Pergamon

Int. Comm. Heat Mass Transfer, Vol. 24, No. 6, pp. 879–888, 1997

0735-1933/97 $17.00 + .00

PII S0735-1933(97)00074-2

IMPROVEMENT ON THE MODELLING OF FILM BOILING ON SPHERES

S.K.W.Tou and C.P.Tso
Nanyang Technological University
School of Mechanical & Production Engineering
Singapore 639798

(Communicated by J.P. Hartnett and W.J. Minkowycz)

ABSTRACT

An analytical model for stable film boiling heat transfer from a sphere is derived following the classical approach of Frederking and Clark but using spherical coordinates instead. The improvement shows more clearly than the previous work that the Nusselt number should approach the value of 2 instead of zero, as the Rayleigh number goes to zero. Furthermore, by consideration of two limits on the liquid-vapor interface boundary conditions based on Bromley's suggestion, the coefficient C_1 is shown to lie in the range 0.586 to 0.828, in the correlation

$$Nu = C_1[\frac{Ra}{Ja}]^{1/4} + 2.$$

This supports the value of 0.67 as proposed by Dhir and Lienhard.

Introduction

Analytical solutions to the problem of natural convection film boiling from spheres have always been based on applying the basic equations in the Cartesian form with the assumption that for thin fluid layers the curvature effect along the sphere can be neglected, as was reported by the analysis of Frederking and Clark[1] and followed by others[e.g. 2-4].

Recently Tso et al.[5] demonstrated that by adopting the spherical coordinates, the results should yield a limiting value of 2 for the average Nusselt number near a low modified Rayleigh number which could not be extracted from the Frederking and Clark model (F-C model). The limiting value of 2 is already widely accepted as a lead term in *single-phase* natural convection correlations, since in the limit of no convection, elementary pure conduction consideration yields this value[6]. In Tso et al.[5] the analytical solution was not carried out to the end, the mid-way manipulation being continued with a numerical solution. It is shown here that by continuing with the analysis the same expression as the F-C model is obtained, except for the second term of 2. Moreover, by considering the F-C model boundary conditions as the lower bound and defining boundary conditions for an upper bound, it is shown that the coefficient C_1 should lie in a specific range.

Model Formulation

Coordinate systems and assumptions

A vertical cross section passing through the center of a sphere of radius R is shown in Fig.1. The origin of a right-handed Cartesian coordinate system (X,Y,Z) is fixed at the sphere center 0 with the Z axis pointing upwards. A spherical coordinate system (r,θ,ϕ) is defined with respect to the Cartesian coordinate to suit the analysis is also shown in Fig. 1. Film boiling heat transfer characteristics are influenced by the boundary layer development which begins at $\theta = 0$ and concludes at $\theta \leq \pi$ with formation of a plume ascending from the sphere.

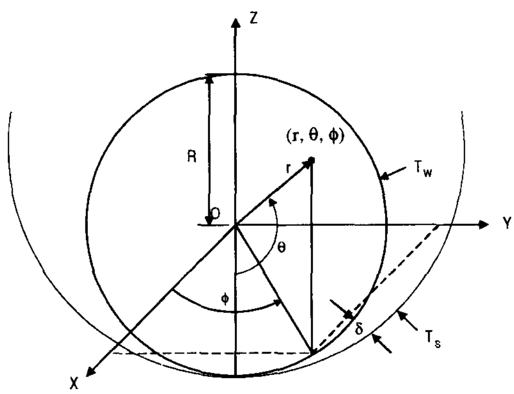

FIG. 1
Boundary layer development and coordinate systems

To develop an analytical model for natural convection heat transfer from a sphere, the following assumptions are employed:

1. Fluid is incompressible and the Boussinesq approximation is applied.
2. Film layer is laminar and fluid motion can be approximated by a boundary layer type.
3. Vapor density is uniform in the film layer and is independent of temperature.
4. Viscous heating is negligible and conduction is dominant in the film.
5. Film layer is stable and thin and wall surface is smooth.
6. The transport process is steady and is axisymmetric with respect to the vertical diameter.
7. For film boiling, $T_W >> T_S$. Both T_W and T_S are constant.
8. Fluid motion is slow and inertia effects are negligible.
9. Fluid properties are constant and surface tension is negligible.
10. Film layer remains attached on the entire sphere surface without separation.
11. Radiative mode of heat transfer is negligible and there is no sub-cooling in the liquid.
12. The effects of waves at the interface are negligible.

Assumption 4 restricts the model application for small to moderate sphere diameters, since heat conduction across the laminar vapor film is the controlling physical mechanism. Whereas for large sphere diameters, the controlling mechanisms are the vapor movement, bubble formation, turbulence and possibly the waves at the liquid-vapor interface. The difference between the present model and that of Frederking and Clark is that the sphere surface is not approximated to be a plane wall. The governing equations for the film layer in spherical coordinate system after taking the above assumptions into consideration are the following:

$$\frac{\partial}{\partial\theta}(v_\theta \sin\theta) = 0 \tag{1}$$

$$\frac{\mu_v}{\rho_v r^2}\frac{\partial}{\partial r}(r^2\frac{\partial v_\theta}{\partial r}) + g\frac{\rho_l - \rho_v}{\rho_v}\sin\theta = 0 \tag{2}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r}) = 0. \tag{3}$$

Boundary conditions

Bromley has suggested, from his observations on film condensation[8], that the velocity boundary conditions at the liquid-vapor interface should be specified to something between a no-slip stagnant fluid and a fluid that exerts no shear. These two different types of boundary conditions represent two extreme cases. *Case 1* assumes that the fluid is immovable and no-slip applies, whereas *Case 2* assumes friction is negligible at the interface. The reason suggested by

Bromley is that the external fluid is not so easily set into motion. It is also noted that Frederking and Clark used only *Case 1* boundary condition in their analysis. The boundary conditions for the film layer are:

At $\theta = 0$, $\delta = 0$

At $r = R$, $v_\theta = 0$

At $r = R$, $T = T_W$

At $r = R+\delta$, $T = T_S$

At $r = R$, $k_v dA \left(-\frac{\partial T}{\partial r}\right)_{r=R} = h_{fg} dw_i,$

where $$dA = 2\pi R^2 \sin\theta d\theta \ . \quad (4)$$

At the vapor-liquid interface, two cases are taken, representing the upper and lower bounds.

When $r = R+\delta$, *Case 1:* $v_{\theta,v} = v_{\theta,l} = 0$

Case 2: $\tau_v = \tau_l = -\nu_v \left(\frac{\partial v_\theta}{\partial r}\right)_i = 0.$

For both cases, $dw_v = dw_l = dw_i$.

Analytical Solution

Since the temperature field has been decoupled from the velocity field, a solution for Eq. (3) can be obtained by direct integration subject to the above boundary conditions. The following results are obtained for the temperature distribution within the film layer[5]:

$$\frac{T - T_s}{T_w - T_s} = 1 + \frac{r^{-1} - R^{-1}}{R^{-1} - (R+\delta)^{-1}} \quad (5)$$

It can be seen that the temperature distribution is non-linear for a sphere whereas it is linear in the F-C model for a plane surface[1]. The temperature gradient at the wall-vapor interface is given by

$$\left(\frac{dT}{dr}\right)_{r=R} = -\frac{\Delta T}{\delta}\left(1 + \frac{\delta}{R}\right). \quad (6)$$

The above shows that heat transfer rate varies with the curvature of a surface. Since $\delta/R > 0$, heat transfer rate per unit area from a convex surface is greater than that from a planar surface which can be obtained by dropping the small term δ/R in comparison with unity, i.e.,

$$(\frac{dT}{dr})_{r=R} = -\frac{\Delta T}{\delta}.$$

This temperature gradient was indeed used in the development of the F-C model. The heat balance at the wall-vapor interface yields Eq. (4) with

$$dw_i = \rho_v d\int_R^{R+\delta} v_\theta 2\pi R \sin\theta dr. \tag{7}$$

The velocity component v_θ is obtained by solving the momentum equation together with the boundary conditions, and the following velocity distributions are obtained across the film layer:

Case 1: $$v_\theta = \frac{-g(\rho_l - \rho_v)\sin\theta}{6\mu_v}[r^2 + \frac{2R^3 + 3R^2\delta + R\delta^2}{r} - 3R^2 - 3R\delta - \delta^2]$$

Case 2: $$v_\theta = \frac{-g(\rho_l - \rho_v)\sin\theta}{3\mu_v}[\frac{r^2}{2} + \frac{(R+\delta)^3}{r} - \frac{R^2}{2} - \frac{(R+\delta)^3}{R}].$$

Substituting the above velocities into Eq. (7) and performing integration, one obtains the following expressions after dropping terms higher than $(\delta/R)^3$.

Case 1: $$dw_i = \frac{2\pi R Ra k_v}{96 c_p} d[\sin^2\theta(\frac{\delta}{R})^3] \tag{8}$$

Case 2: $$dw_i = \frac{2\pi R Ra k_v}{24 c_p} d[\sin^2\theta(\frac{\delta}{R})^3] \tag{9}$$

where the Rayleigh number defined is based on sphere diameter, and is

$$Ra = \frac{c_p g(\rho_l - \rho_v)D^3}{k_v v_v}.$$

For *Case 1* the heat balance equation at the wall-vapor interface reduces to the following form:

$$\frac{\delta}{R(1+\delta/R)}\frac{d}{d\theta}[\sin^2\theta(\frac{\delta}{R})^3] = \frac{96 c_p \Delta T \sin\theta}{(Ra)h_{fg}}. \tag{10}$$

By dropping the second and higher order terms, the following non-linear ordinary differential equation is obtained:

$$\frac{\delta}{R}(1-\delta / R)\frac{d}{d\theta}[\sin^2\theta(\frac{\delta}{R})^3]=\frac{96c_p\Delta T\sin\theta}{(Ra)h_{fg}} \tag{11}$$

By the substitution $z=(\frac{\delta}{R})^{1/4}$, Eq. (11) can be written as

$$\frac{dz}{d\theta}+\frac{8z}{3}\cot\theta-\frac{128c_p\Delta T}{(Ra)h_{fg}}\frac{(1+z^{1/4})}{\sin\theta}=0. \tag{12}$$

To linearize the above ODE, it is assumed that $(1+z^{1/4})$ remains practically unchanged since δ/R is always much less than unity, i.e., $(1+z^{1/4})\equiv$ c, where c is a constant. By making a change of variable and applying the boundary condition, Eq. (12) can be solved and the following result is obtained:

$$\frac{\delta}{R}=2\left[\frac{8c_p\Delta T}{(Ra)h_{fg}}\right]^{1/4}(1+\frac{\delta}{R})^{1/4}\frac{\left[\int_0^\theta\sin^{5/3}\theta d\theta\right]^{1/4}}{\sin^{2/3}\theta}. \tag{13}$$

It is noted that by making the approximation $(1+\frac{\delta}{R})^{1/4}\approx 1$, the same result as that of Frederking and Clark[1] is obtained, that is,

$$\frac{\delta}{R}=2\left[\frac{8c_p\Delta T}{(Ra)h_{fg}}\right]^{1/4}\frac{\left[\int_0^\theta\sin^{5/3}\theta d\theta\right]^{1/4}}{\sin^{2/3}\theta}.$$

The heat transfer from a differential area at the wall-vapor interface is computed from

$$dq=k_v(-\frac{\partial T}{\partial r})_{r=R}2\pi R^2\sin\theta d\theta$$

$$=\frac{k_v\Delta T\pi D\sin^{5/3}\theta(1+\delta / R)\left[\int_0^\theta\sin^{5/3}\theta d\theta\right]^{-1/4}}{2\left[\frac{8c_p\Delta T}{(Ra)h_{fg}}\right]^{1/4}}. \tag{14}$$

The total heat transfer from the sphere is found by integrating the above from θ=0 to θ=π:

$$q_t=\frac{k_v\Delta T\pi D}{2}\left[\frac{8c_p\Delta T}{(Ra)h_{fg}}\right]^{-1/4}\left\{\int_0^\pi\sin^{5/3}\theta\left[\int_0^\theta\sin^{5/3}\theta d\theta\right]^{-1/4}d\theta+\int_0^\pi(\frac{\delta}{R})\sin^{5/3}\theta\left[\int_0^\theta\sin^{5/3}\theta d\theta\right]^{-1/4}d\theta\right\}.$$

Defining the average Nusselt number as $Nu=\frac{q_tD}{\pi D^2\Delta Tk_v}$, it follows that

$$Nu = \frac{2^{1/4}}{4}\left[\frac{c_p \Delta T}{(Ra)h_{fg}}\right]^{-1/4}\left\{\int_0^{\pi} \sin^{5/3}\theta\left[\int_0^{\theta} \sin^{5/3}\theta d\theta\right]^{-1/4} d\theta + \int_0^{\pi}\left(\frac{\delta}{R}\right)\sin^{5/3}\theta\left[\int_0^{\theta}\sin^{5/3}\theta d\theta\right]^{-1/4} d\theta\right\}.$$

Substituting Eq. (13) into the above equation and integrating, yields

$$Nu = 0.586\left[\frac{Ra}{Ja}\right]^{1/4} + 2, \tag{15}$$

where the Jacob number serves to characterize a given fluid and is defined as $Ja = \frac{c_p \Delta T}{h_{fg}}$.

Applying the same solution procedures for *Case 2* boundary condition, the results are

$$\frac{\delta}{R} = \left[\frac{32 c_p \Delta T}{(Ra)h_{fg}}\right]^{1/4} \frac{\left[\int_0^{\theta} \sin^{5/3}\theta d\theta\right]^{1/4}}{\sin^{2/3}\theta}$$

$$Nu = 0.828\left[\frac{Ra}{Ja}\right]^{1/4} + 2. \tag{16}$$

<u>Discussion</u>

It is noted that generally the correlation is of the form

$$Nu = C_1\left[\frac{Ra}{Ja}\right]^n + C_2, \tag{17}$$

where $0.586 < C_1 < 0.828$, $n = 1/4$, and $C_2 = 2$. If effective latent heat of evaporation is used, then

$$Nu = C_1\left[\frac{Ra}{Ja^*}\right]^n + C_2, \tag{18}$$

where $Ja^* = \frac{c_p \Delta T}{h_{fg}^*}$.

For comparison with Eq. (17), the theoretical result of Frederking and Clark is

$$Nu = 0.586 \left[\frac{(Ra) h_{fg}}{c_p \Delta T} \right]^{1/4}$$

$$= 0.586 \left[\frac{Ra}{Ja} \right]^{1/4}, \qquad (19)$$

where the coefficient 0.586 may be regarded as the value for the lower bound in the present model, and the constant 2 is absent. Table 1 shows the comparison with some other correlations. In particular it is noteworthy that the correlation of Dhir and Lienhard[7] has been widely recommended, and it has a coefficient of 0.67, which is roughly a mid-value within the present bounds. Their correlation is based on the approach of film condensation, analogous to Bromley's treatment to the cylinders[8].

It is noted that the constant 2 has not been included in any established correlation, except in single-phase convection. It may be that the value of C_2 is small when compared to the first term in film boiling. However, in view of its theoretical source, there may be a need to correlate experimental data in a more consistent manner. The value of C_2 may also serve to identify the geometry of the boiling surface involved.

Lastly, mention should be made on the exponent 1/4, which is present in most correlations. But Frederking and Clark's empirical correlation[1] actually found 1/3 to be a better fit, and this is supported by others(e.g.[2,4]). Presently, there is no good theoretical justification for accepting 1/3.

TABLE 1
A brief summary of various correlations for natural convection film boiling on spheres.

Source	C_1	n	C_2	**Remarks**
Frederking & Clark, 1963[1]	0.586 0.14	1/4 1/3	0 0	Theoretical Empirical
Dhir & Lienhard, 1971[7]	0.67	1/4	0	Semi-empirical
Farahat & Nasr, 1978[2]	0.77 0.143	1/4 1/3	0 0	Theoretical Empirical
Michiyoshi et al.,1988[3]	~0.5-1.2	1/4	0	Theoretical
Tso et al., 1995[5]	0.586	1/4	2	Numerical
Present study	0.586-0.828	1/4	2	Theoretical

Conclusions

1. An analytical model for film boiling heat transfer from a sphere is derived which improves on the one developed by Frederking and Clark.

2. The present model shows clearly that Nusselt number approaches 2 instead of zero as Rayleigh number goes to zero. It also shows that the constant C_1 varies from 0.586 to 0.828 which agrees with the value of 0.67 proposed by Dhir and Lienhard.

3. There is a need to correlate experimental data in a consistent manner. It is suggested to use a correlation in the form of Eq. (17).

Nomenclature

A	area
C_1, C_2	coefficients in Eq. (17)
c_p	specific heat at constant pressure
D	sphere diameter
g	gravitational acceleration
h_{fg}	latent heat of vaporization
Ja	Jakob number = $c_p \Delta T / h_{fg}$
k_v	vapor thermal conductivity
Nu	average Nusselt number
q_t	total heat transfer from a sphere
R	radius of sphere
Ra	Rayleigh number = $c_p g(\rho_l - \rho_v) D^3 / (k_v \nu_v)$
T	vapor temperature
T_W	wall surface temperature
T_S	vapor saturation temperature
v_θ	vapor velocity component in θ direction
w	interfacial mass flow rate
X,Y,Z	Cartesian coordinates, illustrated in Fig. 1
r,θ,ϕ	spherical coordinates, illustrated in Fig. 1

Greek Letters

δ film layer thickness

ΔT wall superheat = $T_w - T_s$

ρ_v vapor density

ρ_l liquid density

ν_v vapor kinematic viscosity

μ_v vapor dynamic viscosity

τ_i shear stress at liquid-vapor interface

Subscripts

v vapor

l liquid

i interface

Superscript

* effective value

References

1. T.H.K. Frederking and J.A. Clark, *Advanced Cyrogenic Engineering* **8**, 501 (1963).

2. M.M. Farahat and T.N. Nasr, *Int. J. Heat Mass Transfer* **21**, 256 (1978).

3. I. Michiyoshi, O. Takahashi and Y. Kikuchi, *Proc. First World Conf. on Experimental Heat Transfer, Fluid Mechanics and Thermodynamics*, Dubrovnik, Yugosalvia, 1404 (Sept. 1988).

4. C. P. Tso, H.G. Low and S.M. Ng, *Int. J. Heat and Fluid Flow* **11**, 154 (1990).

5. C.P. Tso, K.C. Leong and H.S. Tan, *Int. Commun. Heat Mass Transfer* **22**, 803 (1995).

6. J.H. Lienhard, *A Heat Transfer Textbook*, Prentice-Hall,Inc., New York (1987).

7. V.K. Dhir and J.H. Lienhard, *J. Heat Transfer, Trans. ASME* **93**, 97 (1971).

8. A.L. Bromley, *Chem. Eng. Progr.* **46**, 211 (1950).

Received April 23, 1997