\infty = 0.7982 R^2 \mathrm{Re}^{1/2}. \tag{17}$$

The analysis for spheres gives similar results. Near the surface, the stream function is

$$\Psi = U_\infty (\nu/2u_1)^{1/2} G(x/a) \eta^2/2. \tag{18}$$

From Eq. (6), the definition of u_i for spheres, and tables provided by Rosenhead and Scholkemeier, numerical values for the coefficients in $G(x/a)$ are obtained (Scholkemeier, 1949; Rosenhead, 1963). Retaining the first four terms of the series gives the following polynomial approximation for G:

$$G(x/a) \simeq 1.3916(x/a) - 0.5462(x/a)^3 + 0.0585(x/a)^5 - 0.0036(x/a)^7. \tag{19}$$

Solving $G'(x/a) = 0$ gives the angle of closest approach of streamlines to a spherical surface:

$$\theta_c = 1.009 \text{ rad} = 57.8°. \tag{20}$$

This is the same result as for cylinders, which is consistent with the very similar separation angles calculated for cylinders and spheres using these velocity profiles (Schlichting, 1955, pp. 136–137). The critical streamline for a sphere is then

$$\Psi_c = (\nu a U_\infty/3)^{1/2}(0.9004/2) R^2 (3aU_\infty/\nu). \tag{21}$$

The interception efficiency for a sphere is the ratio of volumetric flow from which particles are removed to the total volumetric flow through the cross-sectional area of the sphere:

$$\eta_R = \frac{2\pi a \Psi_c}{\pi a^2 U_\infty} = \frac{2\Psi_c}{aU_\infty} = 1.1028 R^2 \mathrm{Re}^{1/2}. \tag{22}$$

It can be shown that θ_c corresponds to the distance of closest approach of streamlines to collector surfaces, since $d^2\eta/d(x/a)^2 > 0$.

GENERALIZED CORRELATIONS FOR INTERCEPTION AND DIFFUSION

The total deposition rate J on a cylinder per unit length (particles/sec cm) is given by

$$J = \eta_R (2aU_\infty n_\infty). \tag{23}$$

This expression defines the removal or collection efficiency η_R.

The deposition per unit area of cylinder surface is given by

$$J/2\pi a = \eta_R U_\infty n_\infty/\pi. \tag{24}$$

In terms of the mass transfer coefficient, the following dimensionless relationship is obtained:

$$\frac{kd_p}{D} = \frac{\eta_R R \mathrm{Pe}}{\pi} = \frac{Jd_p}{\pi d\, Dn_\infty} = \frac{0.7982}{\pi} R^3 \cdot \mathrm{Pe} \cdot \mathrm{Re}^{1/2}. \quad (25)$$

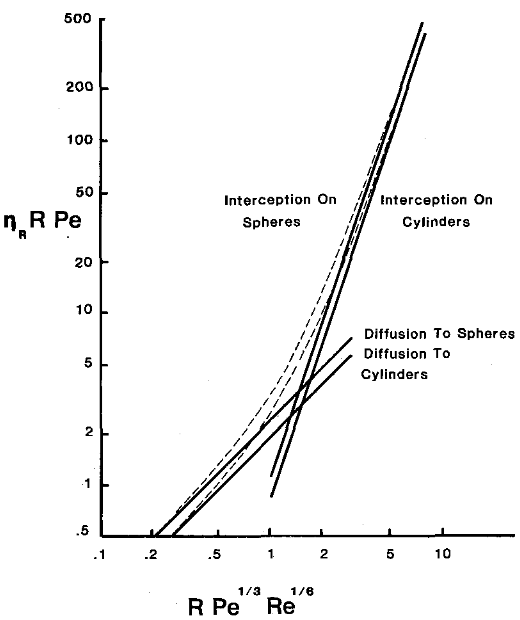
For spheres, the corresponding result is

$$\frac{kd_p}{D} = \frac{\eta_R R \mathrm{Pe}}{4} = \frac{Jd_p}{\pi d^2\, Dn_\infty} = \frac{1.1028}{4} R^3 \cdot \mathrm{Pe} \cdot \mathrm{Re}^{1/2}. \quad (26)$$

Equations (25) and (26) are consistent with the results of Fernandez de la Mora and Friedlander (1982) for blade-shaped elements. In general, the rate of deposition by interception is proportional to $R^3 \cdot \mathrm{Pe} \cdot \mathrm{Re}^{1/2}$.

Figure 2 shows the expressions for diffusion and interception solutions plotted in the manner suggested by Eq. (1). The curves for diffusion are from standard correlations for convective diffusion (e.g., Bird et al., 1960), cast in terms of particle removal efficiency.

FIGURE 2. Removal efficiency for interception and diffusion of particles by cylinders and spheres.



Lines representing the interception efficiency of cylinders and spheres [Eqs. (17) and (22)] are also shown. Thus, interception and diffusion represent asymptotic solutions for deposition. At very small values of $R \cdot \mathrm{Pe}^{1/3} \cdot \mathrm{Re}^{1/6}$ diffusion dominates; interception dominates at higher values of this parameter. For values near unity both mechanisms are important. The diffusion equation with an interception boundary condition [Eq. (3.14) of Fernandez de la Mora and Friedlander (1982)] could be solved numerically to obtain the overall particle removal efficiency when both mechanisms are significant. For approximate calculations, diffusion and interception can be added to give the dotted curves in Figure 2. For cylinders

$$\eta_R R \mathrm{Pe} = 1.88\mu + 0.80\mu^3, \quad (27)$$

and for spheres

$$\eta_R R \mathrm{Pe} = 2.40\mu + 1.10\mu^3. \quad (28)$$

APPLICABILITY OF THE ANALYSIS

This analysis should work best in the range $10^2 < \mathrm{Re} < 10^4$. In this range, boundary layer theory describes the flow over the upstream side of the collector, and the standard correlations for diffusional mass transfer apply. Since the analysis is limited to deposition by diffusion and interception, it should hold for Stokes numbers significantly less than the critical values for impaction, $\frac{1}{8}$ and $\frac{1}{12}$ for cylinders and spheres, respectively.

As an example, the correlation for cylinders should apply to the deposition of unit density particles smaller than about 2 μm from air flowing at a velocity of 10 ft/sec normal to a 1-mm wire. At 20°C, Re = 200, and for 2-μm particles, the Stokes number is about 0.07, $R = 2 \times 10^{-3}$, $\mu = 2.8$, and interception is the dominant mechanism.

The correlation for boundary layer flows leads to prediction of significantly higher deposition rates at high Reynolds numbers than the creeping flow analysis (Friedlander, 1967; Lee and Liu, 1982) because the

Reynolds number appears in the correlation parameter μ. However, at low Reynolds numbers, the results for both correlations are virtually the same, which explains why Friedlander (1958) found the boundary layer form to work reasonably well for data taken at low Reynolds numbers.

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