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Feature Article

Operations Research for Agricultural Pest Management

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It is increasingly important to maximize the productivity of our agricultural land. As concern for efficient food and fiber production increases, the importance of the application of operations research to agricultural problems increases. During the past decade, there has been much effort to combine mathematics and agriculture. This paper presents the role that operations research has played in integrated pest management for the agricultural industry. An effort is made to present the state-of-the-art in biological modeling for agriculture pest management and to provide an understanding of the type of mathematical techniques utilized.

OPERATIONS RESEARCH methodologies are now used in areas which traditionally have not taken advantage of quantitative management techniques. The activity in some areas, like hospital administration or criminal justice systems, has advanced to such an extent that it is not uncommon to see publications relating mathematical definitions for problems which 15 or 20 years ago would not have been considered in an analytical context. One of the newest frontiers for operations research is in agricultural pest control. Many complex interactions are present in these biological systems which can be fully understood only through mathematical modeling. Also, there is a wide range of potential control parameters that must be related mathematically to the underlying biological processes. The obvious benefits of increased productivity and the intelligent use of pesticides make the agricultural applications of operations research an exciting area for current and future research.

Mathematical interest in biological populations has been apparent since Malthus [1798] described population growth as an exponential process. Since that time the interest in biological modeling has increased tremendously so that today there are excellent journals devoted solely to this subject. The range of applications for mathematical modeling and optimization in biology covers the entire biological field. Significant

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mathematical progress has been made in such diverse areas as epidemiology (Bailey [1975]), fisheries (Clark [1976]), ecological assessment (Pielou [1977]), and tracer kinetics (Matis and Wehrly [1979]). There has also been considerable modeling work for pest control within forest systems (Clark et al. [1979], Feldman et al. [1981]). With such a broad spectrum of application areas for operations research, it is necessary to restrict this paper to one area: pest control for agricultural systems. The objective of this paper is to present operations research applications for pest control, especially in the agricultural industry, and to indicate the shortcomings and the need for future research. The paper also attempts to provide the reader with an understanding of the mathematics involved.

The first step in utilizing operations research methodologies is to describe mathematically the underlying biological processes. Only after relevant responses have been described can control measures be investigated. This obvious first step is emphasized because it has so often been ignored. The literature abounds with theoretical studies of mathematical biology, especially the investigation of stability and steady-state questions. For most agricultural control problems, such studies are irrelevant since pest control usually deals with growing populations. These populations, if left uncontrolled, would approach an economically unacceptable steady-state.

Another approach in past studies has been to take well-known models in operations research and change the names of the variables to biologically meaningful terms. This leads to mathematical abstraction for the sake of mathematics and rarely results in applications. Jaquette ([1972], pp. 1144–1145) gives some examples of this in his survey article. During the 1970s, considerable effort went into developing models that were biologically realistic. However, in striving for biological realism, the models often became very complex and were intractable for optimization studies. Thus, it is necessary to simplify these models while retaining their realism to give accurate biological responses to control measures. One of the early characteristics of the field of operations research was its interdisciplinary nature. The interdisciplinary feature is vital in the pest management field where models must be developed in cooperation with an experimental program if results are to be put in practice. Much of the successful mathematical work during the recent decade has been the result of large multi-university research programs funded by the National Science Foundation, the Environmental Protection Agency and the United States Department of Agriculture (Huffaker [1980], Thatcher et al. [1981]). Successful applications of operations research to integrated pest management often involves extensive programming efforts in developing software for a decision support system. Such programming problems, although critical, are beyond the scope of this paper.

World population growth and insufficient food and fiber production are creating severe shortages in many parts of the world. Although the field of operations research seems removed from agricultural production problems, there are several areas where major contributions can be made. The training and orientation of operations research specialists will enhance the descriptive modeling of these systems. Analytical descriptors of agricultural systems are essential for the accelerated refinement and improvement of yields. The use of optimization techniques and parametric analysis will facilitate the improvement of integrated pest management procedures. The decade-plus lead time for experimental research along with the variability of field evaluation procedures makes it extremely difficult to pursue promising avenues without the aid of mathematical guidance. Many alternatives need to be evaluated as to their possible impact both individually and cooperatively. Promising results can then be implemented in a timely fashion.

The operations research analyst can make a major contribution to accelerated agricultural production research. A small start has been made but an increased effort is necessary before the contributions can have an appreciable effect.

This paper begins with a cursory review of agricultural control tactics in Section 1. Section 2, dealing with the development of mathematical models, is the largest section because model development has been the focus of applied agricultural/mathematical research during the past decade. Optimization techniques utilized for agricultural systems are discussed in Section 3. At the end of each major topic discussion, indications are given as to future needs and open research questions. The convention of identifying insects and plants by their scientific names on first use and utilizing their common names subsequently is followed whenever a common name is available.

1. BACKGROUND

Early in this century, agricultural scientists interested in pest control primarily studied cultural methods for control. With the advent of chemical insecticides, a dramatic shift in control practices occurred. In general, cultural practices were replaced by spraying strategies which eventually led to a very narrow crop genetic base. Agricultural systems were heavily energy subsidized and high yield varieties were emphasized along with their inherent increase in pest populations. Thus, an increasing spiral of more and more chemical controls became evident (Huffaker).

In the last two decades the chemical buildup in soil, water, and nontarget species, along with increasing pest resistance, caused a reconsideration of nonchemical control strategies. There have been some studies investigating the buildup of pesticide resistance (Comins [1977], Regev et al. [1977], Plapp et al. [1979]) within a population; however, this work has not been fully incorporated into decision models. An integration of biological and cultural controls with insecticide usage is increasingly emphasized (Huffaker). This integrated pest management approach has the goal of maximizing producer profits while reducing the amount of toxic chemicals utilized. The major components of integrated pest management include biological control agents (predators and parasites), genetic controls (plant breeding for pest resistance, crop timing, and damage tolerance), and cultural practices (row spacing, low inputs, and early harvest), as well as timely applications of pesticides.

Operations research has considerable utility as a research component in pest management studies. Assuming that an adequate model can be developed, the effects of various strategies can be studied at the population level (Curry et al. [1980]). Population level studies are often not immediately applicable in the field environment. For example, the decision to breed a specified trait into a production variety is a commitment of approximately 10 years. Only through a realistic modeling program can an accurate assessment of the effects of a new variety be made within the context of integrated controls.

Biological Control

The study of predator-prey models was one of the first areas to receive biological modeling emphasis (Lotka [1925], Volterra [1926]). Classical biological control involves the regulation of a pest species by an exotic (nonindigenous) natural enemy. There have been many successes, several of which are reported in Caltagirone [1981]. Although not as spectacular, biological control can also involve indigenous species, often in conjunction with pesticides.

Genetic Control

Breeding programs have long been used in agriculture for improved productivity. Breeding for increased pest resistance can either be through systemic or physical changes. Modeling plant-insect systems that include the ability to predict the effects of breeding programs on agricultural productivity is difficult and has not been widespread. The reason is obvious: to reflect the effects of breeding studies, the mathematics must include the relationship between the plant's biological or physical characteristics and the insect's life processes.

Cultural Control

Cultural and physical control has historically been an effective pest management tool. Time of planting and harvesting, crop rotation, and field sanitation have been major contributors throughout the history of crop cultivation. Although the availability of pesticides and chemical fertilizers caused a movement away from these standard measures, research is being redirected toward them.

2. MODELING APPROACHES

Development and Physiological Time Scales

The physiological development of many organisms is through a sequence of clearly defined stages (e.g., egg, larva, pupa, adult). Field populations often involve organisms in overlapping stages which cause difficulties in the statistical analysis of field data. Additional difficulties arise in establishing developmental times because there are two competing causes of an organism leaving a stage: developmental completion and mortality. In a fashion reminiscent of traditional reliability, Read and Ashford [1968] and Kempton [1979] give details for some of the statistical problems and procedures involved in estimating life length distributions from field data. One problem not addressed by their statistical analysis is the dependence between temperature and developmental times.

Most organisms within an agricultural system are poikilotherms (coldblooded), thus their life processes (development, reproduction, mortality, etc.) depend on temperature. The magnitude of changes in the process rates caused by changing temperatures can be very large (Sharpe and DeMichele [1977]); this temperature dependency must be considered. One common procedure for incorporating temperature into a biological model is to use a degree-day time scale instead of chronological time (Candolle [1865], Gilbert et al. [1976]). The degree-day scale accumulates the number of degrees between upper and lower temperature thresholds. For example, if a plant takes 4 days to flower at constant 20°C, then using a 10°C threshold it takes 40 degree-days to flower (because $4 \times (20 - 10) = 40$). Conceptually, this implies that it would take 2 days to flower at 30°C (because $2 \times (30 - 10) = 40$). Frequently the degree-day methodology does not yield acceptable accuracy due to its inherent linearity assumption. To overcome this inaccuracy, "physiological" time scales have been developed (Stinner et al. [1974], Campbell et al. [1974], Sharpe et al. [1977]) which are based on temperature-dependent rate integration methods. (See Equation 1 below.)

Not only are developmental times nonlinear with respect to temperature, they are probabilistic. Monte Carlo simulation is often used to incorporate the randomness into population models (Hardman [1976], Gilbert et al.). A preferred analytical approach is to superimpose a probability density function on the physiological time scale (Stinner et al., Hardman, Sharpe et al.). The probability of emergence is determined by integrating this p.d.f. over the elapsed physiological time interval. To illustrate this mathematically, the physiological time scale must first be determined. Let $r(k)$ denote the developmental rate (inverse of the length of time for the process) at constant temperature k, and let $\psi(t)$ denote the temperature at chronological time t. Physiological time, x, is then given by

$$x(t) = \int_0^t r(\psi(\tau))d\tau \tag{1}$$

where the process is assumed to start at time zero. Superimposed on this time scale is a distribution function $F(\cdot)$. The function F is related to the random variable T^ψ, which denotes the time to complete development under temperature regime ψ, by

$$P\{T^\psi \leq t\} = F(x(t)). \tag{2}$$

The mathematical implications inherent in using Equations 1 and 2 are investigated in Curry et al. [1978a, b]. The utilization of these equations in predicting variable temperature responses is often through an iterative procedure where the temperature function ψ is approximated by a step function.

Several open questions regarding physiological time scales and the associated distribution functions remain. Many organisms experience thermal death when exposed to high or low temperatures for extended periods of time. Such thermal mortality is not incorporated in the above procedure. Another problem is that Equation 2 assumes that the probability distribution functions from different temperature regimes when placed on the physiological time scale are identical. This "same shape" assumption has been shown to hold for many agricultural pests, but no theory has yet been established for pests for which this assumption does not hold.

Utilizing a physiological time scale allows the modeling of populations on a consistent time axis. Many of the historical models for single species (Verhulst [1938], Pearl [1932]) and interacting species (Lotka, and Volterra) are frequently not applicable in agricultural systems because they ignore the age-dependent aspects of critical processes such as reproduction and mortality.

Single Species Models

The incorporation of the age-dependent nature of the processes requires that a two-dimensional age-time scale be used. For single species populations, Lewis [1942] and Leslie [1945, 1948] pioneered a discrete matrix approach which includes both time and age. They represent the population as a vector $\mathbf{n}(t)$, where $n_i(t)$ is the number of individuals alive at time t of age class i. The time dependent dynamics of the population are given by

$$\mathbf{n}(t+1) = \mathbf{A}\mathbf{n}(t) \qquad \text{for } t = 0, 1, \cdots \tag{3}$$

where

$$\mathbf{A} = \begin{bmatrix} \lambda_0 & \lambda_1 & \cdots & 0 & \lambda_n \\ p_0 & 0 & \cdots & 0 & 0 \\ o & p_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & p_{n-1} & 0 \end{bmatrix}$$

and λ_i is the number of births (females occurring while the mother is in the ith age class) and p_i is the conditional probability of surviving age class i given survival through age class $i-1$. Leslie [1959] extended these matrix methods to include time-dependent parameters for birth rates and survival probabilities, while Pollard [1966] extended the approach for determining standard deviations as well as expected population sizes. Usher [1972] and Pielou give excellent summaries and indicate many of the extensions.

An iterative-cohort methodology presented in Curry et al. [1978b] is conceptually similar to Leslie's approach, while being computationally oriented toward time-dependent rate parameters. The iterative-cohort methodology biologically aggregates similar organisms into groups called cohorts. (For example, all eggs oviposited within the same day would be a cohort.) Each cohort is followed through its developmental processes. The procedure describes iteratively the evolution of each cohort, the generation of new cohorts, and the interactions between cohorts and the physical and environmental parameters.

In developing the iterative-cohort approach, the probabilistic aspects and the temperature effects on individual life processes are studied. For example, the developmental completion time distribution function (2) commonly used for emergence times is actually a conditional probability given live emergence. Equation 2 can be applied directly to the final life stage (adult) to predict longevity, but to describe the joint distribution of survival and emergence from one stage to the next (e.g. egg to larvae) (2) should be modified. The modification is necessary because temperature affects the proportion of ultimate survival and thus the effects of temperature cannot be totally incorporated into a physiological time scale. To illustrate these concepts, let the proportion of survival at a fixed temperature k be $c(k)$, let T^{ψ} denote developmental completion time, and let U^{ψ} denote time of death, then

$$P\{T^{\psi} \leq t, U^{\psi} > t\} = \int_0^t c(\psi(\tau))dF(x(\tau)). \tag{4}$$

The assumptions underlying Equations 1–4 are not valid for all poikilotherms. Curry et al. [1978a, b] explicitly give assumptions sufficient for many of the commonly used mathematical procedures and demonstrate their utility. Unlike emergence, reproduction has not been adequately studied as a stochastic process. Current approaches for modeling reproduction are deterministic.

The Leslie matrix and iterative cohort approaches are essentially equivalent to the numerical solution of a continuous description of population dynamics. A continuous time representation of an age-dependent population was formulated by Von Foerster [1959]. His model utilizes a number density function for the population size. Denoting the population density by $n(t, a)$ for $t, a \geq 0$, the population size alive at time t within the age interval (a_1, a_2) is given by $\int_{a_1}^{a_2} n(t, a)da$. Von Foerster assumes that organism age and the time axis are measured on identical scales, i.e. $da/dt = 1$. Additionally, he defined an age-dependent birth rate $\lambda(\cdot)$ and an age-dependent cumulative probability distribution function for life length $F(\cdot)$. An age-dependent mortality rate function, $\mu(\cdot)$, is obtained from the distribution function by

$$\mu(a) = (dF/da)/(1 - F(a)), \qquad \text{for } a \geq 0. \tag{5}$$

Utilizing the rates λ and μ, the population dynamics are described, for a given initial population size $n(0, \cdot)$, by

$$\partial n/\partial t + \partial n/\partial a = -\mu(a)n(t, a), \qquad \text{for } t, a > 0 \tag{6a}$$

with new births being the boundary condition on the time axis

$$n(t, 0) = \int_0^{\infty} \lambda(s)n(t, s)ds, \qquad \text{for } t > 0. \tag{6b}$$

The left-hand side of Equation 6a is the directional derivative along the time-age trajectory of the organism (characteristic line).

It is sometimes convenient to express the system (6a, 6b) as integral equations; that is, integrating along characteristics,

$$n(t, a) = \begin{cases} n(t - a, 0)\exp\left\{-\int_0^a \mu(s)ds\right\} & \text{for } 0 < a < t, \\ \\ n(0, a - t)\exp\left\{-\int_{a-t}^a \mu(s)ds\right\} & \text{for } 0 < t < a, \end{cases} \tag{7}$$

$$n(t, 0) = \int_0^\infty \lambda(s)n(t, s)ds \qquad \text{for } t > 0. \qquad (8)$$

Equations 7 and 8 can now be combined to form a renewal type equation. To see this, let the birth rate function $b(\cdot)$ be defined by $b(t) = n(t, 0)$. Then by substituting (7) into (8),

$$\begin{aligned} b(t) = &\int_t^\infty \lambda(a)n(0, a - t)\exp\left\{-\int_{a-t}^a \mu(s)ds\right\}da \\ &+ \int_0^t \lambda(a)b(t - a)\exp\left\{-\int_0^a \mu(s)ds\right\}da, \end{aligned} \qquad (9)$$

where $n(0, a)$ for $a > 0$ is the known initial population density function. Solutions for renewal type equations are well known (e.g., Feller [1971], p. 468) and thus $b(\cdot)$ can be found, which in turn gives a solution for (7). The Von Foerster equations represented in (5–9) provide a mathematical structure for realistic agricultural models. These systems also provide a rich area for theoretical studies. Some of the basic background is presented in Hoppensteadt [1975] and Oster [1976]. Gurten and MacCamy [1979] consider density dependent mortality and births by extending the theory to nonlinear structures where the birth and death rates are functions of the total population.

One of the first applications of Von Foerster's technique to an agricultural system was the modeling of a cotton crop by Wang et al. [1977]. In this application the time and age scales are both based on day-degrees. The number of fruit and the masses of leaves, stems, and roots are modeled as simultaneous equations. Thus, four separate density functions are defined plus a carbohydrate availability pool is described which links the four components. Using the Von Foerster methodology, Wang et al. were able to present concisely a mathematical model containing a high level of biological realism. To implement their system a finite difference scheme is used to numerically solve the partial differential equation system. Such a scheme is very similar to Leslie's discrete approach (Wang et al., p. 1371).

Although a reasonable level of realism can be mathematically described via Von Foerster's approach, the solution methods for such systems of equations is not advanced. A contractive mapping procedure is utilized in Curry et al. [1981] that obtains an approximate fruiting rate function ($b(\cdot)$ in Eq. 9) for a nonlinear system describing cotton fruiting dynamics. Except for a few cases (Gutierrez et al. [1979]), Von Foerster's equations have not been used for optimization studies, thus the solution procedures have not been developed from an optimization perspective.

From a mathematical modeling perspective, time-dependency as well as age-dependency is an important consideration for reproduction and mortality. A mathematical formulation using time dependent parameters for these processes is presented in Hoppensteadt. Although time-dependent parameters are implicit in many simulations, the mathematical structure of Hoppensteadt is not utilized in describing actual systems. Thus research directed toward the utilization and numerical solution of nonlinear Von Foerster systems containing time- and age-dependent parameters is necessary.

Multiple Species Models

It is sometimes possible to obtain valuable insight using a single species model. For example, Conway et al. [1975] described the sugarcane froghopper (*Aenolamia* var. *saccharina* Distant) as a single species population. Optimal strategies are then based on the cost of spraying, and the pesticide's effect on insect numbers. However, it is usually necessary to model the plant and insect populations simultaneously. Sometimes the pest populations involve multiple species and these must also be modeled.

The original approach for interacting pest populations of the predator-prey type (Lotka, and Volterra) assumed simple proportional removal rates of the prey as a function of predator numbers. Holling [1959, 1966] analyzed the individual predation components of searching, catching, handling, and digestive pause in obtaining the response of a predator to prey density.

Many subsequent studies have investigated parasitism (Royama [1971], Rogers [1972]) and there has been some work on the application of queueing theory to these systems (Taylor [1976], Curry and DeMichele [1977]). Although there have been many theoretical and ecologically oriented studies, effort still needs to be directed toward stochastic modeling of predator-prey systems. Current deterministic approaches have been shown sometimes to yield inexact models of these processes (Oaten [1977], Curry and Feldman [1979]). In a study of the effects of the green rice leafhopper (*Nephotettix cincticeps* Uhler), Kiritani [1977] modeled both the leafhopper and its major predator, a wolf spider (*Lycosa pseudoannulata* Boes). Kiritani's approach is similar to the Lotka-Volterra equations except that mortality of the predator and predation of the prey are functions (via regression) of the predator and prey densities.

Most of the interactions that have been modeled between populations involve the plant-pest interactions. The study by Shoemaker [1982] includes population models of the alfalfa weevil (*Hypera postica* (Gyllenhal)), its parasite (*Bathyplectes curculionis* (Thomson)), and the alfalfa plant (*Medicago sativa* L.). In her work, the plant-insect interactions are primarily in one direction. The plant does not influence the insect's population dynamics (except harvesting of alfalfa removes eggs and small larvae) but the weevil has a significant effect on the plant.

For the cotton-insect pest system, realistic models include two-way interactions. Wang et al. adjust the mortality rate of cotton fruit as a function of the adult boll weevil population and in turn they adjust the weevil's fecundity and mortality rates according to the density of cotton fruit forms.

Two of the major research problems to be addressed are: (1) efficient numerical solution procedures, and (2) mathematical simplification for optimization studies. General computer software for solving two-dimensional nonlinear partial differential equation systems has been developed (Melgaard and Sincovec [1981]). However, it remains to be shown how applicable such general packages are for systems arising from the agricultural setting. As an alternative, research developing specialized numerical procedures for nonlinear Von Foerster systems is needed. In the area of systems optimization, there is a need to obtain simpler mathematical expressions while maintaining the essence of the biology to ensure sufficient responsiveness for decision-making.

3. OPTIMIZATION

Recognition of the need to use realistic biological models together with mathematical and computer based optimization techniques for pest control is often attributed to K. E. F. Watt. Conway [1973] summarizes Watt's early works and contributions. Dynamic programming was first suggested by Watt [1963] as a powerful technique for pest management. One of the first optimization studies was reported by Watt in 1964. Much of his work found immediate application in the research of a Canadian forest pest, the spruce budworm (*Choristoneura fumiferana* (Clemens)), which has been under intensive study for years (Clark et al.).

One of the first applications of dynamic programming in agriculture was by Shoemaker [1973, 1982] for alfalfa harvesting. Alfalfa is a perennial crop which is usually harvested three times per year in New York. Control policies carried out in one year greatly influence the yield in succeeding years. In her study the planning horizon is several years, with each dynamic programming stage representing one year. The two decision variables each year are the timing of the first harvest within the year, h, and a zero-one variable, i, representing whether or not insecticide is applied at harvest time. The state variables in the dynamic programming formulation represent the number of overwintering adult alfalfa weevils, w, and the number of overwintering adult parasites, p.

Shoemaker decomposed the dynamic programming problem into two coupled models: a decision model and a population model. For the decision model, y_m is the yield function in year m with $f_m{}^*$ representing the optimal return from year m through the end of the planning horizon. Furthermore, v_m is the value per ton of alfalfa in year m, c_m is the cost of an insecticide treatment in year m, and g_1 and g_2 are state transition functions. The yearly dependence of the variables is indicated by the subscript. Then, the decision problem is given by

$$f_m{}^*(w_m, p_m) = \max_{h_m, i_m}\{v_m y_m(w_m, p_m, h_m, i_m) - c_m i_m + f^*_{m-1}(w_{m-1}, p_{m-1})\} \tag{10}$$

where

$$\begin{aligned} w_{m-1} &= g_1(w_m, p_m, h_m, i_m) \\ p_{m-1} &= g_2(w_m, p_m, h_m, i_m). \end{aligned} \tag{11}$$

Equations 10 and 11 define the decision model. Detailed population models are used to evaluate the function y_m, g_1, and g_2. The yield function y_m is an extensive simulation of plant growth based on the work of Fick [1975]. The pest and parasite population model (Shoemaker [1982]), mentioned briefly in Section 2, is similar to a Von Foerster formulation and involves age-dependent oviposition, mortality, and parasitism. The many population model variables need not be treated as state variables in the dynamic programming process, since the decision is made only once each year. The intervening winter effectively reduces the insect populations to single aged adults between decisions. Although the population model contains a significant degree of biological realism, the complexity of mathematics does not increase the dimensionality of the decision model. The overwintering population sizes are all that is needed to initiate each year's population growth.

The Shoemaker application of dynamic programming with biological submodels for the state transformations yields a high degree of effectiveness. The method addresses optimal pest management in a dynamic and stochastic environment. For problems with the mathematical structure considered in this analysis (single generation and single pesticide application per year), the computational effort can be reduced by many orders of magnitude.

Many systems require several decisions throughout a single growing season. Shoemaker [1979] applies dynamic programming to the control of the Egyptian alfalfa weevil (*Hypera brunneipennis* (Boheman)) in California. In this control problem there can be several applications of pesticide within one year although the pest population has a single generation. The usual approach in utilizing dynamic programming is to let the state-space represent the pest population discretized by age intervals. Such an approach results in severe dimensionality problems. Shoemaker's approach is to let the timings of the two most recent sprayings be the state space. By observing that few weevils live beyond two pesticide applications (less than 0.25% do so), a two-dimensional state space is maintained and the computations are tractable. An extension of this problem to stochastic environments is found in Shoemaker [1981b].

When there are both multiple treatments and multiple generations, the optimization problem becomes much more difficult. Yu [1978] formulates a dynamic programming problem to optimize sprayings for a cotton-boll weevil system. However, the state space is too large for practical utilization of his methodology. So far, the best applications of dynamic programming have been cases where advantage is taken of the structure of the population process. A survey article by Shoemaker [1981a] discusses in more detail dynamic programming applied to pest management.

Conway et al. avoid the problem of dimensionality of the state space by simplifying the relationships between generations. The sugarcane froghopper has four generations within one growing season. The population growth of the first brood is modeled in detail and the effects on the first brood of insecticide are analyzed. Using a simple relationship between broods, Conway et al. utilize dynamic programming to obtain optimal timing for pesticide applications.

Another approach for optimizing multiple treatments is that of Talpaz et al. [1978]. They consider a large population model of the cotton-boll weevil system with time measured in discrete 4-day intervals. Each time period has an associated variable denoting the amount of pesticide used (possibly equal to zero). For a fixed treatment pattern, the seasonal profit is determined. This optimal control problem is treated as a multidimensional optimization problem with profit as the objective function. The difficulty, of course, is that each evaluation of the objective function is time-consuming and no analytical derivatives are possible. A modified version of the Davidon-Fletcher-Powell method is used for optimization. The Talpaz et al. approach has the same drawbacks as the direct dynamic programming methodology in that the number of functional evaluations required by the procedure is large. For this method each functional evaluation is a numerical solution of the model for the complete growing season.

In an optimization study involving a similar cotton-boll weevil model, Gutierrez et al. utilize the nonderivative Powell's method. The objective is to maximize the total mass of mature fruit at a fixed harvest time. The Gutierrez et al. study is concerned with obtaining optimal values for various plant and insect population parameters. Their results indicate that for the combined system, the optimal parameter values are very close to field measured values. However, the observed value for the rate of fruit production is not optimal for cotton grown independent of the weevil. From this analysis they infer that the two systems evolved jointly to an optimal cohabitation. Thus, their study might have an impact on a plant breeding program, but not on short-term treatment tactics.

Marsolan and Rudd [1976] take a slightly different approach to the problem of optimally determining annual pesticide treatment tactics. They are interested in the control of the southern green stinkbug (*Nezara viridula* L.) in soybean (*Glycine max* L.) fields. Instead of investigating several discrete applications, Marsolan and Rudd consider control policies where pesticide is applied continuously throughout the growing season. They structured the optimization problem using classical control theory and then utilized Pontryagin's maximum principle. The numerical method used to find the optimal control trajectories is referred to as a first-variation gradient technique with a penalty function (Marsolan and Rudd, p. 235). Their study of the soybean/green-stinkbug system is based on actual data and is useful in the general examination of current insecticide treatment tactics. However, because of the unrealistic assumption of continuous control, their procedure does not aid in determining the specific times and quantities of actual pesticide applications. Thus, this approach cannot be directly utilized in an insect pest management program. A review of the general control theory approach for pest problems is in Wickwire [1977]. (See Clark for an extensive treatment of control theory applied to optimal harvesting in fisheries.) Although it has not yet found much practical use in agriculture, control theory does have potential for direct utilization in such "continuous" control areas as trickle irrigation and fertilization.

Another area in which Pontryagin's maximum principle has been utilized is in region-wide pesticide policies where the buildup of pesticide resistance is important. Regev et al. consider pesticide resistance of the Egyptian alfalfa weevil which is the major alfalfa pest in California. In their study, the objective function is total discounted profit. Profit is a function of the number of pesticide applications and the buildup of resistance. Both pesticide application levels and switch-over times to new control technology are variables. The main results of their study are general policy statements. Specifically, they conclude that too much pesticide is used since an individual farmer has negligible effect on buildup of pesticidal resistance. Thus, cooperative control procedures on a regional basis are necessary before individuals can benefit from reduced pesticide utilization.

The need for further research in developing usable optimization techniques for integrated pest management is tremendous. To do this, an understanding of both the biological processes and the mathematics is necessary. Most integrated pest management optimization studies formulate the objective function in such a way that the population model becomes part of its functional evaluation. The major computational time is in the functional evaluations which involve large scale population models. Thus, the studies are not structured as programming problems with constraints. For this reason, techniques using penalty functions or relaxation methods have not been employed. It might be fruitful to consider restructuring the optimization problem, so advanced techniques can be applied. The most promising approach thus far has been in cases where advantages in the biological structure are utilized to simplify the mathematical formulation (as in Shoemaker's dynamic programming examples). The need to include stochastic features in biological models favors dynamic programming, since it can incorporate stochastic features as well as on-line observations of the state of the system. This is not within the capabilities of gradient-based methods. As yet there have been no attempts to provide an overall theoretical framework for optimization of realistic population processes.

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