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Selection of Tyre Sizes for Agricultural Vehicles

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Empirical equations are given from which the tractive performance of tyres in off-road conditions can be predicted. The equations are used to demonstrate the effect of varying tyre diameter and width on a lightly loaded and a heavily loaded axle in different field conditions. The effect of varying tyre flexibility is also demonstrated. The equations are used to derive curves giving the axle load required per unit of available axle power to ensure operation at maximum efficiency in any field condition. The maximum possible tractive efficiency is calculated. The results of validation experiments on the effect of fitting larger tyres and ballasting to the optimum level are given. The relative performance of dual tyres and a single tyre of the same overall dimensions is examined as is the relative performance of a single large tyre and two smaller tyres used as duals. The loss in performance due to the use of high inflation pressure high load carrying capacity tyres such as earthmover tyres, in place of agricultural tyres is shown.

1. Introduction

The correct choice of tyre size is a matter of great importance in the design and operation of off-road wheeled vehicles. Even today, with vehicles which have evolved into their present shape after a comparatively long period of time, there are still a great many vehicles which are fitted with tyres which are much too small.[1,2] The consequences of this, particularly in soft soil conditions, are poor tractive performance and the creation of deep ruts in the soil. Many vehicles, for example combine harvesters and agricultural trailers, still seem to be designed with no thought given to the correct tyre size. Rather, the smallest sized tyres which will carry the necessary load are fitted as a matter of course. That this can be a very short-sighted policy is highlighted later in this paper.

What does an off-road vehicle designer or user need to know before a rational choice of tyre size can be made? Information on the cost and load carrying capacity of various tyres is readily available from manufacturers. Information on the relative wear rates of different tyres is not quite so readily available. The cost to a farmer of reduced crop yields due to soil compaction is a problem which has occupied many research workers in recent years and although there is as yet no reliable way of correlating soil compaction level with crop yield there are indications that this problem will become amenable to analysis in the future.[3,4,5] What remains is information on the tractive performance of tyres so that a vehicle designer or user can balance the cost of providing larger tyres for the vehicle against the benefits to be gained.

A typical statement one often hears when discussing the effect of varying wheel parameters on tractive performance is that tractive performance is increased much more by an increase in wheel diameter than by an equivalent increase in wheel width.[6] This information is useful up to a point but is not sufficiently precise to be generally useful. A more precise and more useful question is "What is the diameter or width of a tyre carrying a given load in given soil conditions beyond which any increase results only in minimal improvements in performance"? What is required to answer this question is an analysis which will allow the tyre performance to be predicted accurately from a knowledge of the tyre parameters and soil properties.

There are at present 3 main ways of approaching this problem. The first is by the use of formal methods of plasticity analysis.[7,8] Whilst this can lead to accurate solutions, a computer analysis

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is required in almost every individual case. This makes the methods of limited use to a vehicle designer who has a multitude of parameters which can be varied. It is of even less use to a vehicle user who almost certainly does not even have ready access to a computer.

The second method is the semi-empirical method begun by Bernstein[9] and further developed by Bekker[10] and Gee-Clough.[11] This method does allow prediction of towed rigid wheel performance with reasonable accuracy and has the attraction of predicting the state of stress at the soil–wheel interface.[12] Unfortunately it cannot yet deal readily with either driven wheels or, without unduly sweeping assumptions, with flexible wheels. Further development of this method should allow these restrictions to be overcome but it is not yet in a form which is readily usable.

The third method is the fully empirical approach begun by Freitag[13] and developed by Turnage,[14] Dwyer *et al.*,[15] Wismer and Luth[16] and Gee-Clough *et al.*[17] This method, whilst giving no information on the state of stress at the soil–wheel interface does allow simple predictive equations to be found which can be used immediately by both the vehicle designer and user. This paper shows how the empirical predictive equations found at N.I.A.E. can be used to make a rational choice of tyre sizes for off-road vehicles.

2. Empirical equations and field conditions

The prediction of 3 parameters is all that is necessary to fully describe the tractive performance of a tyre in off-road conditions. The first is the maximum coefficient of traction $(C_T)_{max}$, the second the rate constant (k) defined in Eqn (1), and the third the coefficient of rolling resistance (C_{RR}).

The coefficient of traction versus slip curve can then be drawn by using the expression:

$$C_T = (C_T)_{max}\,(1 - \exp\,(-ks)) \qquad \ldots(1)$$

where C_T is the coefficient of traction and s the slip.

The tractive efficiency versus slip curve can be drawn using the expression:

$$\eta = \frac{C_T(1-s)}{C_T + C_{RR}} \qquad \ldots(2)$$

where η is the tractive efficiency.

Figs 1(a)–(c) show the empirical relationships found at N.I.A.E. between $(C_T)_{max}$, k, C_{RR} and the tyre mobility number, M. The equations to the relationships are:

$$(C_T)_{max} = 0{\cdot}796 - \frac{0{\cdot}92}{M}, \qquad \ldots(3)$$

$$k\,(C_T)_{max} = 4{\cdot}838 + 0{\cdot}061\,M, \qquad \ldots(4)$$

$$C_{RR} = 0{\cdot}049 + \frac{0{\cdot}287}{M}. \qquad \ldots(5)$$

The mobility number is defined as:

$$M = \frac{Cbd}{W}\sqrt{\frac{\delta}{h}}\left(\frac{1}{1+b/2d}\right) \qquad \ldots(6)$$

where C = soil cone index value,
b = tyre width,
d = tyre diameter,
W = load on the tyre,
δ = tyre deflection under load,
h = tyre section height.

LIST OF SYMBOLS

C	soil cone index value, kPa	d	tyre diameter, m
C_{RR}	coefficient of rolling resistance	g	gravitational constant, m/s^2
C_T	coefficient of traction	h	tyre section height, m
$(C_T)_{max}$	maximum coefficient of traction	k	rate constant
M	tyre mobility number	s	slip
P	power, kW	w	width of ploughing, m
V	forward speed, m/s	γ	soil specific weight, kN/m^3
W	load on the tyre, kN	δ	tyre deflection under load, m
a	depth of ploughing, m	η	tractive efficiency
b	tyre width, m	σ	plough specific resistance, kPa

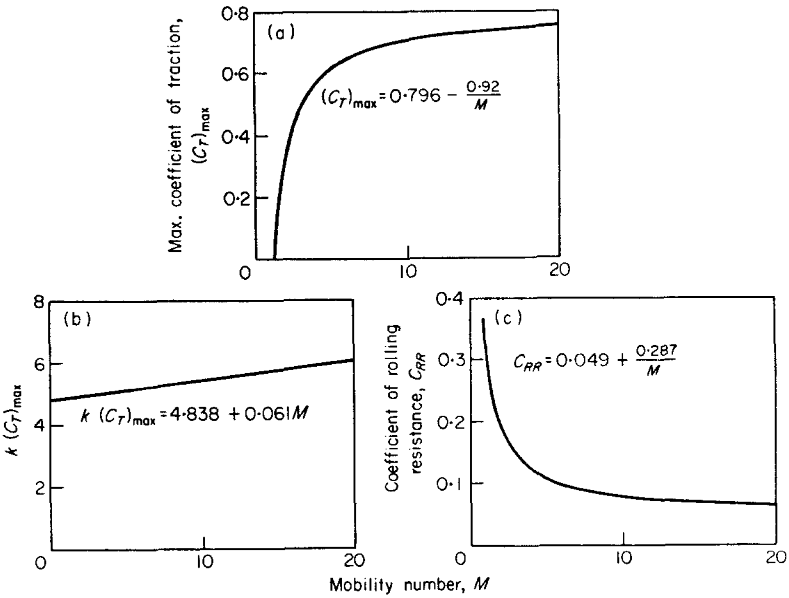

Fig. 1. Variation of main traction parameters with tyre mobility number: (a) maximum coefficient of traction, (b) rate constant multiplied by maximum coefficient of traction, (c) coefficient of rolling resistance

δ is normally measured statically on a hard surface and a typical value of δ/h at a manufacturer's recommended load and inflation pressure is 0·2.

The relationships were found from experiments in more than 150 different field conditions over several years and apply to stubble, ploughed and cultivated fields only. Fields with a firm binding surface layer such as hard, dry grass fields gave significantly higher tractive performance than would be predicted using these equations. Fields with a slippery surface layer such as sugar beet fields after harvest gave significantly lower tractive performance.

In a series of verification experiments the relationships were used by Gee-Clough *et al.*[17] to predict tractor–plough field performance in 14 different field conditions. The data from which *Figs 1(a)–(c)* were produced is given in Reference (17) as are the 95% confidence limits for the data analysed. It was found that the forward speed and work-rate of the tractor–plough combination were predicted quite well using the empirical relationships, 86% of the predicted values being within ±20% of the measured values.

TABLE 1
Correction factors for tractive performance parameters

Effect	*Traction conditions*	*Correction factor*		
		$(C_T)_{max}$	k	C_{RR}
Radial-ply tyres (at low inflation pressure) compared to cross-ply tyres	Bad	0·95	1·38	1·00
	Average	0·95	1·38	1·00
	Good	0·95	1·38	1·00
High lugged tyres (e.g. 75 mm) compared to medium lugged tyres (35 mm)	Bad	1·10	0·92	1·03
	Average	1·10	0·50	1·21
	Good	1·10	0·40	1·32
Forward speed increased to 6·4 km/h from 3·2 km/h	Bad	1·01	1·00	0·98
	Average	1·01	1·00	0·98
	Good	1·01	1·00	0·98
Running in the furrow bottom rather than on the field surface	Bad	—	—	—
	Average	1·25	1·00	1·00
	Good	1·23	0·69	1·80

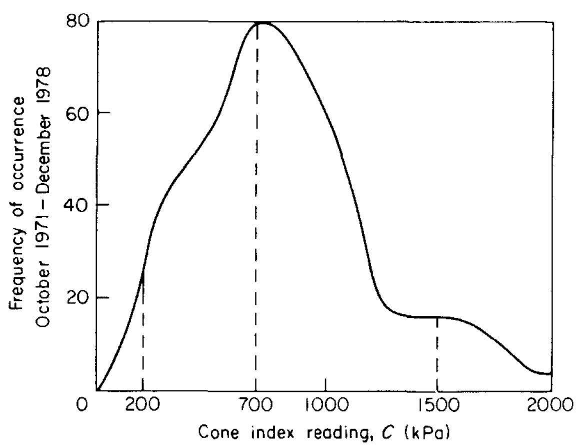

Fig. 2. Frequency of occurrence of soil cone index values

Figs 1(a) and (*c*) approach asymptotic values at high mobility numbers and very little increase in maximum coefficient of traction or decrease in coefficient of rolling resistance is obtained by increasing mobility number beyond 15. The product $k\,(C_T)_{max}$ is the slope of the coefficient of traction versus slip curve at the origin. As mobility number increases, this product increases linearly with it. Eqn (6) shows that mobility number may be increased by increasing tyre diameter, width and deflection/section height ratio for any value of soil cone index and load. However, *Figs 1(a)*, *1(c)* and Eqn (6) imply that there are limits beyond which increases in these parameters will produce only small improvements in performance.

Fig. 2 shows the frequency distribution of soil cone index readings obtained in our field experiments since October 1971. From this figure, a cone index reading of 200 kPa was taken to represent bad field conditions, 700 kPa to represent average and 1500 kPa to represent good field conditions. All the experiments were carried out in actual farmers fields, with a wide range of soil types, strengths, moisture contents and surface conditions, in the period September of one year to July of the following year and these readings can therefore be taken as indicative of field conditions in British farming.

3. Other factors affecting tyre tractive performance

Figs 1(a)–(c) apply to cross-ply drive tyres with a lug height of 35 mm, an aspect ratio (section height)/(section width) of 0·75, at a forward speed of 3·2 km/h, running on the field surface rather than in the furrow bottom and not running in a rut made previously by a leading tyre. More recent work has indicated that *Fig. 1(c)* is adequate to describe the rolling resistance of tractor front tyres, implement tyres etc. in addition to drive tyres.[18]

All the factors mentioned above will affect tractive performance[19–24] and it has been found that some of the factors can be taken into account by calculating the main performance parameters and applying correction factors. These factors are given in Table I and are used to multiply the performance parameters calculated from Eqns (3) to (6) to allow for the added effect. They should not be used in conjunction with each other, for example the k value for a high-lugged radial tyre in bad conditions compared to a cross-ply tyre with average height lugs cannot be found by multiplying 1·38 by 0·92. The correction factors for a high lugged radial tyre can only be obtained when such tyres have been compared directly to medium lugged cross-ply tyres.

The factors for radial-ply tyres apply for low inflation pressures (80 to 100 kPa) only. It was found that when the inflation pressure was increased to the maximum permissible value (160 kPa), there was no difference in tractive performance between radial and cross-ply tyres. The factor for C_{RR} when running in the furrow bottom is dependent on whether the tyre overlaps the furrow or not. Slight overlapping of the furrow (a 16·9×34 tyre rather than a 13·6×38 tyre running in a 356 mm (14 in) wide furrow) caused the correction factor for C_{RR} to increase to 2·4 in good conditions but the factor remained the same in average conditions. Although aspect ratio is known to affect tractive performance the range of aspect ratios available in commercial tyres is small and when a tyre of aspect ratio 0·69 (advertized as a low aspect ratio tyre) was compared to one with an aspect ratio of 0·75 there was no significant difference in performance. Running in a rut made previously by a leading tyre will definitely increase tractive performance. Unfortunately the experiment on this phenomenon was carried out early in our series of traction performance

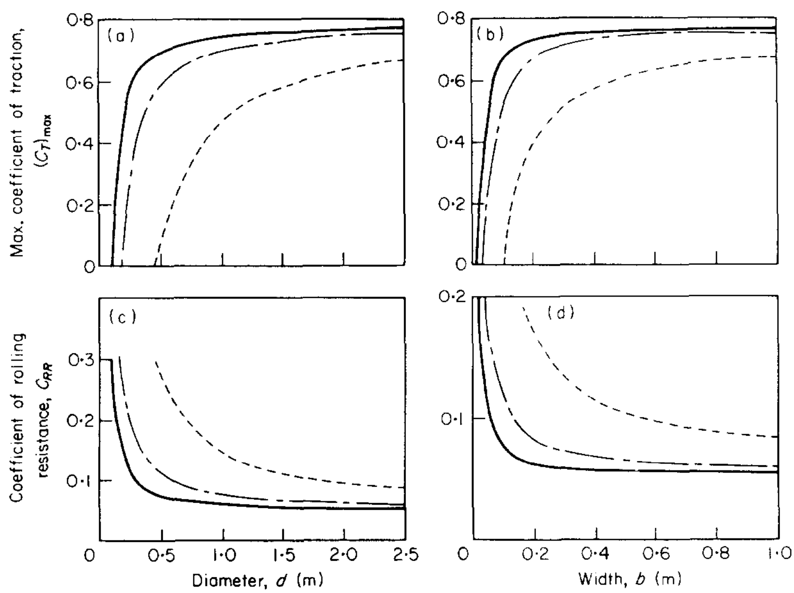

Fig. 3. Variation of traction parameters for a tyre carrying 7 kN load at δ/h = 0·2. (a) $(C_T)_{max}$ against d for b = 0·25 m; (b) $(C_T)_{max}$ against b for d = 1 m; (c) C_{RR} against d for b = 0·25 m; (d) C_{RR} against b for d = 1 m. ——, Good conditions; — — —, average conditions; - - -, bad conditions

experiments and the data are not in a suitable form to quantify the effect on the main performance parameters given in Table I. It was found that the coefficient of traction at 20% slip was increased by an average of 7% and the coefficient of rolling resistance decreased by an average of 11% when running in a rut made previously by a leading tyre of the same size. The factors for the effect of speed will obviously be dependent on the actual increase in forward speed. The N.I.A.E. single wheel tester[25] on which these experiments were carried out has a maximum field speed of 6·4 km/h. Increasing field speed beyond this value will increase the value of the correction factors given in Table I. There has not been found to be any significant difference in performance between similar tyres made by different manufacturers, even when there are small differences in tread pattern and lug angle.[26]

4. Drive tyre sizes for a lightly loaded axle

As a first example of how Eqns (3)–(6) can be used to examine the effect of parameter changes on a wheel's performance in different soil conditions, we can take a lightly loaded axle carrying 14 kN (i.e. 7 kN per wheel). As a first step we might wish to examine the effect of a change of diameter at fixed width and deflection/section height.

Fig. 3(a) shows the effect of changing wheel diameter on $(C_T)_{max}$ at $b = 0{\cdot}25$ m and $\delta/h = 0{\cdot}2$. Clearly in good conditions there is very little benefit from increasing wheel diameter beyond 1 m and in average conditions beyond 1·5 m. However, in bad conditions performance is still increasing appreciably at a wheel diameter of 2·5 m. A wheel diameter of 1 m would give values of $(C_T)_{max}$ of 0·95 of the maximum possible in good conditions, 0·88 of the maximum possible in average conditions and 0·59 of the maximum possible in bad conditions. Increasing the diameter to 1·5 m would improve these figures to 0·96 in good conditions, 0·93 in average and 0·74 in bad conditions. A wheel diameter of 1 m would therefore be an appropriate solution for a vehicle expected to spend most of its working life in average to good field conditions and a diameter of 1·5 m an appropriate solution for a vehicle expected to spend most of its life in average to bad conditions.

Assume now that there is some constraint on increasing diameter beyond 1 m. *Fig. 3(b)* shows the effect of changing wheel width on $(C_T)_{max}$ at $d = 1$ m and $\delta/h = 0{\cdot}2$. Little benefit is obtained by increasing width beyond 0·25 m in good conditions and 0·4 m in average conditions. In bad conditions performance is still increasing noticeably at a width of 1 m. Increasing width to 0·4 m at a diameter of 1 m gives the same $(C_T)_{max}$ in bad conditions as increasing diameter to 1·5 m at a width of 0·25 m. To get the same improvement in performance therefore the diameter has to be increased by 50% but the width by 60%.

Eqn (5) may now be used to calculate the effect on coefficient of rolling resistance of changing diameter and width. *Fig. 3(c)* shows the effect of changing wheel diameter on C_{RR} at $b = 0{\cdot}25$ m and $\delta/h = 0{\cdot}2$. As with $(C_T)_{max}$ the major benefits have been gained at a diameter of 1 m in good conditions and 1·5 m in average conditions with performance still increasing up to 2·5 m diameter in poor conditions. C_{RR} approaches its asymptotic value more slowly than $(C_T)_{max}$ however. At a diameter of 1 m C_{RR} is at 1·27 of its minimum possible value in good conditions, at 1·59 of its minimum in average conditions and at 3·06 of its minimum in bad conditions. Increasing diameter to 1·5 m decreases these figures to 1·18 in good conditions, 1·39 in average and 2·32 in bad conditions.

Fig 3(d) shows the effect of changing width on C_{RR}. Again, as with $(C_T)_{max}$ there is little benefit in increasing b beyond 0·25 m in good conditions and 0·4 m in average conditions but performance is still improving at a width of 1 m in bad conditions. Increasing width to 0·45 m at a diameter of 1 m gives the same coefficient of rolling resistance as increasing diameter to 1·5 m at a width of 0·25 m. This constitutes an increase of 50% in diameter but 80% in width for the same increase in performance. As with $(C_T)_{max}$, increasing diameter is more effective than increasing width. However both diameter and width have limiting values for any given load and field condition beyond which improvements in performance become small. Graphs such as *Fig. 3* enable these limiting values to be found.

Eqns (1), (2) and (4) allow curves of coefficient of traction against slip and tractive efficiency against slip to be drawn to illustrate the effect of changing diameter and width on these parameters. However, tyre selection can be made on the basis of *Fig. 3*.

The options which have emerged as possibilities are a tyre of 1 m diameter and 0·25 m width for a vehicle expected to spend most of its life in average to good field conditions and a tyre of 1·5 m diameter and 0·25 m width for a vehicle expected to spend most of its life in average to bad field conditions. If some vehicle constraint prevents tyre diameter being increased beyond 1 m, then the latter tyre could be replaced by one of 1 m diameter and 0·4 m width. Which of these options is chosen for a particular vehicle will, of course, depend on the vehicle itself, the relative cost of the various tyres and other constraints on the vehicle. Table II shows some commerical tyres roughly corresponding in dimensions with those found from these calculations.

To complete the calculations a designer or user would now take the actual tyre dimensions from Table II, actual values of δ/h and, using Eqns (1)–(6), calculate values of the major tractive performance parameters in different field conditions. This is done in Table III for the 3 tyres

TABLE II

Optional tyres for an axle carrying 14 kN load

Dimensions				*Load carrying capacity of tyre*		*Comments*
Calculated		*Actual*				
b *(m)*	d *(m)*	b *(m)*	d *(m)*	W *(kN)*	*Inflation pressure (kPa)*	
0·25	1·0	0·24	1·04	7	97*	9·5 × 24 agricultural drive tyre
0·25	1·5	0·24	1·69	7	200	9·5 × 48 rowcrop tyre
0·40	1·0	0·47	0·98	13	103†	38 × 20–16·1 terra-tyre

* Using 20% overload factor allowed by manufacturers for low speed operation
† Lowest recommended inflation pressure

TABLE III

Predicted performance parameters for optional tyres for 14 kN axle load

Tyre	*Tractive conditions*	*Parameter*			
		$(C_T)_{max}$	$k(C_T)_{max}$	C_{RR}	$(\eta)_{max}(\%)$
9·5 × 24 agricultural drive tyre	Bad	0·46	5·01	0·15	61
	Average	0·70	5·43	0·08	74
	Good	0·75	6·10	0·06	78
9·5 × 48 rowcrop tyre	Bad	0·59	5·11	0·11	68
	Average	0·74	5·79	0·07	77
	Good	0·77	6·87	0·06	80
38 × 20–16·1 terra-tyre	Bad	0·57	5·08	0·12	67
	Average	0·73	5·69	0·07	77
	Good	0·77	6·67	0·06	80

Note: A 9·5 × 24 tyre has a tyre section width of 9·5 in., and a rim diameter of 24 in. A 38 × 20–16·1 terra-tyre has an overall diameter of 38 in, a section width of 20 in, and a rim diameter of 16·1 in

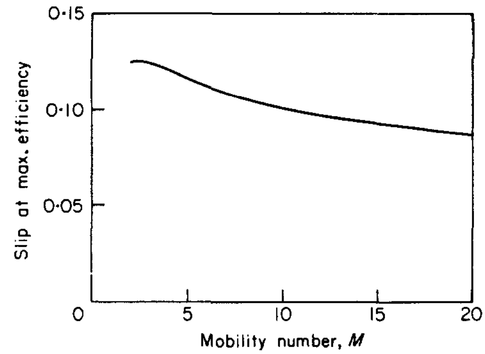

Fig. 4. Slip at maximum efficiency against mobility number

tabulated in Table II. Maximum tractive efficiency is calculated from a curve of slip at maximum efficiency against mobility number (*Fig. 4*) which will be derived later in this paper.

The axle could, of course, be a towed axle instead of a driven one. In that case decisions on tyre sizes would be made on the basis of *Figs 3(c)* and *3(d)* only. However, as can be seen from these calculations, the optimum tyre size would be virtually the same whether the axle was driven or towed.

5. Drive tyre sizes for a heavily loaded axle

To illustrate the effect of load on tyre selection, the calculations of section 4 were repeated for an axle carrying a load of 50 kN (25 kN per tyre). *Fig. 5(a)* shows the effect on $(C_T)_{max}$ of varying diameter with $b = 0{\cdot}35$ m and $\delta/h = 0{\cdot}2$. There are few agricultural drive tyres made which are larger than 1·9 m dia. and a typical diameter is 1·6 m. *Fig. 5(a)* shows that at a diameter of 1·9 m and width of 0·35 m, $(C_T)_{max}$ is almost at its maximum possible level in good conditions.

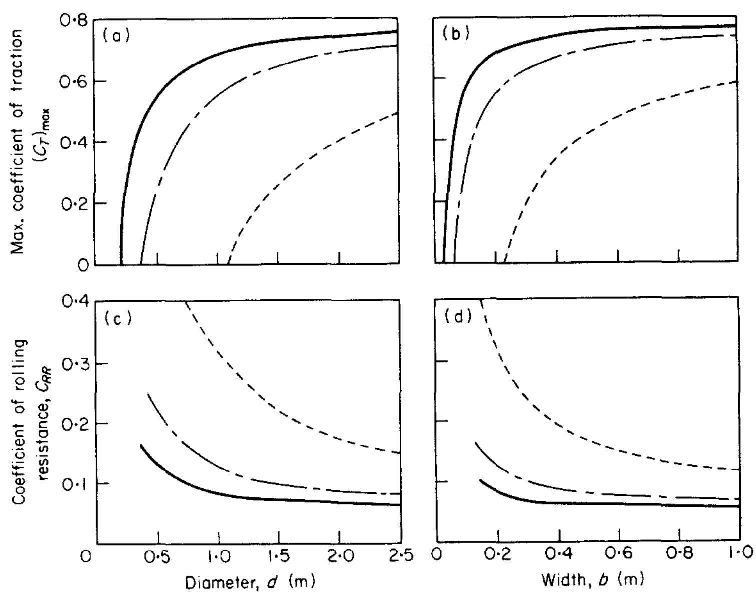

Fig. 5. Variation of traction parameters for a tyre carrying 25 kN load at $\delta/h = 0{\cdot}2$. (a) $(C_T)_{max}$ against d for $b = 0{\cdot}35$ m; (b) $(C_T)_{max}$ against b for $d = 1{\cdot}6$ m; (c) C_{RR} against d for $b = 0{\cdot}35$ m; (d) C_{RR} against b for $d = 1{\cdot}6$ m; ——, Good conditions; — - —, average conditions; - - -, bad conditions

This is not true in average to bad conditions however. To improve tractive performance in average to bad conditions an increase in diameter to 2·5 m would be desirable. The width of 0·35 m was chosen because this fits conveniently into the furrow bottom made by typical mouldboard ploughs used in Britain. Most mouldboard ploughing in Britain with low to medium powered tractors (up to 75 kW) is carried out with the furrow-side wheels in the furrow bottom. Overlapping the furrow causes part of the freshly ploughed land to be re-compacted and reduces tractive performance.[23] It should therefore be avoided where possible. *Fig. 5(a)* shows that if tyre width is restricted to 0·35 m to accomplish this then tyre diameter has to be increased to at least 2·5 m in order to obtain near maximum possible values of $(C_T)_{max}$ in average to bad conditions. As will be shown in section 7, an axle load of 50 kN is the load required to enable operation at maximum efficiency of a 50 p.t.o. kW tractor travelling at 6·4 km/h (a typical ploughing speed).

As mentioned previously a typical diameter for an agricultural drive tyre for a medium power tractor is 1·6 m. *Fig. 5(b)* shows the effect on $(C_T)_{max}$ of varying width with d = 1·6 m and δ/h = 0·2. In good conditions a width of 0·4 m is enough to ensure near-maximum possible values of $(C_T)_{max}$. In average conditions this width is increased to 0·8 m and in bad conditions performance is still increasing rapidly with width at a width of 1 m.

TABLE IV

Optional tyres for an axle carrying 50 kN load

Dimensions				*Load carrying capacity of tyre*		*Comments*
Calculated		*Actual*				
b (*m*)	d (*m*)	b (*m*)	d (*m*)	W (*kN*)	*Inflation pressure* (*kPa*)	
0·35	1·90	0·53	1·83	28	110†	20·8 × 38 agricultural drive tyre (increase in width necessary)
0·35	2·50	N.A.	N.A.	—	—	No tyre available at 2·5 m diameter
0·40	1·60	0·39	1·56	25	180*	15·5 × 38 agricultural drive tyre
1·00	1·60	1·05	1·69	43	100†	66 × 43·00–25 terra-tyre

* Highest inflation pressure recommended for 8 ply-rating tyre. 20% overload factor used
† Lowest recommended inflation pressure
N.A. = Not available

TABLE V

Predicted performance parameters for optional tyres for 50 kN axle load

Tyre	*Tractive conditions*	*Parameter*			
		$(C_T)_{max}$	$k(C_T)_{max}$	C_{RR}	$(\eta)_{max}(\%)$
20·8 × 38 agricultural drive tyre	Bad	0·49	5·02	0·15	62
	Average	0·71	5·48	0·08	75
	Good	0·76	6·21	0·06	79
15·5 × 38 agricultural drive tyre	Bad	0·35	4·96	0·19	53
	Average	0·67	5·27	0·09	70
	Good	0·74	5·77	0·07	77
66 × 43·00–25 terra-tyre	Bad	0·60	5·12	0·11	68
	Average	0·74	5·82	0·07	77
	Good	0·77	6·94	0·06	80

Figs 5(c) and *5(d)* show the effect on C_{RR} of varying diameter and width respectively.

The tyre sizes which have emerged as options are a diameter of 1·9 m in good conditions and 2·5 m in average to bad conditions if width is restricted to 0·35 m. Unfortunately there are at present no agricultural drive tyres with these dimensions. At a width of 0·35 m the maximum tyre diameter which can be obtained is 1·6 m (13·6 × 38 tyre) and this tyre will not carry the required load. To obtain a diameter of 1·9 m it is necessary to accept an increase in width to 0·53 m (20·8 × 38 tyre). If diameter is restricted to 1·6 m the options are a width of 0·4 m in good conditions and 1·0 m in average to bad conditions. The first option roughly corresponds to a 15·5 × 38 tyre which will just carry the required load but only at its highest recommended inflation pressure. The second option is satisfied by a 66 × 43·00–25 terra-tyre if an increase in diameter to 1·69 m can be tolerated. Dimensions of the tyres are shown in Table IV and predicted performance parameters in Table V.

6. The effect of changing tyre stiffness

The calculations of tractive performance parameters in sections 4 and 5 have been made using the assumption that the tyre deflection/section height was constant at 0·2. This is a reasonable figure to use with agricultural drive tyres since currently available tyres used at recommended inflation pressures for a given load give hard surface δ/h values close to 0·2. Tyre inflation pressure will of course have a strong effect on tyre deflection as will tyre side wall stiffness. Both of these factors are known to affect tractive performance. It is interesting therefore to examine predictions made by the empirical equations on the effect of increasing tyre deflection beyond the levels commonly encountered with current tyres.

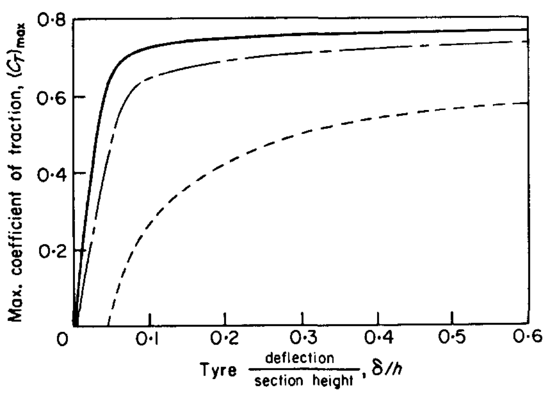

Fig. 6. Variation of $(C_T)_{max}$ with δ/h at d = 1·6 m, b = 0·5 m, W = 25 kN. ——, Good conditions; — - —, average conditions; - - -, bad conditions

Fig. 6 shows the effect on $(C_T)_{max}$ of increasing δ/h at a wheel diameter of 1·6 m, width of 0·5 m and load of 25 kN. The dimensions of the tyre are similar to a commercial tyre large enough to carry a 25 kN load. In good conditions little will be gained by increasing δ/h beyond the current level of 0·2. At this value $(C_T)_{max}$ is already at 0·94 of its maximum possible value and increasing δ/h to 0·6 only increases this to 0·96. In average conditions $(C_T)_{max}$ is at 0·87 of its maximum possible value at δ/h = 0·2 and at 0·92 of the maximum at δ/h = 0·6. The major benefits would once again be in average to bad conditions. In bad conditions $(C_T)_{max}$ would increase from 0·53 of its maximum possible value at δ/h = 0·2 to 0·73 of the maximum at δ/h = 0·6.

From *Fig. 2* the area of the curve between the cone index reading of 700 kPa and the origin is 39 % of the total area. Thus 39 % of all the fields used in our experiments had traction conditions which are classified here as average or worse. This seems to be a large enough percentage to warrant more consideration being given to designing tyres to operate efficiently in these conditions than is the case at present.

7. Loading for maximum efficiency

Eqns (2) to (5) may be used to calculate the load which must be on a driving axle per unit of available power at the axle to ensure that the tyres operate at maximum efficiency at any field speed.

From Eqn (2) the tractive efficiency η is given by:

$$\eta = \frac{C_T\,(1-s)}{C_T + C_{RR}}.$$

Differentiating and equating $d\eta/ds$ to zero it is found that the slip at maximum efficiency can be found from the equation

$$(1-s)\frac{dC_T}{ds} = \frac{C_T\,(C_T + C_{RR})}{C_{RR}}. \qquad \ldots(7)$$

From (1), $C_T = (C_T)_{max}\,(1-\exp(-ks))$, i.e.

$$\frac{dC_T}{ds} = k\,(C_T)_{max}\,(\exp(-ks)). \qquad \ldots(8)$$

Substituting for C_T and dC_T/ds in Eqn (7):

$$(1-s)\,k(C_T)_{max}\,(\exp(-ks)) = \frac{(C_T)_{max}\,(1-\exp(-ks))\,\{(C_T)_{max}\,(1-\exp(-ks)) + C_{RR}\}}{C_{RR}}. \qquad \ldots(9)$$

$(C_T)_{max}$, k and C_{RR} are all, of course, functions of mobility number and therefore the slip at maximum efficiency is also a function of mobility number. The solution to Eqn (9) is shown in

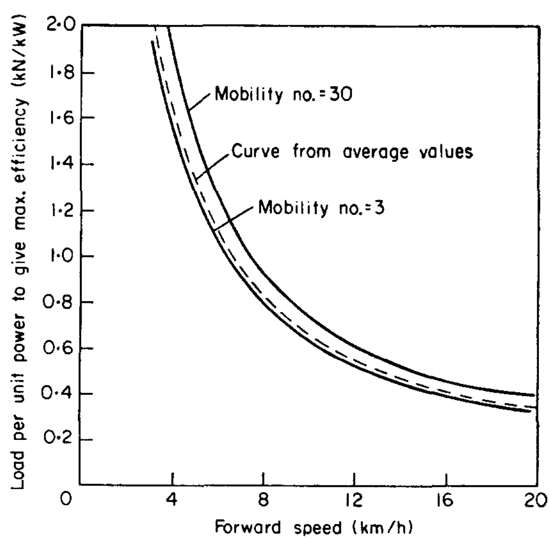

Fig. 7. Axle load for maximum efficiency at different forward speeds

Fig. 4 which shows that, fortunately, slip at maximum efficiency does not vary strongly with mobility number in the range normally encountered in practice. A value of 0·10 would seem to be a good average figure for slip at maximum efficiency. To find the load required per unit of available power to ensure operation at maximum efficiency the following equality is used:

$$\text{Power} = C_T \times W \times \text{forward speed} = \eta \times \text{available power.} \qquad \ldots(10)$$

Using the solutions to Eqn (9), values of C_T at maximum efficiency and maximum efficiency itself are calculated and inserted into Eqn (10). Curves may then be drawn of the axle load per unit of available power at the axle to ensure operation at maximum efficiency at any forward speed. Total axle power may be taken to be equivalent to p.t.o. power for most agricultural tractors. These curves are shown in *Fig. 7*. As can be seen, the variation with mobility number is small, a mobility number of 30 represents very good traction conditions and one of 3, very bad conditions. It would seem to be justified therefore to condense these curves into a single average curve to simplify calculations. The dashed line in *Fig. 7* shows the curve calculated by Dwyer[1] from average values of the traction parameters. This curve corresponds approximately to the curve for a mobility number of 5 and is a suitable curve to use for these calculations. From this curve the load per unit power required to ensure operation at maximum efficiency at 6·4 km/h (a typical field speed for heavy cultivations) is 1kN/kW. A 50 p.t.o. kW 2-wheel-drive tractor therefore requires a total rear axle load (including weight transfer from an implement where applicable) of 50 kN or 25 kN per wheel. This is the load used for the heavily loaded axle calculations in section 5.

To optimize performance of the vehicle, the draught force must then be adjusted so that the coefficient of traction produced by the driving wheels is the value for maximum efficiency. C_T at maximum efficiency against mobility number is shown in *Fig. 8*. This curve is remarkably flat for values of mobility number greater than 5. The value of C_T in this region is 0·38. Therefore to optimize the field performance of, say, a 2-wheel-drive tractor when ploughing at a particular speed, the rear axle load is first calculated from *Fig. 7* and the plough draught force adjusted until it is 0·38 of the rear axle load or the wheel-slip is approximately 10%.

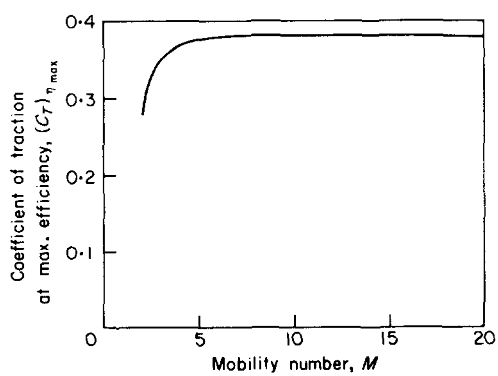

Fig. 8. Variation of coefficient of traction at maximum efficiency with mobility number

The effect of operating with less than the optimum load on the driving wheels has been investigated by Dwyer.[27] He showed that a reduction of load to 0·8 kN/kW at 6·4 km/h was not too detrimental to efficiency, particularly if its effect was at least partially offset by an increase in speed. However, a reduction to the commonly used value of 0·65 kN/kW could lead to losses in efficiency of up to 30%, depending on speed and soil conditions.

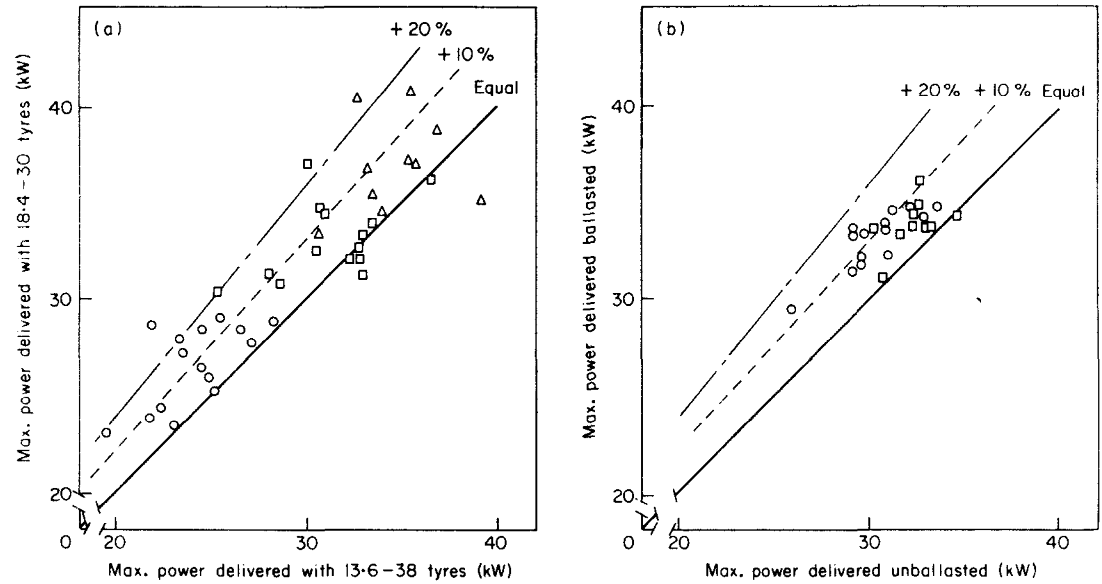

Fig. 9. Increase in power delivered by 50 p.t.o. kW. 2-wheel-drive tractor due to (a) fitting larger drive tyres ; ○, 3rd gear; □ 4th gear; △, 5th gear: (b) ballasting to the optimum level; ○, 3rd gear; □, 4th gear

Validation experiments on the benefits from fitting larger tyres and ballasting to the optimum level have been carried out at N.I.A.E. in recent years. *Fig.* 9(*a*) shows the maximum power delivered by 2 matched 50 p.t.o. kW 2-wheel-drive tractors when ploughing. One tractor was fitted with the "standard" 13·6 × 38 drive tyres and the other with 18·4 × 30 drive tyres which are the same diameter but almost 40% wider than the 13·6 × 38 tyres. The load on the driving wheels was approximately the same for both tractors. The tractor with 18·4 × 30 tyres delivered an average 10% more power in 3rd gear and 6% more in 4th and 5th gears. Pull at maximum power was increased an average of 14% in 3rd gear, 7% in 4th and 9% in 5th gear.[28] When the axle load on the tractor with 18·4 × 30 drive tyres was increased by ballasting to near the 1 kN/kW level and the performance compared with unballasted 18·4 × 30 tyres, the results shown in *Fig.* 9(*b*) were obtained.

Mean power increase due to ballasting was 9% in 3rd gear and 5% in 4th gear.

Note that these rules are for maximizing the power output of the tractor which is not necessarily the same as maximizing the work-rate of the tractor–implement combination. For example, the useful power delivered by a tractor when ploughing will be given by:

$$P = awV\sigma \qquad \ldots(11)$$

where P = power,
a = depth of ploughing,
w = width of ploughing,
V = forward speed,
σ = plough specific resistance.

The product wV is the work rate, i.e.

$$\text{work rate} = \frac{P}{a\sigma}. \qquad \ldots(12)$$

P and a may be regarded as constants since ploughing has to be carried out to a fixed depth and we wish to utilize all the power available from the tractor. Plough specific resistance for one particular plough was found by Gee-Clough *et al.*[17] to be given by the expression:

$$\sigma = 13{\cdot}3\ \gamma a + 3{\cdot}06\,\frac{\gamma V^2}{g} \qquad \ldots(13)$$

where γ is the soil specific weight and g the gravitational constant. Since this expression increases as speed squared the work rate will be maximized at the lowest speed at which it is practically possible to ballast to the correct axle load and handle a plough wide enough to produce the required pull.

The maximum possible tractive efficiency in field conditions may be calculated from Eqns (2)–(5). By setting M to an infinitely high value, the maximum possible tractive efficiency is given as

$$(\eta)_{\text{max. possible}} = \frac{0{\cdot}796}{0{\cdot}796+0{\cdot}049} = 0{\cdot}942. \qquad \ldots(14)$$

8. Single tyre compared to duals of same overall dimensions

Eqn (6) defines mobility number as

$$M = \frac{Cbd}{W}\sqrt{\delta/h}\left(\frac{1}{1+b/2d}\right).$$

Figs (1)–(3) show that, as mobility number increases, then tractive performance also increases. Eqn (6) may therefore, by itself, be used to make predictions as to the comparative tractive performance of different tyre combinations.

As a first example the performance of a single tyre may be compared to duals of the same overall dimensions.

For the duals, if they are assumed to act independently of each other, each tyre has width b, diameter d and carries a load of $W/2$. Therefore the mobility number will be:

$$M_1 = \frac{Cbd}{(W/2)}\sqrt{\delta/h}\left(\frac{1}{1+(b/2d)}\right). \qquad \ldots(15)$$

The single tyre has width $2b$, diameter d and carries a load of W. Its mobility number will be

$$M_2 = \frac{C(2b)d}{W}\sqrt{\delta/h}\left(\frac{1}{1+(2b/2d)}\right). \qquad \ldots(16)$$

Assume that δ/h is the same for the duals and single tyre and that they are operating in the same soil.

$$\frac{M_1}{M_2} = \frac{1+(b/d)}{1+(b/2d)}. \qquad \ldots(17)$$

This ratio will always be greater than unity therefore the prediction is that duals will always perform better than a single tyre of the same overall dimensions. Preliminary results from experiments at N.I.A.E. in progress at the moment confirm this prediction. Gee-Clough[29] performed experiments on dual rigid wheels in sand in which the spacing between wheels was varied from zero to 3 wheel widths. The coefficient of rolling resistance fell steadily as separation increased and at 3 wheel widths separation was 12% below that at zero separation. However, the wheels were not acting independently of each other even at 3 wheel widths separation and since, in practice, the allowable separation will almost always be less than this, the likelihood is that although duals will perform better than a single tyre of the same overall dimensions, the improvement will be slightly less than that predicted by these empirical equations.

9. Single tyre compared to duals of different dimensions

A question often asked is "Which will give better performance, a single large diameter tyre or two smaller tyres used as duals?". The answer of course is dependent on the relative dimensions of the tyres.

If the duals each have a diameter d_1, width b_1 and each carry a load of $W/2$, the mobility number will be:

$$M_1 = \frac{Cb_1d_1}{(W/2)}\sqrt{\frac{\delta}{h}}\left(\frac{1}{1+(b_1/2d_1)}\right). \qquad \ldots(18)$$

If the single tyre has diameter d_2, width b_2 and carries a load W, the mobility number will be:

$$M_2 = \frac{Cb_2d_2}{W}\sqrt{\frac{\delta}{h}}\left(\frac{1}{1+(b_2/2d_2)}\right). \qquad \ldots(19)$$

Assuming C and δ/h are the same for the duals and single tyre:

$$\frac{M_1}{M_2} = \frac{2b_1d_1}{b_2d_2}\left[\frac{1+(b_2/2d_2)}{1+(b_1/2d_1)}\right]. \qquad \ldots(20)$$

This will be greater than unity if

$$\frac{2b_1d_1}{b_2d_2} > (1+b_1/2d_1)(1+b_2/2d_2). \qquad \ldots(21)$$

If the inequality of Eqn (21) is satisfied (i.e. M_1 is greater than M_2) the duals will have better tractive performance than the single tyre. If it is not satisfied the single tyre will perform better.

Fig. 10 shows the results of calculating the relative performance of a single 13·6×38 size tyre (dia. = 1·6 m, width = 0·35 m) compared to different sized tyres used as duals. If the width of each of the dual tyres is 0·4 m then diameter has to be 0·77 m or more before the duals are better than the single tyre. If the width of each of the duals is reduced to 0·2 m the diameter necessary for the duals to be better is 1·32 m or more.

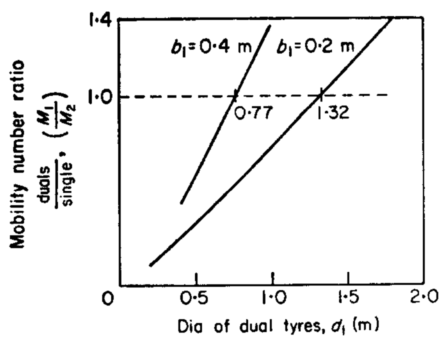

Fig. 10. Mobility number ratio of dual tyres of different dimensions compared to a single 13·6×38 tyre carrying the same load

10. Use of high inflation pressure, high load carrying capacity tyres instead of agricultural tyres

There is a temptation on some agricultural vehicles, particularly harvesting machines, to either grossly overload the drive tyres or to replace agricultural drive tyres with high inflation pressure, high load carrying capacity tyres such as earthmover tyres. The earthmover tyres are generally much smaller than an agricultural drive tyre which can carry the same load. They are therefore easier to fit to a machine which has been designed allowing only a limited space for drive wheels. The consequences of fitting these smaller tyres can have a drastic effect on the tractive performance of the vehicle as is illustrated by the following example.

A load of 53 kN can be safely carried by dual 18·4×38 agricultural drive tyres at 124 kPa inflation pressure using the 20% overload factor for low speed operation. The same load can be safely carried by a single 16·00×25 earthmover tyre at 517 kPa inflation pressure. The maximum coefficient of traction which would be delivered by these different tyres is shown in *Fig. 11(a)*.

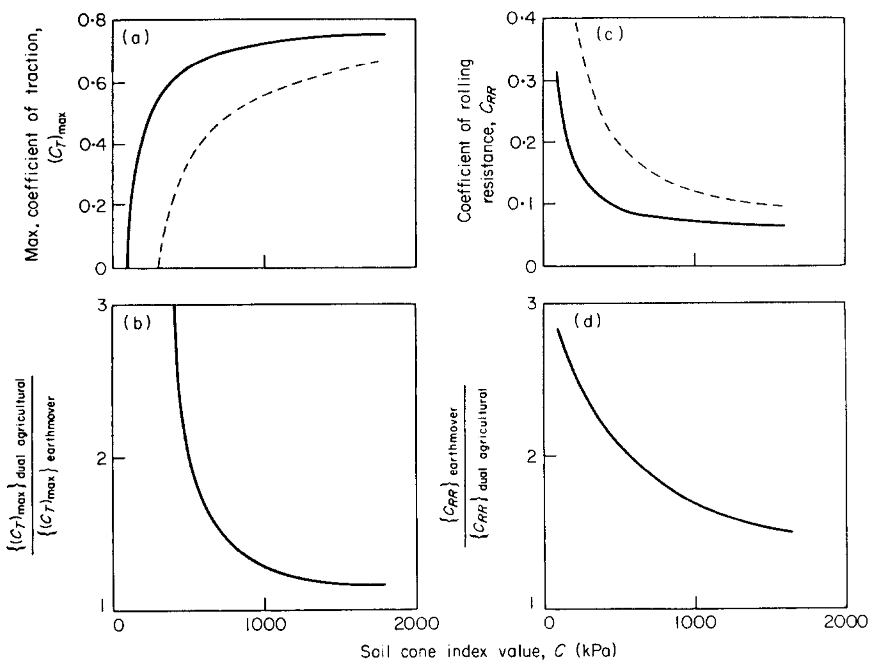

Fig. 11. Relative performance of dual 18·4×38 agricultural drive tyres and a single 16·00×25 earthmover tyre. (a) $(C_T)_{max}$ against cone index: (b) $(C_T)_{max}$ ratio against cone index: (c) C_{RR} against cone index: (d) C_{RR} ratio against cone index. ——, Dual agricultural drive tyres; - - -, single earthmover tyre

The dual 18·4×38 tyres would deliver considerably higher values of $(C_T)_{max}$ than the earthmover tyre, particularly at low soil cone index values (soft soil conditions). A vehicle fitted with the earthmover tyres would become immobilised in fields with a cone index value of 300 kPa or less while a vehicle fitted with the dual agricultural tyres would not become immobilized until a cone index value of 100 kPa or less was reached. *Fig. 11(b)* shows the ratio of $(C_T)_{max}$ values of the duals compared to the single earthmover. At a cone index value of 400 kPa the duals would deliver 3 times the $(C_T)_{max}$ value delivered by the earthmover tyre. At a cone index value of 700 kPa the value would be 1·52 times the earthmover tyre value and at 1500 kPa, 1·08 times. Thus only in very good conditions would the earthmover tyre deliver a similar pull to the dual agricultural tyres and as field conditions deteriorate its relative performance would deteriorate badly.

Figs 11(c) and *11(d)* show a similar result for coefficient of rolling resistance. Thus, although the earthmover tyre could carry the load safely, its field performance would be considerably inferior to that of the dual agricultural tyres carrying the same load. A small initial saving on tyre cost for a vehicle fitted with the earthmover tyres would be quickly lost in increased operating costs, increased soil damage and reduced vehicle efficiency.

11. Conclusions

The simple empirical equations given in this paper can readily be used by both vehicle designers and vehicle users to obtain predictions of the effect of varying tyre parameters on tractive performance in any field condition. The accuracy of prediction has been checked in validation experiments and has been found to be satisfactory. Further refinement of the equations may allow prediction to become even more accurate but the equations can be used in their present form. Measurement of the tractive performance of agricultural vehicles indicates that there is considerable scope for improvement.[30] The use of these equations will indicate what is necessary to obtain the required improvement.

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